

Impact factor for high-energy quark-antiquark-gluon jet production in DDIS

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Diffractionn 2014

[RB, A.Grabovsky, L.Szymanowski, S.Wallon (arXiv:1405.7676)]
[To be published in JHEP]

Overview

1 Diffractive DIS

- Rapidity gap events at HERA
- Collinear factorization approach
- k_T -factorization approach : two exchanged gluons
- Confrontation of the two approaches with HERA data

2 Diffractive production of jets : our approach

- $q\bar{q}$ production
- $q\bar{q}g$ production
- Linear approximation : 2 and 3 exchanged gluons

3 Conclusion

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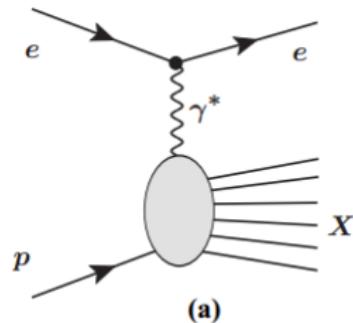
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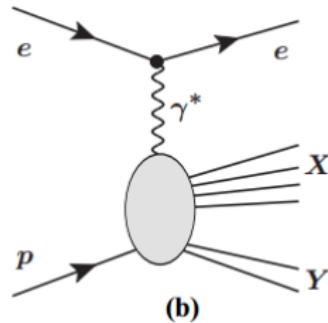
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a rapidity gap



DIS events

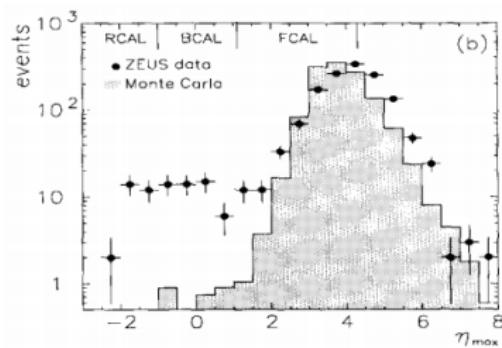


DDIS events

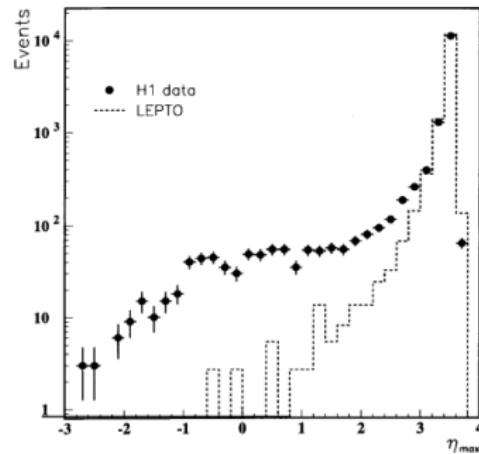
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of events reveal a rapidity gap



