Proton spin in leading order of the covariant approach

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(inspired by the collaboration and discussions with A.Efremov, O.Teryaev and P.Schweitzer)

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Outline

- Introduction
- System of *non-interacting* fermions (J=1/2)
  - Eigenstates of angular momentum (relativistic case)
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- The use for description of the proton spin structure in DIS conditions & comparison with the DIS spin data
- Summary

Remark: *Since we work with the covariant representation, 3D description is obtained automatically.*

Introduction

Covariant approach has been discussed in the former studies, main results:

- ❑ Sum rules: Wanzura-Wilczek (WW), Burhardt-Cottinngham (BC) and Efremov-Leader-Teryaev (ELT)

- ❑ Relations between TMDs, PDFs and TMDs (giving predictions for TMDs)

- ❑ Study and prediction of the role of OAM

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The aim of this talk is to further develop and extend the study of common role of the spin and OAM of quarks.

For details see P.Z. Phys. Rev. ***D*** *89, 014012 (2014).*

Non-interacting fermions

Eigenstates of angular momentum

Usual plane-wave spinors are replaced by spinor spherical harmonics (both in momentum representation):

$$u(\mathbf{p},\lambda\mathbf{n}) = \frac{1}{\sqrt{N}}\begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}} \end{pmatrix}$$

$$\frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}}\,, \qquad N = \frac{2p_0}{p_0+m}$$

$$|j,j_z\rangle = \Phi_{jl_pj_z}(\omega) = \frac{1}{\sqrt{2\epsilon}}\begin{pmatrix} \sqrt{\epsilon+m}\,\Omega_{jl_pj_z}(\omega) \\ -\sqrt{\epsilon-m}\,\Omega_{j\lambda_pj_z}(\omega) \end{pmatrix}$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}Y_{l_p,j_z-1/2}(\omega) \\ \sqrt{\frac{j-j_z}{2j}}Y_{l_p,j_z+1/2}(\omega) \end{pmatrix}; \quad l_p = j-\frac{1}{2},$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}Y_{l_p,j_z-1/2}(\omega) \\ \sqrt{\frac{j+j_z+1}{2j+2}}Y_{l_p,j_z+1/2}(\omega) \end{pmatrix}; \quad l_p = j+\frac{1}{2}$$

where ω represents the polar and azimuthal angles (θ,φ) of the momentum p with respect to the quantization axis, $l_p = j \pm 1/2$ and $\lambda_p = 2j - l_p$ (l_p defines parity).

New representation is convenient for general discussion about role of OAM.

Spinor spherical harmonics $|j,j_z>$

- SSH represent solutions of the free Dirac equation, which reflects the known QM rule:

In relativistic case spin and OAM are not separately conserved, but only sums j and $j_z = s_z + l_z$ are conserved.

- However, one can always calculate the mean values of corresponding operators:

$$s_z = \frac{1}{2}\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}, \qquad l_z = -i\left(p_x \frac{\partial}{\partial p_y} - p_y \frac{\partial}{\partial p_x}\right)$$

and get the result

$$\langle s_z \rangle_{j,j_z} = \frac{1+(2j+1)\,\mu}{4j\,(j+1)} j_z, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1+(2j+1)\,\mu}{4j\,(j+1)}\right)$$

where $\mu = m/\varepsilon$.

Non-relativistic limit (*μ=1*):

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{2j}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{2j}\right) j_z$$

μ=m/ε

j≥1/2

$l_p = j-1/2$

Relativistic case (*μ → 0*):

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{4j\,(j+1)}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{4j\,(j+1)}\right) j_z$$

$$\Downarrow$$

$$\left|\langle s_z \rangle_{j,j_z}\right| \leq \frac{1}{4\,(j+1)} \leq \frac{1}{6}, \qquad \frac{\left|\langle s_z \rangle_{j,j_z}\right|}{\left|\langle l_z \rangle_{j,j_z}\right|} \leq \frac{1}{4j^2 + 4j - 1} \leq \frac{1}{2}$$

