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New approach to QCD factorization

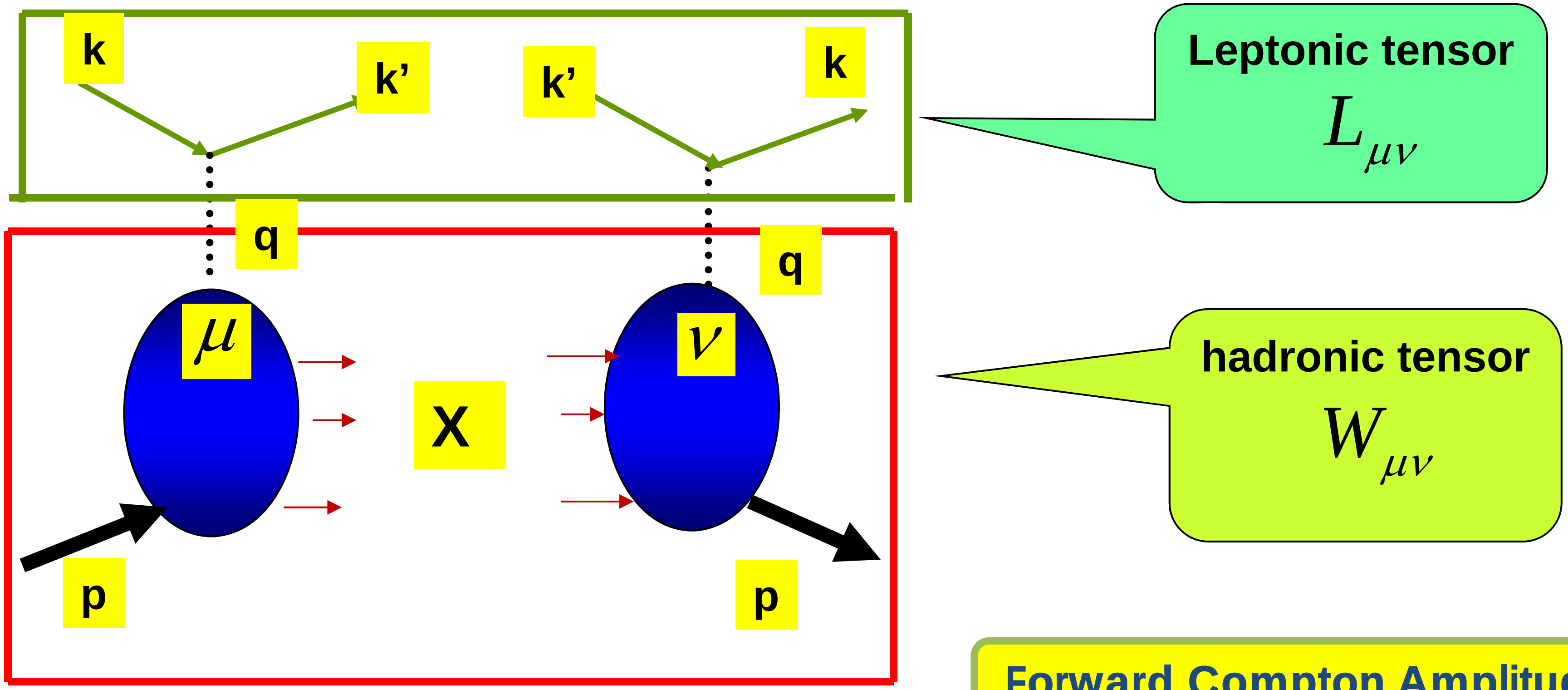
talk based on results obtained in collaboration with
M. Greco and S.I. Troyan

Factorization is the key concept in applied QCD. It makes possible to apply perturbarive QCD to description of hadronic reactions. Factorization is approximation and it proved to be quite efficient

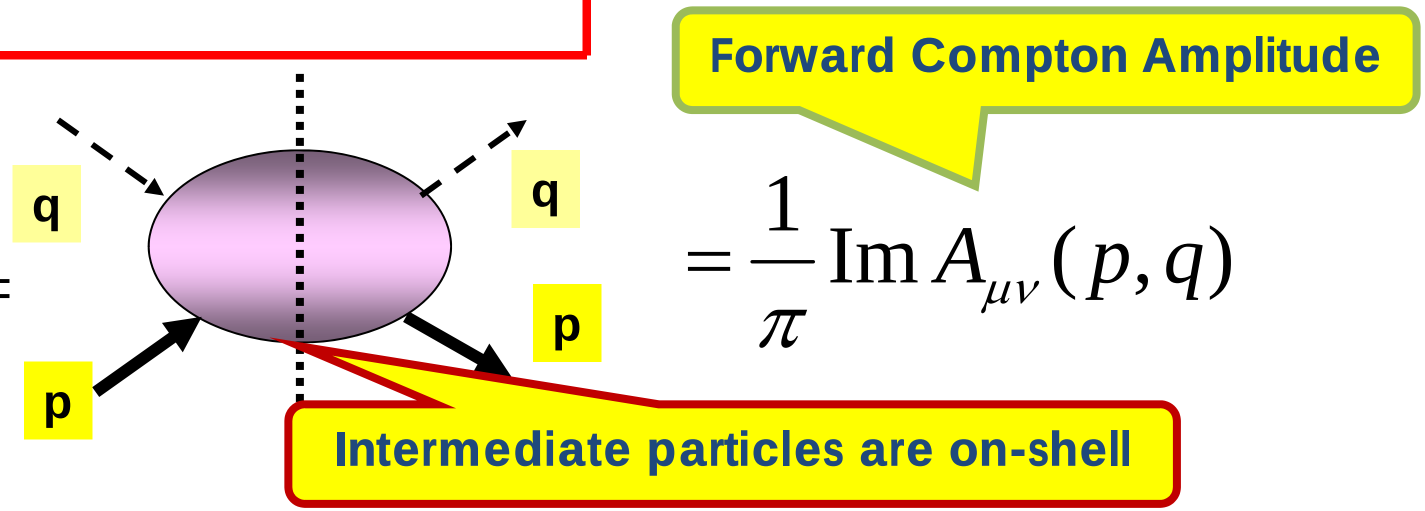
The need for Factorization: QCD is poorly known (does not exists as a regular science) in the infrared region (at large distances), so lack of such knowledge should be approximated/mimicked somehow and the most popular way to do it is QCD Factorization

For simplicity, I focus on the simple and at the same time important example of hadronic reaction:
Deep-Inelastic lepton-hadron Scattering

DIS Inclusive cross-section:



$$W_{\mu\nu}(p,q) =$$



$$= \frac{1}{\pi}\,\mathrm{Im}\,A_{\mu\nu}(p,q)$$

For instance, Hadronic tensor for unpolarized electron-proton DIS is conventionally parameterized as follows:

Projection operators

$$W_{\mu\nu} = \left(g_{\mu\nu} - q_\mu q_\nu / q^2\right) F_1(x, Q^2) + \left(p_\mu - q_\mu \frac{pq}{q^2}\right)\left(p_\nu - q_\nu \frac{pq}{q^2}\right) \frac{1}{pq} F_2(x, Q^2)$$