ZEUS, 1993



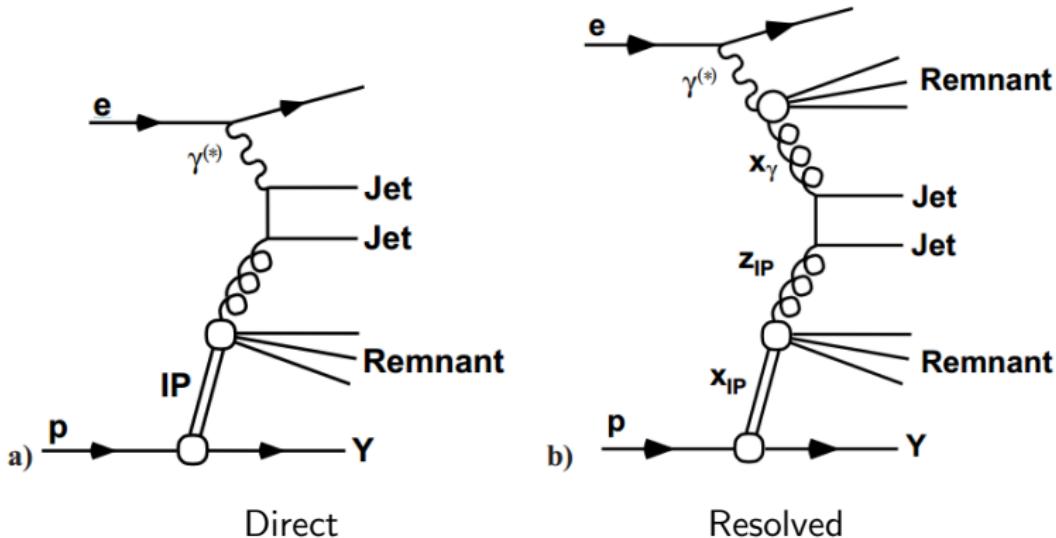
H1, 1994

Theoretical approaches for DDIS using pQCD

- Collinear factorization approach
 - Relies on QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
 - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
 - low- x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
 - The pomeron is described as a two-gluon color-singlet state

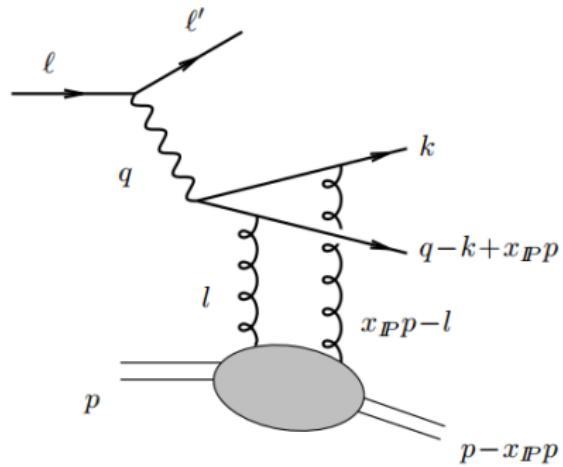
Theoretical approaches for DDIS using pQCD

Collinear factorization approach



Theoretical approaches for DDIS using pQCD

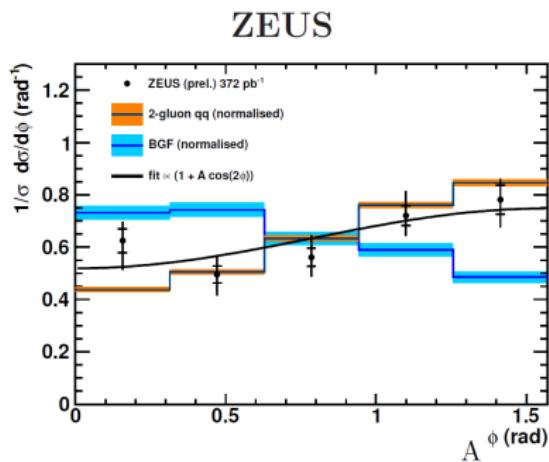
k_T -factorization approach : two gluon exchange



Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data

The k_T -factorization approach gives a better description of diffractive events in the very low x kinematic regime :



Valkárová, low-x 2014, Kyoto

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Assumptions

- Regge limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- No approximation for the outgoing gluon
- Lightcone coordinates :

$$p^+ = n_2 \cdot p = \frac{1}{2} (p^0 + p^3),$$

$$p^- = n_1 \cdot p = p^0 - p^3,$$

$$n_1 = (1, 0, 0, 1),$$

$$n_2 = \frac{1}{2} (1, 0, 0, -1)$$

- Shockwave (Wilson lines) approach

Shockwave approach

One decomposes the gluon field \mathcal{A} into an internal field A and an external field b :

$$\mathcal{A}^\mu = \textcolor{red}{A}^\mu + \textcolor{green}{b}^\mu$$

The internal one contains the gluons with rapidity $p^+ > e^\eta = \sigma$ and the external one contains the gluons with rapidity $p^+ < \sigma$. One writes :

