Some news about GPDs & hard exclusive processes

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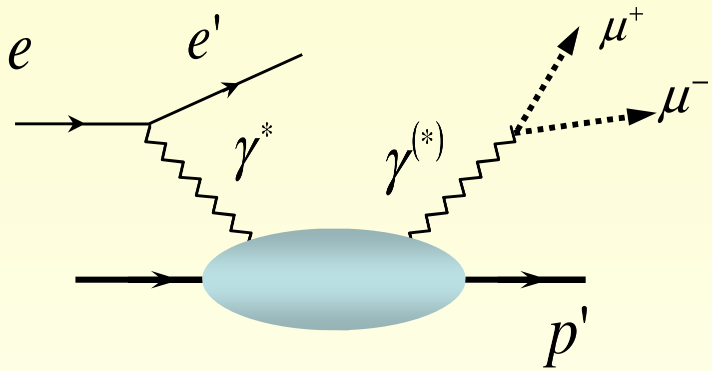
- *From quasi-real to deeply virtual Compton scattering*
- *Higher twist contributions to DVCS*
- *One-to-one maps among GPD representations*
- *Data description/predicitions (partially includes DVMP)*
- *Unifying the partonic picture*
- *Conclusions*

A. Belitsky, Y. Ji;
V. Braun, A. Manashov; B. Pirnay,
M. Polyakov, K. Semenov-Tian-Shansky; O. Teryaev
K. Kumerički (KK), E. Aschenauer, S. Firzo, M. Murray, …
K. Passek-Kumerički (KP-K), T. Lautenschlager, A. Schäfer; M. Meskauskas
D.S. Hwang

GPD related hard exclusive processes

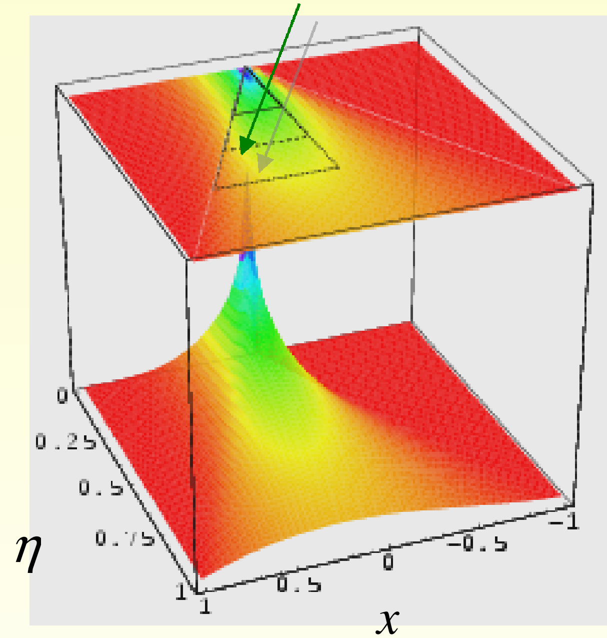
- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^-e^+$$



factorization proof for transversal cross sections
[Collins Freund (99)]

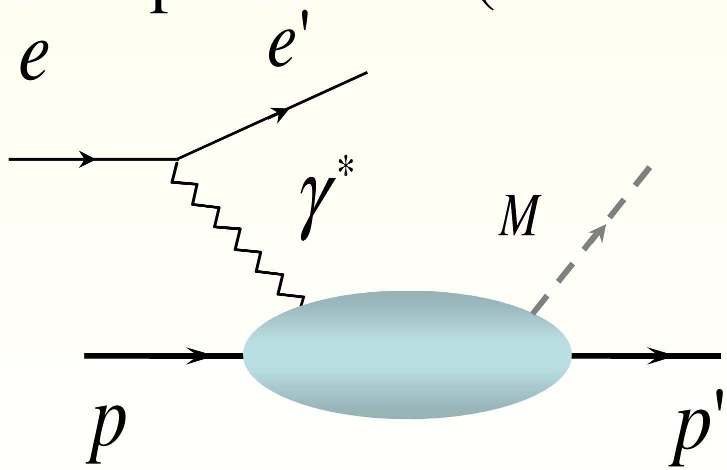
scanned area of the surface as a functions of lepton energy



$ep \to e'p'\mu^+\mu^-$

- Deeply virtual meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$



twist-two observables:

longitudinal cross sections

transverse target spin asymmetries

- etc.

