

# pp interaction at LHC energies and beyond



(A. K. Kohara, E. Ferreira, T. Kodama)  
**IF/UFRJ**

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# Outlook

Description of pp elastic scattering

b-space asymptotic behaviour

p-air cross sections (Glauber)

Conclusions

# Elastic Scattering Amplitudes

pp differential cross section

A. Kendi Kohara, E. Ferreira and T. Kodama , *Eur. Phys. J. C* ,**73**, 2326 (2013).  
A. K. Kohara , E. Ferreira and T. Kodama , *Phys. Rev. D* **87** , 054024 (2013).

$$\frac{d\sigma}{dt} = (\hbar c)^2 |T_R(s, t) + iT_I(s, t)|^2$$

Real and imaginary amplitudes ,  $K = R, I$

Regge like term

Stochastic Vacuum Model\*

$$T_K^N(s, t) = \underbrace{\alpha_K(s)e^{-\beta_K(s)|t|}}_{\text{Regge like term}} + \underbrace{\lambda_K(s)\Psi_K(\gamma_K(s), t)}_{\text{Stochastic Vacuum Model*}}$$

$$\Psi_K(\gamma_K(s), t) = 2 e^{\gamma_K} \left[ \frac{e^{-\gamma_K \sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - e^{\gamma_K} \frac{e^{-\gamma_K \sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}} \right]$$

\* H.G. Dosch, *Phys. Lett. B* **190**, 177 (1987); H.G. Dosch, E. Ferreira, A. Kramer *Phys. Rev. D* **50**, 1992 (1994).

# Elastic Scattering Amplitudes

pp differential cross section

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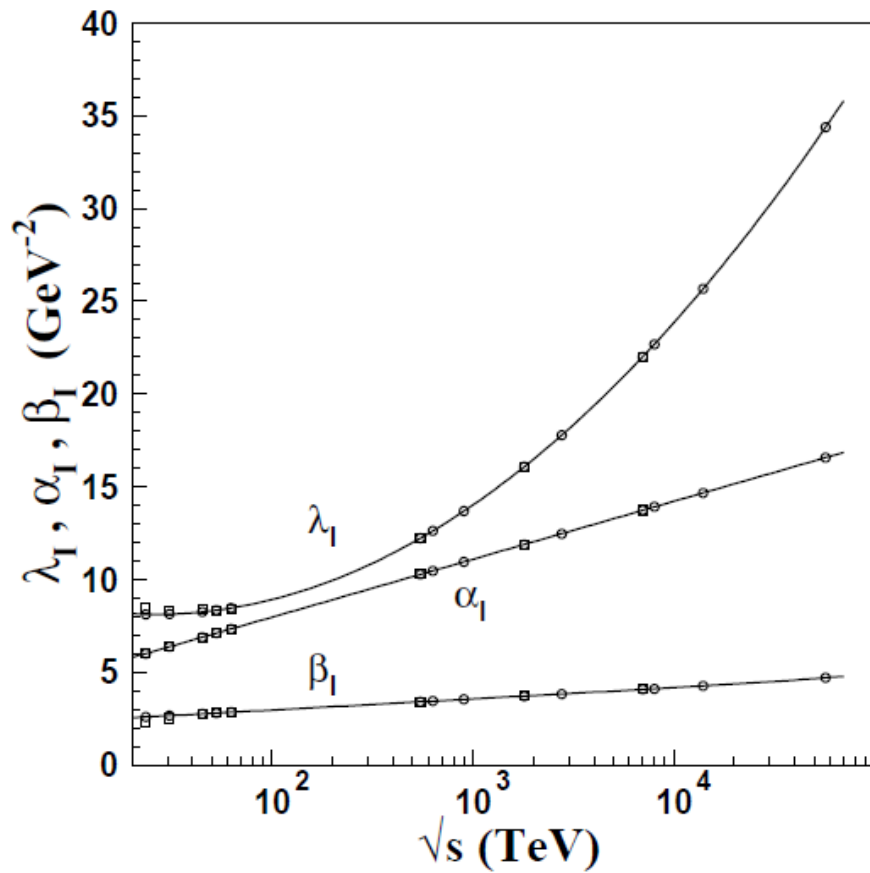
Real and imaginary amplitudes ,  $K = R, I$

$$T_K^N(s, t) = \alpha_K(s) e^{-\beta_K(s)|t|} + \lambda_K(s) \Psi_K(\gamma_K(s), t)$$

$$\Psi_K(\gamma_K(s), t) = 2 e^{\gamma_K} \left[ \frac{e^{-\gamma_K \sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - e^{\gamma_K} \frac{e^{-\gamma_K \sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}} \right]$$

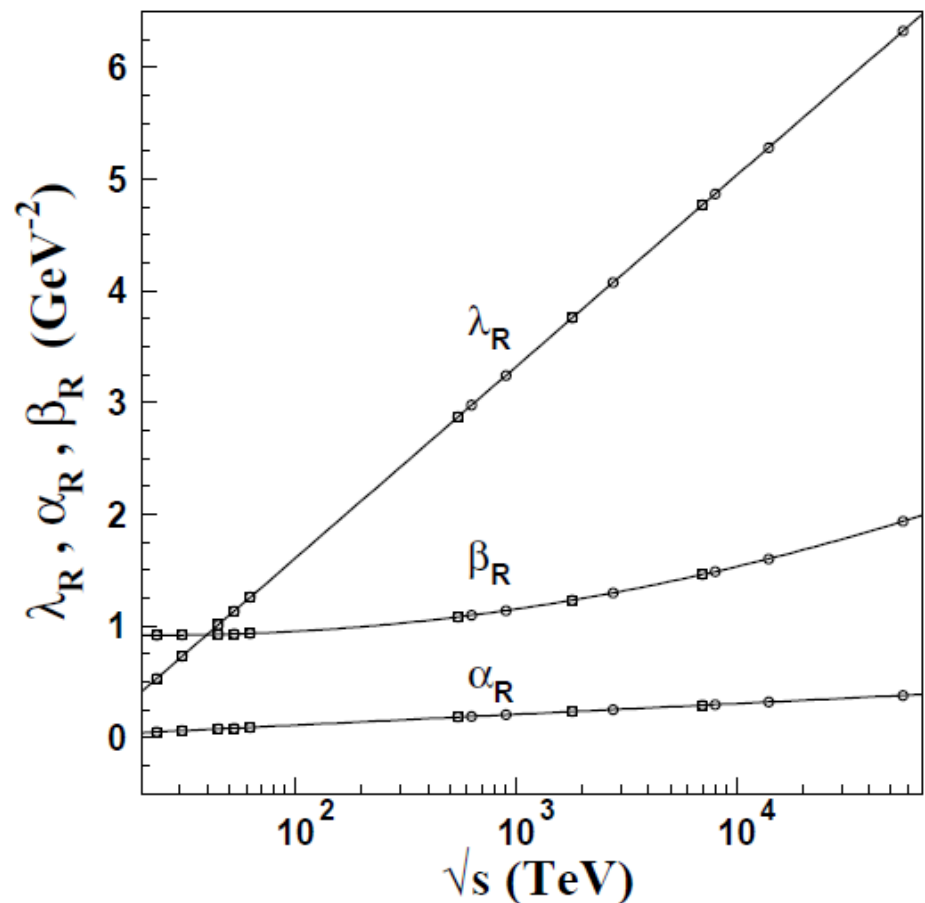
with 4 parameters for each amplitude

# Energy dependence of parameters



Parameters of imaginary part

Parameters of real part



# quantities in forward scattering

$$\sigma(s) = 4\sqrt{\pi} (\hbar c)^2 (\alpha_I(s) + \lambda_I(s))$$

Total cross section

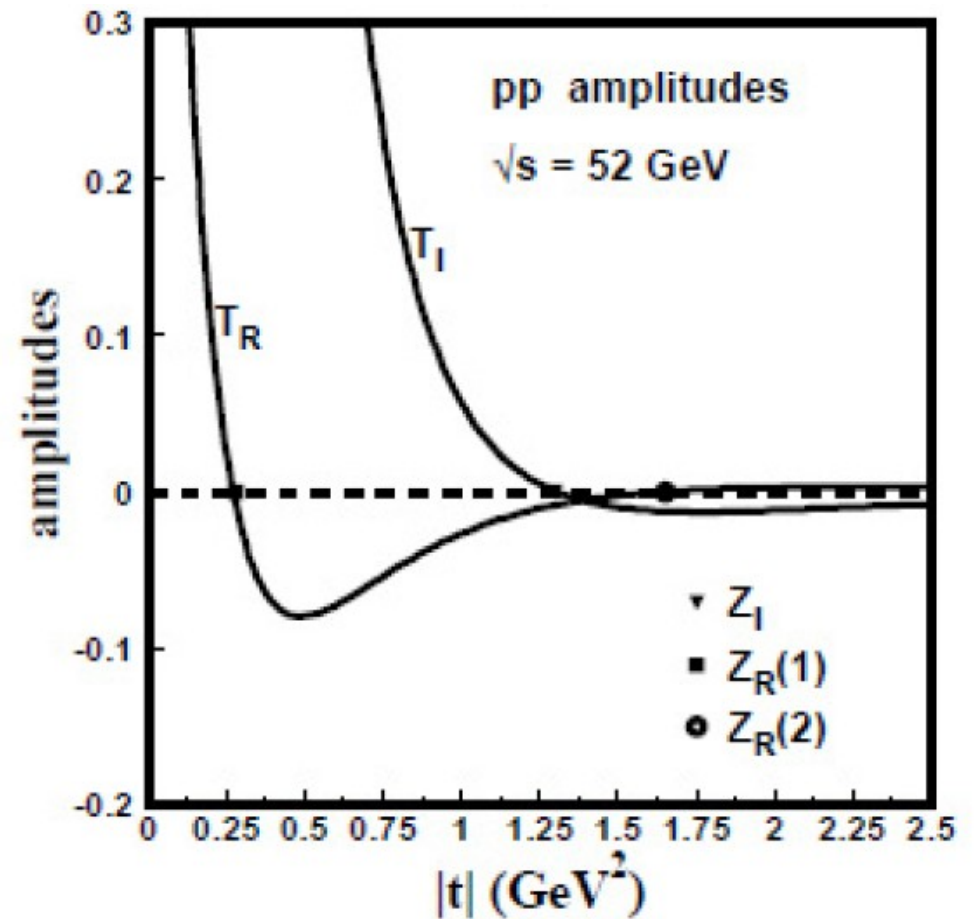
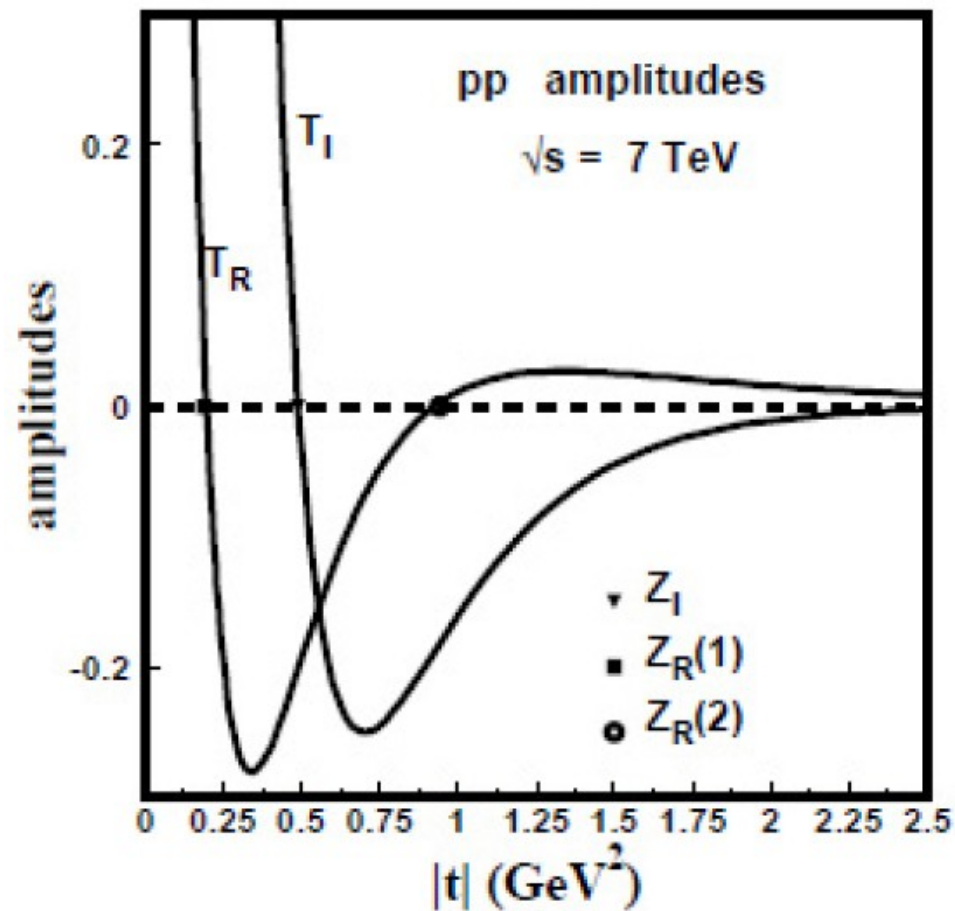
$$\rho(s) = \frac{T_R^N(s, t=0)}{T_I^N(s, t=0)} = \frac{\alpha_R(s) + \lambda_R(s)}{\alpha_I(s) + \lambda_I(s)}$$