$$q_\mu W_{\mu\nu} = q_\nu W_{\nu\mu} = 0$$

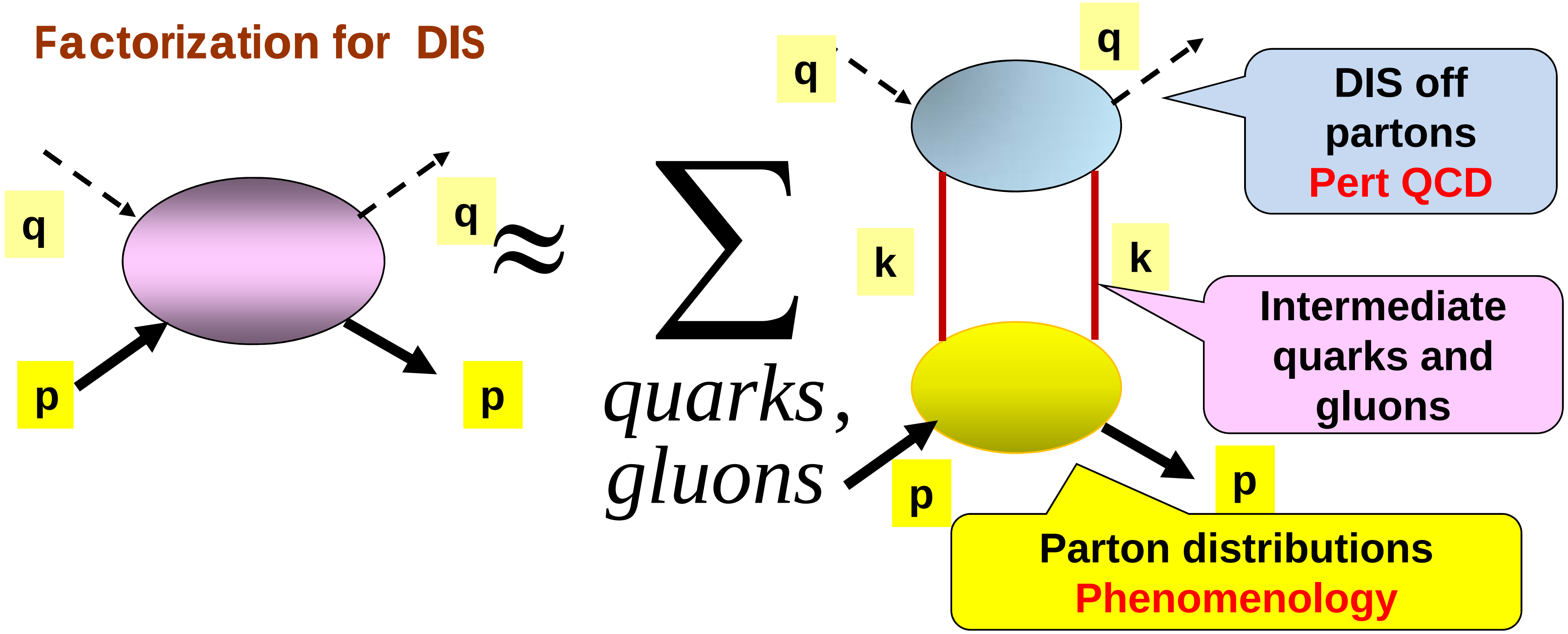
Structure functions

Conventional option of arguments of F_1, F_2

$$Q^2 = -q^2 > 0, \quad x = Q^2 / 2pq, \quad 0 < x < 1$$

In order to calculate structure functions, one should know both Perturbative and Non-Perturbative QCD but Non-Perturbative QCD is known poorly, so straightforward calculation of structure functions cannot be performed Instead of straightforward calculations there is conventionally used approximation of **FACTORIZATION**

Factorization for DIS



There is no theory whatsoever to calculate parton distributions.
The fits for them are made from purely phenomenological considerations.
Any formula for them is welcome providing it explains available experimental data
I am presenting theoretical restrictions on the fits

There are well-known the following kinds of factorization in the literature:

Collinear Factorization

Amati-Petronzio-Veneziano, Efremov-Ginzburg-Radyushkin, Libby-Sterman, Brodsky-Lepage, Collins-Soper-Sterman

K_T- factorization

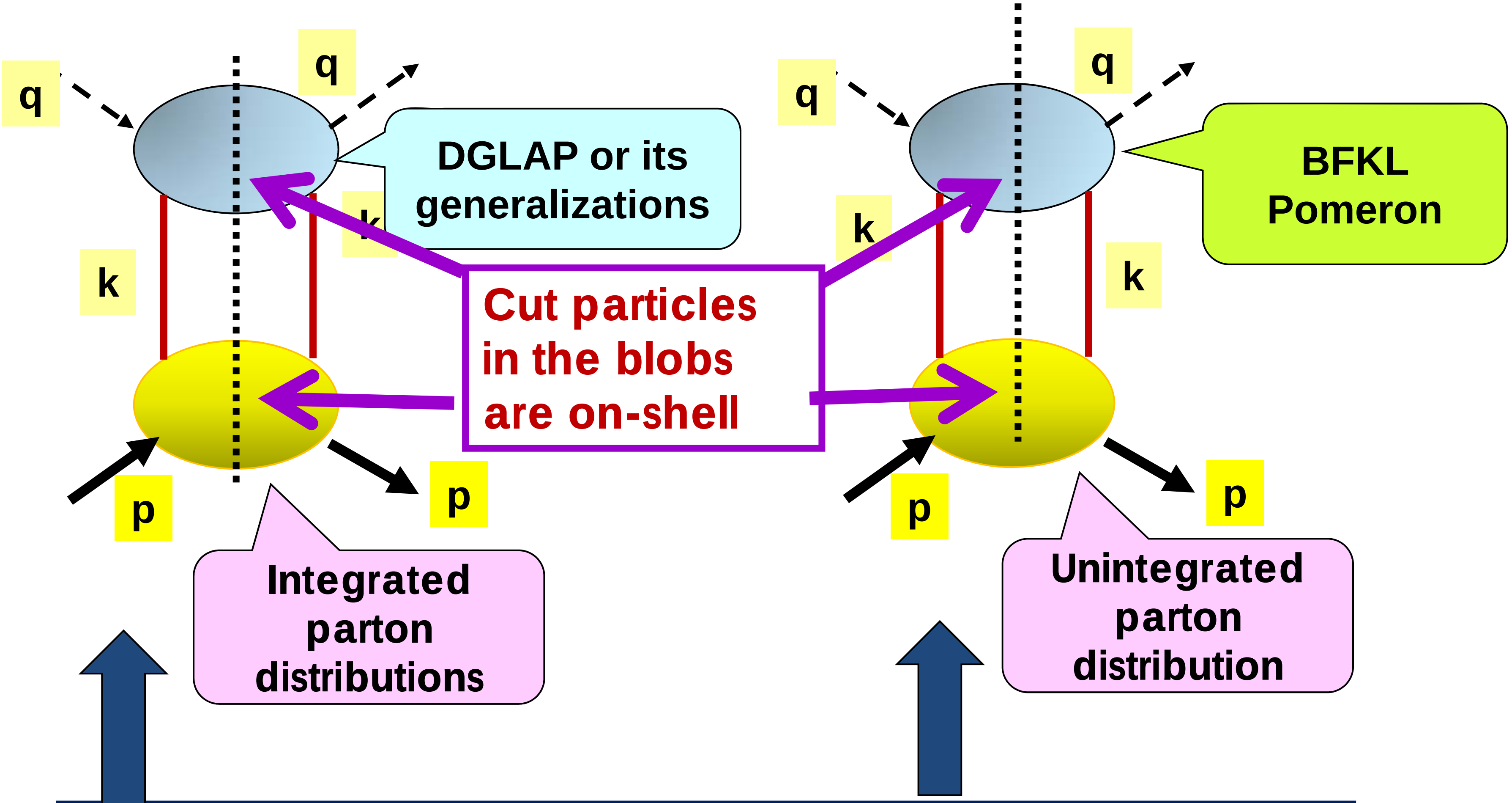
S. Catani - M. Ciafaloni – F. Hautmann

These kinds of factorization were introduced from different considerations and are used for different perturbative approaches, so they look absolutely unrelated to each other.