Many-fermion states

Composition of one-particle states (SSH) representing composed particle with spin ***J=J$_z$=1/2:***

$$\left|(j_1, j_2, \ldots j_n)_c J, J_z\right\rangle = \sum_{j_{z1}=-j_1}^{j_1} \sum_{j_{z2}=-j_2}^{j_2} \cdots \sum_{j_{zn}=-j_n}^{j_n} c_j \left|j_1, j_{z1}\right\rangle \left|j_2, j_{z2}\right\rangle \ldots \left|j_n, j_{zn}\right\rangle$$

where c_j's consist of Clebsch-Gordan coeficients:

$$c_j = \left\langle j_1, j_{z1}, j_2, j_{z2} \,|\, J_3, J_{3z} \right\rangle \left\langle J_3, J_{z3}, j_3, j_{z3} \,|\, J_4, J_{z4} \right\rangle \ldots \left\langle J_n, J_{zn}, j_n, j_{zn} \,|\, J, J_z \right\rangle$$

What can be said about the mean values:

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} = \langle s_{z1} + s_{z2} + \ldots + s_{zn} \rangle_c \,, \qquad \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \langle l_{z1} + l_{z2} + \ldots + l_{zn} \rangle_c$$

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} + \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \frac{1}{2},$$

?

Comment

Algebra of many-particle states ***J=1/2*** is rather complex. Their discussion in this talk is correspondingly simplified. For more details see *Phys. Rev.* ***D****89, 014012 (2014)* and citations therein. Some results has been obtained or verified with the help of Wolfram Mathematica.

Examples for *n=3*

Composition pattern symbolically:

$$\boxed{((j_a \oplus j_b)_{J_c} \oplus j_c)_{1/2}; \quad abc = 123, 312, 231.}$$

Constraint:

$$\boxed{J_c = j_c \pm 1/2, \qquad |j_a - j_b| \leq J_c \leq j_a + j_b.}$$

The results on $\langle \mathbb{S}_z \rangle$ and $\langle \mathbb{L}_z \rangle$ depend on the composition pattern (order of composition and intermediate J_c)

Examples: $\langle \mathbb{S}_z \rangle$ for $((j_a \oplus j_b)_{J_c} \oplus j_c)_{1/2}$; $\quad abc = 123, 312, 231.$

j_1	j_2	j_3	$J_c = j_c - 1/2$: $\langle S_z \rangle_3$	$\langle S_z \rangle_2$	$\langle S_z \rangle_1$	$J_c = j_c + 1/2$: $\langle S_z \rangle_3$	$\langle S_z \rangle_2$	$\langle S_z \rangle_1$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1+2\tilde{\mu}}{6}$	$\frac{1+2\tilde{\mu}}{6}$	$\frac{1+2\tilde{\mu}}{6}$	$\frac{1+2\tilde{\mu}}{6}$	$\frac{1+2\tilde{\mu}}{6}$	$\frac{1+2\tilde{\mu}}{6}$
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\times$	$\times$	$\frac{-1}{18}$	$\frac{-1}{18}$	$\frac{-1}{18}$	$\times$
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1+2\tilde{\mu}}{6}$	$\frac{1+3\tilde{\mu}}{18}$	$\frac{1+3\tilde{\mu}}{18}$	$\frac{-1+6\tilde{\mu}}{90}$	$\frac{3+7\tilde{\mu}}{30}$	$\frac{3+7\tilde{\mu}}{30}$
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1+4\tilde{\mu}}{30}$	$\frac{1+4\tilde{\mu}}{30}$	$\frac{1+4\tilde{\mu}}{30}$	$\frac{1+4\tilde{\mu}}{30}$	$\frac{1+4\tilde{\mu}}{30}$	$\frac{1+4\tilde{\mu}}{30}$
$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\times$	$\times$	$\frac{-5-4\tilde{\mu}}{90}$	$\frac{-5-4\tilde{\mu}}{90}$	$\frac{-5-4\tilde{\mu}}{90}$	$\times$
$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{5+17\tilde{\mu}}{90}$	$\frac{5+17\tilde{\mu}}{90}$	$\frac{-1+2\tilde{\mu}}{90}$	$\frac{-1+29\tilde{\mu}}{630}$	$\frac{-1+29\tilde{\mu}}{630}$	$\frac{41+134\tilde{\mu}}{630}$
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{29+104\tilde{\mu}}{630}$	$\frac{23+152\tilde{\mu}}{1890}$	$\frac{23+152\tilde{\mu}}{1890}$	$\frac{-1+8\tilde{\mu}}{210}$	$\frac{55+232\tilde{\mu}}{1890}$	$\frac{55+232\tilde{\mu}}{1890}$
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{1+6\tilde{\mu}}{70}$	$\frac{1+6\tilde{\mu}}{70}$	$\frac{1+6\tilde{\mu}}{70}$	$\frac{1+6\tilde{\mu}}{70}$	$\frac{1+6\tilde{\mu}}{70}$	$\frac{1+6\tilde{\mu}}{70}$
$\frac{7}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\times$	$\times$	$\frac{-7-8\tilde{\mu}}{126}$	$\frac{-7-8\tilde{\mu}}{126}$	$\frac{-7-8\tilde{\mu}}{126}$	$\times$
$\frac{7}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{7+25\tilde{\mu}}{126}$	$\frac{25+102\tilde{\mu}}{630}$	$\frac{-20-11\tilde{\mu}}{1260}$	$\frac{-35-19\tilde{\mu}}{1890}$	$\frac{-1+10\tilde{\mu}}{378}$	$\frac{40+149\tilde{\mu}}{756}$
$\frac{7}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{133+668\tilde{\mu}}{5670}$	$\frac{133+668\tilde{\mu}}{5670}$	$\frac{-1+\tilde{\mu}}{210}$	$\frac{1+44\tilde{\mu}}{1134}$	$\frac{1+44\tilde{\mu}}{1134}$	$\frac{11+52\tilde{\mu}}{378}$
$\frac{7}{2}$	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{43+218\tilde{\mu}}{1890}$	$\frac{4+41\tilde{\mu}}{756}$	$\frac{4+41\tilde{\mu}}{756}$	$\frac{-1+10\tilde{\mu}}{378}$	$\frac{56+331\tilde{\mu}}{3780}$	$\frac{56+331\tilde{\mu}}{3780}$
$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{1+8\tilde{\mu}}{126}$	$\frac{1+8\tilde{\mu}}{126}$	$\frac{1+8\tilde{\mu}}{126}$	$\frac{1+8\tilde{\mu}}{126}$	$\frac{1+8\tilde{\mu}}{126}$	$\frac{1+8\tilde{\mu}}{126}$