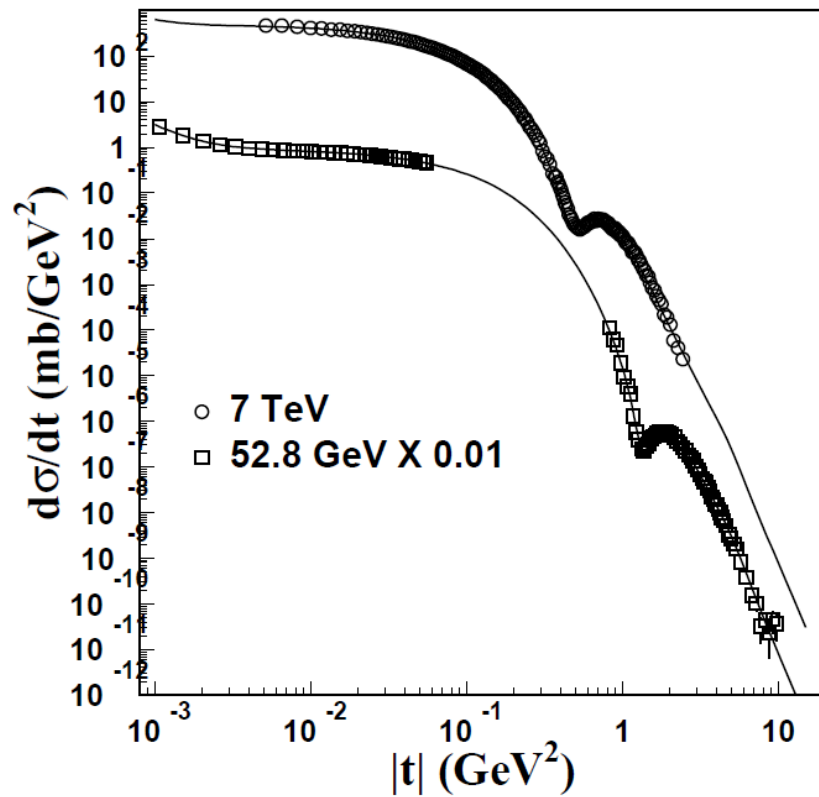
Real/Imaginary

$$\begin{aligned} B_K(s) &= \frac{2}{T_K^N(s, t)} \frac{dT_K^N(s, t)}{dt} \Big|_{t=0} \\ &= \frac{1}{\alpha_K(s) + \lambda_K(s)} \left[ \alpha_K(s) \beta_K(s) + \frac{1}{8} \lambda_K(s) a_0 (6\gamma_K(s) + 7) \right] \end{aligned}$$

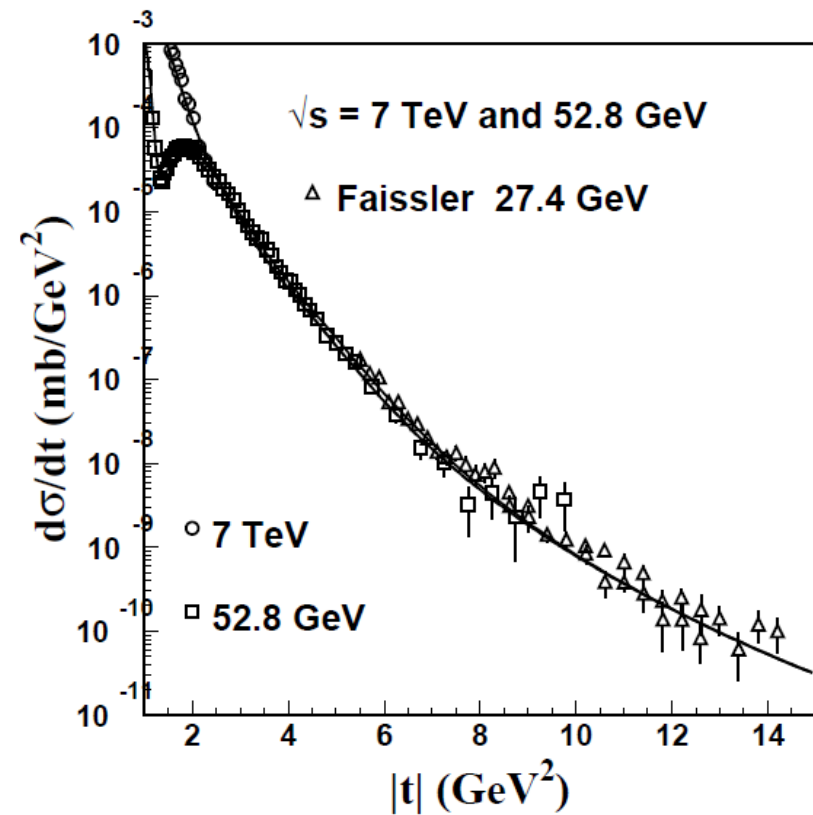
Real and Imaginary slopes

Real and imaginary amplitudes together determined the dip/bump structure of the data.



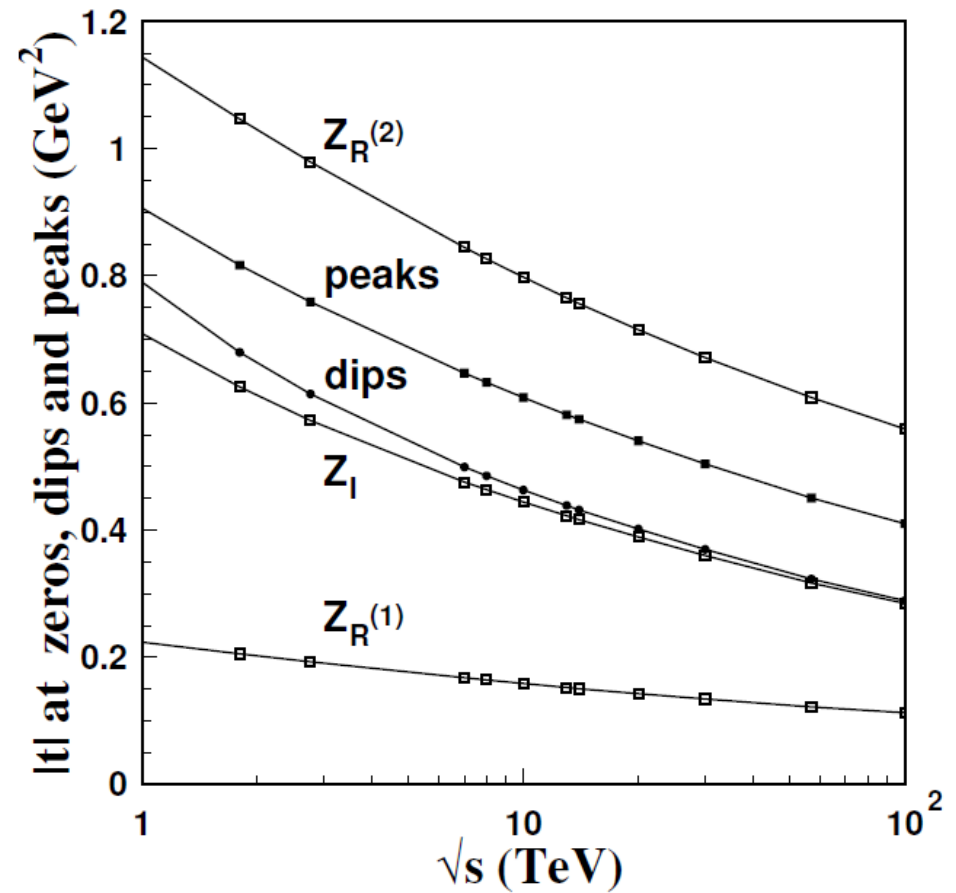
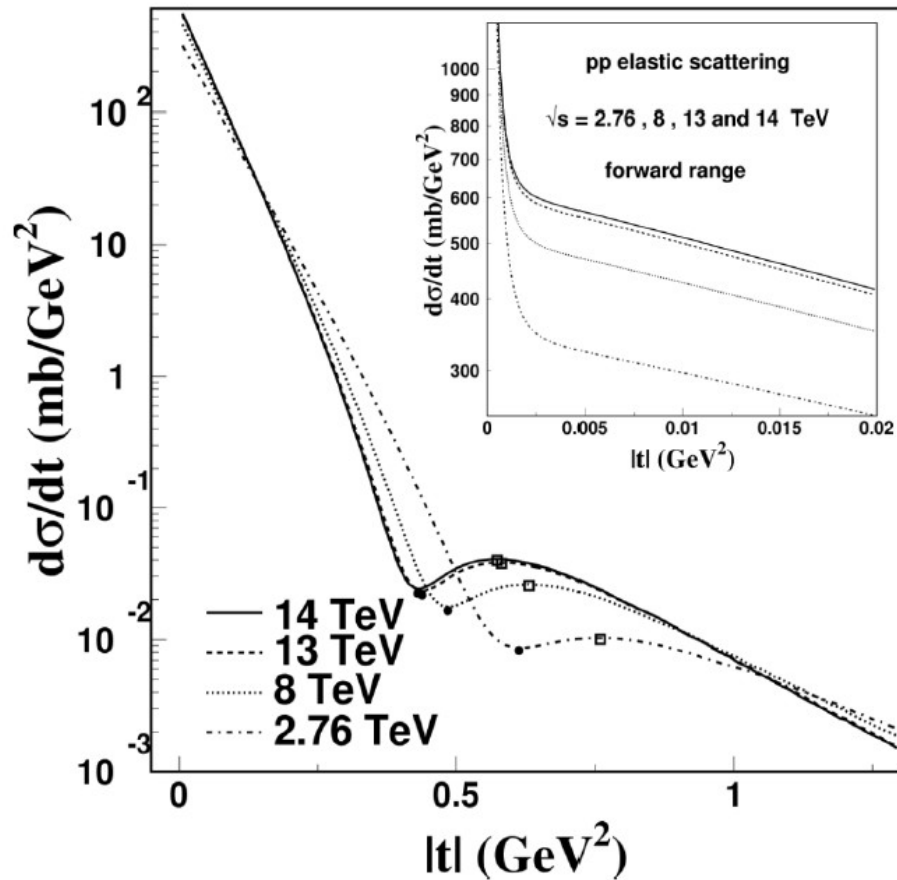


pp at 52 GeV (ISR-CERN)  
and pp at 7 TeV (TOTEM-CERN)



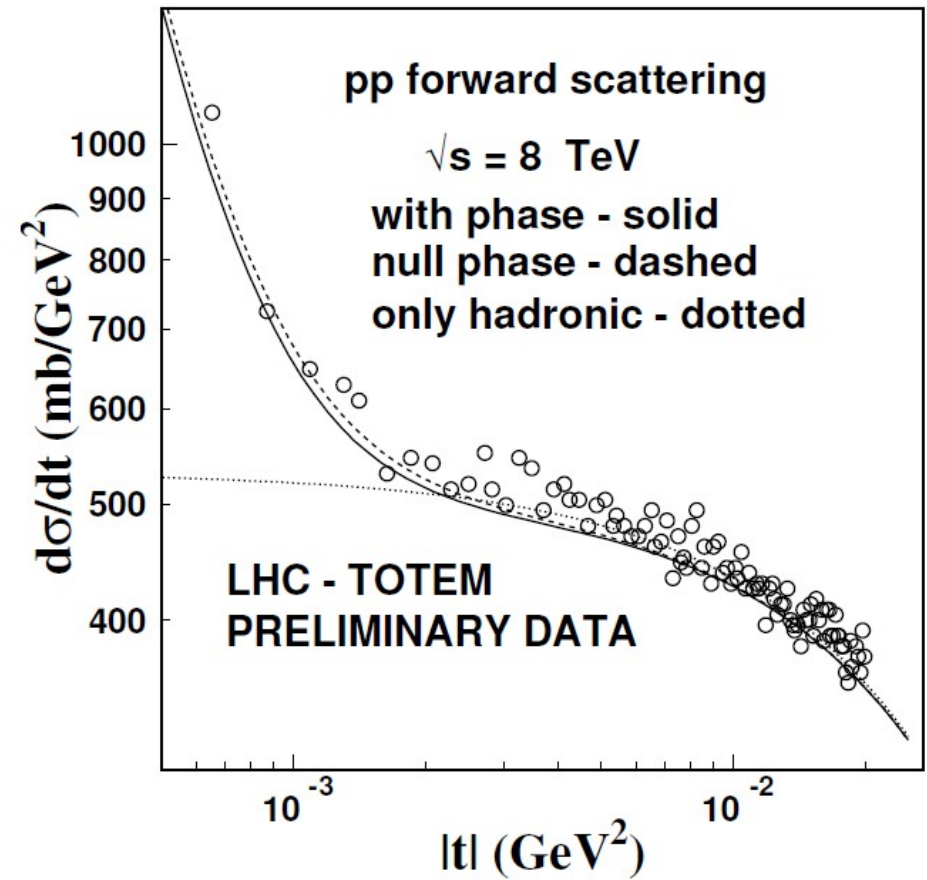
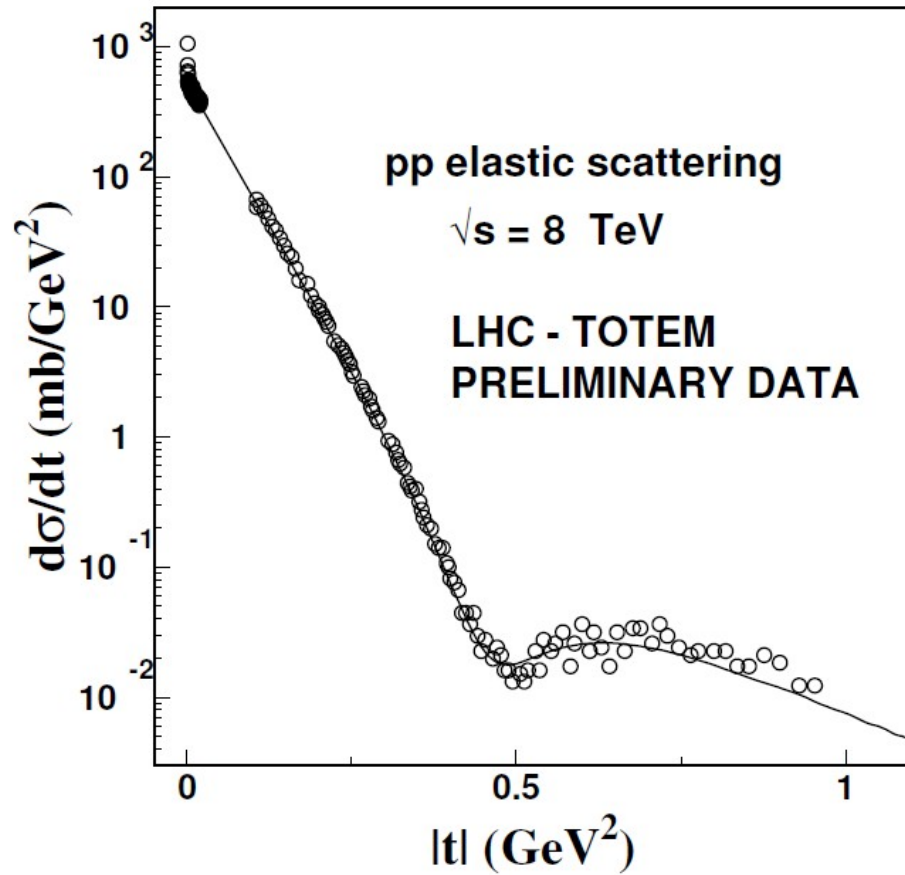