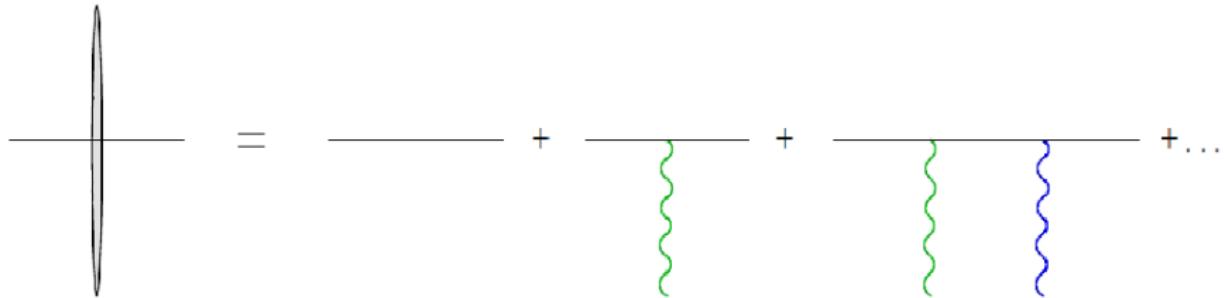
$$\textcolor{green}{b}^\mu(z) = \delta(z^+) B(\vec{z}) n_2^\mu$$

We introduce the Wilson lines :

$$U_i = U_{\vec{z}_i} = U(\vec{z}_i, \eta) = P \exp \left[ig \int_{-\infty}^{+\infty} \textcolor{green}{b}_\eta^-(z_i^+, \vec{z}_i) dz_i^+ \right]$$

Shockwave approach

$$U_i = 1 + ig \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) b_\eta^-(z_j^+, \vec{z}_j) \theta(z_{ji}^+) dz_i^+ dz_j^+$$



Dipole operator :

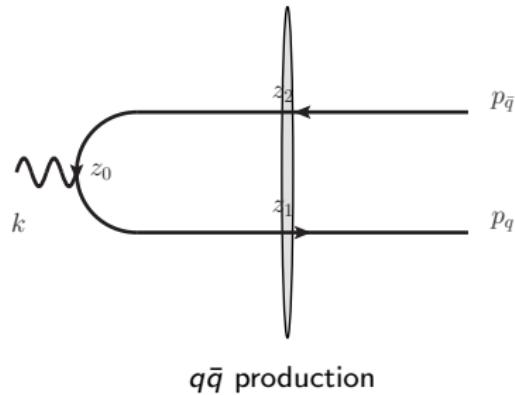
$$\mathbf{U}_{12} = \frac{1}{N_c} \text{Tr} \left(U_1 U_2^\dagger \right) - 1.$$

BK equation :

$$\frac{d\mathbf{U}_{12}}{d \ln \sigma} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13}\mathbf{U}_{32}]$$

$q\bar{q}$ production

Matrix element for EM current



$$\begin{aligned} M_0^\alpha &= \delta_I^n \frac{\langle 0 | b_{p_{\bar{q}}}^I (a_{p_q})_n \bar{\psi}(z_0) \gamma^\alpha \psi(z_0) e^{i \int \mathcal{L}_i(z) dz} | 0 \rangle}{\langle 0 | e^{i \int \mathcal{L}(z) dz} | 0 \rangle} \\ &\rightarrow \int d\vec{z}_1 d\vec{z}_2 F(p_q, p_{\bar{q}}, z_0, \vec{z}_1, \vec{z}_2)^\alpha N_c \mathbf{U}_{12} \end{aligned}$$

$q\bar{q}$ production

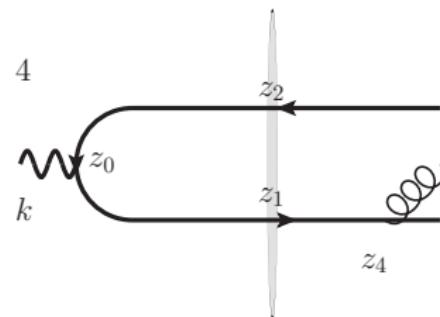
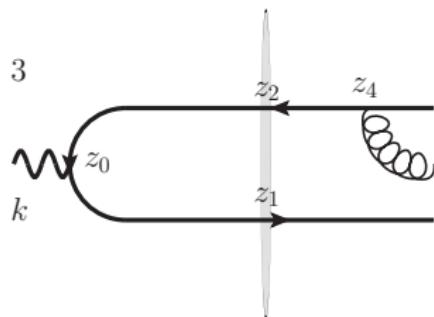
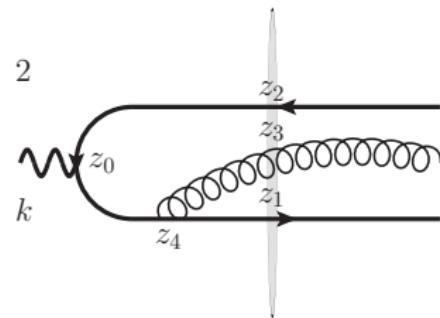
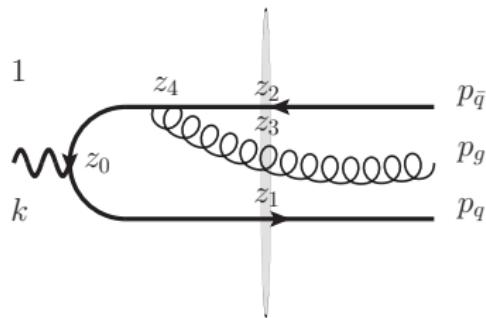
We recover the well-known results :