x =not allowed

Other patterns, n=4,5..:

$$(((j_1 \oplus j_2)_{J_1} \oplus j_3)_{J_2} \oplus j_4)_J,$$

$$(((j_1 \oplus j_2)_{J_1} \oplus (j_3 \oplus j_4)_{J_2})_{J_3} \oplus j_5)_J$$

Complementary tab. for $\langle \mathbb{L}_z \rangle$ satisfies:

$$J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}.$$

Comment:

Composition $((j_a \oplus j_b)_{J_c} \oplus j_c)_{1/2}$ for $j_a=j_b=j_c=1/2$ and $J_c=1, 0$ gives the states:

$$\Psi_{abc,1/2,1/2} = \frac{\phi_{abc}}{\sqrt{6}}(|-1/2,1/2,1/2\rangle + |1/2,-1/2,1/2\rangle - 2|1/2,1/2,-1/2\rangle)$$

$$\Psi_{abc,1/2,1/2} = \frac{\phi_{abc}}{\sqrt{2}}(|1/2,-1/2,1/2\rangle - |-1/2,1/2,1/2\rangle)$$

$$\phi_{abc} = \phi_a(\epsilon_a)\phi_b(\epsilon_b)\phi_c(\epsilon_c)$$

Comparison with SU(6),

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{6}}|duu + udu - 2uud\rangle\frac{1}{\sqrt{6}}|\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow\rangle + \frac{1}{\sqrt{2}}|duu - udu\rangle\frac{1}{\sqrt{2}}|\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow\rangle\right\}$$

suggests this state can be generated by the superposition

$$((u_1 \oplus u_2)_J \oplus d)_{1/2}, \qquad ((d \oplus u_1)_J \oplus u_2)_{1/2}, \qquad ((u_2 \oplus d)_J \oplus u_1)_{1/2}$$

Comments

- Regardless of complexity, in relativistic case (*μ=0*), we obtain (like for one-fermion state):

$$\boxed{|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},} \quad \boxed{\frac{|\langle \mathbb{S}_z \rangle|}{|\langle \mathbb{L}_z \rangle|} \leq \frac{1}{2}} \quad \text{AND} \quad \boxed{J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}}$$

- n-dimensional angular distribution $\boxed{P(\omega_1, \omega_2, ..\omega_n) = \Phi^{+}_{1/2} \Phi_{1/2}}$

after contraction to 1D gives:

$$\boxed{p_k(\omega_k) = \int P(\omega_1, \omega_2, ..\omega_n) \prod_{i \neq k}^{n} d\omega_i = \frac{1}{4\pi}}$$

i.e. rotational symmetry. It is another similarity to one-fermion state *j=1/2.* In this sense any system **J=1/2** should be rot. symmetric.