# Predictions for LHC energies

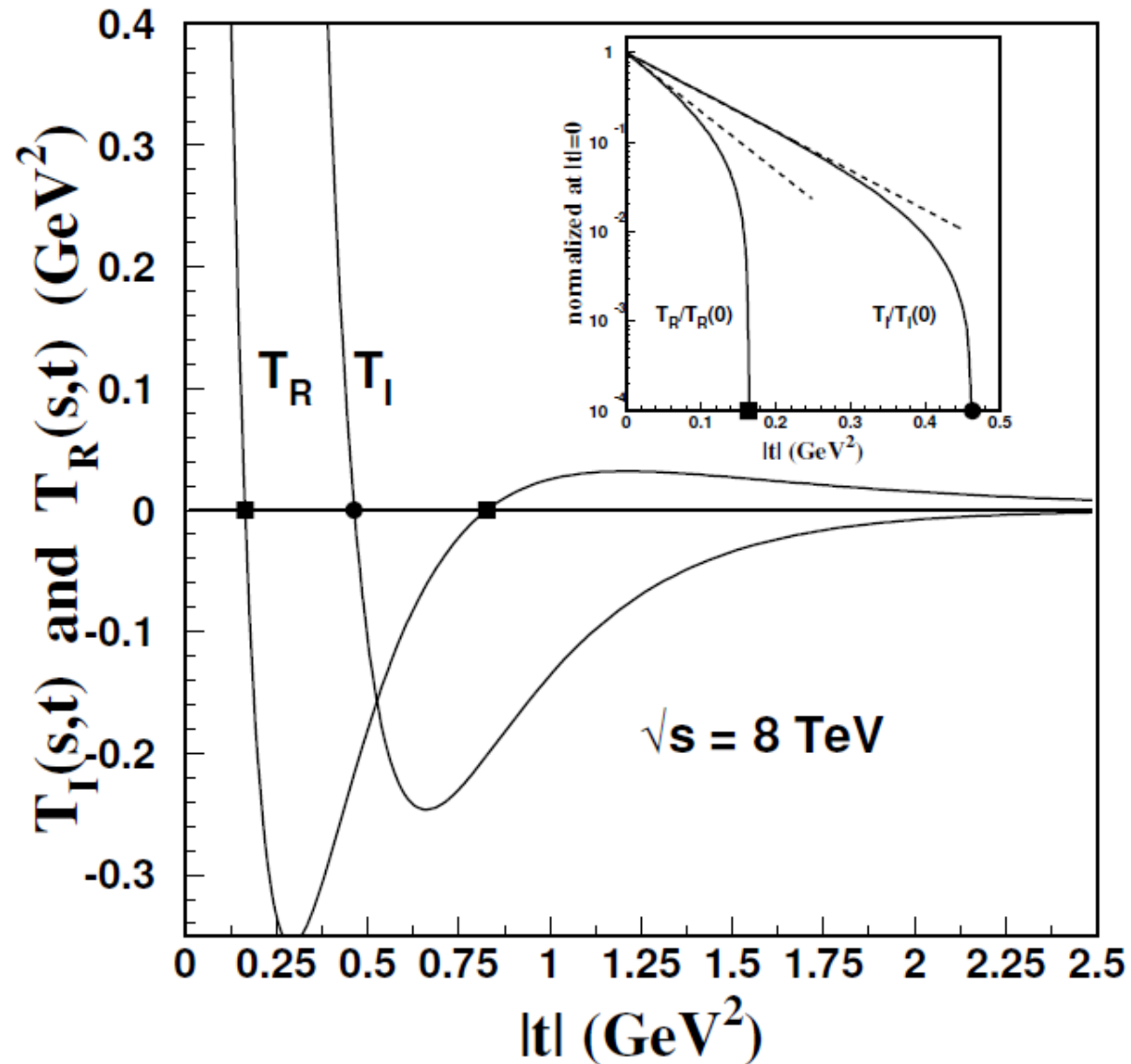


zeros, dips, peaks  $\rightarrow A + \frac{1}{a + b \log \sqrt{s} + c \log^2 \sqrt{s}}$

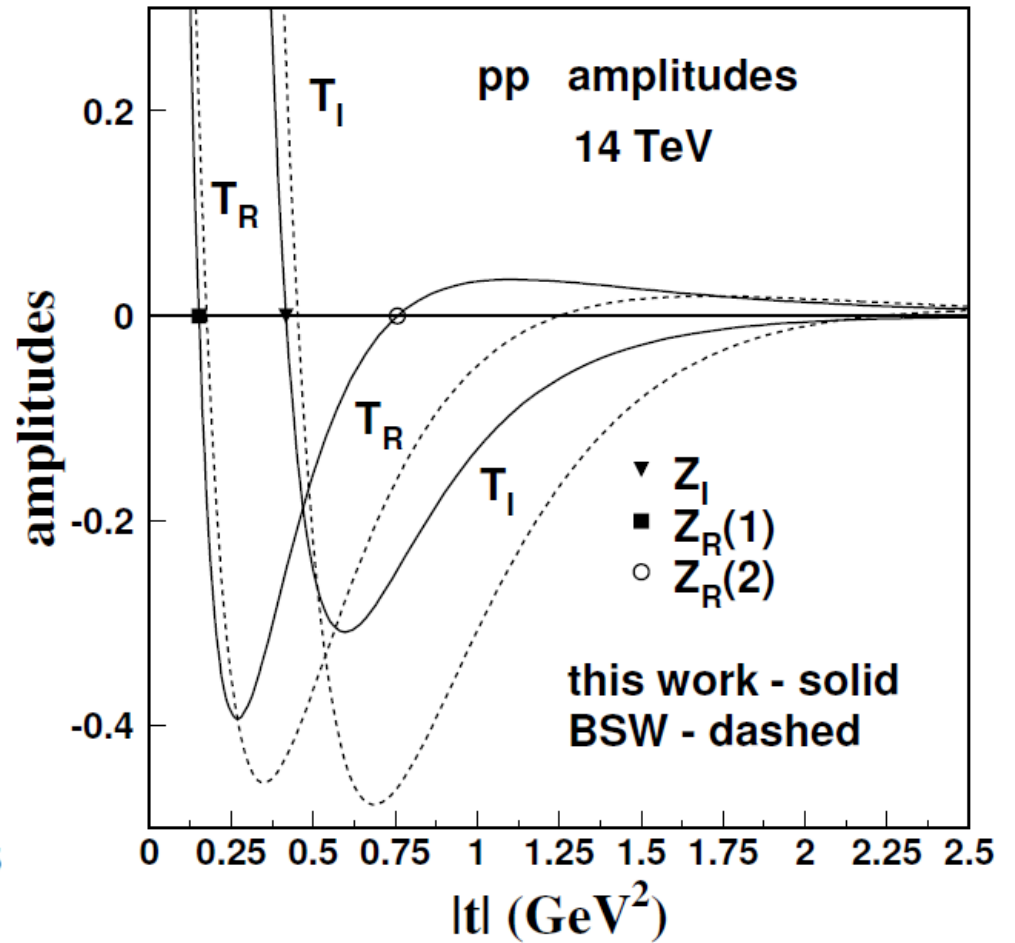
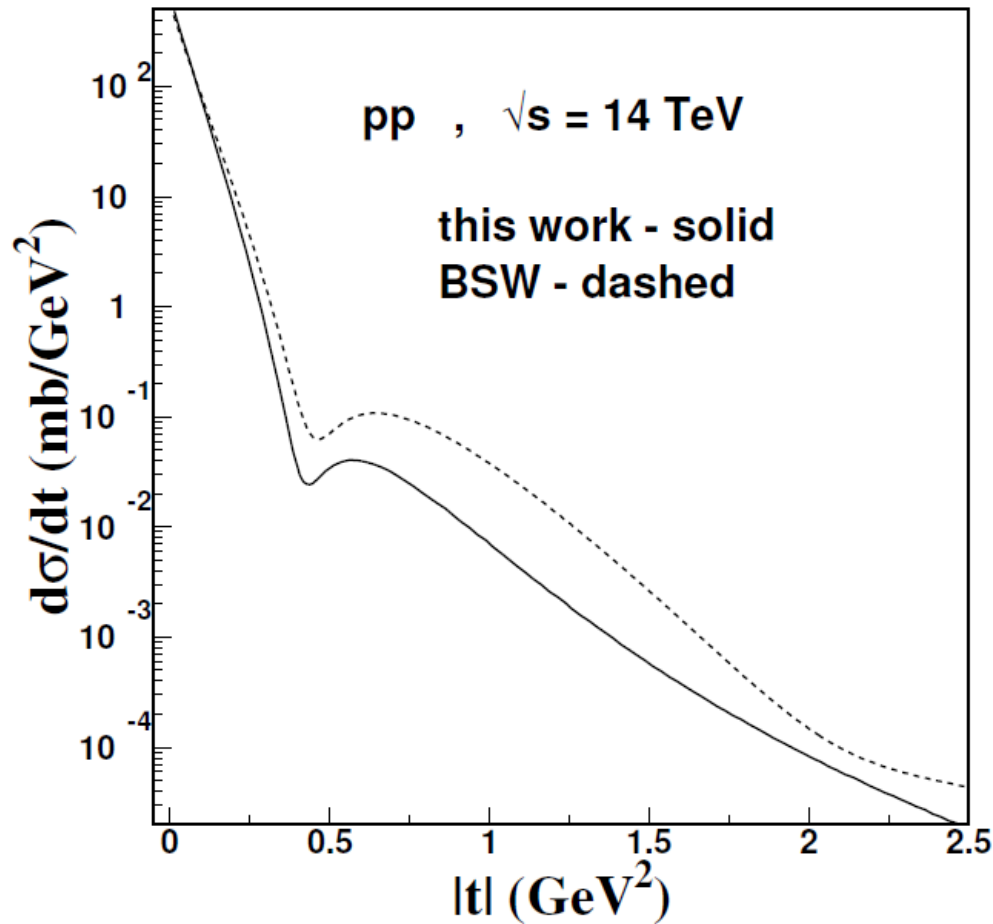
# 8 TeV case