I will show they are related and introduce a new kind of factorization

Conventional illustrations of Factorizations

Collinear Factorization | **K_T- factorization**



Pictures look identically but formulae are quite different

Factorization representations for DIS structure functions

fraction of longitudinal moment

factorization scale

Collinear Factorization

$$f(x,Q^2)=\int_x^1 \frac{d\beta}{\beta} f^{(pert)}(x/\beta, Q^2/\mu^2)\,\varphi(\beta,\mu^2)$$

integrated parton distribution

Perturbative terms are calculated with evolution equations

Parton distributions found from phenomenological considerations

K_T-Factorization

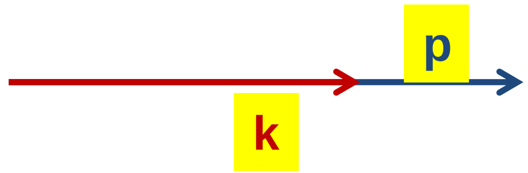
$$f(x,Q^2)=\int \frac{d\beta}{\beta}\frac{dk_\perp^2}{k_\perp^2} f^{(pert)}(x/\beta, Q^2/k_\perp^2)\,\Phi(\beta,k_\perp^2)$$

Unintegrated parton distribution

Different Factorizations imply different parameterizations of momenta of the connecting partons

Collinear Factorization

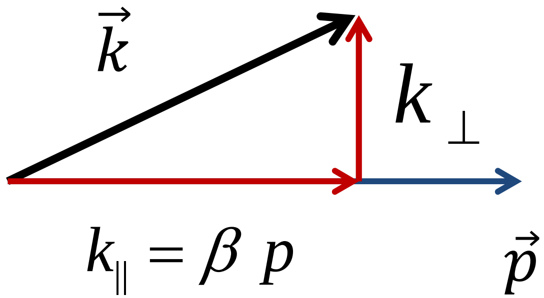
$$\vec{k} = \beta\, \vec{p} \qquad (0 < \beta < 1)$$



momentum fraction

K_T -Factorization

$$\vec{k} = \beta\, \vec{p} + \vec{k}_\perp$$



Actual situation is more involved: k =[k_0, k_x, k_y, k_z]

All components of k should be accounted for

Sudakov parametrization $\quad k = \alpha q + \beta p + k_\perp$

$$d^4k = d^2k_{||}d^2k_\perp = (s/2)d\alpha d\beta d^2k_\perp \approx \pi s d\alpha d\beta k_\perp dk_\perp$$

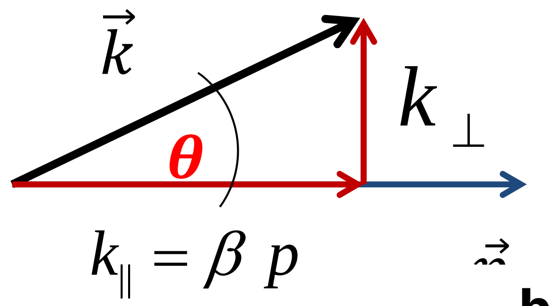
$$\alpha = \frac{2pk}{2pq} \approx \frac{2E\omega}{4E^2}(1-\cos\theta) \quad \beta = \frac{2qk}{2pq} \approx \frac{2E\omega}{4E^2}(1+\cos\theta)$$

ϑ is angle between $\vec{k}$ and $\vec{p}$, $\quad E = p_0, \ \omega = k_0$

when $\theta \ll 1$

$$\alpha \approx \frac{\omega}{4E}\theta^2 \qquad \beta \approx \frac{\omega}{E}$$

$$\alpha << \beta$$



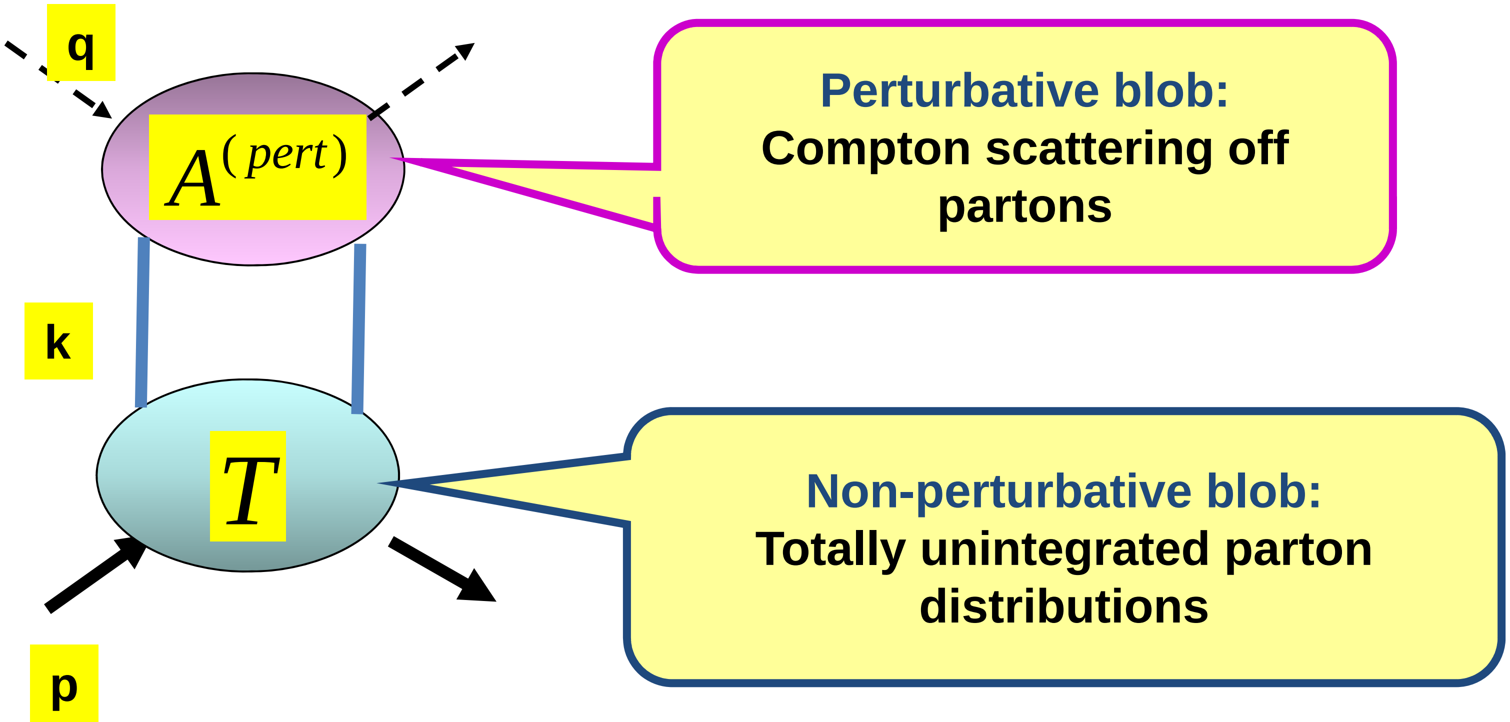
because of this reason α is often neglected and

$$\vec{k} \approx \beta\vec{p} + k_\perp$$

parameterization used in K_T- factorization

When the α -dependence is accounted for, we arrive at a more general factorization:

Basic factorization Ermolaev_Greco-Troyan



In contrast to the cases of Collinear and K_T –factorization, here one can apply the standard Feynman rules to the convolution new integration

$$A(x,Q^2) = \int \frac{d\beta}{\beta}\, d\,k^2\; d\alpha\, A^{(pert)}(\beta, Q^2, k^2)\, \frac{1}{k^2}\, T\;(\alpha, k^2)$$

Integration in collinear factorization

integrations in K_T-factorization

Integration runs over the whole phase space

There can be IR and UV singularities in the integrands
They can be different for different amplitudes

IR singularities arrive from the region of small k^2
UV singularities come from large α

Guiding idea: integration over the loop momentum k must yield a finite result: no IR and UV divergences

PIECE OF TERMINOLOGY: Singlet and non-singlet amplitudes:

Non-singlet amplitudes $(1/\pi)\,\mathrm{Im}\,A_{NS} = F_1^{NS}, F_2, g_1^{S}, g_1^{NS},$ etc

$$A_{NS} = M_{NS}\left(\ln\,(s\beta/k^2),\ \ln\,(Q^2/k^2)\right)$$

IR-sensitive terms

$$A_S = (s\beta/k^2)\,M_S\left(\ln\,(s\beta/k^2),\ \ln\,(Q^2/k^2)\right)$$

Singlet amplitude $(1/\pi)\,\mathrm{Im}\,A_S = F_1^S$

Such terminology is wide-spread but not altogether correct:

For instance, F_2 and flavour singlet g_1^S are referred as non-singlets

Regulation of IR divergences

$$A = \int_{-\infty}^{\infty} \frac{d\beta}{\beta} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\,k^2}{k^2} A_{pert}\,(x, Q^2, \beta, k^2)\; T\;\; (\alpha, k^2)$$

