

On the BLM optimal renormalization scale setting for semihard processes



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...BLM method!

Outline

1 Theoretical setup

- BFKL approach
- BLM method

2 Semihard processes

- Mueller-Navelet jets
- Electroproduction of two light vector mesons
- $\gamma^*\gamma^*$ forward scattering

3 Conclusions

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The BFKL approach

Total cross section $A + B \rightarrow X$, via the optical theorem, $\sigma = \frac{\text{Im}_s \mathcal{A}}{s}$

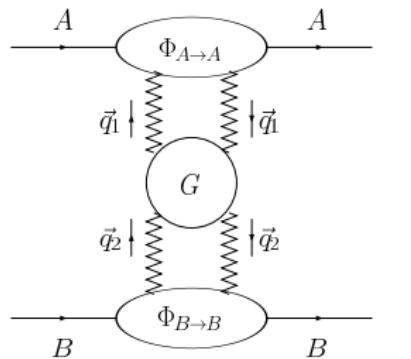
- Regge limit ($s \rightarrow \infty$)

⇒ BFKL factorization for $\text{Im}_s \mathcal{A}$: convolution of the **Green's function** of two interacting Reggeized gluons and of the **impact factors** of the colliding particles.

- Valid both in

LIA (resummation of all terms $(\alpha_s \ln s)^n$)

NLA (resummation of all terms $\alpha_s (\alpha_s \ln s)^n$).



$$\text{Im}_s \mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^2} \Phi_A(\vec{q}_1, \textcolor{blue}{s}_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^2} \Phi_B(-\vec{q}_2, \textcolor{blue}{s}_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{\textcolor{blue}{s}_0}\right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

- The **Green's function** is process-independent and is determined through the **BFKL equation**.
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

Solution of the BFKL equation

- The Green's function obeys the **BFKL equation**

$$\delta^2(\vec{q}_1 - \vec{q}_2) = \omega G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2\vec{q} K(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_2)$$

- Transverse momentum space definition

$$\hat{\vec{q}} |\vec{q}_i\rangle = \vec{q}_i |\vec{q}_i\rangle, \quad \langle \vec{q}_1 | \vec{q}_2 \rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2), \quad \langle A | B \rangle = \langle A | \vec{k} \rangle \langle \vec{k} | B \rangle = \int d^2k A^*(\vec{k}) B(\vec{k})$$

- The **kernel operator** \hat{K} is

$$K(\vec{q}_2, \vec{q}_1) = \langle \vec{q}_2 | \hat{K} | \vec{q}_1 \rangle$$

- Straightforward solution in the transverse momentum space

$$\hat{1} = (\omega - \hat{K}) \hat{G}_\omega \quad \longrightarrow \quad \hat{G}_\omega = (\omega - \hat{K})^{-1}, \quad \hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1, \quad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

- Solution for the \hat{G}_ω operator with NLA accuracy

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} \left(\bar{\alpha}_s^2 \hat{K}^1 \right) (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + \mathcal{O} \left[\left(\bar{\alpha}_s^2 \hat{K}^1 \right)^2 \right]$$

- Basis of eigenfunctions of the LO kernel

$$\hat{K}^0 |n, \nu\rangle = \chi(n, \nu) |n, \nu\rangle, \quad \chi(n, \nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right)$$

$$\langle \vec{q} | n, \nu \rangle = \frac{1}{\pi\sqrt{2}} (\vec{q}^2)^{i\nu - \frac{1}{2}} e^{in\theta}, \quad \cos\theta \equiv q_x$$

$$\langle n', \nu' | n, \nu \rangle = \int \frac{d^2 q}{2\pi^2} (\vec{q}^2)^{i\nu - i\nu' - 1} e^{i(n-n')\theta} = \delta(\nu - \nu') \delta_{nn'}$$