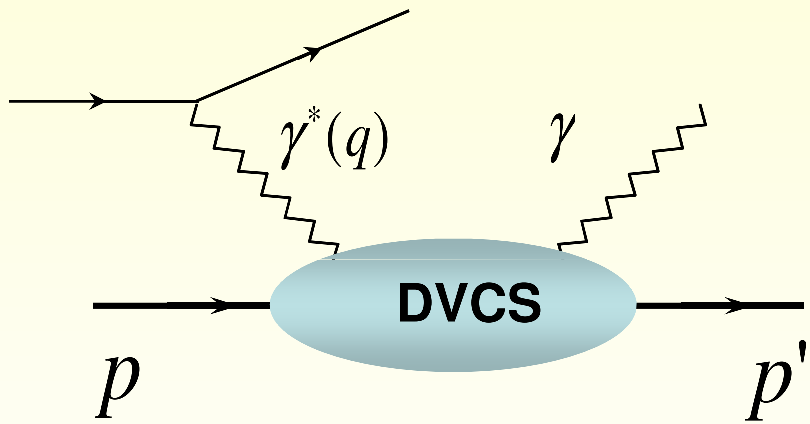
factorization proof for longitudinal cross sections
[Collins, Frankfurt, Strikman (96)]

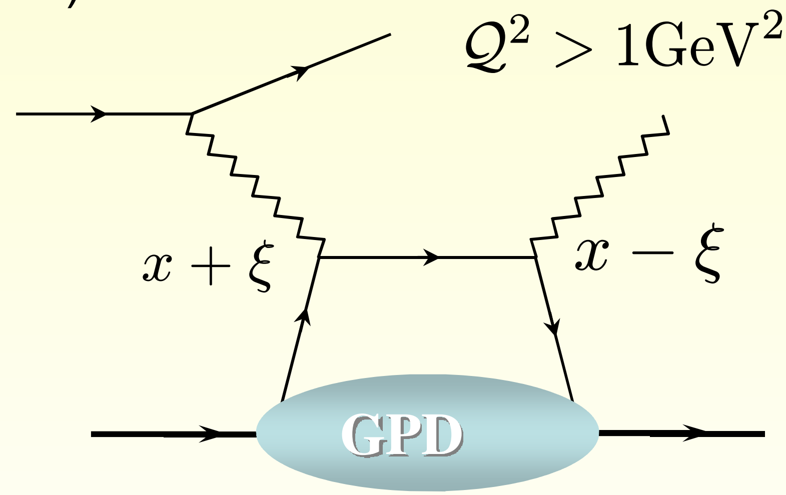
GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,

[DM et. al (91/94)
Radyushkin (96)
Ji (96)]

e.g., hard electroproduction of photons (DVCS)





$t = \Delta^2$ – fix

$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

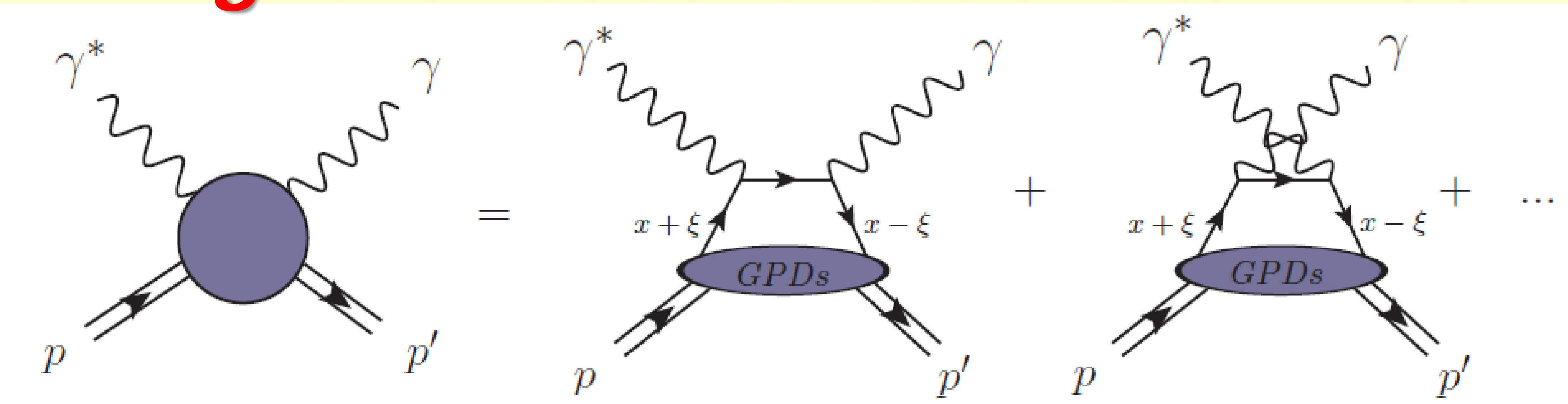
CFF
Compton form factor
observable

hard scattering part
perturbation theory
(our conventions/microscope)