No physical reasons to introduce IR cut-offs

To regulate IR divergences, we Impose requirements

when $k^2 \to 0$

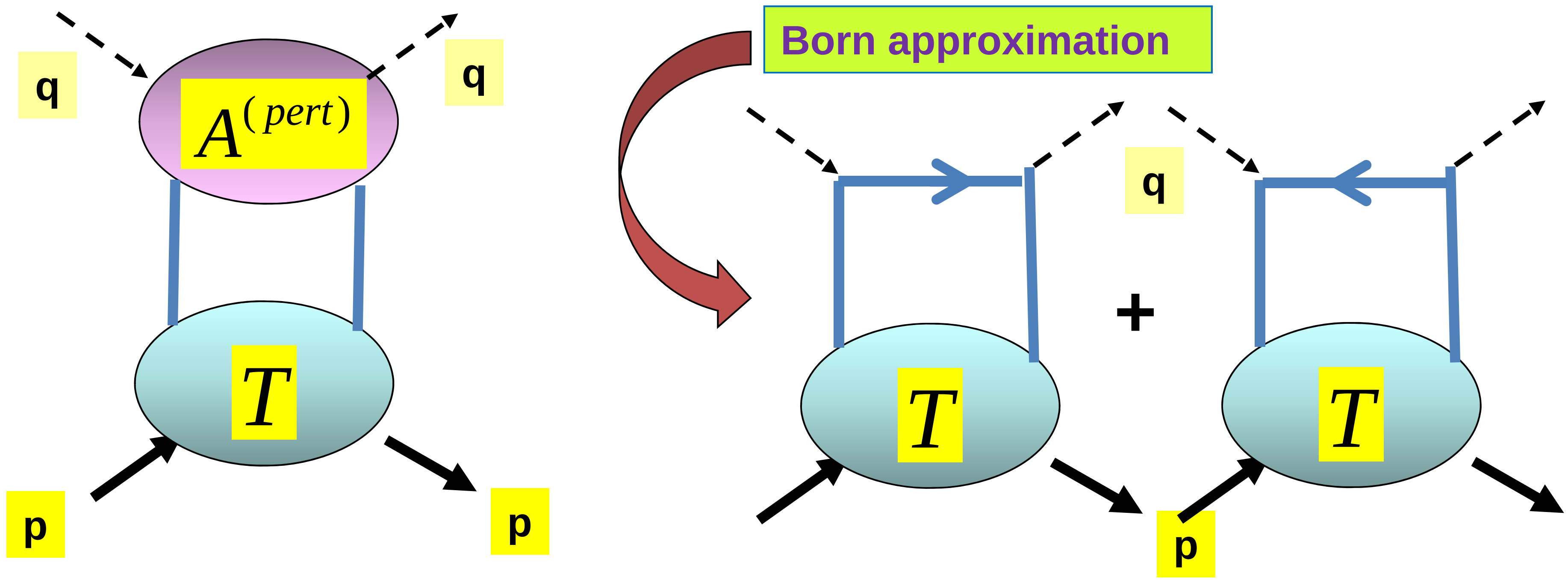
$$T_{NS} \sim (k^2)^{\gamma}$$

with $\gamma > 0$

$$T_S \sim (k^2)^{1+\gamma}$$

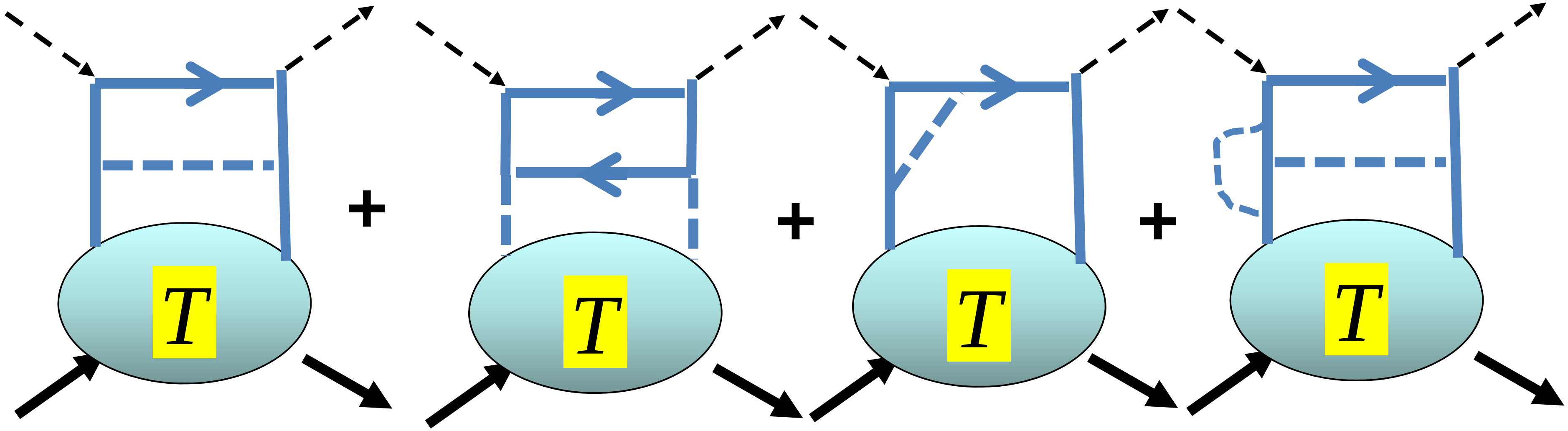
IR singularities are cut out

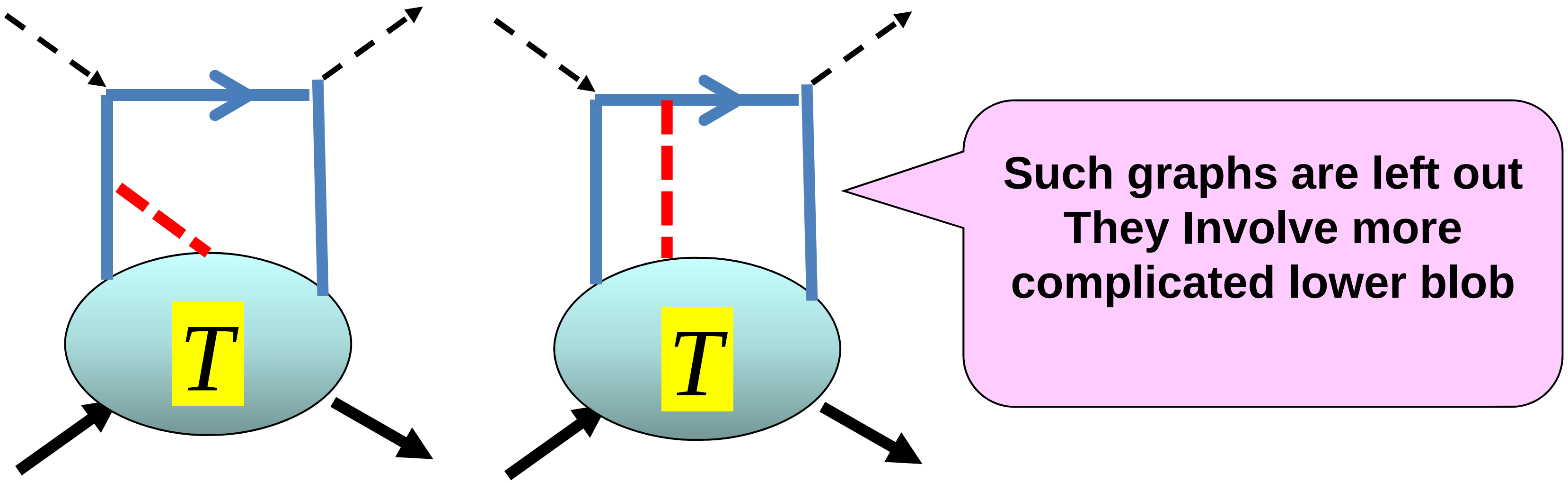
now the upper blobs are within the Perturbative QCD domain