# 8 TeV amplitudes



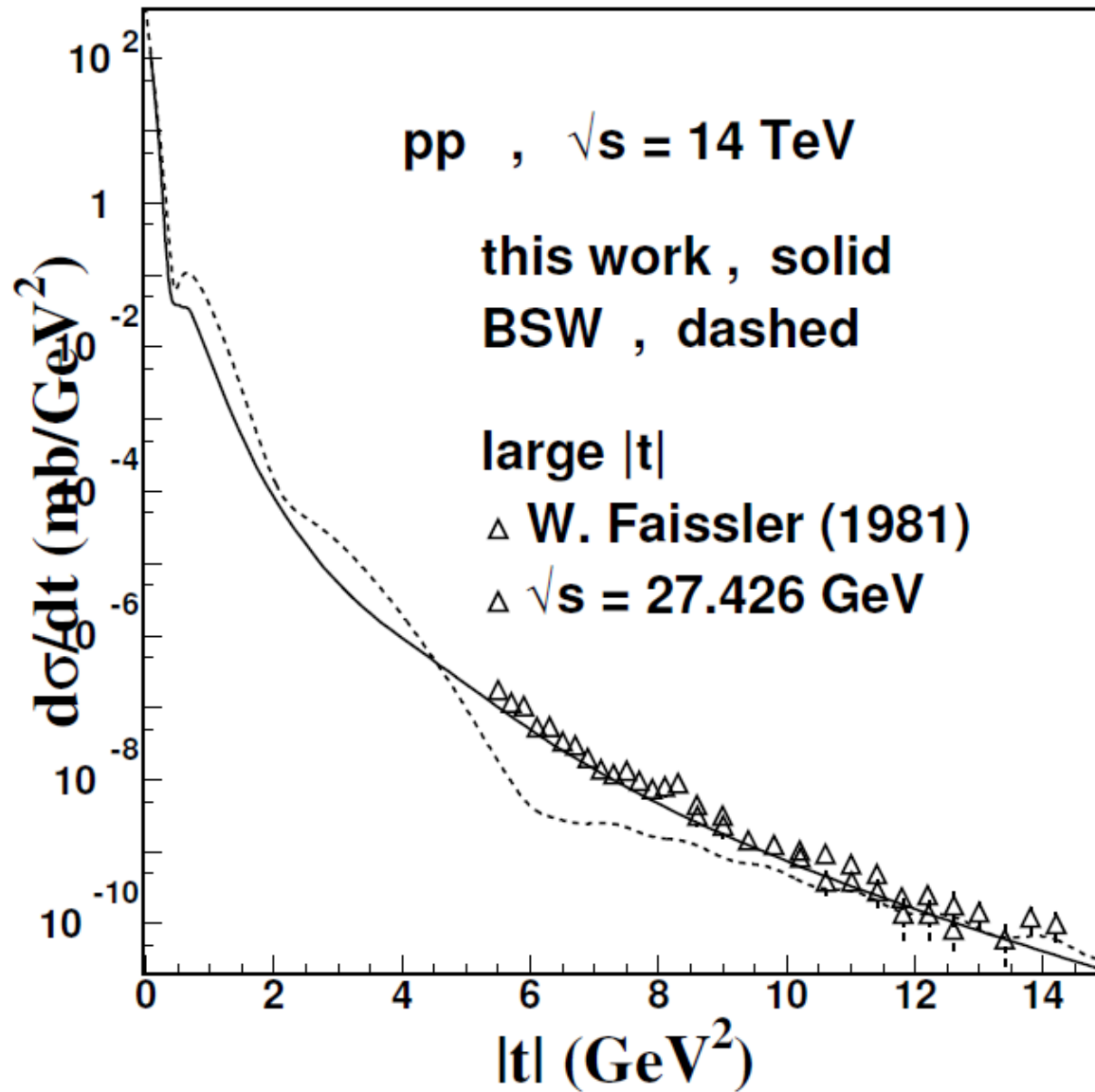
# 14 TeV



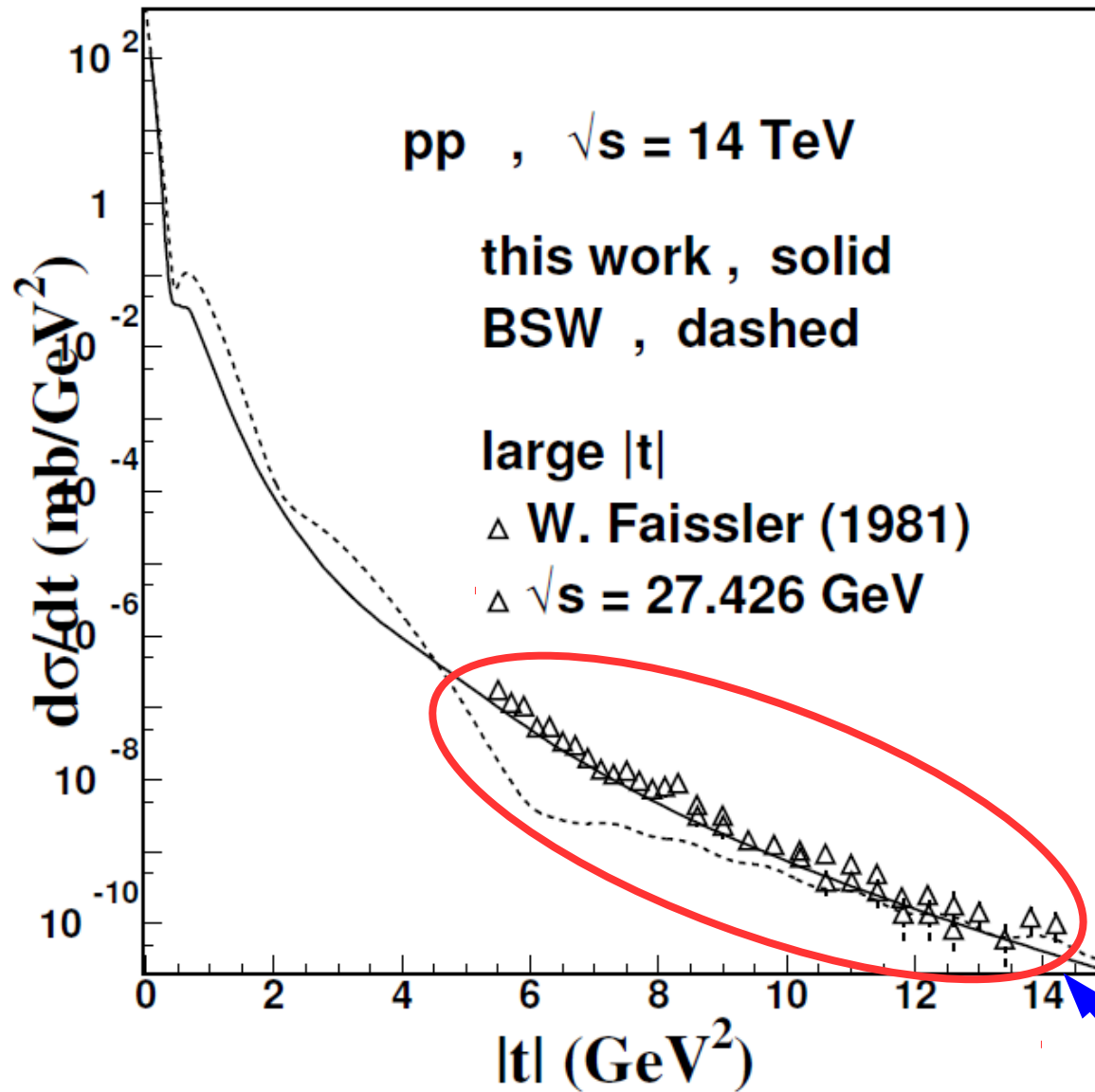
BSW model

C. Bourrely, J.M. Myers, J.Soffer and T.T. Wu , *Phys. Rev.D* **85**, 096009 (2012).

# Large t behaviour

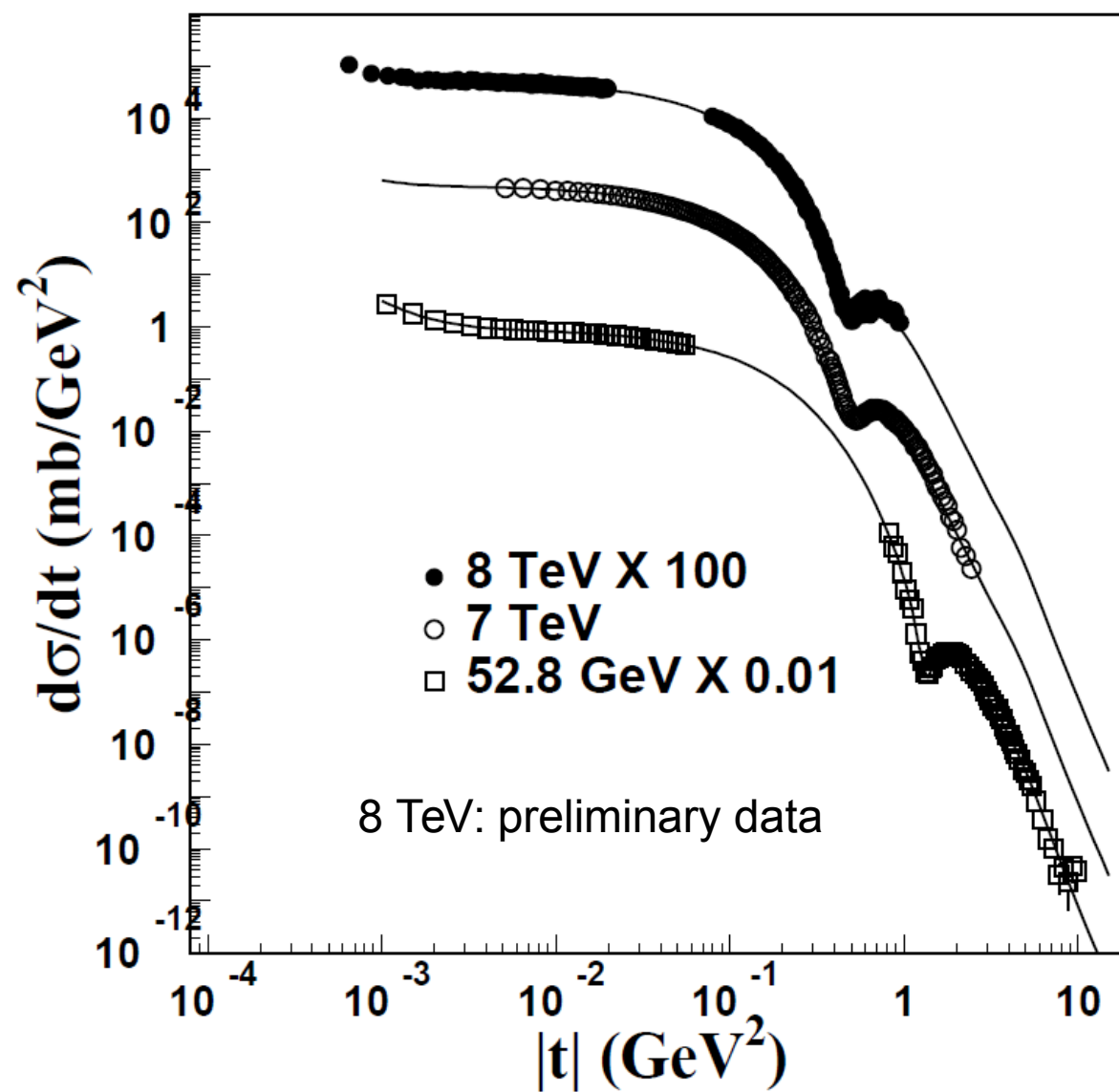


# Large t behaviour



Tri-gluon exchange - DL

$$R_{ggg}(t) \equiv \pm 0.45 t^{-4} (1 - e^{-0.005|t|^4}) (1 - e^{-0.1|t|^2})$$



# numbers of the model

$\sqrt{s}$ TeV	imaginary amplitude				real amplitude			
	$\sigma$ mb	$B_I$ GeV <sup>-2</sup>	$\alpha_I$ GeV <sup>-2</sup>	$\beta_I$ GeV <sup>-2</sup>	$\rho$	$B_R$ GeV <sup>-2</sup>	$\lambda_R$ GeV <sup>-2</sup>	$\beta_R$ GeV <sup>-2</sup>
1.8	77.21	17.17	11.8898	3.7175	0.1427	24.63	3.7566	1.2304
2.76	83.47	17.96	12.4689	3.8293	0.1431	26.08	4.0745	1.2959
7	98.65	19.77	13.7298	4.0745	0.1415	29.65	4.7667	1.4599
8	101.00	20.21	13.9107	4.1100	0.1411	30.21	4.8660	1.4858
13	109.93	21.35	14.5685	4.2409	0.1392	32.35	5.2271	1.5852
14	111.34	21.53	14.6689	4.26123	0.1389	32.68	5.2822	1.6011

$\sqrt{s}$ TeV	$Z_I$ GeV <sup>2</sup>	$Z_R(1)$ GeV <sup>2</sup>	$Z_R(2)$ GeV <sup>2</sup>	$ t _{\text{dip}}$ GeV <sup>2</sup>	$d\sigma/dt _{\text{dip}}$ mb/GeV <sup>2</sup>	$ t _{\text{peak}}$ GeV <sup>2</sup>	$d\sigma/dt _{\text{peak}}$ mb/GeV <sup>2</sup>	ratio R	$\sigma_{\text{inel}}$ mb	$\sigma_{\text{el}}$ mb	$\sigma_{\text{el}}^I$ mb	$\sigma_{\text{el}}^R$ mb	$\sigma_{\text{el}}/\sigma$
1.8	0.6250	0.2052	1.0464	0.6798	0.005832	0.8170	0.006627	1.1362	58.97	18.24	18.00	0.24	0.24
2.76	0.5723	0.1925	0.9788	0.6138	0.008248	0.7587	0.010080	1.2221	63.13	20.33	20.07	0.27	0.24
7	0.4757	0.1673	0.8445	0.4988	0.015339	0.6465	0.022841	1.4891	73.28	25.37	25.05	0.32	0.26
8	0.4635	0.1639	0.8267	0.4850	0.016571	0.6319	0.025466	1.5368	74.85	26.16	25.83	0.33	0.26
13	0.4225	0.1522	0.7654	0.4385	0.021558	0.5816	0.037378	1.7338	80.76	29.17	28.82	0.35	0.27
14	0.4166	0.1505	0.7565	0.4319	0.022397	0.5743	0.039593	1.7678	81.69	29.65	29.29	0.35	0.27



# numbers of the model

## INPUTS

$\sqrt{s}$ TeV	imaginary amplitude				real amplitude			
	$\sigma$ mb	$B_I$ GeV <sup>-2</sup>	$\alpha_I$ GeV <sup>-2</sup>	$\beta_I$ GeV <sup>-2</sup>	$\rho$	$B_R$ GeV <sup>-2</sup>	$\lambda_R$ GeV <sup>-2</sup>	$\beta_R$ GeV <sup>-2</sup>
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## DERIVED QUANTITIES

$\sqrt{s}$ TeV	$Z_I$ GeV <sup>2</sup>	$Z_R(1)$ GeV <sup>2</sup>	$Z_R(2)$ GeV <sup>2</sup>	$ t _{\text{dip}}$ GeV <sup>2</sup>	$d\sigma/dt _{\text{dip}}$ mb/GeV <sup>2</sup>	$ t _{\text{peak}}$ GeV <sup>2</sup>	$d\sigma/dt _{\text{peak}}$ mb/GeV <sup>2</sup>	ratio R	$\sigma_{\text{inel}}$ mb	$\sigma_{\text{el}}$ mb	$\sigma_{\text{el}}^I$ mb	$\sigma_{\text{el}}^R$ mb	$\sigma_{\text{el}}/\sigma$
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14	0.4166	0.1505	0.7565	0.4319	0.022397	0.5743	0.039593	1.7678	81.69	29.65	29.29	0.35	0.27

# energy dependence of the inputs in forward scattering

Forward differential cross section

$$\frac{d\sigma}{dt} = \pi(\hbar c)^2 \left\{ \left[ \frac{\rho\sigma}{4\pi(\hbar c)^2} e^{B_R t/2} \right]^2 + \left[ \frac{\sigma}{4\pi(\hbar c)^2} e^{B_I t/2} \right]^2 \right\}$$