GPD
universal
(conventional)

higher twist
depends on
approximation

Higher twist contributions to DVCS



$$T_{\mu\nu} = i \int d^4x \, \mathrm{e}^{\frac{i}{2}(q_1+q_2)\cdot x} \langle p_2 | T\{ j_\mu(x/2) j_\nu(-x/2) \} | p_1 \rangle$$

- collinear factorization approach (calculating Feynman diagrams on partonic level)
- operator product expansion (in terms of light-ray operators)

$$T j_\mu(x/2) j_\nu(-x/2) \overset{\mathrm{LO}}{=} \frac{S_{\mu\nu\alpha\beta} i x^\alpha}{(x^2 - i\epsilon)^2} \left[\overline{\psi}(x/2)\gamma^\beta \psi(-x/2) - \overline{\psi}(-x/2)\gamma^\beta\psi(x/2) \right]$$
$$+ \frac{i\epsilon_{\mu\nu\alpha\beta} i x^\alpha}{(x^2 - i\epsilon)^2} \left[\overline{\psi}(x/2)\gamma^\beta \gamma^5 \psi(-x/2) + \overline{\psi}(-x/2)\gamma^\beta \gamma^5 \psi(x/2) \right]$$

- expansion in leading $1/x^2$ singularities is easily done by projection on the light cone $n_\mu \sim q_\mu + \ldots$ and $n^*_\mu \sim P_\mu + \ldots$ or $n_\mu = q_{2\mu}$ and $n^*_\mu = q_{1\mu} + \ldots q_{2\mu}$

with $q_\mu = (q_{1\mu} + q_{2\mu})/2$ and $P_\mu = p_{1\mu} + p_{2\mu}$

$$T_{\mu\nu} \overset{\rm LO}{=} -g^{\perp}_{\mu\nu} \sum_q \int_{-1}^{1} dx \left[\frac{e_q^2}{\xi - x - i\epsilon} - \frac{e_q^2}{\xi + x - i\epsilon} \right] q(x,\xi,t,\mathcal{Q}^2|s_1,s_2)$$

$$-i\epsilon^{\perp}_{\mu\nu} \sum_q \int_{-1}^{1} dx \left[\frac{e_q^2}{\xi - x - i\epsilon} + \frac{e_q^2}{\xi + x - i\epsilon} \right] \widetilde{q}(x,\xi,t,\mathcal{Q}^2|s_1,s_2)$$

consequences of *1/Q* truncation and restriction to leading order in pQCD

- DVCS tensor structure depends on the choice of *n*
- scaling variable $\xi \sim x_B/(2-x_B)$ depends on the choice of *n*
- gauge and translation invariance holds only to leading power accuracy
- DVCS tensor structure is not complete

to overcome these problems one should go

- to twist-3 accuracy, yields 4 other GPDs (LT photon helicity flips) **[done, 2000]**
- to NLO, yields 4 gluon transversity GPDs (TT photon helicity flips) **[done, 2000]**
- twist-4 accuracy pushes ambiguity to the $1/Q^4$ level **[Braun, Manashov 2011]** but yields new parton correlation functions, however, no new structures

What is the problem in calculating higher twist contributions?

`correct' decomposition of $\partial^\mu \mathcal{O}_{\mu\mu_1\mu_2\cdots} \overset{\rm tree}{=} 0 \quad \text{and} \quad \partial^2 \mathcal{O}_{\mu\mu_1\mu_2\cdots}$

guiding principle conformal covariance (same evolution as twist-2 operators)

Conformal PWE of GPDs

- GPD support is a consequence of Poincaré covariance (polynomiality)

$$H_j(\eta,t,\mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomously (to LO and beyond in a special scheme)

$$\mu \frac{d}{d\mu} H_j(\eta,t,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \gamma_j^{(0)} H_j(\eta,t,\mu^2)$$

- inverse relation is given as series of (mathematical) generalized distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) H_j(\eta,t)\,,\ p_j(x,\eta) \propto \theta(|x| \le \eta) \frac{\eta^2 - x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:

- smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- `dual' parameterization [M. Polyakov, A. Shuvaev (02), Polyakov (07), Semenov-Tian-Shansky]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