- The action of the full NLO BFKL kernel on these functions may be expressed as follows:

$$\begin{aligned} \hat{K} |n, \nu\rangle &= \bar{\alpha}_s(\mu_R) \chi(n, \nu) |n, \nu\rangle + \bar{\alpha}_s^2(\mu_R) \left(\chi^{(1)}(n, \nu) + \frac{\beta_0}{4N_c} \chi(n, \nu) \ln(\mu_R^2) \right) |n, \nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R) \frac{\beta_0}{4N_c} \chi(n, \nu) \left(i \frac{\partial}{\partial \nu} \right) |n, \nu\rangle, \end{aligned}$$

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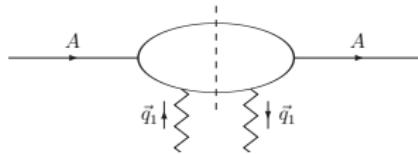
Impact factors

- Projection of the impact factors $\phi_i(\vec{q})$ onto the eigenfunctions of LO BFKL kernel, i.e. the transfer to the (n, ν) -representation

$$c_i(n, \nu) = \int d^2 \vec{q} \frac{\phi_i(\vec{q})}{\vec{q}^2} \frac{1}{\pi\sqrt{2}} (\vec{q}^2)^{i\nu - \frac{1}{2}} e^{in\phi}.$$

Here ϕ is the azimuthal angle of the vector \vec{q} counted from some fixed direction in the transverse space.

- Impact factors are process-dependent;



only very few have been calculated in the NLA:

- colliding partons

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
 [M. Ciafaloni and G. Rodrigo (2000)]

- $\gamma^* \rightarrow V$, with $V = \rho^0, \omega, \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

- forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]

[F. Caporale, D.Yu. Ivanov, B. M., A. Papa, A. Perri (2012)]
 (small-cone approximation) [D.Yu. Ivanov, A. Papa (2012)]

- forward identified hadron production

[D.Yu. Ivanov, A. Papa (2012)]

- $\gamma^* \rightarrow \gamma^*$

[J. Bartels *et al.* (2001) →]

[V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2002)-(2003)]
 [I. Balitsky, G.A. Chirilli (2011)-(2014)]
 [G.A. Chirilli, Yu.V. Kovchegov (2014)]

To study a process...

$$\begin{aligned} \mathcal{C}_n = & \int_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0} \right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu) + \bar{\alpha}_s^2(\mu_R)K^{(1)}(n,\nu)} \alpha_s^2(\mu_R) c_1(n,\nu) c_2(n,\nu) \\ & \times \left[1 + \bar{\alpha}_s(\mu_R) \left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)} \right) \right] \end{aligned}$$

with

$$K^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c} \chi(n,\nu) \left(-\chi(n,\nu) + \frac{10}{3} + i \frac{d}{d\nu} \ln \left(\frac{c_1(n,\nu)}{c_2(n,\nu)} \right) + 2 \ln(\mu_R^2) \right)$$

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- Optimization method

BLM scale setting: in an observable the scale is chosen such that it makes vanish the β_0 -dependent part.

[S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]

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It is supposed to mimic the effect of the most relevant unknown subleading terms.

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Optimization method

- **Step 1:** change of renormalization scheme

$$\alpha_s^{\overline{\text{MS}}}(\mu_R) = \alpha_s^{\text{MOM}}(\mu_R) \left(1 + \frac{\alpha_s^{\text{MOM}}(\mu_R)}{\pi} T \right) \quad \text{with} \quad T = T^\beta + T^{\text{conf}},$$

$$T^\beta = -\frac{\beta_0}{2} \left(1 + \frac{2}{3} I \right) \quad \text{and} \quad T^{\text{conf}} = \frac{C_A}{8} \left[\frac{17}{2} I + \frac{3}{2} (1-I) \xi + \left(1 - \frac{1}{3} I \right) \xi^2 - \frac{1}{6} \xi^3 \right]$$
$$I = -2 \int_0^1 dx \frac{\ln(x)}{[x^2 - x + 1]} \simeq 2.3439, \quad \xi = 0.$$