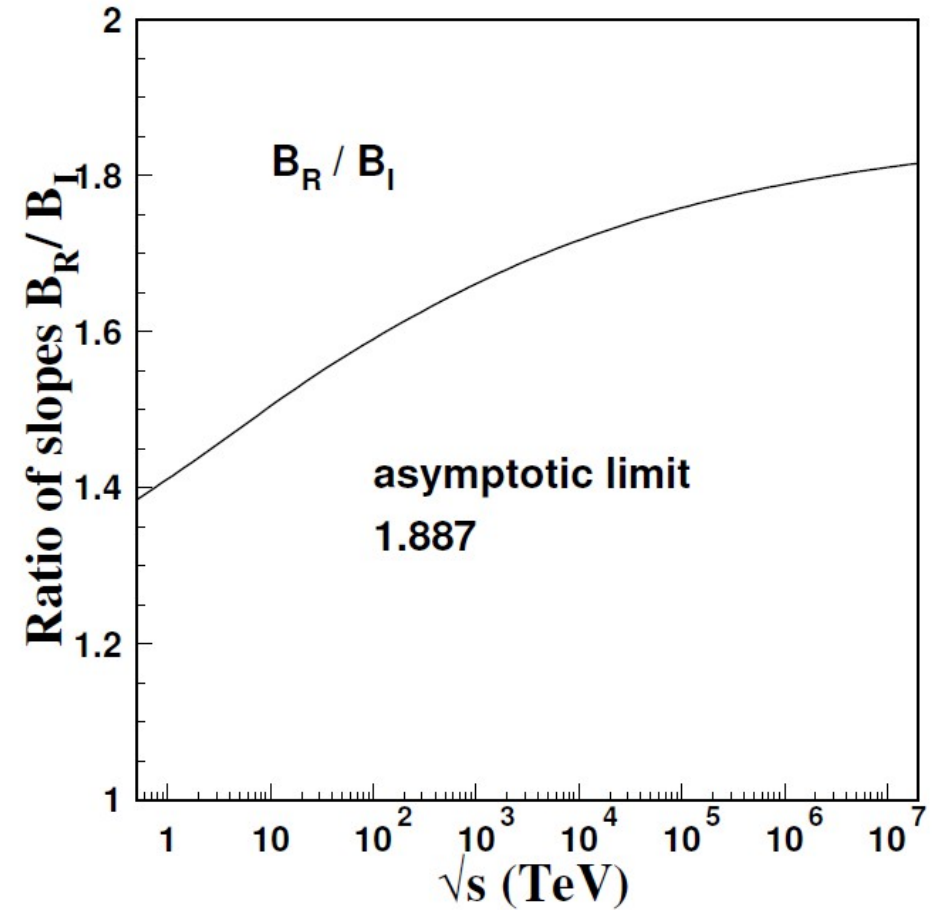
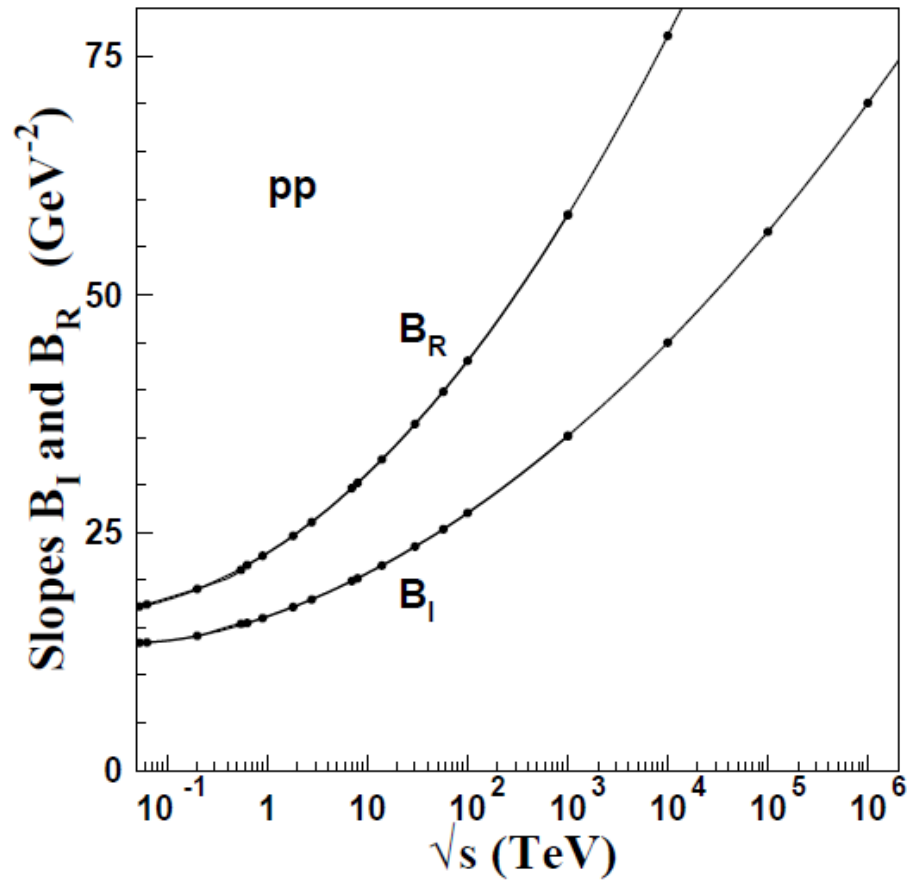
$$\sigma(s) = 69.3286 + 12.6800 \log \sqrt{s} + 1.2273 \log^2 \sqrt{s}$$

$$B_I(s) = 15.7848 + 1.75795 \log \sqrt{s} + 0.149067 \log^2 \sqrt{s}$$

$$\rho(s) = \frac{3.528018 + 0.7856088 \log \sqrt{s}}{25.11358 + 4.59321 \log \sqrt{s} + 0.444594 \log^2 \sqrt{s}}$$

$$B_R(s) = 22.8365 + 2.86093 \log \sqrt{s} + 0.329886 \log^2 \sqrt{s}$$


# Slopes of real and imaginary amplitudes



# b-space (geometric space)

Fourier transform of the amplitudes

$$i\sqrt{\pi} (1 - e^{i\chi(s, \vec{b})}) \equiv \tilde{T}(s, \vec{b}) = \tilde{T}_R(s, \vec{b}) + i\tilde{T}_I(s, \vec{b})$$

  
in eikonal formalism →  $\chi(s, \vec{b}) = \chi_R(s, \vec{b}) + i\chi_I(s, \vec{b})$

with


$$\tilde{T}_K(s, \vec{b}) = \frac{\alpha_K}{2\beta_K} e^{-b^2/4\beta_K} + \lambda_K \tilde{\psi}_K(s, b)$$

$$\tilde{\psi}_K(s, b) = \frac{2e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}}}{a_0 \sqrt{\gamma_K^2 + b^2/a_0}} \left[ 1 - e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}} \right]$$

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with

$$\tilde{T}_K(s, \vec{b}) = \frac{\alpha_K}{2\beta_K} e^{-b^2/4\beta_K} + \lambda_K \tilde{\psi}_K(s, b)$$


Large b (Yukawa like)

$$\tilde{\psi}_K(s, b) = \frac{2e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}}}{a_0 \sqrt{\gamma_K^2 + b^2/a_0}} \left[ 1 - e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}} \right] \quad \xrightarrow{\text{red arrow}} \quad \sim \frac{1}{b} e^{-b/b_0}$$

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unitarity conditions

$$\frac{\tilde{T}_R^2}{\pi} \leq e^{-2\chi_I(s, \vec{b})} \leq 1 \quad \text{or} \quad 0 \leq \chi_I \leq -\frac{1}{2} \log(\tilde{T}_R^2/\pi)$$

satisfied by our solutions

In this space the cross sections are written

$$\sigma_{\text{el}}(s) = \frac{(\hbar c)^2}{\pi} \int d^2\vec{b} |\tilde{T}(s, \vec{b})|^2 \equiv \int d^2\vec{b} \frac{d\tilde{\sigma}_{\text{el}}(s, \vec{b})}{d^2\vec{b}}$$

$$\sigma(s) = \frac{2}{\sqrt{\pi}} (\hbar c)^2 \int d^2\vec{b} \tilde{T}_I(s, \vec{b}) \equiv \int d^2\vec{b} \frac{d\tilde{\sigma}_{\text{tot}}(s, \vec{b})}{d^2\vec{b}}$$

$$\sigma_{\text{inel}} = (\hbar c)^2 \int d^2\vec{b} \left( \frac{2}{\sqrt{\pi}} \tilde{T}_I(s, \vec{b}) - \frac{1}{\pi} |\tilde{T}(s, \vec{b})|^2 \right) \equiv \int d^2\vec{b} \frac{d\tilde{\sigma}_{\text{inel}}(s, \vec{b})}{d^2\vec{b}}$$

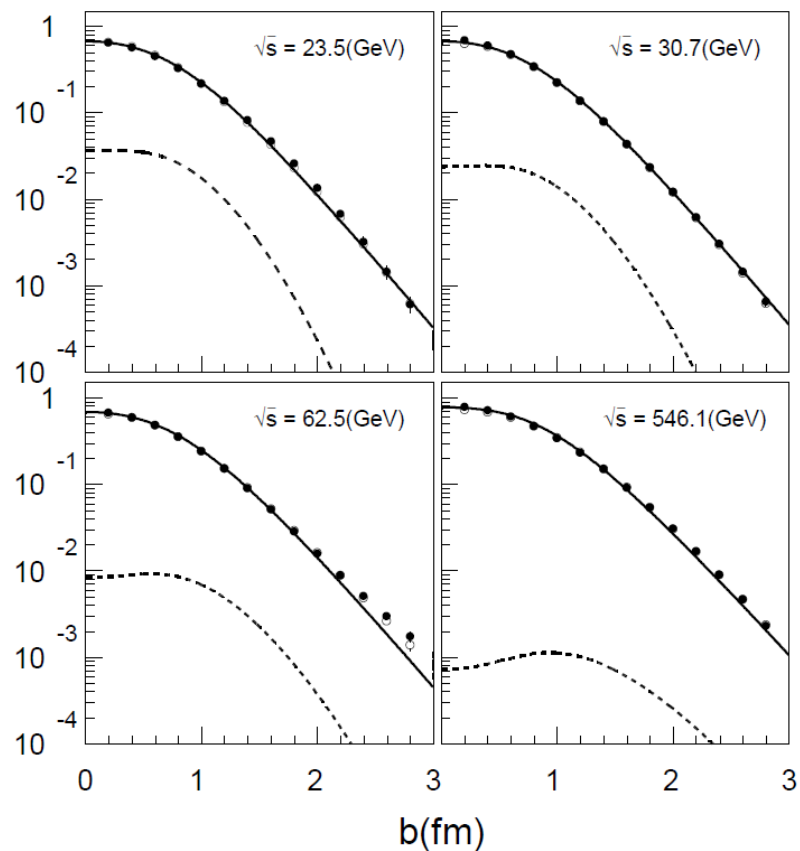
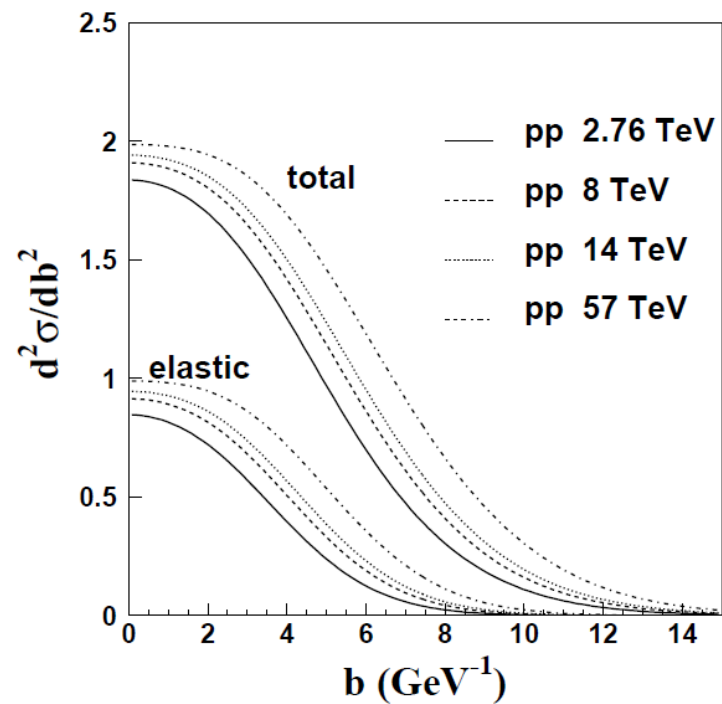
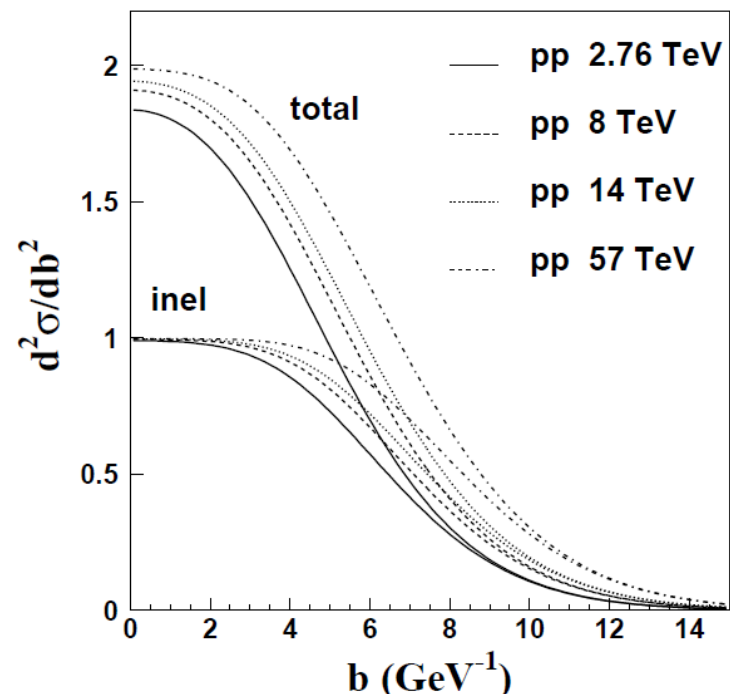
differential cross sections in terms of eikonal functions

$$\frac{d\tilde{\sigma}_{\text{el}}(s, \vec{b})}{d^2\vec{b}} = 1 - 2 \cos \chi_R e^{-\chi_I} + e^{-2\chi_I}$$

$$\frac{d\tilde{\sigma}(s, \vec{b})}{d^2\vec{b}} = 2 (1 - \cos \chi_R e^{-\chi_I})$$

$$\frac{d\tilde{\sigma}_{\text{inel}}(s, \vec{b})}{d^2\vec{b}} = 1 - e^{-2\chi_I}$$