Purely mathematical problems/exercises:

- show that GPD representations are in one-to-one correspondence
- give inverse transformation (in principle done for Mellin-Barnes integral)
- support properties of Shuvaev`s `forward-like GPDs' is not known
  (i.e. Shuvaev`s claim is wrong -- known since more than one decade that)

new: **[DM, M. Polyakov, K. Semenov-Tian-Shansky (??)]**

i. it is explicitly shown that `dual parameterization' and
Mellin-Barnes integral + SO(3) PWE are equivalent

$$H(x \geq -\eta, \eta, t)) = \sum_{\nu=0}^{\infty} \int_0^1 dy K_{2\nu}(x, \eta|y) y^{2\nu} Q_{2\nu}(y, t) + \text{`}D - term' \,,$$

$$H(x \geq -\eta, \eta, t) = \sum_{\nu=0}^{\infty} \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \, \frac{\eta^{2\nu} p_{j+2\nu}(x, \eta)}{\sin(\pi[j+1])} H_{j+2\nu, j+1}(t) \, \hat{d}_{00}^{j+1}(\eta) + \text{`}D - term'$$

(state-of-art formalism (NLO/NNLO, twist-4) is worked out in MB representation)

ii. (numerical) map of double distributions to conformal moments
by means of Appell`s F_4 function

Strategies to analyze DVCS data

(ad hoc) modeling: **VGG** code [**Goeke et. al (01)** based on **Radyushkin's** DDA]
BMK model [**Belitsky, DM , Kirchner (01)** based on RDDA]
`aligned jet' model [**Freund, McDermott, Strikman (02)**]
Goloskokov/Kroll (05) based on RDDA (pinned down by DVMP)
`dual' model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07**]
" -- " [**KMP-K (07)** in MBs-representation]
polynomials [**Belitsky et al. (98), Liuti et. al (07), Moutarde (09)**]

dynamical models: not applied **[Radyushkin et.al (02); Tiburzi et.al (04); Hwang DM (07)]…**
(respecting Lorentz symmetry)

flexible models: any representation by including ***unconstrained*** degrees of freedom
(for fitting) **KMP-K (07/08)** for H1/ZEUS in MBs-integral-representation

CFFs (real and imaginary parts) and GPD fits/predictions

i. CFF extraction with formulae (local) **[BMK (01), HALL-A (06)]** and **[KK,DM, Murray]**
least square fits (local) **[Guidal, Moutarde (08…)]**
neural networks – a start up **[KMS (11)]**
ii. `dispersion integral' fits **[KMP-K (08),KM (08…)]**
iii. **flexible GPD model fits** **[KM (08…), AFKM (13), KMM (13), LSM (13)]**
vi. model comparisons **VGG code, however also BMK01 (up to 2005)**
& predictions **Goloskokov/Kroll (07) model** based on RDDA
[DVCS: Kroll, Moutarde, Sabatie (13)]

Status of theory

[Belitsky, DM (97); Mankiewicz et. al (97); Ji,Osborne (97/98); Pire, Szymanowski, Wagner (11); DM, Pire, Szymanowski, Wagner (11)]

✓ ***twist-two*** DVCS coefficients at ***next-to-leading*** order

✓ twist-two DVMP coefficients at ***next-to-leading*** order

new

[Belitsky, DM (01); Ivanov, Szymanowski,Krasnikov (04)]

NLO effects are well understood generically

large-ξ: logarithmical enhancement

valence region: weak evolution implies moderate effects

small-ξ: model dependence

DM, T. Lautschlager, K. Passek-Kumericki. A. Schaefer (13)

✓ anomalous dimensions and evolution kernels at ***next-to-leading*** order

evolution effects can be called moderate, except for H/E at small- ξ

NLO analyses have to include NLO evolution

[Belitsky, DM (98) + Freund (01)]

✓ gluon transversity at ***next-to-leading*** order [Belitsky, DM (00)]

[DM (06); KMP-K, Schaefer (06)]

✓ ***next-to-next-to-leading*** order for DVCS in a specific conformal subtraction scheme

NLO → NNLO corrections can be called moderate w.r.t. LO → NLO

[Anikin,Teryaev, Pire (00); Polyakov et. al (00), Belitsky DM (00); Kivel et. al, Weiss, Radyushkin (00)]

✓ ***twist-three*** including quark-gluon-quark correlation at LO

✓ partially, ***twist-three*** sector at ***next-to-leading*** order [Kivel, Mankiewicz (03)]