Structure functions:

Invariants by definition. Its measuring gives <u>invariant</u> representation of DIS data and/or state of the target in terms of parameters x_B, Q^2, $\boldsymbol{S}$

Distribution functions are extracted by model-dependent way.

Spin structure functions

Generation of spin structure functions from many-fermion states
J=1/2 (still non-interacting mutally, only with the probing photon)

Procedure:

1) Spin structure functions are obtained from antisym. tensor:

$$T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} \left(M S^{\sigma} G_1 + ((Pq)S^{\sigma} - (qS)P^{\sigma}) \frac{G_2}{M} \right)$$

2) Antisym. tensor corresponding to the free-fermion vertex:

$$t^{(A)}_{\alpha\beta} = m \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} w^{\sigma}(p)$$

3) Integral over phase space of all fermions allows to extract spin SFs:

$$T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} m \int w^{\sigma}(p) \delta((p+q)^2 - m^2) \frac{d^3p}{\epsilon}$$

task: $|(j_1, j_2, \ldots j_n)_c J, J_z\rangle$ $\Rightarrow$ $\boldsymbol{w^{\sigma}}$

Spin vector $w^\sigma(p)$

1. Projection operators:

$$\mathcal{P}_{\lambda,\pm} = \begin{pmatrix} \sigma_{\lambda,\pm} & 0 \\ 0 & \frac{\mathbf{p}\sigma}{\epsilon+m}\sigma_{\lambda,\pm}\frac{\mathbf{p}\sigma}{\epsilon-m} \end{pmatrix},$$

where

$$\sigma_{\lambda,\pm} = \frac{1}{2}\left(\mathbf{1} \pm \sigma_\lambda\right)$$

and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices. Obviously

$$\mathcal{P}_{\lambda,+} + \mathcal{P}_{\lambda,-} = \mathbf{1}, \qquad \mathcal{P}_{\lambda,+}\mathcal{P}_{\lambda,-} = \mathcal{P}_{\lambda,-}\mathcal{P}_{\lambda,+} = \mathbf{0}, \qquad \left(\mathcal{P}_{\lambda,\pm}\right)^2 = \mathcal{P}_{\lambda,\pm},$$

$$\Delta\mathcal{P}_\lambda \equiv \mathcal{P}_{\lambda,+} - \mathcal{P}_{\lambda,-} = \begin{pmatrix} \sigma_\lambda & 0 \\ 0 & \frac{\mathbf{p}\sigma}{\epsilon+m}\sigma_\lambda\frac{\mathbf{p}\sigma}{\epsilon-m} \end{pmatrix}.$$

ΔP_λ define components of the spin vector ***w*** in the fermion rest frame

2. Contribution of one fermion (from many-fermion state J=1/2):

$$h_{\lambda,c,k}(\omega_k) = \int \Phi^{+}_{c,1/2,1/2} \Delta\mathcal{P}_{\lambda,k} \Phi_{c,1/2,1/2} \prod_{i\neq k}^{n} d\omega_i$$

has (regardless of complexity of Φ) a simple form:

$$h_{x,c,k}(\omega) = \frac{1}{4\pi}\beta_{c,k}\sin 2\theta\cos\varphi, \qquad h_{y,c,k}(\omega) = \frac{1}{4\pi}\beta_{c,k}\sin 2\theta\sin\varphi,$$

$$h_{z,c,k}(\omega) = \frac{1}{4\pi}\left(\alpha_{c,k} + \beta_{c,k}\cos 2\theta\right),$$

where the constants α and β depend on the pattern of composition and absorb corresponding Clebsch-Gordan coefficients entering matrix elements