# Monotonic behaviour of differential cross sections

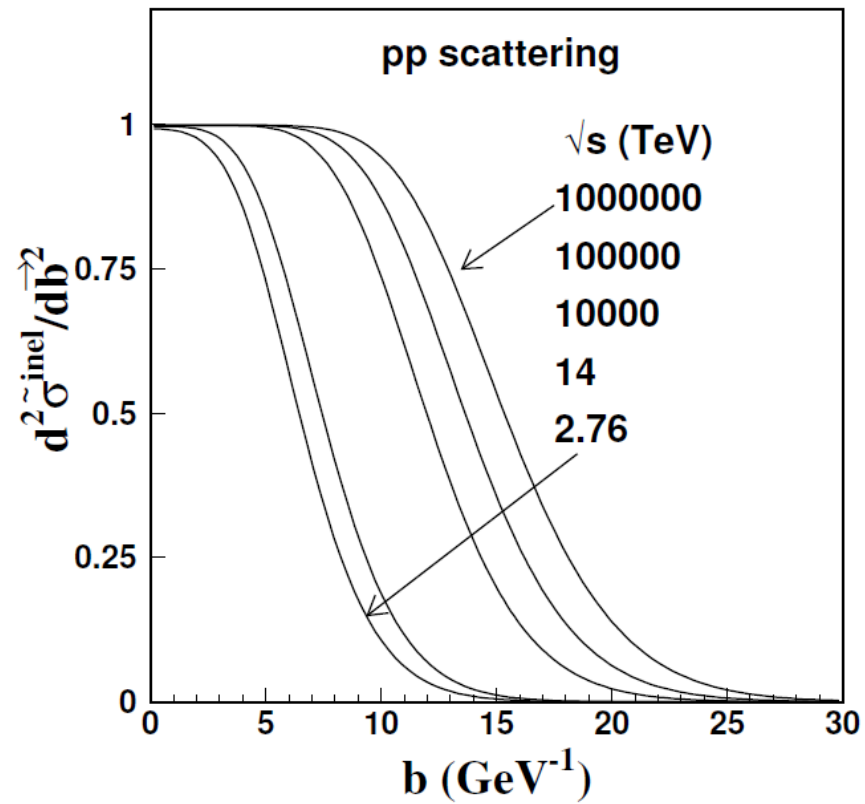


From 50 to 500 GeV: same behaviour

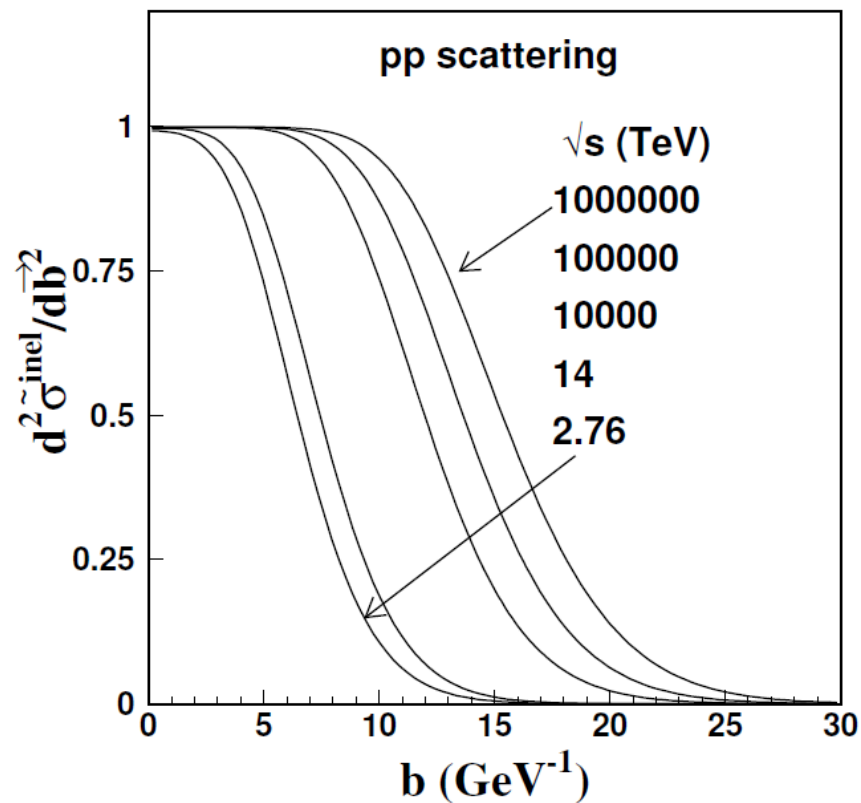
B.Z. Kopeliovich, I.K. Potashnikova, B. Povh and E. Predazzi, Phys. Rev. Lett. 85, 507 (2000) ; ibid. Phys. Rev. D 63, 054001 (2001).



...and this regular behaviour continues to asymptotic energies



...and this regular behaviour continues to asymptotic energies

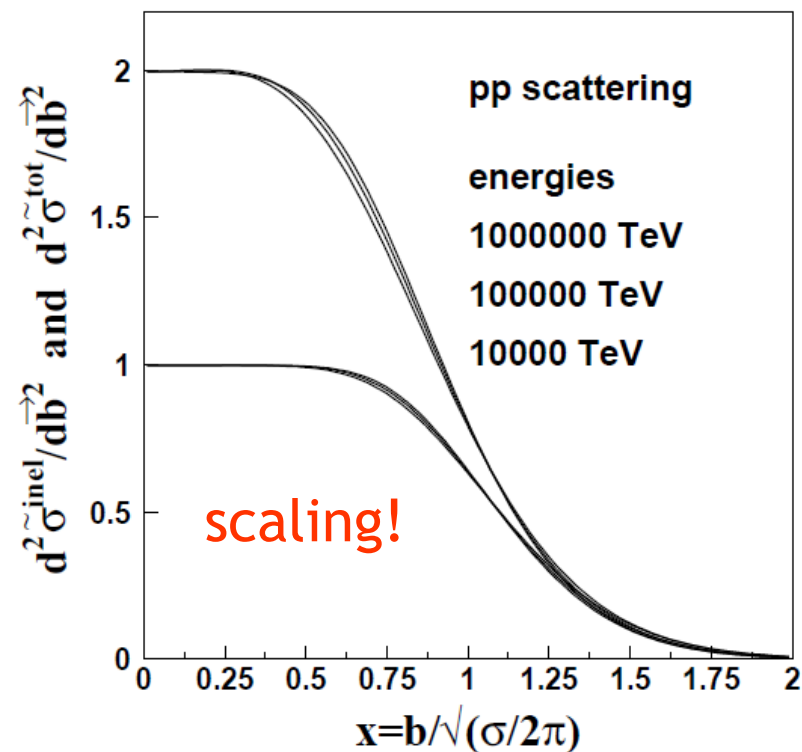


This property determines asymptotic behaviour

with a decreasing range

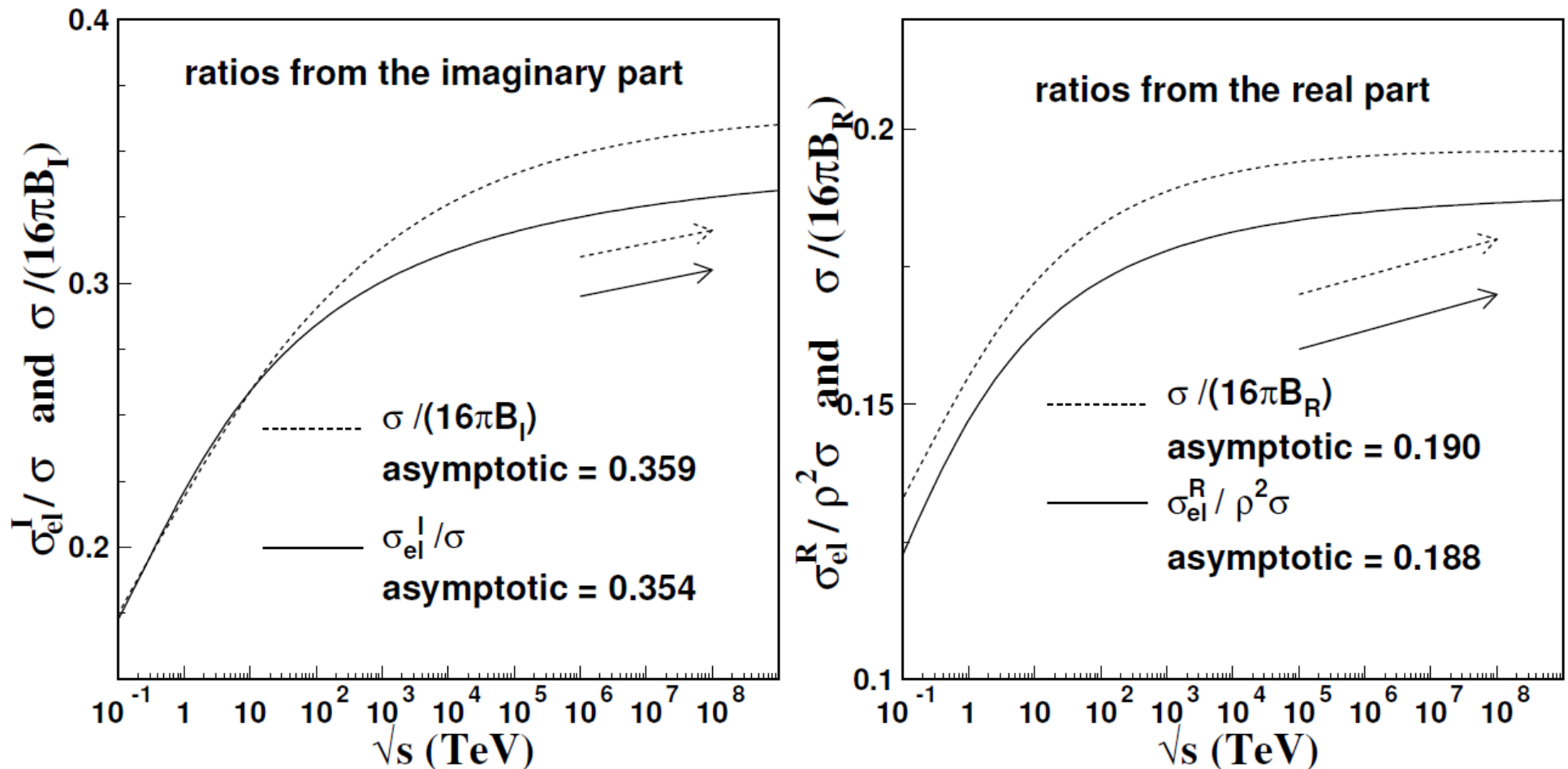
Observe a scaling variable

$$x = b / \sqrt{\sigma(\sqrt{s}) / 2\pi}$$



For asymptotic energies proton scattering is different from a black disk

Dimensionless ratios for imaginary and real parts

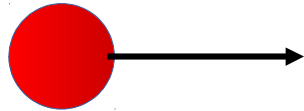


# COSMIC RAY MEASUREMENTS

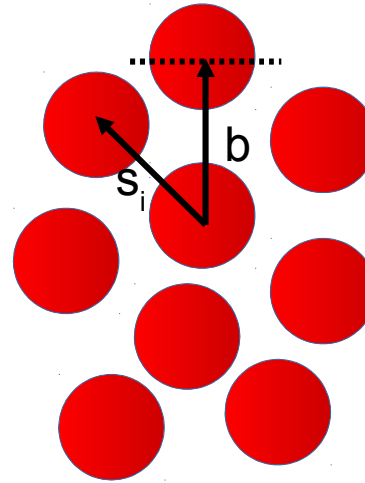
using our pp input, calculate p-air  
cross sections



# p-air cross sections measurements in EAS (extended air showers)

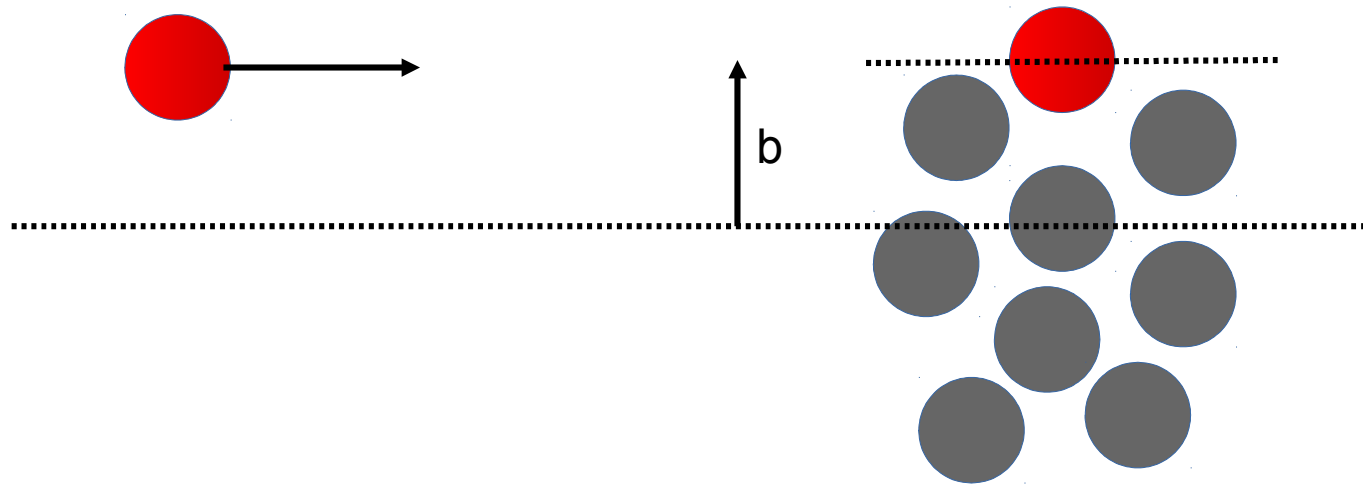


proton from cosmic ray



atom of atmosphere

...considering the nucleus composed by uncorrelated nucleons distributed according a wave function





# Glauber framework

R. Engel and R. Ulrich , Internal Pierre Auger Note  
GAP-2012, March 2012

forward amplitudes for pp elastic scattering

$$\begin{aligned}\hat{T}_{pp}(s, \vec{b}) &= \hat{T}_R(s, \vec{b}) + i\hat{T}_I(s, \vec{b}) \\ &= \frac{\sigma_{pp}^{\text{tot}}}{4\pi(\hbar c)^2} \left[ \frac{\rho}{B_R} e^{-\frac{b^2}{2B_R}} + i \frac{1}{B_I} e^{-\frac{b^2}{2B_I}} \right]\end{aligned}$$

In terms of eikonal functions

$$-i \hat{T}_{pp}(s, \vec{b}) = 1 - \underbrace{e^{i\chi_{pp}(s, \vec{b})}}_{\text{S-matrix in b space}} \equiv \Gamma_{pp}(s, \vec{b})$$

Optical theorem

$$\sigma_{pp}^{\text{tot}}(s) = 2 (\hbar c)^2 \Re \int d^2\vec{b} \Gamma_{pp}(s, \vec{b})$$

Analogous optical theorem for p-Air

$$\sigma_{pA}^{\text{tot}}(s) = 2 (\hbar c)^2 \Re \int d^2\vec{b} \Gamma_{pA}(s, \vec{b})$$