? \`target mass corrections' (not understood) [Belitsky DM (01)]

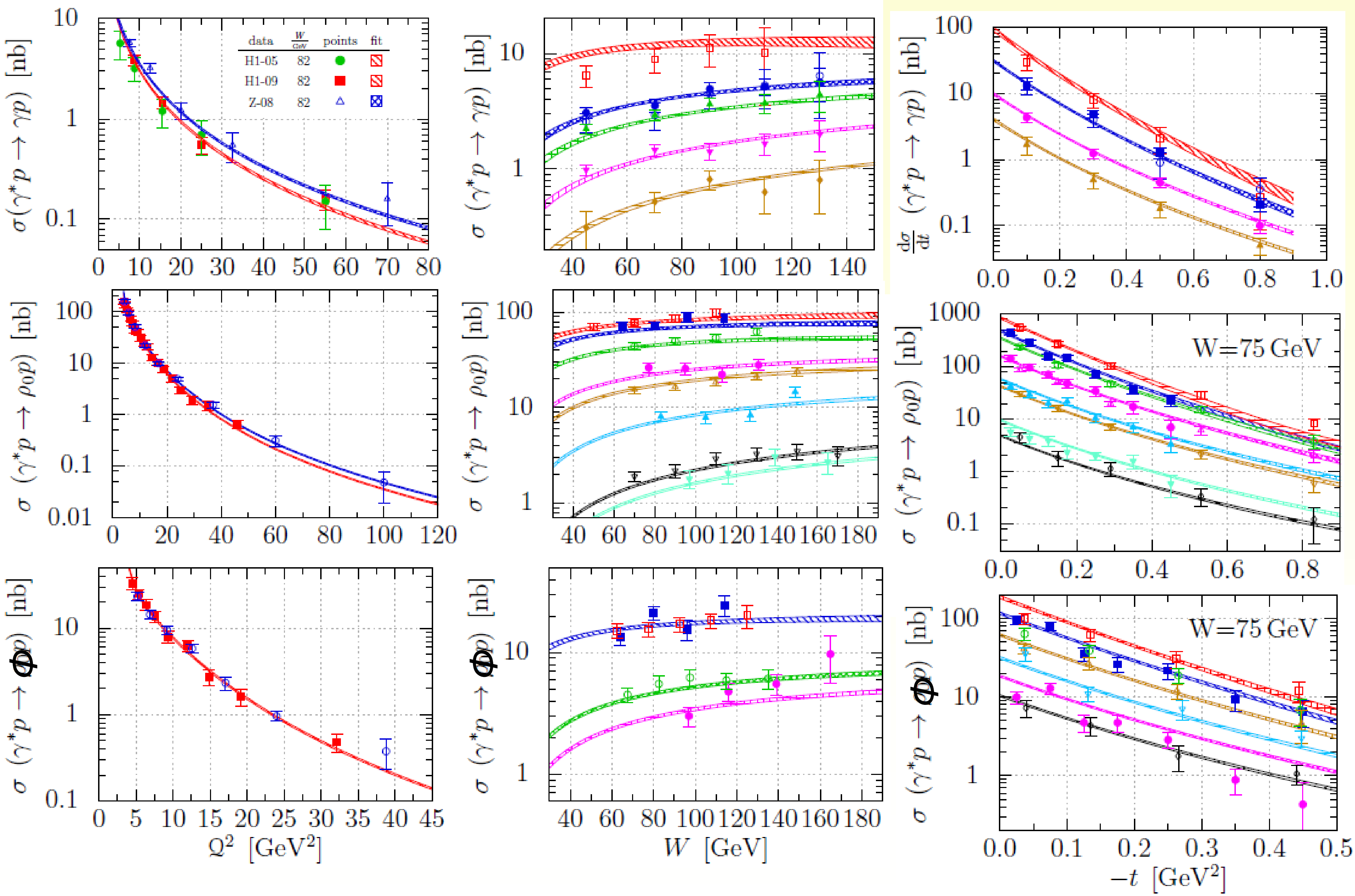
new

✓ **kinematical twist-four** corrections [Braun, Manashov (11)]