$$F(p_q, p_{\bar{q}}, k, \vec{z}_1, \vec{z}_2)^\alpha \varepsilon_{L\alpha} = \theta(p_q^+) \theta(p_{\bar{q}}^+) \frac{\delta(k^+ - p_q^+ - p_{\bar{q}}^+)}{(2\pi)^2} e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2} \\ \times (-2i) \delta_{\lambda_q, -\lambda_{\bar{q}}} x_q x_{\bar{q}} Q K_0 \left(Q \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2} \right)$$

and

$$F(p_q, p_{\bar{q}}, k, \vec{z}_1, \vec{z}_2)^j \varepsilon_{Tj} = \theta(p_q^+) \theta(p_{\bar{q}}^+) \frac{\delta(k^+ - p_q^+ - p_{\bar{q}}^+)}{(2\pi)^2} e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2} \\ \times \delta_{\lambda_q, -\lambda_{\bar{q}}} (x_q - x_{\bar{q}} + s\lambda_q) \frac{\vec{z}_{12} \cdot \vec{\epsilon}_T}{\vec{z}_{12}^2} Q \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2} K_1 \left(Q \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2} \right).$$

$q\bar{q}g$ production



$q\bar{q}g$ production

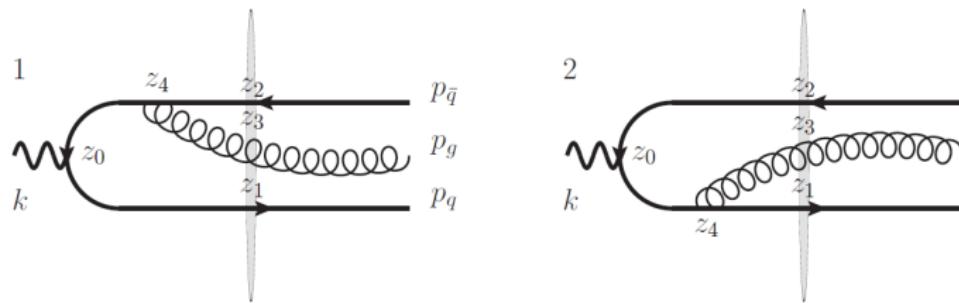
As before :

$$\tilde{M}^\alpha = (t^b)_I^n \frac{\langle 0 | c_{p_g}^b b_{p_{\bar{q}}}^I (a_{p_q})_n \bar{\psi}(z_0) \gamma^\alpha \psi(z_0) e^{i \int \mathcal{L}_i(z) dz} | 0 \rangle}{\langle 0 | e^{i \int \mathcal{L}(z) dz} | 0 \rangle}$$

Projection on color singlet and subtraction of non-interacting terms give :

$$\begin{aligned} M^\alpha &= \frac{N_c^2}{2} \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha (\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} + \mathbf{U}_{13}\mathbf{U}_{32}) \\ &+ \int d\vec{z}_1 d\vec{z}_2 F_2(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2)^\alpha (N_c^2 - 1) \mathbf{U}_{12} \end{aligned}$$