3. Contribution of all fermions (from the state J=1/2)

-is given by their sum: $$\boxed{H_{\lambda,c}(\omega) = \sum h_{\lambda,c,k}(\omega)}$$

$$\Rightarrow \boxed{\begin{aligned} H_{x,c}(\omega) &= b_c \sin 2\theta \cos\varphi, \\ H_{y,c}(\omega) &= b_c \sin 2\theta \sin\varphi, \\ H_{z,c}(\omega) &= a_c + b_c \cos 2\theta, \end{aligned}}$$

from which the final form of spin vector ***w*** is obtained:

$$\boxed{\mathbf{w}(\omega,\epsilon) = (\mathrm{u}(\epsilon) - \mathrm{v}(\epsilon))\,\mathbf{S} + 2\mathrm{v}(\epsilon)\,(\mathbf{nS})\,\mathbf{n}}$$

$$\boxed{\mathrm{u}(\epsilon) = \sum \alpha_{c,k} a^*_{j_k}(\epsilon)\, a_{j_k}(\epsilon), \qquad \mathrm{v}(\epsilon) = \sum \beta_{c,k} a^*_{j_k}(\epsilon)\, a_{j_k}(\epsilon)}$$

where **n** = **p/|p|** and **S** is the unit vector defining the axis of ***j_z*** projections, which is identical to the proton spin vector in the proton rest frame.

Example: $H_Z(\omega)$ for $((j_a \oplus j_b)_{J_c} \oplus j_c)_{1/2}$; $abc = 123, 312, 231.$

			$J_c = j_c - 1/2$			$J_c = j_c + 1/2$		
j_1	j_2	j_3	H_3	H_2	H_1	H_3	H_2	H_1
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	1
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\times$	$\times$	$\frac{-1-\cos 2\theta}{6}$	$\frac{-1-\cos 2\theta}{6}$	$\frac{-1-\cos 2\theta}{6}$	$\times$
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	1	$\frac{5-\cos 2\theta}{12}$	$\frac{5-\cos 2\theta}{12}$	$\frac{1-2\cos 2\theta}{15}$	$\frac{13-\cos 2\theta}{20}$	$\frac{13-\cos 2\theta}{20}$
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3-\cos 2\theta}{10}$	$\frac{3-\cos 2\theta}{10}$	$\frac{3-\cos 2\theta}{10}$	$\frac{3-\cos 2\theta}{10}$	$\frac{3-\cos 2\theta}{10}$	$\frac{3-\cos 2\theta}{10}$
$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\times$	$\times$	$\frac{-7-3\cos 2\theta}{30}$	$\frac{-7-3\cos 2\theta}{30}$	$\frac{-7-3\cos 2\theta}{30}$	$\times$
$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{27-7\cos 2\theta}{60}$	$\frac{27-7\cos 2\theta}{60}$	$\frac{-\cos 2\theta}{15}$	$\frac{27-31\cos 2\theta}{420}$	$\frac{27-31\cos 2\theta}{420}$	$\frac{54-13\cos 2\theta}{105}$
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{81-23\cos 2\theta}{210}$	$\frac{99-53\cos 2\theta}{630}$	$\frac{99-53\cos 2\theta}{630}$	$\frac{3-5\cos 2\theta}{70}$	$\frac{171-61\cos 2\theta}{630}$	$\frac{171-61\cos 2\theta}{630}$
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{6-3\cos 2\theta}{35}$	$\frac{6-3\cos 2\theta}{35}$	$\frac{6-3\cos 2\theta}{35}$	$\frac{6-3\cos 2\theta}{35}$	$\frac{6-3\cos 2\theta}{35}$	$\frac{6-3\cos 2\theta}{35}$
$\frac{7}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\times$	$\times$	$\frac{-11-3\cos 2\theta}{42}$	$\frac{-11-3\cos 2\theta}{42}$	$\frac{-11-3\cos 2\theta}{42}$	$\times$
$\frac{7}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{39-11\cos 2\theta}{84}$	$\frac{38-13\cos 2\theta}{105}$	$\frac{-51-29\cos 2\theta}{840}$	$\frac{-89-51\cos 2\theta}{1260}$	$\frac{2-3\cos 2\theta}{63}$	$\frac{229-69\cos 2\theta}{504}$
$\frac{7}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{467-201\cos 2\theta}{1890}$	$\frac{467-201\cos 2\theta}{1890}$	$\frac{1-3\cos 2\theta}{70}$	$\frac{23-21\cos 2\theta}{378}$	$\frac{23-21\cos 2\theta}{378}$	$\frac{37-15\cos 2\theta}{126}$
$\frac{7}{2}$	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{76-33\cos 2\theta}{315}$	$\frac{49-33\cos 2\theta}{504}$	$\frac{49-33\cos 2\theta}{504}$	$\frac{2-3\cos 2\theta}{63}$	$\frac{443-219\cos 2\theta}{2520}$	$\frac{443-219\cos 2\theta}{2520}$
$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{5-3\cos 2\theta}{42}$	$\frac{5-3\cos 2\theta}{42}$	$\frac{5-3\cos 2\theta}{42}$	$\frac{5-3\cos 2\theta}{42}$	$\frac{5-3\cos 2\theta}{42}$	$\frac{5-3\cos 2\theta}{42}$

x =not allowed

one can check: $\frac{1}{2}\int H_{z,c}(\omega)d\omega = \langle \mathbb{S}_z \rangle_{c,\mathrm{NR}}$