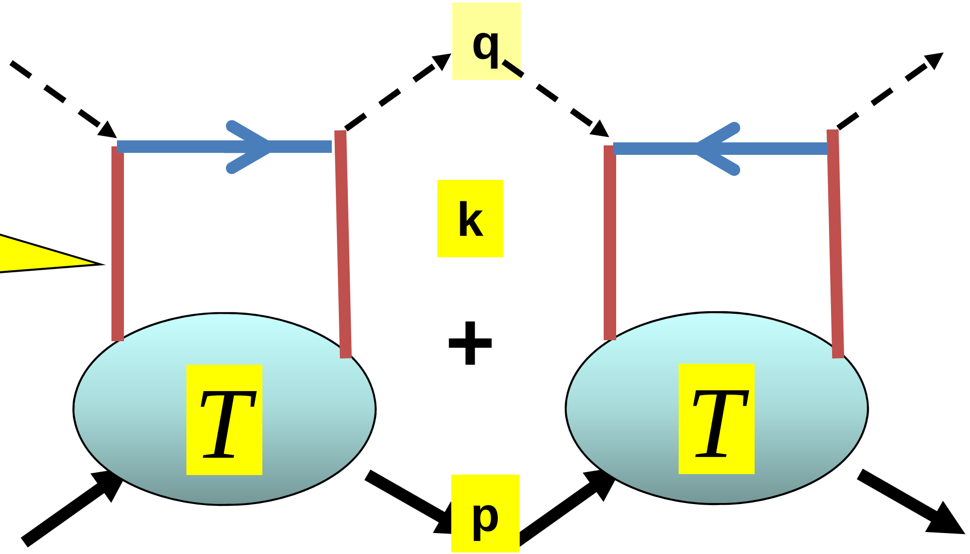
Radiative corrections are absent, so blob T contains
non-perturbative contributions only

Beyond the Born approximation





Remark on gauge invariance



Gauge invariance is broken when intermediate partons are off-shell

$$k^2 \neq m^2_{quark}$$

However, typically the violation is proportional to

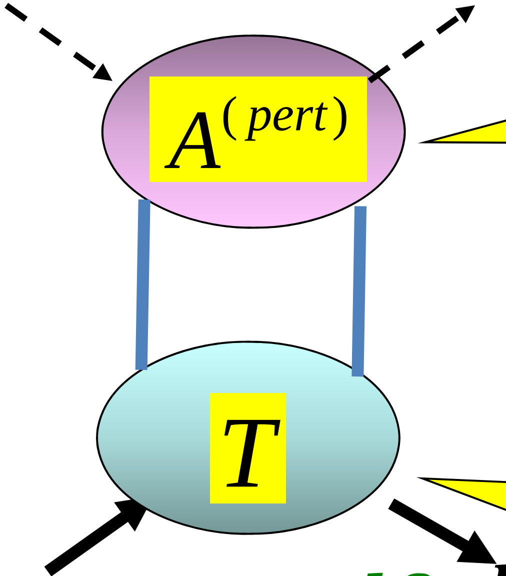
$$\frac{1}{1-x+z} - \frac{1}{1+x-z}$$

$$x = Q^2/2pq, \;\; z = k^2/2pq$$

Gauge invariance is restored at small x and small z in Born approximation
Accounting for logarithmic rad corrections does not violate the invariance

This is also the applicability region for K_T - factorization: small x and accounting for logarithmic contributions

Adding more virtual partons leads to the convolutions without IR and UV divergences and where pert and non-pert contributions are in different blobs



Perturbative blob

Non-perturbative blob

this construction is a QCD factorization but it differs from K_T - and collinear factorizations

We call it

BASIC FACTORIZATION

$$A(x,Q^2) = \int \frac{d\beta}{\beta} \frac{d\,k^2_\perp}{k^2_\perp} \, d\alpha \, A^{(pert)}(\beta, Q^2\ k^2\ )\ \Phi(\alpha, k^2\ )$$

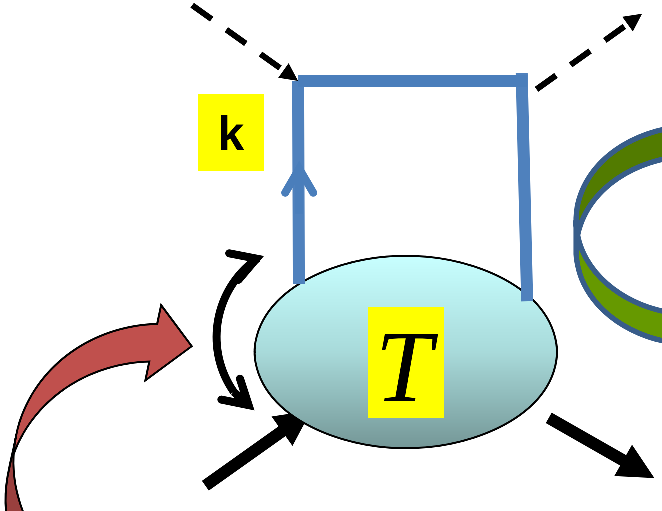
Integration in collinear factorization

integrations in K_T-factorization

New integration

Ultraviolet stability

Born approximation



at large k, each propagator $\sim 1/\hat{k}$

$$d^4k = \mathrm{d}k\, k^3 \mathrm{d}\Omega$$

$$A \sim \int d^4k \, \frac{T(k)}{k^3} = \int d\Omega \, dk \, T(k)$$

at large k $\quad T \sim k^{-1-h}$

$h > 0$

invariant energy

$$s_1 = (p-k)^2 \approx -w\alpha$$

$$w = 2pq$$

In terms of invariant energies

$$T \sim (s_1)^{-1-h} \approx (w\alpha)^{-1-h}$$

Suppression of UV divergences beyond Born approximation:

$$T_{NS} \sim (w\alpha)^{-1-h} \sim (s')^{-1-h}$$

$$T_S \sim (w\alpha)^{-h} \sim (s')^{-h}$$

$s' \approx s\alpha$ invariant energy of the lowest blob

Applying Optical theorem, we arrive at basic factorization for DIS structure functions:

$$f_{NS}(x,Q^2) = \int \frac{d\beta}{\beta}\, d\,k^2{}_{\perp}\, d\alpha\, f_{NS}^{(pert)}(w\beta, Q^{2\prime}\, k^2)\, \frac{1}{k^2}\, \Psi_{NS}\,(\alpha, k^2)$$

$$f_S(x,Q^2) = \int \frac{d\beta}{\beta}\, d\,k^2{}_{\perp}\, d\alpha\, f_S^{(pert)}(w\beta, Q^{2\prime}\, k^2)\, \frac{1}{k^2}\, \Psi_S\,(\alpha, k^2)$$

with totally unintegrated singlet and non-singlet parton distributions

$$\Psi_{NS} = \mathrm{Im}\, T_{NS}(w\alpha, k^2), \quad \Psi_S = \mathrm{Im}\, T_S(w\alpha, k^2)$$

$$f(x,Q^2) = \int \frac{d\beta}{\beta}\, d\,k^2{}_\perp\, d\alpha\, f^{(pert)}(w\beta, Q^{2\prime}\, k^2)\, \frac{1}{k^2}\, \Psi(\alpha, k^2)$$

Basic factorization

Reduction to K_T-factorization:

integration over $\boxed{\alpha}$ should be performed without dealing with $f^{(pert)}(x, \beta, k^2)$ which is impossible to be done exactly:

$$k^2 = -w\,\alpha\,\beta - k^2{}_\perp$$

Approximation: $\boxed{w\,\alpha\,\beta \ll k^2{}_\perp}$

Sense: virtualities of intermediate partons are generated by their transverse momenta

$$f(x,Q^2) = \int \frac{d\beta}{\beta} \frac{d\,k^2{}_\perp}{k^2{}_\perp}\, f^{(pert)}(w\beta, Q^{2\prime}\, k^2{}_\perp)\, \Phi(\beta, k^2{}_\perp)$$