$q\bar{q}g$ production : first kind



$q\bar{q}g$ production : first kind

$q\bar{q}g$ production : first kind

Result for a longitudinal photon

$$F_{1L}(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2, \vec{z}_3)$$

$$= \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \theta(p_g^+ - \sigma) 2Qg \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2 - i\vec{p}_g \cdot \vec{z}_3}}{\pi \sqrt{2p_g^+}} K_0(QZ_{123})$$

$$\times \delta_{\lambda_q, -\lambda_{\bar{q}}} \left\{ (x_{\bar{q}} + x_g \delta_{-s_g \lambda_q}) x_q \frac{\vec{z}_{32} \cdot \vec{\varepsilon}_g^*}{\vec{z}_{32}^2} - (x_q + x_g \delta_{-s_g \lambda_{\bar{q}}}) x_{\bar{q}} \frac{\vec{z}_{31} \cdot \vec{\varepsilon}_g^*}{\vec{z}_{31}^2} \right\}$$

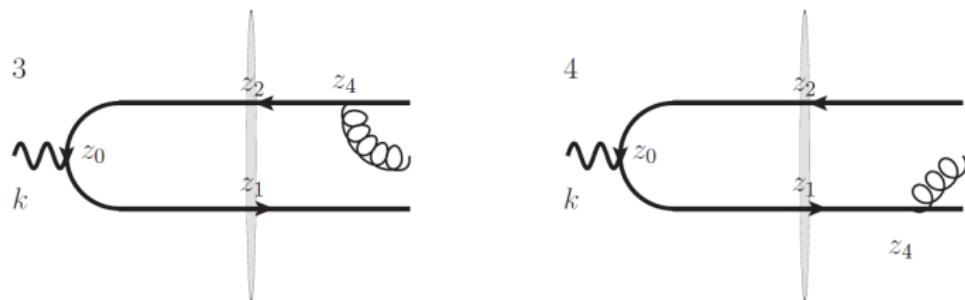
$$Z_{123} = \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2 + x_q x_g \vec{z}_{13}^2 + x_{\bar{q}} x_g \vec{z}_{23}^2}$$

$q\bar{q}g$ production : first kind

Result for a transverse photon

$$F_{1T}(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2, \vec{z}_3) = 2igQ\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\theta(p_g^+ - \sigma) \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2 - i\vec{p}_g \cdot \vec{z}_3}}{\pi Z_{123}\sqrt{2p_g^+}}$$
$$\times \delta_{\lambda_q, -\lambda_{\bar{q}}} K_1(QZ_{123}) \left\{ -\frac{(\vec{z}_{23} \cdot \vec{\varepsilon}_g^*) (\vec{z}_{13} \cdot \vec{\varepsilon}_T)}{\vec{z}_{23}^2} x_q (x_q - \delta_{s\lambda_{\bar{q}}}) (x_{\bar{q}} + x_g \delta_{-s_g \lambda_q}) \right.$$
$$\left. - \frac{(\vec{z}_{23} \cdot \vec{\varepsilon}_g^*) (\vec{z}_{23} \cdot \vec{\varepsilon}_T)}{\vec{z}_{23}^2} x_q x_{\bar{q}} (x_{\bar{q}} + x_g \delta_{-s_g \lambda_q} - \delta_{s\lambda_q}) \right\} - (q \leftrightarrow \bar{q})$$

$q\bar{q}g$ production : second kind



$q\bar{q}$ production : second kind

$q\bar{q}g$ production : second kind

Result for a longitudinal photon

$$\begin{aligned}\tilde{F}_{2L}(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2) &= 4ig Q \theta(p_g^+ - \sigma) \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2}}{\sqrt{2p_g^+}} \\ &\times \delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{x_q (x_g + x_{\bar{q}}) (\delta_{-s_g \lambda_q} x_g + x_{\bar{q}})}{x_{\bar{q}} x_g} \frac{\vec{P}_{\bar{q}} \cdot \vec{\varepsilon}_g^*}{\vec{P}_{\bar{q}}^2} e^{-i\vec{p}_g \cdot \vec{z}_2} K_0(QZ_{122}) - (q \leftrightarrow \bar{q})\end{aligned}$$