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↓

$$\begin{aligned} C_n \simeq & \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s^{\text{MOM}}\chi(\nu,n)} \left(\alpha_s^{\text{MOM}} \right)^2 c_1 c_2 \left\{ 1 + \bar{\alpha}_s^{\text{MOM}} \left(\frac{\bar{c}_1^{(1)}}{c_1} + \frac{\bar{c}_2^{(1)}}{c_2} \right) \right. \\ & + \bar{\alpha}_s^{\text{MOM}} \left(\frac{\bar{c}_1^{(1)}}{c_1} + \frac{\bar{c}_2^{(1)}}{c_2} \right) + \bar{\alpha}_s^{\text{MOM}} \frac{2T^{\text{conf}}}{C_A} + \left(\bar{\alpha}_s^{\text{MOM}} \right)^2 (Y - Y_0) \left(\frac{T^{\text{conf}}}{C_A} \chi(\nu,n) + \bar{\chi}(\nu,n) \right) \\ & + \bar{\alpha}_s^{\text{MOM}} \left[\frac{2T^\beta}{C_A} + \bar{\alpha}_s^{\text{MOM}} (Y - Y_0) \left(\frac{T^\beta}{C_A} \chi(\nu,n) + \frac{\beta_0}{8C_A} \left(-\chi(\nu,n) + \frac{10}{3} \right. \right. \right. \\ & \quad \left. \left. \left. + i \frac{d}{d\nu} \ln \left(\frac{c_1}{c_2} \right) + 2 \ln(\mu_R^2) \right) \right) \right] \end{aligned}$$

- **Step 2:** choice of the BLM scale

$$\frac{\beta_0}{2C_A} \left\{ \left[-2 \left(1 + \frac{2}{3} I \right) + \frac{5}{3} + f(\nu) + \ln \left(\frac{\mu_R^2}{k_1 k_2} \right) \right] + \bar{\alpha}_s^{MOM}(\mu_R) (Y - Y_0) \chi(\nu, n) \right. \\ \left. \times \left[-1 - \frac{2}{3} I - \frac{1}{4} \chi(\nu, n) + \frac{5}{6} + f(\nu) + \frac{i}{4} \frac{d}{d\nu} \ln \left(\frac{c_1}{c_2} \right) + \frac{1}{2} \ln(\mu_R^2) \right] \right\} = 0$$

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Partial BLM:

a) $(\mu_R^{BLM})^2 = k_1 k_2 \exp [2(1 + \frac{2}{3}I) - f(\nu) - \frac{5}{3}]$

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$f(\nu)$ is a function that depends on the considered process.

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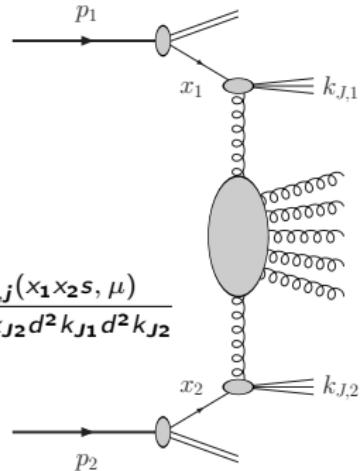
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Mueller-Navelet jets

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow \text{jet}_1(k_1) + \text{jet}_2(k_2) + X$$

$$\frac{d\sigma}{dx_{J1} dx_{J2} d^2 k_{J1} d^2 k_{J2}} = \sum_{i,j=\mathbf{q},\bar{\mathbf{q}},\mathbf{g}} \int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1, \mu) f_j(x_2, \mu) \frac{d\hat{\sigma}_{i,j}(x_1 x_2 s, \mu)}{dx_{J1} dx_{J2} d^2 k_{J1} d^2 k_{J2}}$$



Sudakov decomposition:

$$k_{J,1} = x_{J,1} p_1 + \frac{\vec{k}_{J,1}^2}{x_{J,1} s} p_2 + k_{J,1\perp}, \quad k_{J,1\perp}^2 = -\vec{k}_{J,1}^2, \quad s = 2p_1 \cdot p_2$$

$$k_{J,2} = x_{J,2} p_2 + \frac{\vec{k}_{J,2}^2}{x_{J,2} s} p_1 + k_{J,2\perp}, \quad k_{J,2\perp}^2 = -\vec{k}_{J,2}^2$$