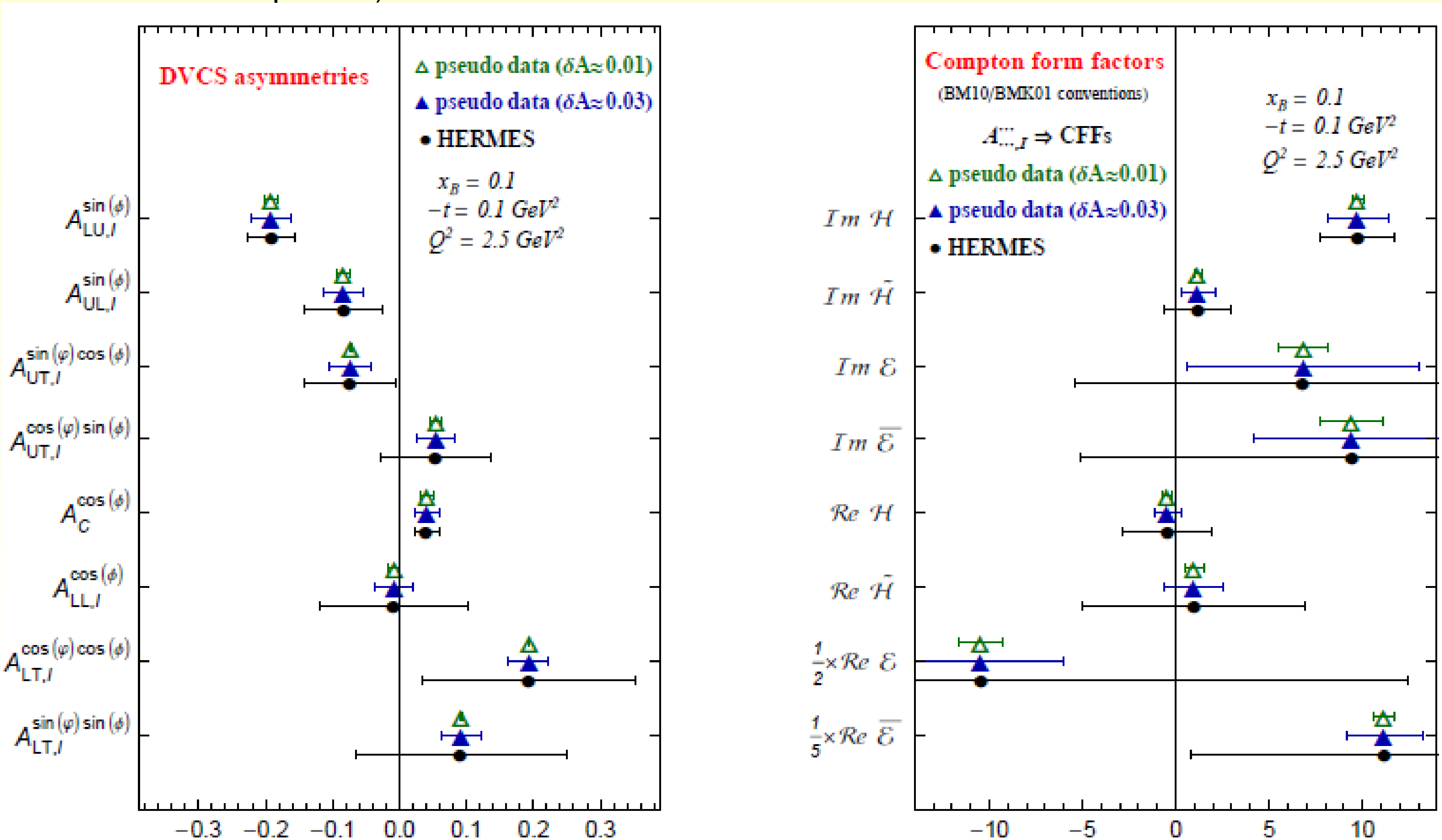
DIS+DVCS+DVMP phenomenology at small-x_B (H1,ZEUS)

works somehow without DIS at LO **[T. Lautenschlager, DM, A. Schäfer (13)]**

works at NLO ($Q^2 > 4\ GeV^2$), done with Bayes theorem (probability distribution function)



- a complete measurement allows in principle to pin down all CFFs **KK, DM, Murray (13)**
- missing information in incomplete measurements can be filled with noise
  (Guidal`s philosophy: use noise together with hypotheses and model constraints, our results are compatible)



- larger statistics:
  some CFF *E* constraint might have been obtained by HERMES

