

# The photon PDF of the proton

Alan Martin & Misha Ryskin

arXiv:1406.2118

Present precision of PDFs (NNLO in  $\alpha_s$ ) implies that we should study the QED corrections (LO in  $\alpha$ ).

We show how the photon input PDF can be calculated with good accuracy, and used in DGLAP global analyses in which the photon is treated as an additional point-like parton.

Diffraction 2014, Primosten, Croatia, Sept.10-16

## Evolution equations to LO in $\alpha$ and $\alpha_s$

$$\frac{\partial q_i(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} (P_{qq} \otimes q_i + P_{qg} \otimes g) + \\ + \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} \left( \sum_i \frac{e_i^2}{C_F} P_{qq} \otimes q_i + \sum_i e_i^2 P_{q\gamma} \otimes \gamma \right)$$

$$\frac{\partial g(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left( \sum_i P_{gq} \otimes q_i + P_{gg} \otimes g \right)$$

$$\frac{\partial \gamma(x, Q^2)}{\partial \log Q^2} = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} \left( P_{\gamma\gamma} \otimes \gamma + \sum_i e_i^2 P_{\gamma q} \otimes q_i \right)$$

but input for  $\gamma(x, Q_0^2)$  ?

## Existing determinations of $\gamma(x, Q_0^2)$

**MRST(2004)** – input given by emission from val. quark

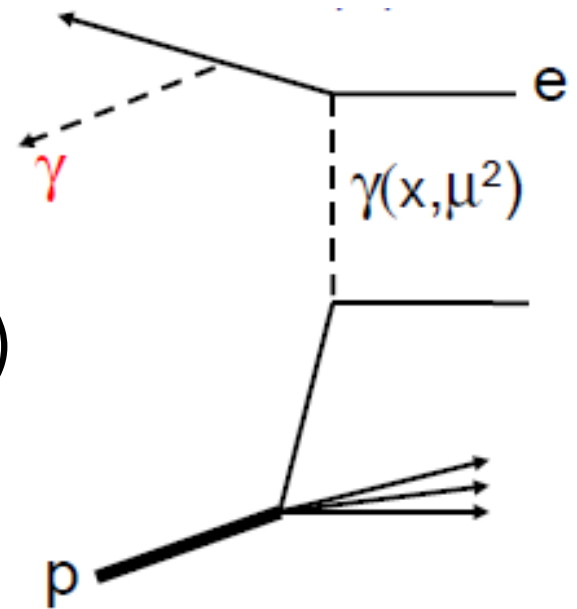
$$\gamma^p(x, Q_0^2) = \frac{\alpha}{2\pi} \int \frac{dz}{z} \left[ \frac{4}{9} \log \left( \frac{Q_0^2}{m_u^2} \right) u_0 \left( \frac{x}{z} \right) + \frac{1}{9} \log \left( \frac{Q_0^2}{m_d^2} \right) d_0 \left( \frac{x}{z} \right) \right] \frac{1 + (1-z)^2}{z}$$

using current quark masses.