$$Z_{122} = \sqrt{x_q (1 - x_q) \vec{z}_{12}^2}$$

$q\bar{q}g$ production : second kind

Result for a transverse photon

$$\begin{aligned}\tilde{F}_{2T}(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2) &= -4g \theta(p_g^+ - \sigma) \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2}}{\sqrt{2p_g^+}} \delta_{\lambda_q, -\lambda_{\bar{q}}} \\ &\times \frac{(\delta_{\lambda_{\bar{q}} s} - x_q) (\delta_{-\epsilon_g \lambda_q} x_g + x_{\bar{q}})}{x_{\bar{q}} x_g} \frac{\vec{P}_{\bar{q}} \cdot \vec{\epsilon}_g^*}{\vec{P}_{\bar{q}}^2} \frac{\vec{z}_{12} \cdot \vec{\epsilon}_T}{\vec{z}_{12}^2} Q Z_{122} K_1(Q Z_{122}) e^{-i\vec{p}_g \cdot \vec{z}_2} - (q \leftrightarrow \bar{q})\end{aligned}$$

$$Z_{122} = \sqrt{x_q (1 - x_q) \vec{z}_{12}^2}$$

Linear approximation

Back to the general expression for the matrix element M^α :

$$\begin{aligned} M^\alpha &= \frac{N_c^2}{2} \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha (\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} + \mathbf{U}_{13}\mathbf{U}_{32}) \\ &+ \int d\vec{z}_1 d\vec{z}_2 F_2(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2)^\alpha (N_c^2 - 1) \mathbf{U}_{12} \end{aligned}$$

For 2 or 3 gluon exchange, one can linearize this expression by neglecting the $\mathbf{U}_{13}\mathbf{U}_{32}$ term.

Linear approximation

After linearization one gets :

$$M^\alpha \stackrel{g^3}{=} \frac{1}{2} \int d\vec{z}_1 d\vec{z}_2 \mathbf{U}_{12} \left\{ \tilde{F}_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2)^\alpha + (N_c^2 - 1) \tilde{F}_2(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2)^\alpha \right\}$$

$$\begin{aligned} \tilde{F}_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2)^\alpha &= \int d\vec{z}_3 [N_c^2 F_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_3, \vec{z}_2)^\alpha \\ &+ N_c^2 F_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_3, \vec{z}_2, \vec{z}_1)^\alpha - F_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha] \end{aligned}$$

⇒ One has to integrate the previously derived expressions

Linear approximation

No analytical expression for most of the integrals. BUT :

- For null transverse momenta \vec{p}_q , $\vec{p}_{\bar{q}}$ and \vec{p}_g they can be performed
- In any case, they can be reduced to convergent integrals over a real parameter in $[0,1]$ so a numerical calculation can be done.

For example :

$$\begin{aligned} & \int d\vec{z}_3 e^{-i\vec{p}_g \cdot \vec{z}_3} \frac{\vec{z}_{32}}{\vec{z}_{32}^2} K_0(QZ_{123}) \\ &= -\frac{\pi e^{-i\vec{p}_g \cdot \vec{z}_2}}{(1-x_g)x_g} \int_0^1 d\alpha e^{\alpha \frac{i x_q (\vec{z}_{21} \cdot \vec{p}_g)}{x_{\bar{q}} + x_q}} \left(\frac{i \vec{p}_g Z_{q\bar{q}g}}{Q_g(\alpha)} K_1(Q_g(\alpha) Z_{q\bar{q}g}) + x_g x_q \vec{z}_{21} K_0(Q_g(\alpha) Z_{q\bar{q}g}) \right) \\ & \int d\vec{z}_3 \frac{\vec{z}_{32}}{\vec{z}_{32}^2} K_0(QZ_{123}) = -\frac{2\pi}{x_g x_q Q} \frac{\vec{z}_{21}}{\vec{z}_{21}^2} (Z_{q\bar{q}} K_1(QZ_{q\bar{q}}) - Z_{122} K_1(QZ_{122})) \end{aligned}$$

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Further analysis from our results

- A lot of phenomenology can be done from HERA data :
 - From $q\bar{q}$ production to jet production
 - Is our approach better than the 2-gluon approximation for H1 and ZEUS data?
- The same calculation can be done again for massive quarks
- The calculation of virtual correction for the cross section is still to be performed (Work in progress)