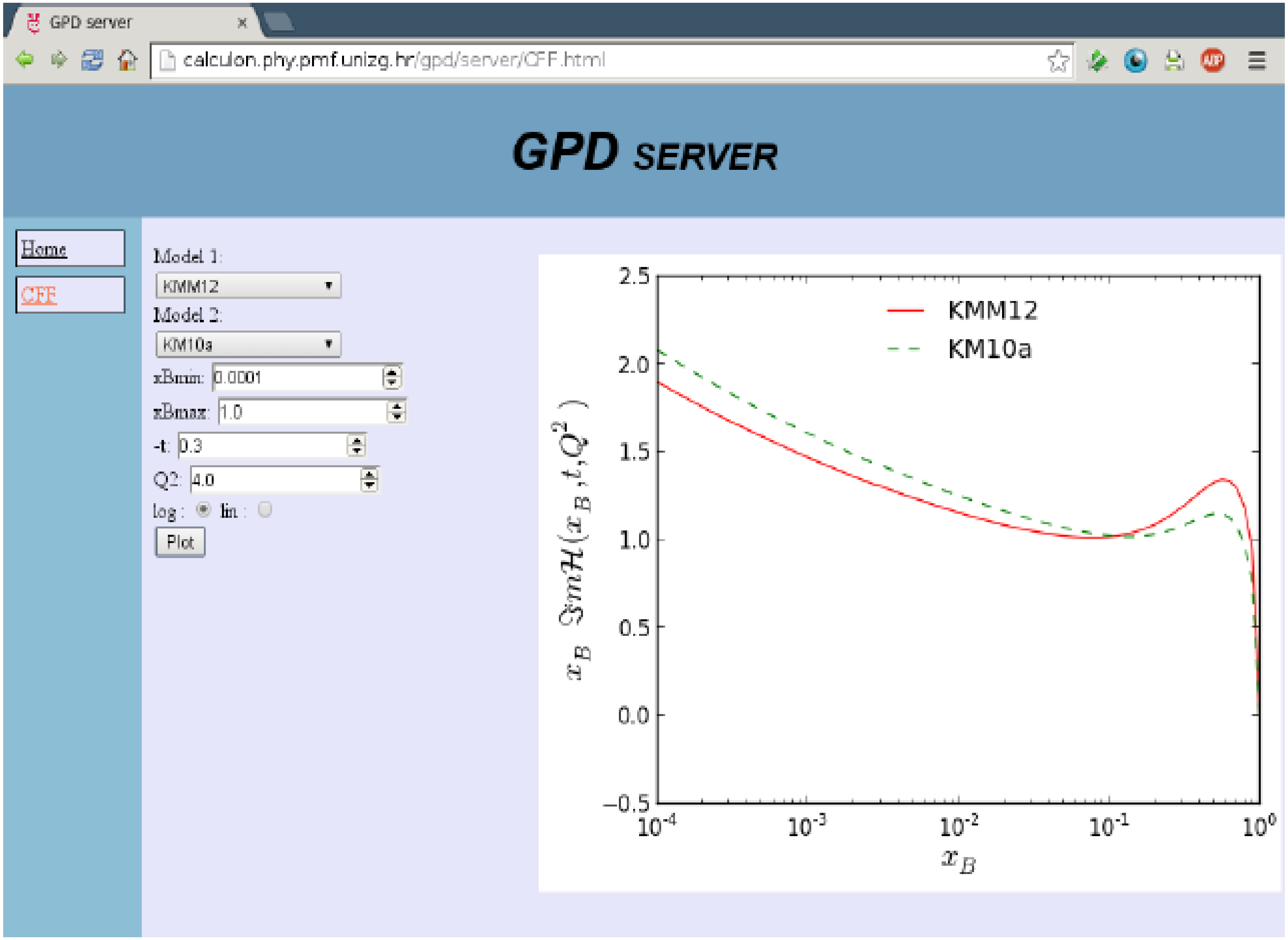
KM models are available at WWW

- http://calculon.phy.hr/gpd/ — binary code for cross sections

```
% xs.exe

  xs.exe  ModelID  Charge  Polarization  Ee  Ep  xB  Q2  t  phi

returns cross section (in nb) for scattering of lepton of energy Ee
on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
   0 debug, always returns 42,
   1 KM09a - arXiv:0904.0458 fit without Hall A,
   2 KM09b - arXiv:0904.0458 fit with Hall A,
   3 KM10  - preliminary hybrid fit with LO sea evolution, from Trento presentation,
   4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
   5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi  -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938  0.111 3. -0.3 0

  0.18584386497251
```