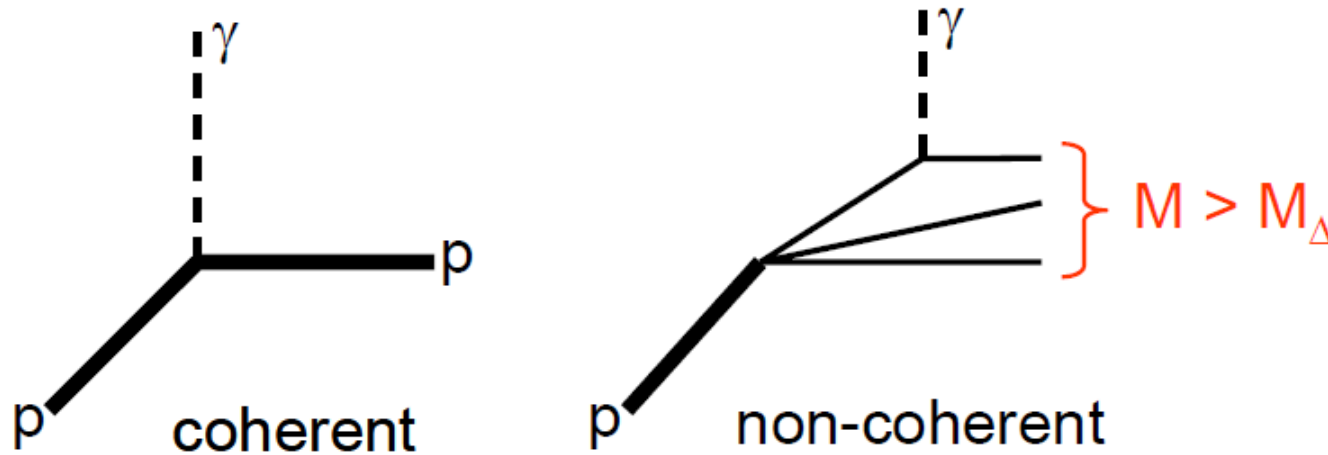
Compare with data for electroprod<sup>n</sup>  
of isolated photon:  $ep \rightarrow e\gamma X$

**NNPDF(2013)** freely parametrise  $\gamma(x, Q_0^2)$   
and attempt to determine it from LHC  
Drell-Yan data  $\rightarrow$  large uncertainties.

**CTEQ(prelim)** use MRST form, but  
with arbitrary normalisation factor,  $p_0(\gamma)$ ,  
fitted to  $ep \rightarrow e\gamma X$  data. Again large uncertainties.



**Here** we emphasize that the major part of  $\gamma(x, Q_0^2)$  comes from coherent  $\gamma$  emission from proton which stays intact  
 (Previous analyses based on incoherent emission from quark)



$$\gamma(x, Q_0^2) = \gamma_{\text{coh}} + \gamma_{\text{incoh}}$$

major part---  
 well known to  
 good accuracy

relatively small --- QED excit<sup>ns</sup> of p –  
 here we estimate it from quark viewpt.  
 $\gamma_{\text{incoh}}$  gives main uncertainty of  $\gamma(x, Q^2)$

$$\gamma_{\text{coh}}(x, Q_0^2) = \frac{\alpha}{2\pi} \frac{[1 + (1-x)^2]}{x} \int_0^{|t| < Q_0^2} dq_t^2 \frac{q_t^2}{(q_t^2 + x^2 m_p^2)^2} F_1^2(t)$$

$$\gamma_{\text{incoh}}(x, Q_0^2) = \frac{\alpha}{2\pi} \int_{|t_{\min}|}^{Q_0^2} \frac{dt}{t - m_q^2} (1 - F_1^2(t)) \int_x^1 \frac{dz}{z} \times$$

$$\times \left[ \frac{4}{9} u_0\left(\frac{x}{z}\right) + \frac{1}{9} d_0\left(\frac{x}{z}\right) \right] \frac{1 + (1-z)^2}{z}$$

upper limit of  $\gamma_{\text{incoh}}$ : use current q masses; for  $Q < Q_0$   
freeze  $u_0$ ,  $d_0$  at their values at  $Q_0$ .

lower limit of  $\gamma_{\text{incoh}}$ : use constituent q masses; with  
non-rel  $u_0 = 2\delta(x-1/3)$ ,  $d_0 = \delta(x-1/3)$