Backup slides : Results in momentum space

Results in momentum space

First kind, longitudinal photon

$$F_{1L}(p_q, p_{\bar{q}}, p_g, z_0, \vec{p}_1, \vec{p}_2, \vec{p}_3) = \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} + \vec{p}_{3g}) \theta(p_g^+ - \sigma) \\ \times \frac{\delta_{\lambda_q, -\lambda_{\bar{q}}}}{\sqrt{2p_g^+}} \frac{4iQ g (x_q + x_g \delta_{-s_g \lambda_{\bar{q}}}) ((\vec{p}_{2\bar{q}} \cdot \vec{\varepsilon}_g^*) x_q + (\vec{p}_{1q} \cdot \vec{\varepsilon}_g^*) (1 - x_{\bar{q}}))}{(1 - x_{\bar{q}}) x_g x_q \left(Q^2 + \frac{\vec{p}_{2\bar{q}}^2}{x_{\bar{q}}(1-x_{\bar{q}})}\right) \left(Q^2 + \frac{\vec{p}_{1q}^2}{x_q} + \frac{\vec{p}_{2\bar{q}}^2}{x_{\bar{q}}} + \frac{\vec{p}_{3g}^2}{x_g}\right)} - (q \leftrightarrow \bar{q}),$$

Results in momentum space

Second kind, longitudinal photon

$$\begin{aligned}\tilde{F}_{2L}(p_q, p_{\bar{q}}, p_g, k, \vec{p}_1, \vec{p}_2) &= \theta(p_g^+ - \sigma)\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g) \\ &\times \frac{4igQ}{\sqrt{2p_g^+}}\delta_{\lambda_q, -\lambda_{\bar{q}}}\frac{x_{\bar{q}} + \delta_{-s_g, \lambda_q}x_g}{x_{\bar{q}}x_g}\frac{2\pi}{Q^2 + \frac{\vec{p}_{1q}^2}{x_q(1-x_q)}}\frac{\vec{P}_{\bar{q}} \cdot \vec{\varepsilon}_g^*}{\vec{P}_{\bar{q}}^2} - (q \leftrightarrow \bar{q}).\end{aligned}$$

Results in momentum space

First kind, transverse photon

$$F_{1T} (p_q, p_{\bar{q}}, p_g, z_0, \vec{p}_1, \vec{p}_2, \vec{p}_3) = \frac{2ig}{\sqrt{2p_g^+}} \frac{\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} + \vec{p}_{3g}) \theta(p_g^+ - \sigma) \delta - \lambda_{\bar{q}} \lambda_q}{Q^2 (1 - x_q) \left(\frac{\vec{p}_{1q}^2}{x_q} + \frac{\vec{p}_{2\bar{q}}^2}{x_{\bar{q}}} + \frac{\vec{p}_{3g}^2}{x_g} + Q^2 \right)} \left\{ \delta_{ssg} \delta_{s\lambda_q} \right. \\ \left. + 2(\vec{p}_{1q} \cdot \vec{\varepsilon}_T)((\vec{p}_{2\bar{q}} \cdot \vec{\varepsilon}_g^*) (x_{\bar{q}} + x_g) + (\vec{p}_{1q} \cdot \vec{\varepsilon}_g^*) x_{\bar{q}}) \frac{(x_q - \delta_s \lambda_{\bar{q}})(x_g \delta - s_g \lambda_q + x_{\bar{q}})}{(1 - x_q) x_q x_{\bar{q}} x_g \left(Q^2 + \frac{\vec{p}_{1q}^2}{(1 - x_q) x_q} \right)} \right\} - (q \leftrightarrow \bar{q}).$$

Results in momentum space

Second kind, transverse photon

$$\tilde{F}_{2T}(p_q, p_{\bar{q}}, p_g, k, \vec{p}_1, \vec{p}_2) = -\theta(p_g^+ - \sigma) \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g) \frac{\delta_{\lambda_q, -\lambda_{\bar{q}}}}{\sqrt{2p_g^+}} \\ \times 4g \frac{(\delta_{\lambda_{\bar{q}} s} - x_q) (\delta_{-s_g \lambda_q} x_g + x_{\bar{q}})}{x_{\bar{q}} x_g} \frac{2\pi i (\vec{p}_{1q} \cdot \vec{\varepsilon}_T)}{x_q (1 - x_q) Q^2 + \vec{p}_{1q}^2} \frac{\vec{P}_{\bar{q}} \cdot \vec{\varepsilon}_g^*}{\vec{P}_{\bar{q}}^2} - (q \leftrightarrow \bar{q})$$