K_T-factorization

unintegrated parton distribution

$$\Phi(\beta, k^2{}_\perp) = \int_{w_0/w}^{k^2{}_\perp/w\beta} d\alpha\, \Psi(\alpha, k^2)$$

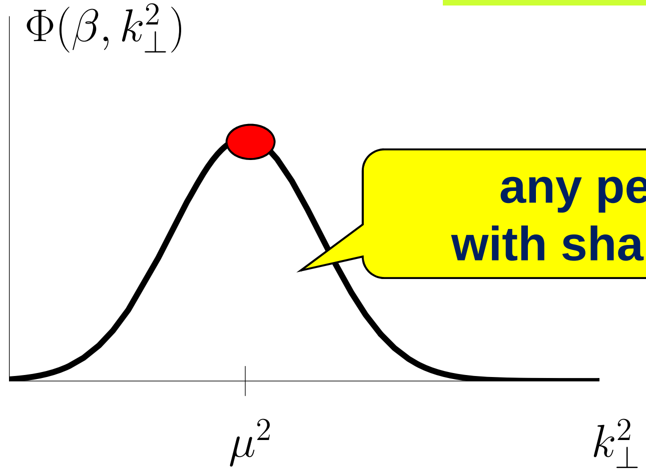
Reduction of k_T – factorization to collinear factorization

$$f(x,Q^2) = \int \frac{d\beta}{\beta} \frac{d\,k^2_\perp}{k^2_\perp} f^{(pert)}(w\beta, Q^2\ \ k^2_\perp)\, \Phi(\beta, k^2_\perp)$$

K_T-factorization

integration over $k_\perp^2$ should be done without integration of $f^{(pert)}(x, \beta, k^2_\perp)$

Assumption:



any peaked form with sharp maximum

Integrated parton distribution

$$f(x,Q^2) = \int \frac{d\beta}{\beta} f^{(pert)}(w\beta, Q^2, \mu^2)\, \varphi(\beta, \mu^2)$$

intrinsic factorization scale

$$\varphi\left(\beta, \mu^2\right) = \int_D \frac{dk^2{}_\perp}{k^2{}_\perp} k^2{}_\perp \Phi\left(\beta, k^2{}_\perp\right)$$

intrinsic factorization scale

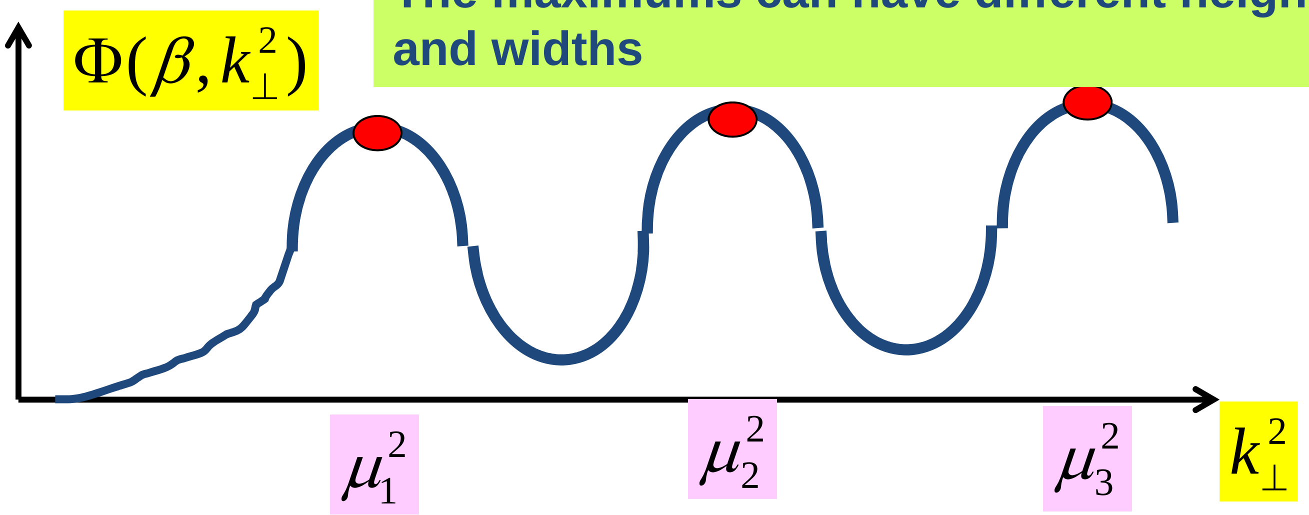
Integration runs in vicinity of maximum

Intrinsic scale has the physical meaning: it is maximum of parton distribution in K_T –factorization and this maximum is associated with non-perturbative physics entirely: $\mu \sim \Lambda$

The sharper the maximum is, the better is accuracy of the transition from K_T to collinear factorization

More involved picture is possible: several maximums

The maximums can have different heights and widths



If several maximums, then:

$$f(x,Q^2) = \sum_{k} \int \frac{d\beta}{\beta} f^{(pert)}(w\beta, Q^2, \mu^2{}_k)\, \varphi_k(\beta, \mu^2{}_k)$$

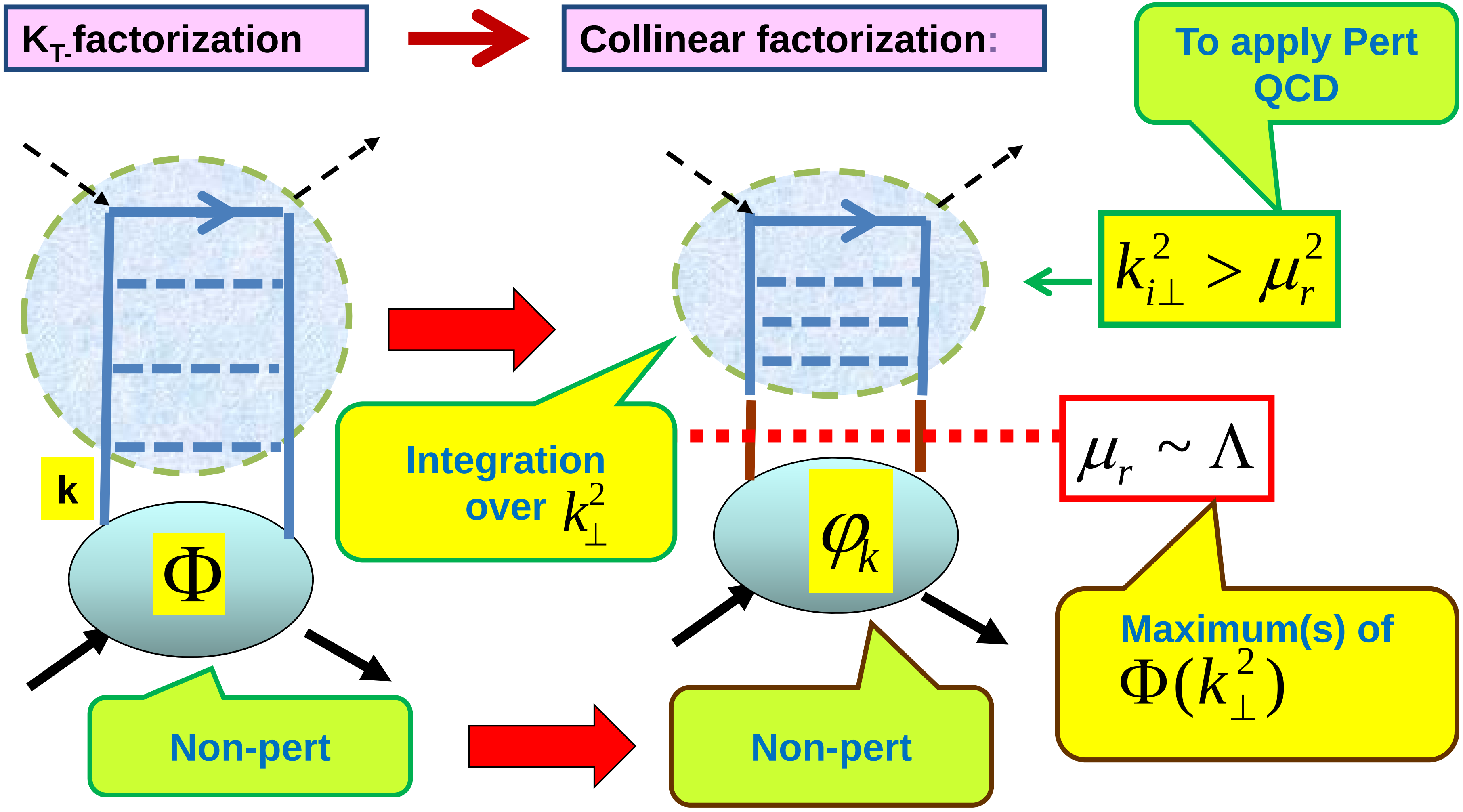
intrinsic scales

This form of collinear factorization looks totally incompatible with the conventional form where