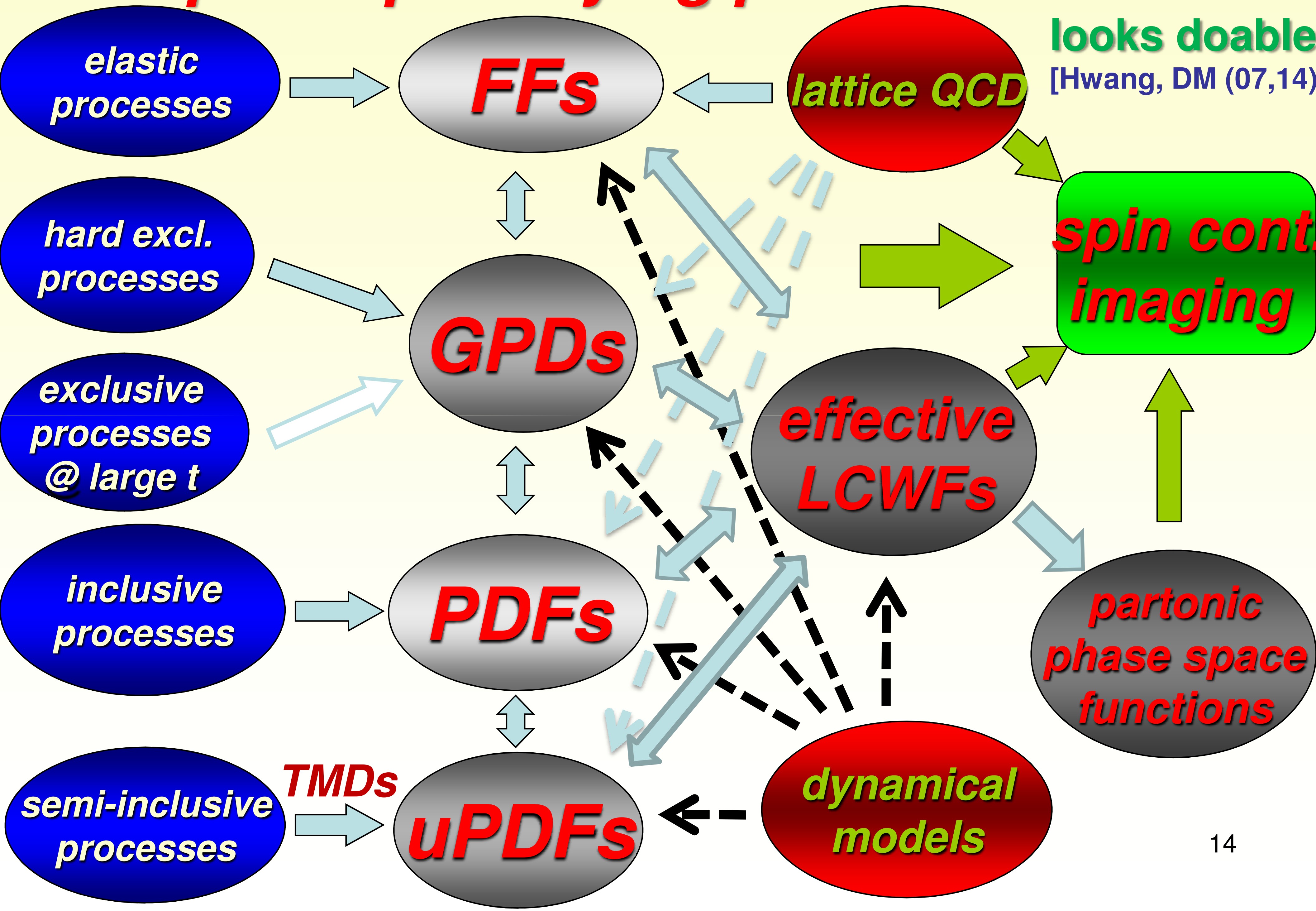
GPD page and server

- Durham-like CFF/GPD server page



- Do we need "Les Houches Accord" CFF/GPD interface?

Prospect: quantifying partonic content



Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons (to some extend done at small x_B)
- to address the spin content of the nucleon (not possible at present in pheno.)
- providing a bridge to LCWFs & non-perturbative methods (e.g., lattice)
- modeling in terms of effective LCWFs is doable (requires efforts)

first decade of hard exclusive leptoproduction measurements

- CFFs have their own interest, bridging low and high virtuality regimes
- should be straightforward to improve global (flexible) model fits to DVCS
- DVCS and DVMP data are describable in global NLO fits at small x
- moving on: to NLO, kinematical twist, full GPD models, DVCS+DVMP+...
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena: handle on GPD E & 3D
- some kind of education is desired before one can enter GPD phenomenology
- theory development is needed to address phenomenological goals