Glauber method introduces the p-A amplitude for A independent nucleons

$$\Gamma_{pA}(s, \vec{b}, \vec{s}_1, \dots, \vec{s}_A) = 1 - \prod_{j=1}^A \left[ 1 - \Gamma_{pp}(s, |\vec{b} - \vec{s}_j|) \right]$$



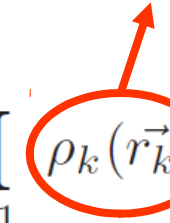
We want to compute the production cross section defined by

$$\sigma_{\text{p-air}}^{\text{prod}} = \sigma_{\text{p-air}}^{\text{tot}} - (\sigma_{\text{p-air}}^{\text{el}} + \sigma_{\text{p-air}}^{\text{q-el}})$$

with

$$\sigma_{\text{pA}}^{\text{el}} + \sigma_{\text{pA}}^{\text{q-el}} = (\hbar c)^2 \int d^2\vec{b} \int \left| 1 - \prod_{j=1}^A \left[ 1 - \Gamma_{\text{pp}}(s, |\vec{b} - \vec{s}_j|) \right] \right|^2 \prod_{k=1}^A \rho_k(\vec{r}_k) d^3\vec{r}_k$$

Nuclear density



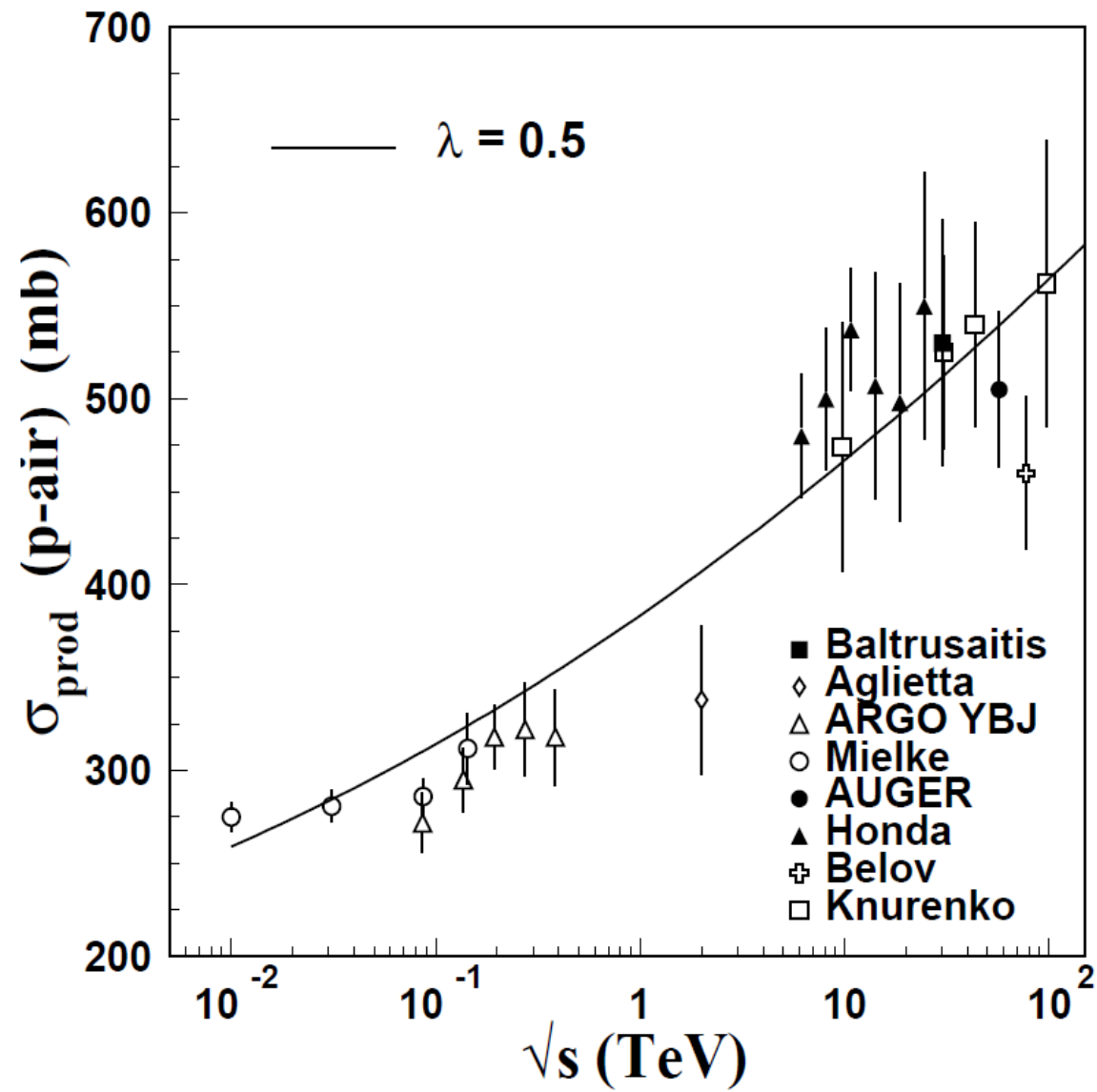
We also test the effects of the diffractive intermediate states according to the Good Walker framework (with a parameter  $\lambda$ ) that modifies the p-air amplitude to

$$\Gamma_{\text{pA}}(s, \vec{b}, \vec{s}_1, \dots, \vec{s}_A) = 1 - \frac{1}{2} \prod_{j=1}^A \left[ 1 - (1 + \lambda) \Gamma_{\text{pp}}(\vec{b} - \vec{s}_j) \right] - \frac{1}{2} \prod_{j=1}^A \left[ 1 - (1 - \lambda) \Gamma_{\text{pp}}(\vec{b} - \vec{s}_j) \right]$$

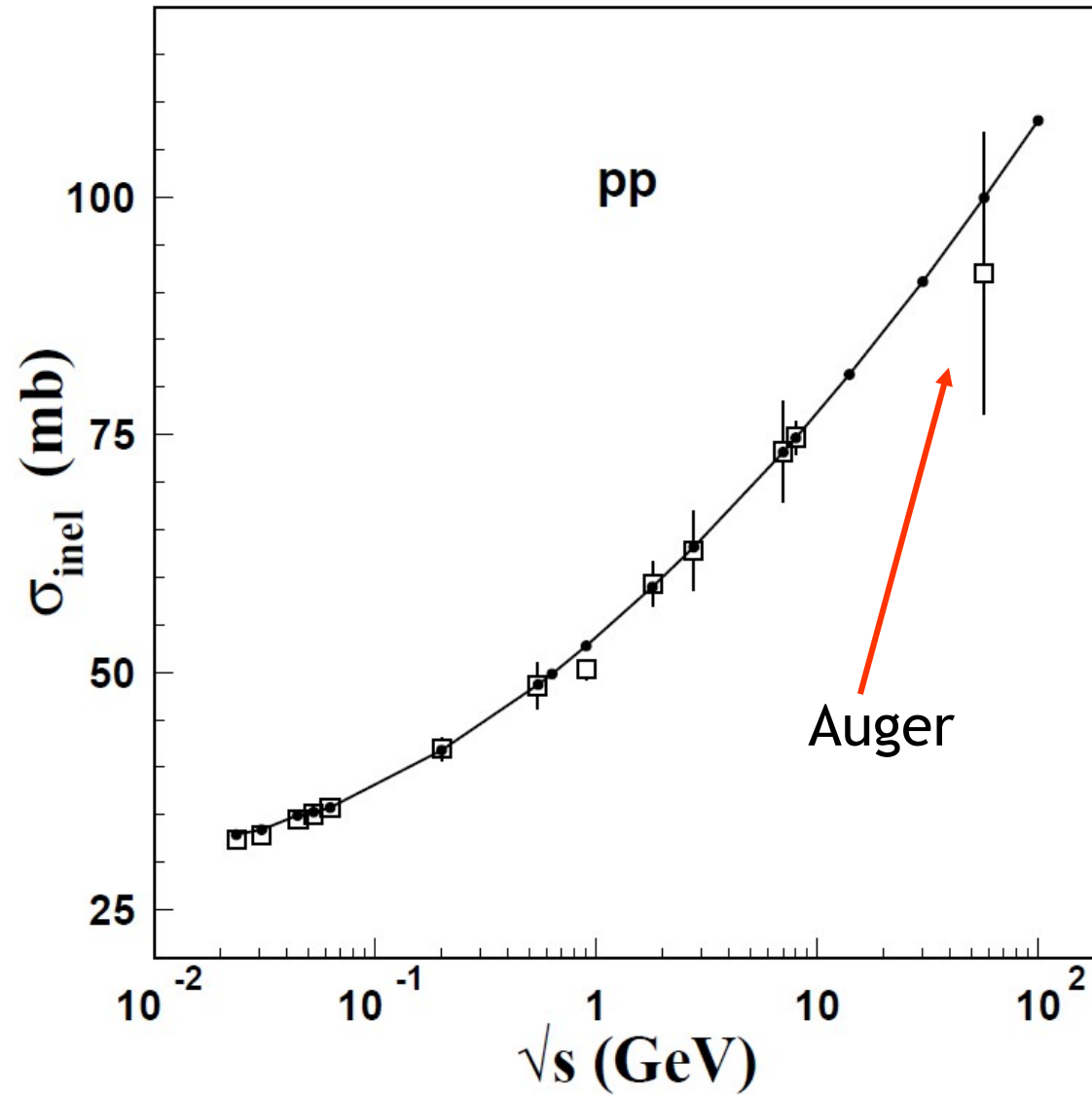
From the energy dependences in our input we obtain the parametrization for p-air production cross section with powers of  $\log \sqrt{s}$ .

$$\sigma_{\text{p-air}}^{\text{prod}}(s) = 383.474 + 33.158 \log \sqrt{s} + 1.3363 \log^2 \sqrt{s}$$

## Comparison with p-air cosmic ray measurements



# pp inelastic cross section



# p-air in b-space

p-air elastic scattering amplitude

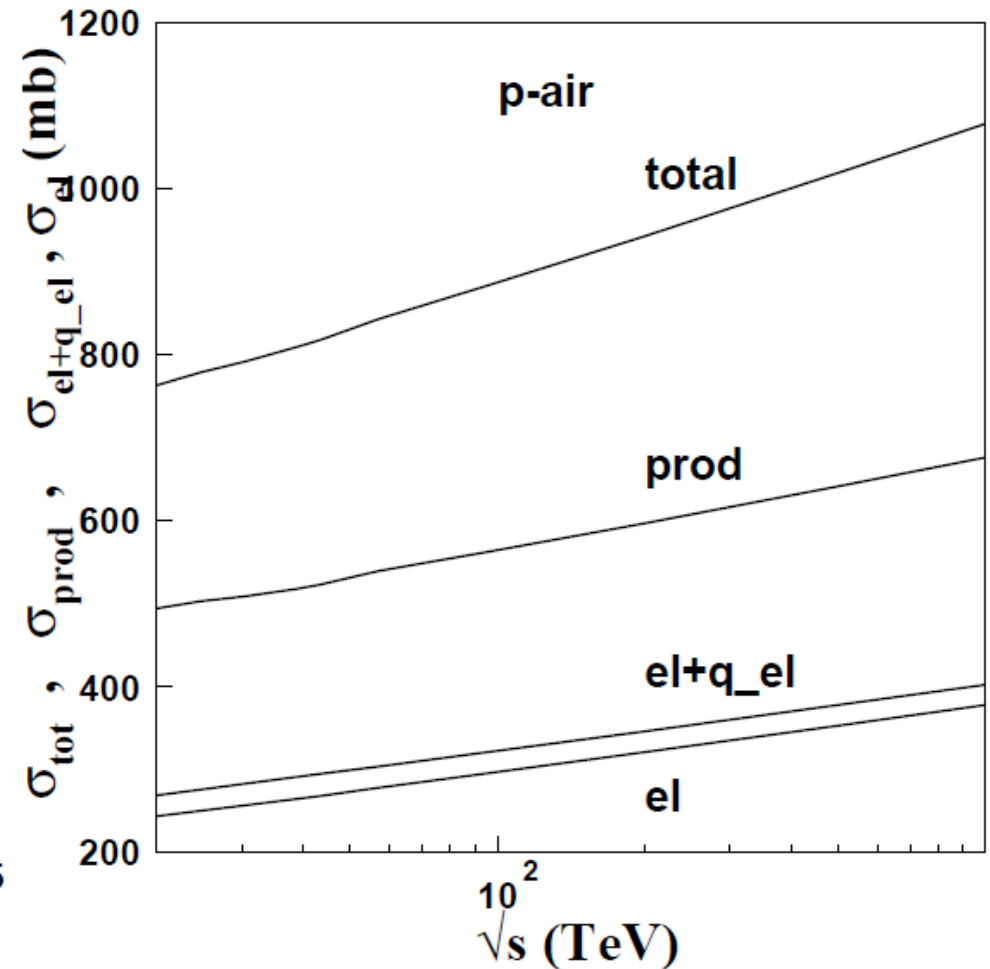
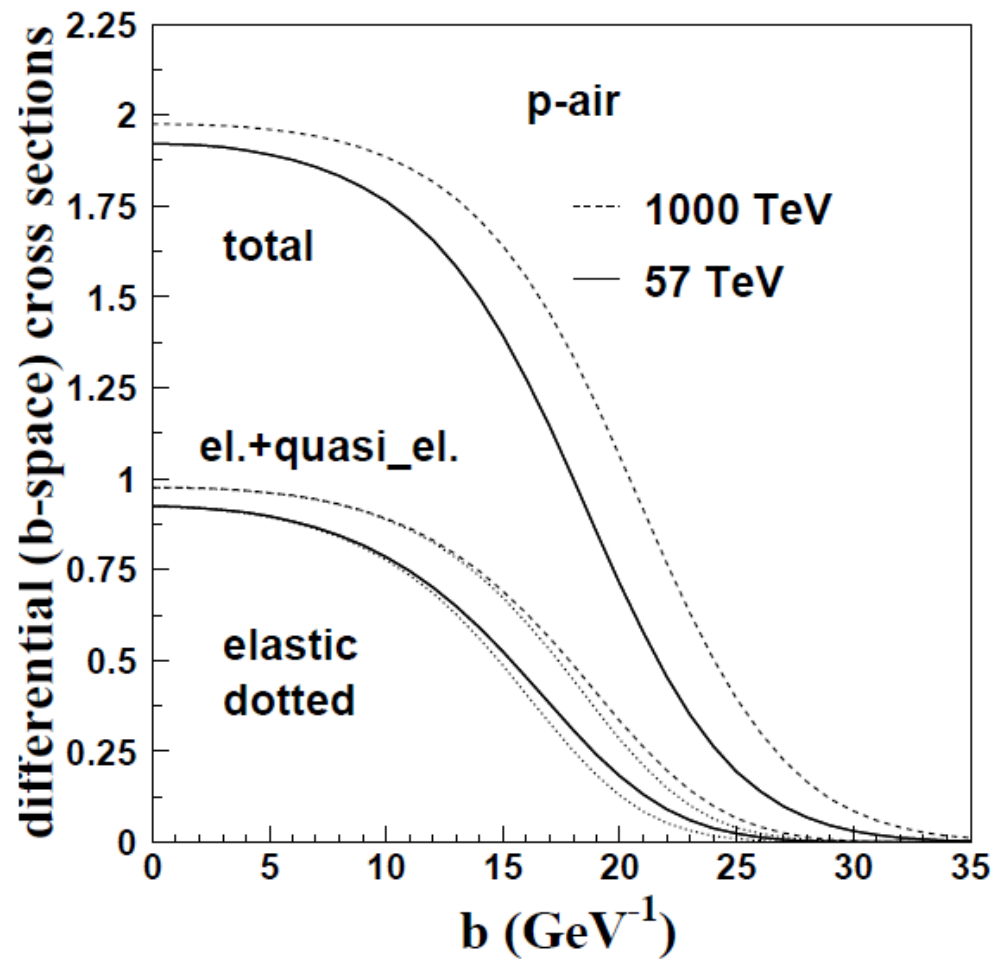
$$-i\hat{T}_{pA}(\vec{b}) = 1 - e^{i\chi_{pA}} \simeq 1 - \left\langle \prod_{j=1}^A e^{i\chi_{pN_j}} \right\rangle = 1 - \left\langle \prod_{j=1}^A \left( 1 + i\hat{T}_{pN}(\vec{b}) \right) \right\rangle$$