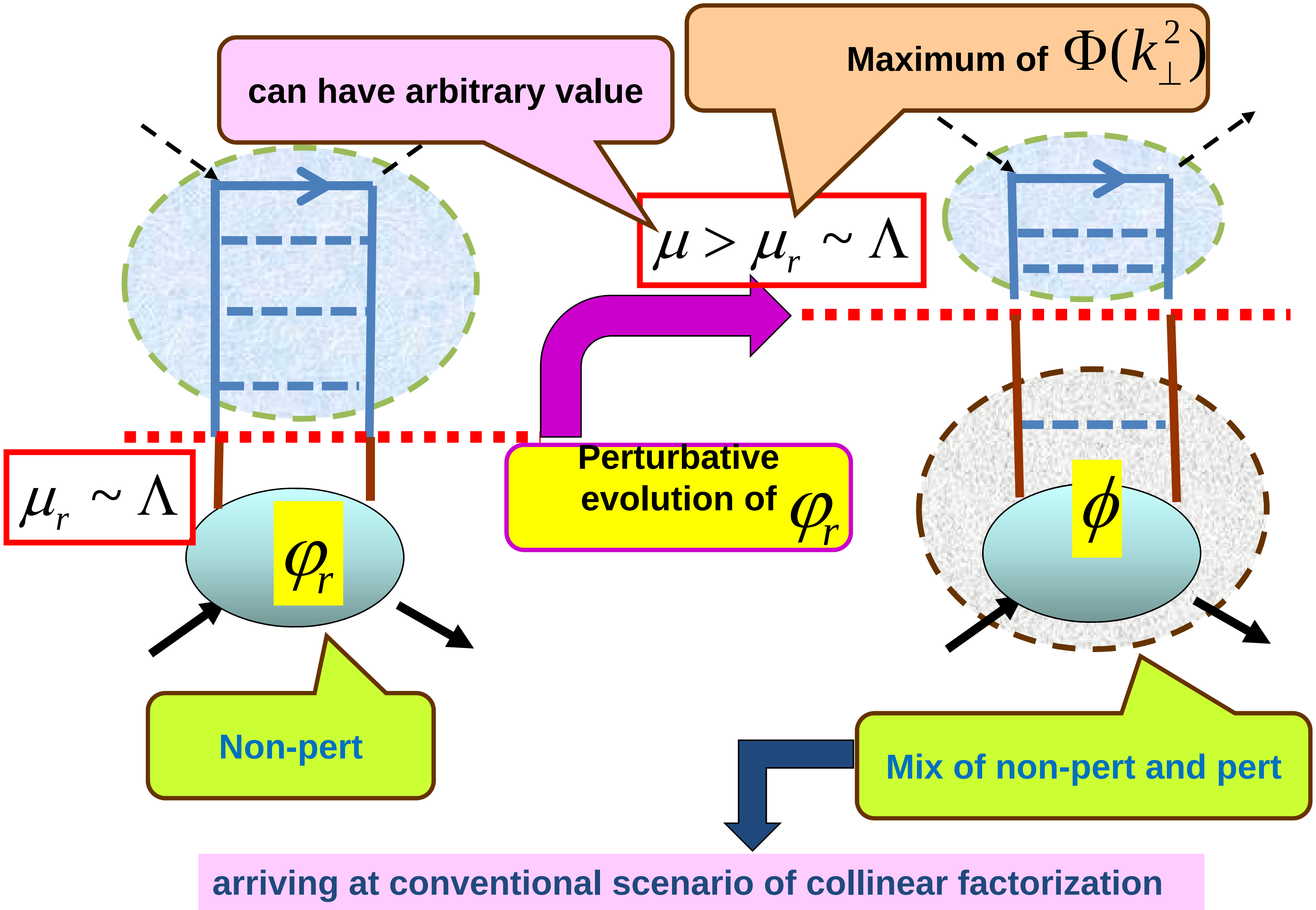
$$f(x,Q^2) = \int \frac{d\beta}{\beta} f^{(pert)}(w\beta, Q^2, \mu^2)\, \phi(\beta, \mu^2)$$

arbitrary value

K_T-factorization → Collinear factorization:

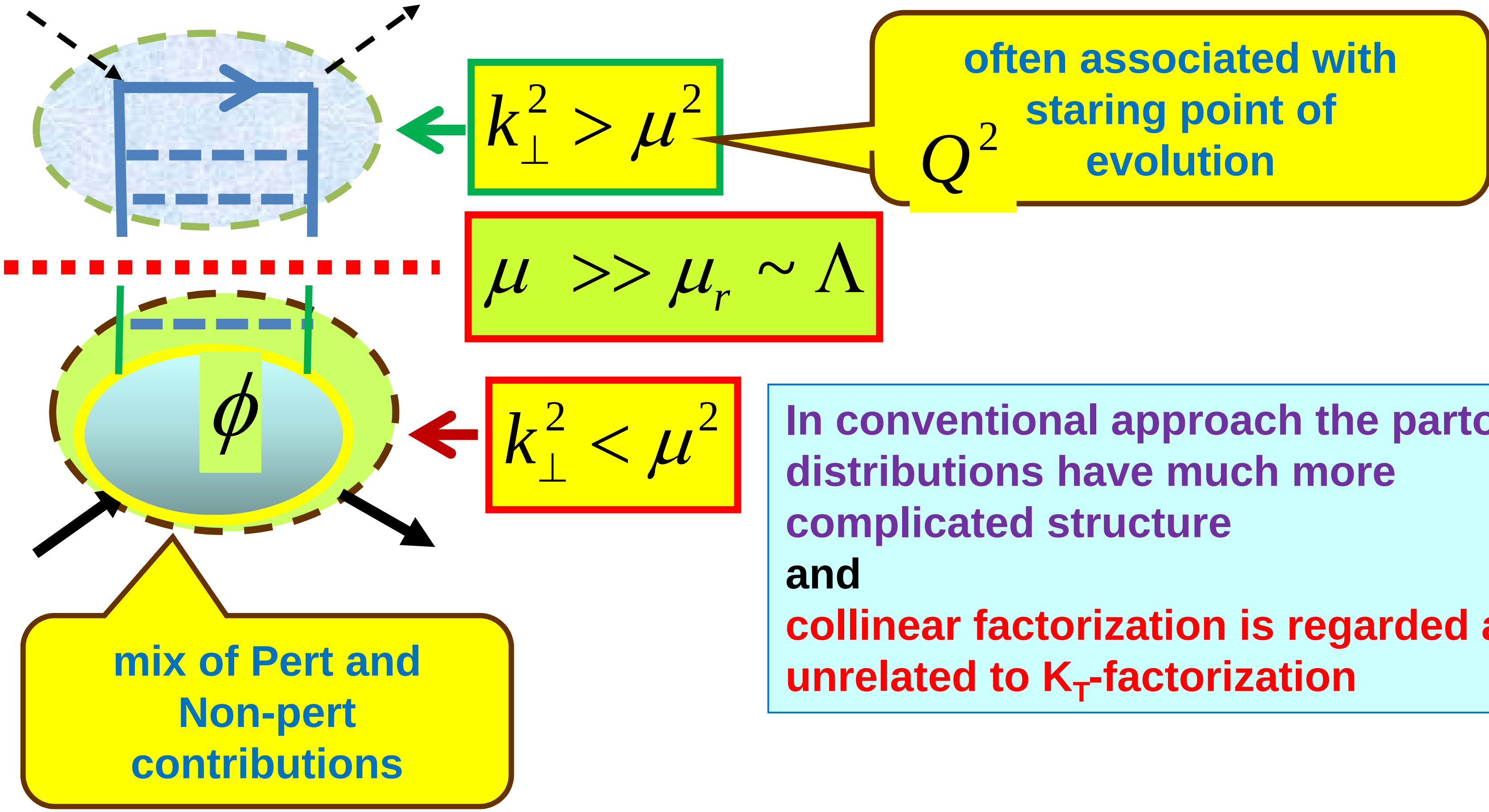


We obtain collinear factorization through the reduction of K_T-factorization
The lower blob was and is totally non-perturbative


can have arbitrary value
Maximum of $\Phi(k_\perp^2)$
$\mu > \mu_r \sim \Lambda$
$\mu_r \sim \Lambda$
φ_r
Perturbative evolution of φ_r
ϕ
Non-pert
Mix of non-pert and pert
arriving at conventional scenario of collinear factorization