also take a, physics-motivated, linear interpol<sup>n</sup> of 2 limits

$Q^2=200 \text{ GeV}^2$

$Q^2=1 \text{ GeV}^2$

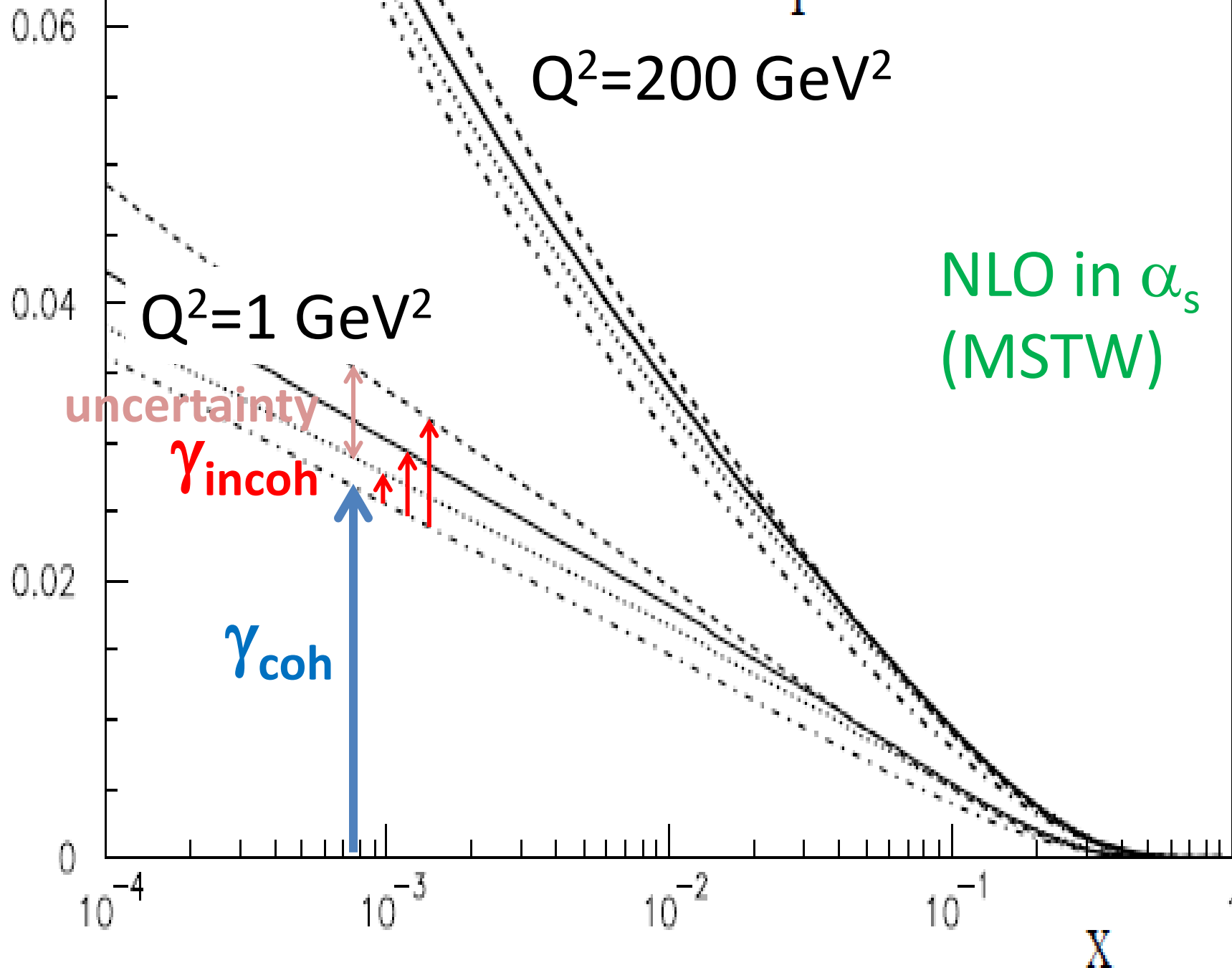
NLO in  $\alpha_s$   
(MSTW)