Comments

- **The form**

$$\boxed{H_z(\omega,\epsilon) = \mathrm{u}(\epsilon) + \mathrm{v}(\epsilon)\cos 2\theta}$$

corresponds to the state J=1/2. The function *v(ϵ)* is generated by an admixture of the states *j>1/2.*

- **The higher J would generate additional terms, e.g. for J=3/2:**

$$\boxed{H_z(\omega,\epsilon) = \mathrm{u}_1(\epsilon) + \mathrm{u}_2(\epsilon)\cos 2\theta + \mathrm{u}_3(\epsilon)\cos 4\theta}$$

Spin vector $w^\sigma(p)$ - manifestly covariant form

w ➡

$$\boxed{w^\sigma = AP^\sigma + BS^\sigma + Cp^\sigma}$$

$$A = -pS\left(\frac{\mathrm{u}(\epsilon)}{pP + mM} - \frac{\mathrm{v}(\epsilon)}{pP - mM}\right),$$

$$B = \mathrm{u}(\epsilon) - \mathrm{v}(\epsilon),$$

$$C = -pS\frac{M}{m}\left(\frac{\mathrm{u}(\epsilon)}{pP + mM} + \frac{\mathrm{v}(\epsilon)}{pP - mM}\right).$$

$w^\sigma(p)$ ➡

$$\boxed{T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma}q^\lambda m \int w^\sigma(p)\delta((p+q)^2 - m^2)\frac{d^3p}{\epsilon}}$$

➡ ***g_1 & g_2***

From this tensor spin structure functions are extracted

Spin structure functions: explicit form

For $Q^2 \gg 4M^2x^2$ we get (in terms of rest frame variables)

$$x = Q^2/2Pq$$

$$g_1(x) = \frac{1}{2}\int \left(\mathrm{u}(\epsilon)\left(p_1 + m + \frac{p_1^2}{\epsilon + m}\right) + \mathrm{v}(\epsilon)\left(p_1 - m + \frac{p_1^2}{\epsilon - m}\right)\right)\delta\left(\frac{\epsilon + p_1}{M} - x\right)\frac{d^3p}{\epsilon},$$

$$g_2(x) = -\frac{1}{2}\int \left(\mathrm{u}(\epsilon)\left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon + m}\right) + \mathrm{v}(\epsilon)\left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon - m}\right)\right)\delta\left(\frac{\epsilon + p_1}{M} - x\right)\frac{d^3p}{\epsilon}.$$

This result is exact for SFs generated by (free) many-fermion state **J=1/2** represented by the spin spherical harmonics.

For given state $\Psi_{1/2}$ we have checked calculation:

$$\langle \mathbb{S}_z \rangle = \left\langle \Psi_{1/2} \left| \mathbb{S}_z \right| \Psi_{1/2} \right\rangle = \langle s_{z1} + s_{z2} + \ldots + s_{zn} \rangle$$

$$\Gamma_1 = \int_0^1 g_1(x)\,dx$$

give equivalent results! ✓

Quasi-free quarks
in conditions of DIS

Basic inputs

❑ **Large Q²:** In the rest frame we have

$$|\mathbf{q}_R|^2 = Q^2 + \nu^2 = Q^2\left(1 + \frac{Q^2}{(2Mx)^2}\right) \;\Rightarrow\; |\mathbf{q}_R| \gtrsim \nu = \frac{Q^2}{2Mx} \geq \frac{Q^2}{2M}$$

$$\Rightarrow\; \Delta\lambda \lesssim \Delta\tau \approx \frac{2Mx}{Q^2}$$

So a space-time domain of lepton -quark QED interaction is limited.