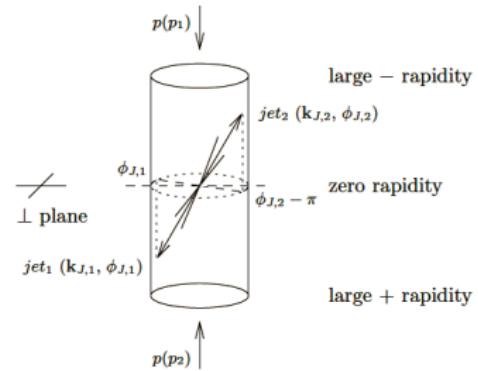
- large jet transverse momenta: $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{\text{QCD}}^2 \longrightarrow$ perturbative QCD applicable
- large rapidity gap between jets, $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}| |\vec{k}_{J,2}|}, \longrightarrow s = 2p_1 \cdot p_2 \gg \vec{k}_{J,1}^2 \longrightarrow$ BFKL resummation: $\sum_n c_n \alpha_s^n \ln^n s + d_n \alpha_s^n \ln^{n-1} s$

Observables of interest

Moments of the azimuthal decorrelations

$$\langle \cos [n(\phi_{J_1} - \phi_{J_2} - \pi)] \rangle \equiv \frac{C_n}{C_0},$$

$$\mathcal{R}_{m,n} = \frac{\langle \cos(m\Delta\phi) \rangle}{\langle \cos(n\Delta\phi) \rangle}$$



Picture from

[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

$$C_n = \int_0^{2\pi} d\phi_{J_1} \int_0^{2\pi} d\phi_{J_2} \cos [n(\phi_{J_1} - \phi_{J_2} - \pi)] \frac{d\sigma_n}{dJ_1 dJ_2}, \text{ with } dJ_i = dx_{J_i} dk_{J_i}$$

\Rightarrow

$$\begin{aligned} C_n &= \int_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0} \right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^2(\mu_R)K^{(1)}(n,\nu)} \alpha_s^2(\mu_R) c_1(n,\nu) c_2(n,\nu) \\ &\quad \times \left[1 + \bar{\alpha}_s(\mu_R) \left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)} \right) \right] \end{aligned}$$

Our analysis

In order to compare our predictions with the **LHC data** [CMS Collaboration (2013)]

- **Observables:**

$$\langle \cos(\pi - \Delta\phi) \rangle = \frac{C_1}{C_0} , \quad \langle \cos[2(\pi - \Delta\phi)] \rangle = \frac{C_2}{C_0} , \quad \langle \cos[3(\pi - \Delta\phi)] \rangle = \frac{C_3}{C_0} ,$$

$$\frac{\langle \cos[2(\pi - \Delta\phi)] \rangle}{\langle \cos(\pi - \Delta\phi) \rangle} = \frac{C_2}{C_1} , \quad \frac{\langle \cos[3(\pi - \Delta\phi)] \rangle}{\langle \cos[2(\pi - \Delta\phi)] \rangle} = \frac{C_3}{C_2} ,$$

with $\Delta\phi = \phi_{J_2} - \phi_{J_1}$.

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$$\langle \cos(\pi - \Delta\phi) \rangle = \frac{C_1}{C_0}, \quad \langle \cos[2(\pi - \Delta\phi)] \rangle = \frac{C_2}{C_0}, \quad \langle \cos[3(\pi - \Delta\phi)] \rangle = \frac{C_3}{C_0},$$

$$\frac{\langle \cos[2(\pi - \Delta\phi)] \rangle}{\langle \cos(\pi - \Delta\phi) \rangle} = \frac{C_2}{C_1}, \quad \frac{\langle \cos[3(\pi - \Delta\phi)] \rangle}{\langle \cos[2(\pi - \Delta\phi)] \rangle} = \frac{C_3}{C_2},$$