p-air distributions:

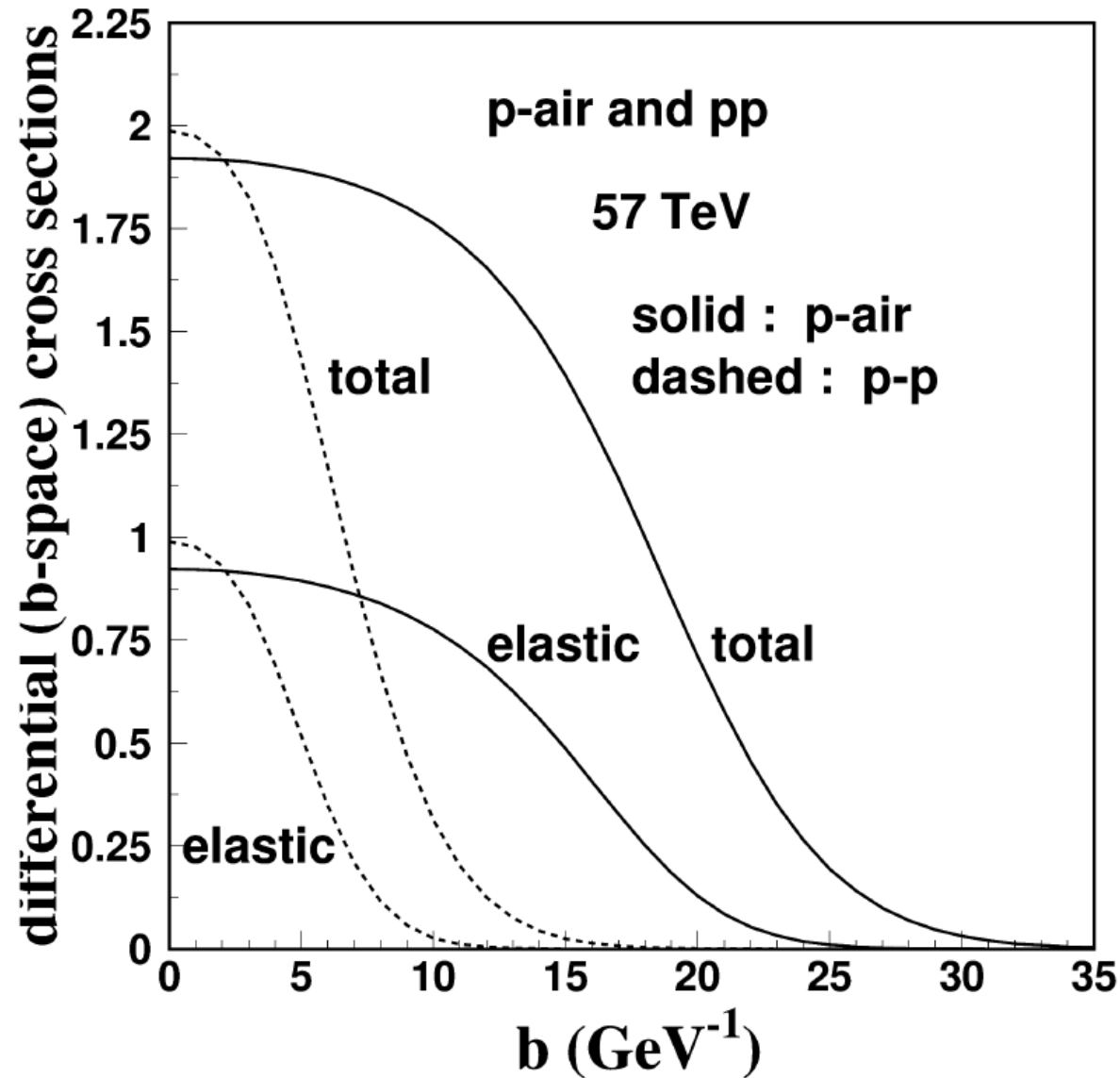
$$\frac{1}{2} \frac{d^2 \sigma_{pA}^{\text{tot}}}{d^2 \vec{b}}(s, \vec{b}) = \left\langle 1 - \prod_{i=1}^A \left( 1 - \frac{1}{2} \frac{d^2 \sigma_{pp}^{\text{tot}}}{d^2 \vec{b}_i}(s, \vec{b} - \vec{b}_i) \right) \right\rangle$$

$$\frac{d^2 \sigma_{pA}^{\text{el}}}{d^2 \vec{b}}(s, \vec{b}) = \left\langle \left[ 1 - \prod_{i=1}^A \left( 1 - \frac{d^2 \sigma_{pp}^{\text{tot}}}{d^2 \vec{b}_i}(s, \vec{b} - \vec{b}_i) \right) \right]^2 \right\rangle$$

# p-air cross section predictions

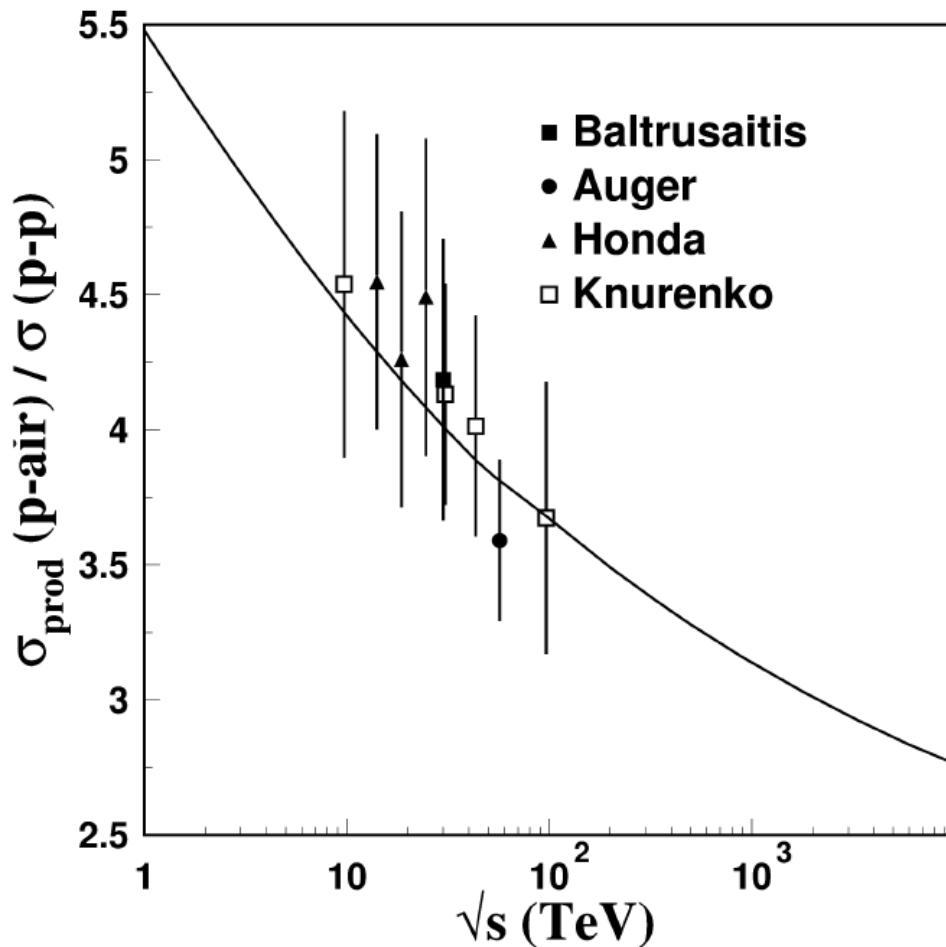


# Comparison between p-air and pp in b-space



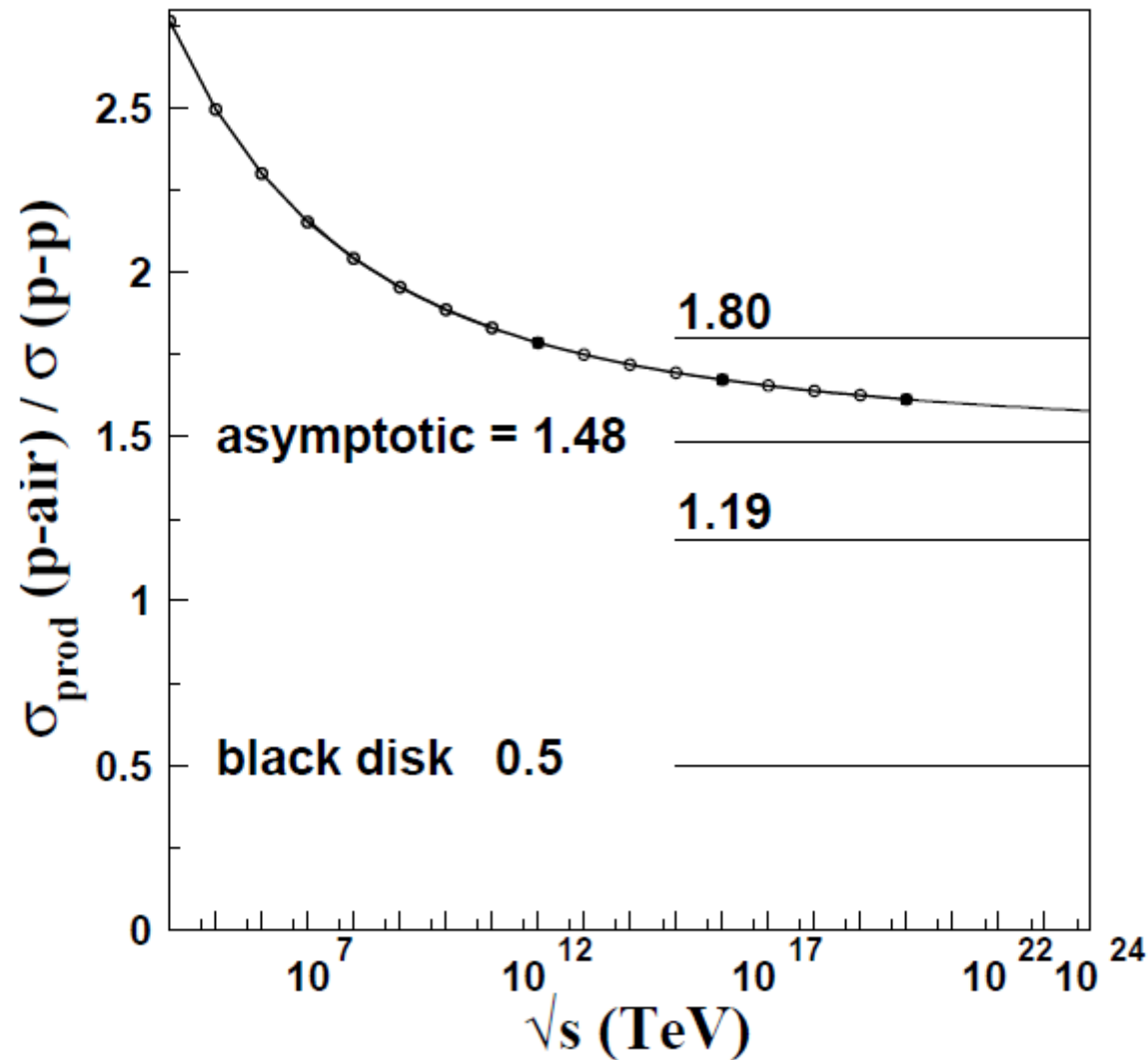
# ratio of p-air/pp cross sections

$$\frac{\sigma_{\text{p-air}}^{\text{prod}}(s)}{\sigma_{\text{pp}}(s)} = \frac{383.474 + 33.158 \log \sqrt{s} + 1.3363 \log^2 \sqrt{s}}{69.3286 + 12.6800 \log \sqrt{s} + 1.2273 \log^2 \sqrt{s}}$$



The ratio tends to a finite value in the asymptotic limit

# asymptotic limit of ratio p-air / pp





# Conclusions

- We believe that we have realistic pp inputs, with energy dependence.
- The simplest Glauber calculation accounts for the C.R measurements of p-air production cross section at all energies 1 TeV – 100 TeV.
- We give predictions for LHC energies and beyond present experiments and for an asymptotic regime.

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# Thanks