❑ **Effect of asymptotic freedom:** Limited extend of this domain prevent the quark from an interaction with the rest of nucleon during the lepton-quark interaction – **in any reference frame.**

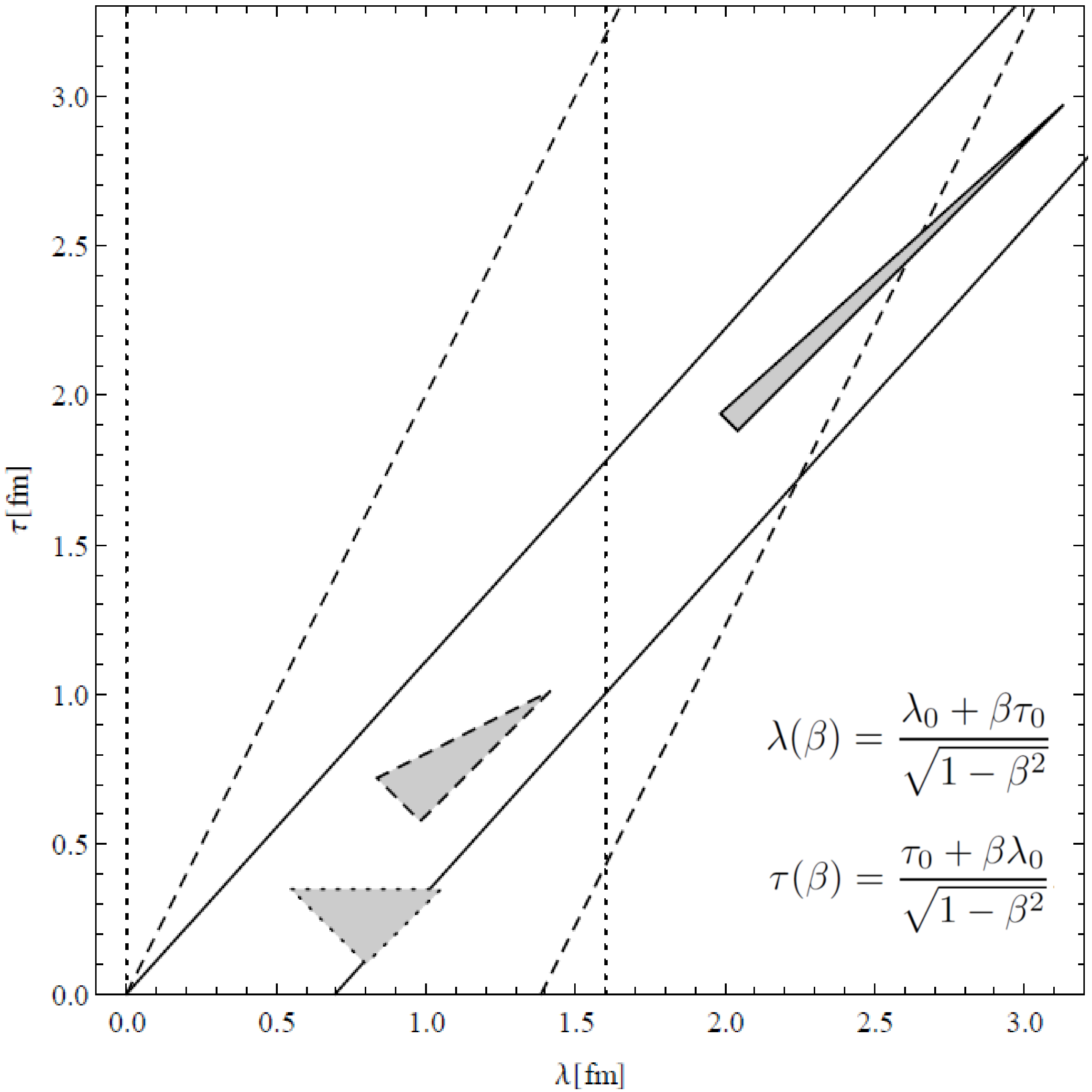

FIG. 1: The space-time domain of the photon momentum transfer to the quark in different Lorentz frames. The different styles of lines and triangles represent the proton boundary and the domain for: rest frame, $\beta = 0$ *(dotted)*, $\beta = 0.5$ *(dashed)*, $\beta = 0.9$ *(solid)*. Note that Lorentz boosts does not change the area of the domain $\Delta\lambda \times \Delta\tau$.