$$\text{with } \Delta\phi = \phi_{J_2} - \phi_{J_1}.$$

- **Kinematic settings**

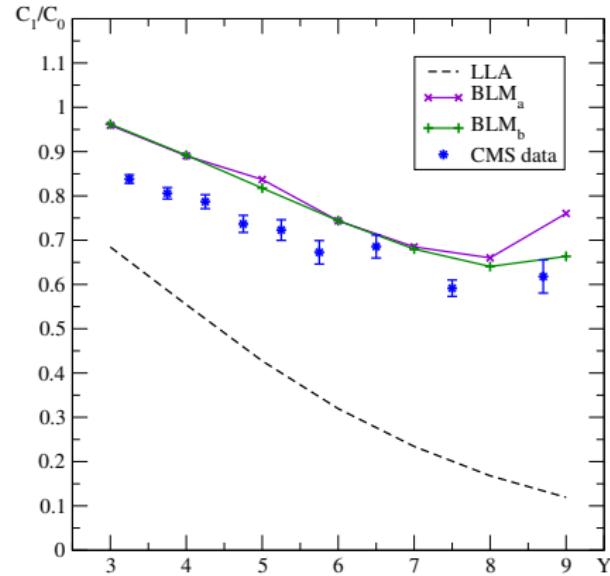
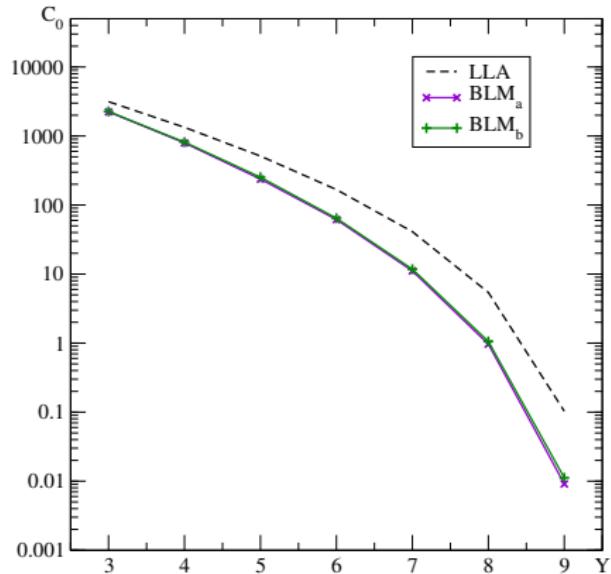
- $R = 0.5$
- $\sqrt{s} = 7 \text{ TeV}$