Conventional approach:

First, arbitrary factorization scale μ is chosen
Then, rad corr are distributed between two blobs



In conventional approach the parton distributions have much more complicated structure
and
collinear factorization is regarded as unrelated to K_T-factorization

Restrictions on fits for parton distribution

k_T -factorization

$$\Phi(\beta, k^2{}_\perp) = \int_{w_0/w}^{k^2{}_\perp/w\beta} d\alpha \Psi(\alpha, k^2)$$

Parton distribution in k_T-factorization

Parton distribution in basic factorization

At small k^2

$$\Psi_{NS}(\alpha, k^2) \sim (k^2)^\gamma$$

$$\Psi_S(\alpha, k^2) \sim (k^2)^{1+\gamma}$$

At large α

$$\Psi(\alpha, k^2) \sim \alpha^{-1-h}$$

General structure of fits in k_T -factorization

$$\Phi_{NS} = (k^2{}_\perp)^{(\gamma-h)} \beta^h D_{NS}(\beta, k^2{}_\perp) + (k^2{}_\perp)^{\gamma} B_{NS}(\beta, k^2{}_\perp)$$

$$\Phi_S = (k^2{}_\perp)^{(1+\gamma-h)} \beta^h D_S(\beta, k_\perp{}^2) + (k^2{}_\perp)^{(1+\gamma)} B_S(\beta, k^2{}_\perp)$$

$\gamma - h > 0$; $D_{S,NS}$ $B_{S,NS}$ have sharp maximums in $k_T{}^2$

$$\Phi(x, k_\perp{}^2) = (k^2{}_\perp)^a x^h D(x, k_\perp{}^2) + (k^2{}_\perp)^b B(x, k_\perp{}^2)$$

with $0<a<h<b$

Sharp maximums in k_T

This form of the fits has recently been used by Grinyuk-Jung-Lykasov-Lipatov-Zotov

Simplest:

$$\Phi(x, k_\perp{}^2) = [(k^2{}_\perp)^a x^h + (k^2{}_\perp)^b]\, B(k_\perp{}^2)$$

$$B(k_\perp{}^2) = \exp\left[\left((k^2{}_\perp - \mu^2)\right)^2 / c\right]$$

Restrictions on DGLAP fits in collinear factorization

Typical structure of DGLAP-fits for initial parton densities:

$$\delta q, \delta g = \underbrace{N}_{\text{normalization}} \; \underbrace{x^{-a}}_{\text{singular factor}} \; \underbrace{(1-x)^{b} \; (1+c\, x^{d})}_{\text{two regular terms}}$$

$N, a, b, c, d > 0$

UV-stability of Compton amplitude
In basic factorization

$\Downarrow$

No singular factors in $\delta q, \; \delta g$

Necessity to use singular factors in DGLAP

When non-singular fits are used, the DGLAP structure functions grow too slow, eventually becoming the very well-known DGLAP small-x asymptotics

$$f \sim \exp\left[\sqrt{\ln(1/x)\, ln\left(\frac{ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right)}\right]$$

It grows not fast enough. In order to get a faster growth, they introduce the singular factors These factors change the DGLAP small-x asymptotics for the Regge asymptotics

$$\delta q \sim x^{-a} \xrightarrow{\text{Mellin transform}} 1/(\omega - a)$$

$$f = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} x^{-\omega} \delta q(\omega) C(\omega)\, e^{\gamma(\omega)\ln(Q^2/\mu^2)} \quad \sim x^{-a}$$

intercept

we suppress the use of singular factors but the fast growth of the structure functions at small x is mandatory

WAY OUT

Total resummation of leading logs of x automatically leads to the Regge asymptotics When the resummation is accounted for, the singular factors can be dropped

WARNING

If a parton distribution needs singular factor to match exp data, it means that important log contributions are not accounted for

Structure functions	F_1^{NS}	g_1^{NS}	g_1^{S}
Their intercepts	0.38	0.42	0.86

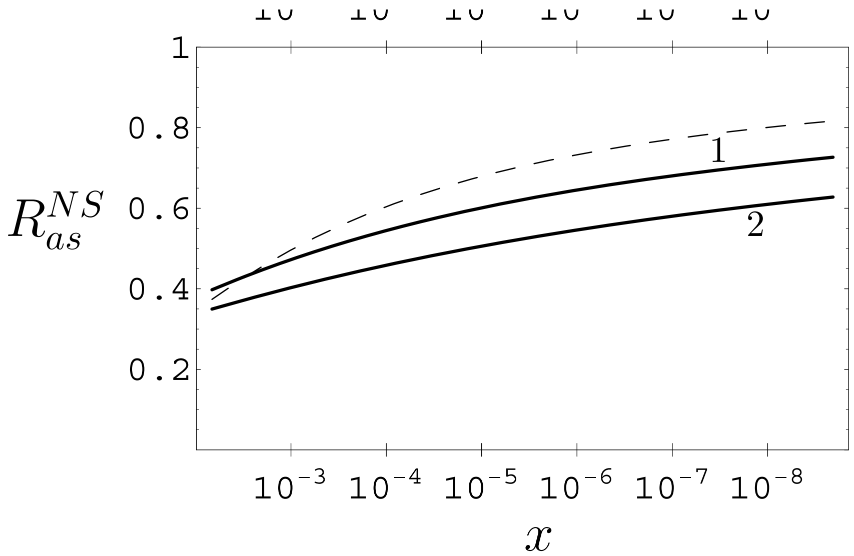
Ermolaev-Greco-Troyan

$$\boxed{f \sim x^{-\Delta}(Q^2/\mu^2)^{\Delta/2} \sim \left(\frac{Q^2}{x^2}\right)^{\Delta/2}}$$

Asymptotic scaling

WARNING: **Asymptotics reliably represent structure functions at extremely small x only**

$$R_{as}^{NS} = \frac{Asymptf_{NS}}{f_{NS}}$$



dashed line $Q^2 = \mu^2$

line1 $Q^2 = 10\mu^2$

line2 $Q^2 = 100\mu^2$

Situation for the singlets is even worse

One more look at the DGLAP-fits

normalization

Small-x asymptotics

asymptotics for x->1

interpolation term

$$\delta q, \delta g = N \; x^{-a} \; (1-x)^{b} \; (1+c\, x^{d})$$

Dropped when logs of x are resummed

Dropped when logs of (1-x) are resummed

May become unnecessary when logs of both x and (1-x) are resummed

Therefore, the fits can be simplified down to Normalizations

CONCLUSION

We obtained a new, more general kind of factorization.
We call it Basic QCD Factorization

IR and UV stability of the convolutions in Basic Factorization allowed us to impose restrictions on fits for parton distributions

Basic factorization can be reduced first to K_T- and then to collinear factorizations

Using the relations between Basic factorization and K_T – and Collinear Factorizations, we obtain the following restrictions on the fits for initial parton distributions:

Fits in K_T –factorization should include two terms, each with factor $(k^2{}_\perp)^a$, with different exponents These factors are multiplied by functions with peaked dependence on $k^2{}_\perp$

Fits used in DGLAP in collinear factorization should not involve singular factors x^{-a}