In fact we assume characteristic time of QCD process accompanying γ absorption is much greater than absorption time itself: $\boxed{\Delta\tau \ll \Delta\tau_{QCD}}$

Since Lorentz time dilation is universal, the first relation holds in any reference frame. This is essence of our covariant leading order approach.

$$\boxed{\Delta T(\beta) = \frac{\Delta T_0}{\sqrt{1-\beta^2}}}$$

Remarks:

- We suppose $\Delta\tau_{QCD}$ has a good sense in any reference frame - even if we cannot transform QCD corrections...

- We do not aim to describe complete nucleon dynamic structure, but only a short time interval corresponding to DIS.

- We assume the approximation of quarks by free waves in limited space-time domain is acceptable for description of DIS regardless of the reference frame.

Proton spin structure

- The proton state can be formally represented by a superposition of the Fock states: $\boxed{\Psi = \sum_{q,g} a_{qg} \left|\varphi_1, ...\varphi_{n_q}\right\rangle \left|\psi_1, ...\psi_{n_g}\right\rangle}$

- We ignore possible contribution of gluons: $\boxed{\Psi = \sum_{q} a_{q} \left|\varphi_1, ...\varphi_{n_q}\right\rangle}$

  where the states $\boxed{\left|\varphi_1, ...\varphi_{n_q}\right\rangle}$ are represented by eigenstates:

$$\boxed{J = J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}}$$

- We assume this approximation (effectively free quarks) is valid at a limited space-time domain corresponding to DIS.

Comparison with polarized DIS data

Burkhardt-Cottingham sum rule can be easily obtained:

$$\boxed{\Gamma_2 = \int_0^1 g_2(x)\, dx = 0}$$

cf. experiments [25,26,29]

To simplify discussion, in the next we assume $m \longrightarrow 0$:

$$\boxed{\begin{aligned} g_1(x) &= \frac{1}{2}\int (\mathrm{u}(\epsilon) + \mathrm{v}(\epsilon)) \left(p_1 + \frac{p_1^2}{\epsilon}\right) \delta\left(\frac{\epsilon + p_1}{M} - x\right) \frac{d^3p}{\epsilon}, \\ g_2(x) &= -\frac{1}{2}\int (\mathrm{u}(\epsilon) + \mathrm{v}(\epsilon)) \left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon}\right) \delta\left(\frac{\epsilon + p_1}{M} - x\right) \frac{d^3p}{\epsilon} \end{aligned}}$$

The sum $\mathrm{u}(\epsilon) + \mathrm{v}(\epsilon)$ can be identified with our former phenomenological distribution $H(\epsilon)$. The functions satisfy the Wanzura-Wilczek (WW), Efremov-Leader-Teryaev (ELT) and other rules that we proved for massless quarks. Cf. experiments [25,26,29]. Also our transversity and TMDs relations keep to be valid.

Remark

WW validity follows also from the further approaches [23, 24] that are based on the Lorentz invariance. The possible breaking of the WW and other so-called Lorentz invariance relations were discussed in [27, 28]. In our approach this relation is violated by the mass term.

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Proton spin content

We have shown the system **J=1/2** composed of (quasi) free fermions *$m \to 0$* satisfies:

$$\boxed{|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},}$$

(or the same in terms of Γ_1)

Reduced spin is compensated by OAM $\boxed{\langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}}$

and equality takes place for a simplest configuration:

$$\boxed{j_1 = j_2 = j_3 = \ldots = j_{n_q} = \frac{1}{2}}$$

Conditions of this system fit to our simplified proton.
If we change notation

$$\boxed{|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},} \quad \Rightarrow \quad \boxed{\Delta\Sigma \lesssim 1/3}$$

this result is well compatible with the data
(cf. experiments [30-32]):

$$\boxed{\Delta\Sigma = 0.32 \pm 0.03(stat.)}$$

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Summary

- In the framework of the covariant QPM (spin spherical harmonics representation) we have studied the interplay between the spins and OAMs of the quarks, which collectively generate the proton spin.
- We have shown the ratio $\mu=m/\varepsilon$ plays a crucial role, since it controls a "contraction" of the spin component which is compensated by the OAM. It is effect of relativistic kinematics.
- We have shown the resulting quark spin vector obtained from composition of the spins of contributing quarks is a quantity of key importance. It is a basic input for calculation of the proton spin content and the related SFs.
- A very good agreement with the data, particularly as for $\Delta\Sigma$ is a strong argument in favor of this approach.
- *Open question*: how do the functions u and v defining the spin vector w and corresponding spin SFs depend on the scale Q^2? Is such task calculable in terms of the pQCD?