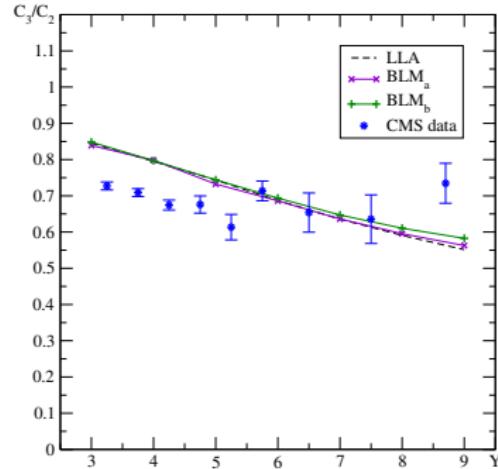
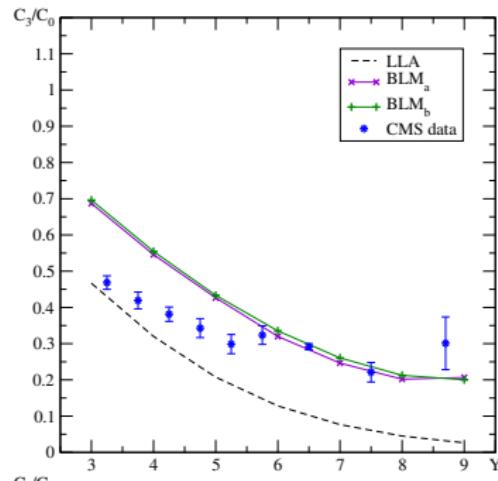
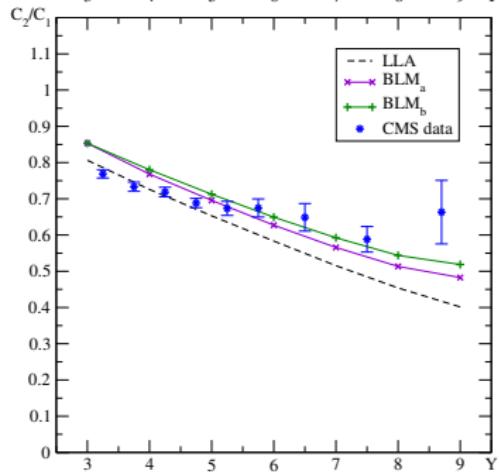
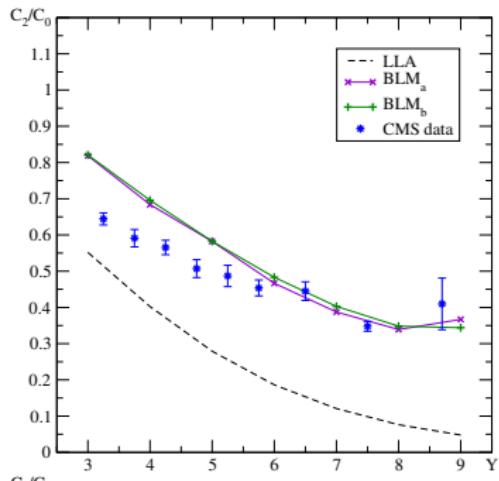
C_n is averaged over the following intervals:

- $35 \text{ GeV} \leq k_{J_i} \leq 60 \text{ GeV}$, with $i = 1, 2$
- $0 \leq y_{J_i} \leq 4.7$, with $i = 1, 2$

$$\Delta y \equiv Y = y_1 + y_2, \quad 3 \leq Y \leq 9$$

$$f(\nu) = 0$$





Outline

1 Theoretical setup

- BFKL approach
- BLM method

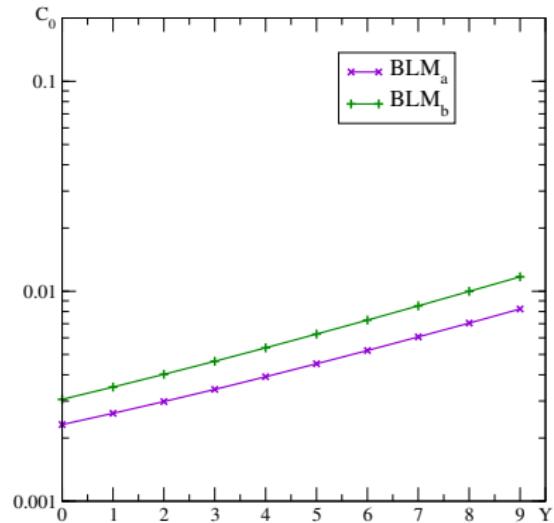
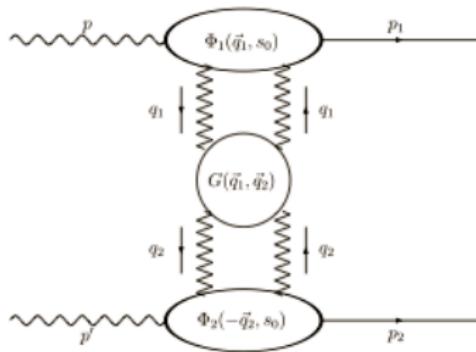
2 Semihard processes

- Mueller-Navelet jets
- Electroproduction of two light vector mesons
- $\gamma^*\gamma^*$ forward scattering

3 Conclusions

Electroproduction of two light vector mesons

$$\gamma^* \gamma^* \rightarrow V_1 V_2$$



Picture from [D.Yu. Ivanov, A. Papa (2007)]

$$Q_1 = Q_2 = Q, \text{ with } Q^2 = 2500 \text{ GeV}^2$$

$$f(\nu) = \psi(3 + 2i\nu) + \psi(3 - 2i\nu) - \psi\left(\frac{3}{2} + i\nu\right) - \psi\left(\frac{3}{2} - i\nu\right)$$

Outline

1 Theoretical setup

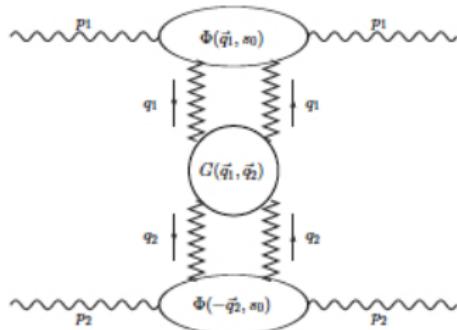
- BFKL approach
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2 Semihard processes

- Mueller-Navelet jets
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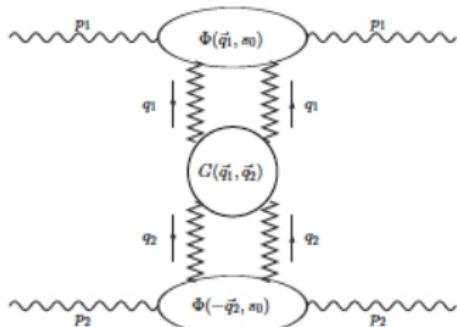
3 Conclusions

$\gamma^* \gamma^*$ total cross section

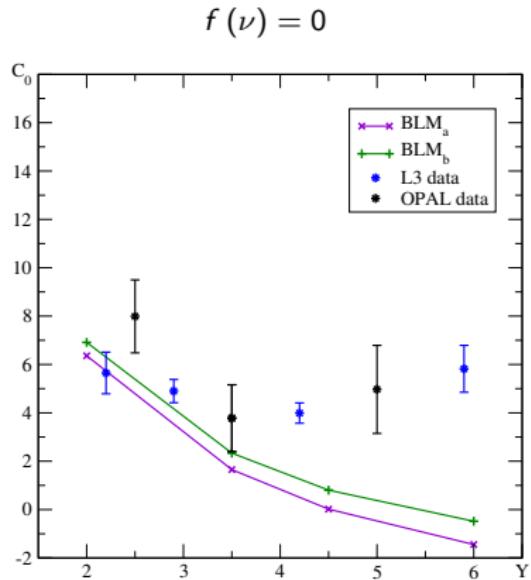


- NLO impact factor
[I. Balitsky, G.A. Chirilli (2013)]
+ quark box LO diagrams
[V.M. Budnev, I.F. Ginzburg,
G.V. Meledin, V.G. Serbo (1974);
I. Schienbein (2002)]

$\gamma^* \gamma^*$ total cross section



- NLO impact factor
[I. Balitsky, G.A. Chirilli (2013)]
- + quark box LO diagrams
[V.M. Budnev, I.F. Ginzburg,
G.V. Meledin, V.G. Serbo (1974);
I. Schienbein (2002)]



$$Q_1 = Q_2 = Q, \text{ with } Q^2 = 17 \text{ Gev}^2$$

Comparison with LEP2 data [P. Achard *et al.* (2002); G. Abbiendi *et al.* (2002)]

For more details, see Ivanov's talk!

[D.Yu. Ivanov, B. M., A. Papa (2014)]

Conclusions

Implementation of a partial **BLM method** for NLO BFKL \Rightarrow two different choices of scale!

We tested the two solutions on different semihard processes:

- **Mueller-Navelet jets** \Rightarrow agreement between the two different choices and with data at large energy;
- **electroproduction of two light vector mesons** \Rightarrow very small constant discrepancy;
- **collision of two highly-virtual photons** \Rightarrow small constant discrepancy and no agreement with data \Rightarrow see Ivanov's talk!