uncertainty

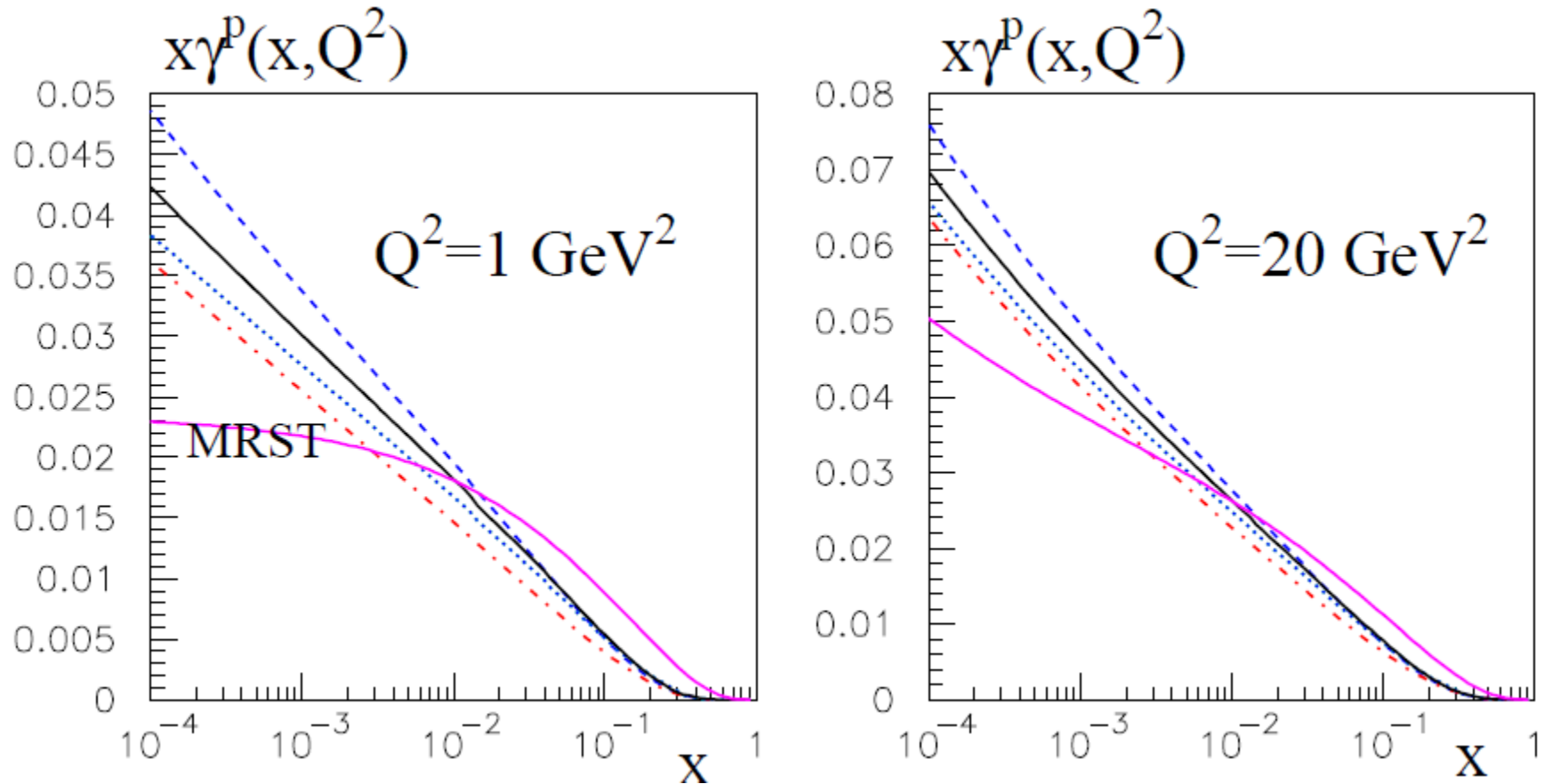
$\gamma_{\text{incoh}}$

$\gamma_{\text{coh}}$

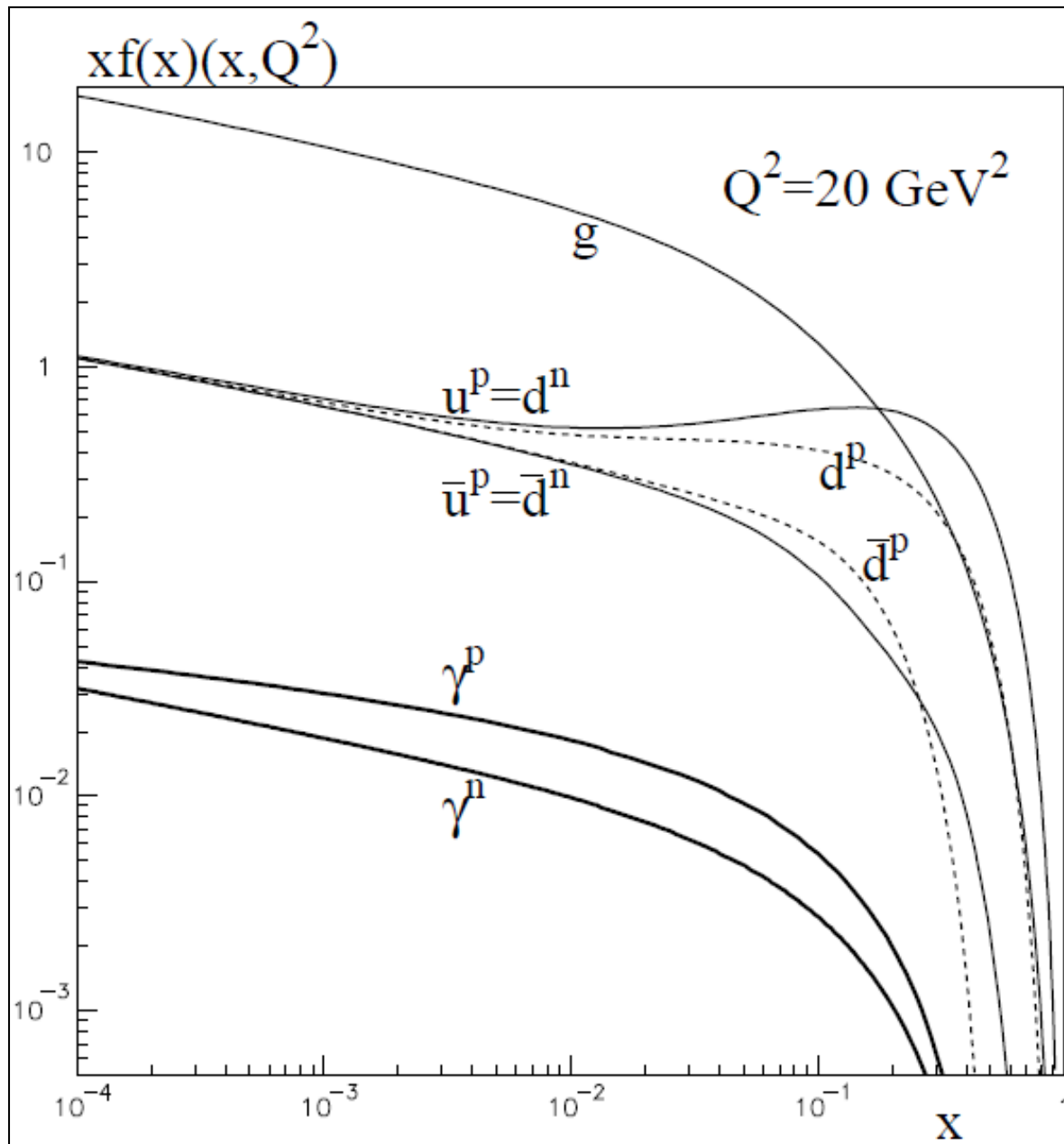
$X$



## Comparison with MRST(2004)



Note MRST is based solely on  $\gamma_{\text{incoh}}$ , whereas here it comes mainly from  $\gamma_{\text{coh}}$ . MRST should have been suppressed by  $t_{\text{min}}$  and by  $1-F_1^2$  --- so, at large  $x$ , where  $|t_{\text{min}}|$  is large, MRST exceeds the present input.



Comparison of  $\gamma^p, \gamma^n$  PDFs with the other PDFs.

There will be a small breaking of isospin symmetry,  $u^p \neq d^n$ , coming from  $P_{q\gamma}$  in the evolution eq. for quarks.



## Evolution equations to LO in $\alpha$ and $\alpha_s$

$$\frac{\partial q_i(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} (P_{qq} \otimes q_i + P_{qg} \otimes g) +$$

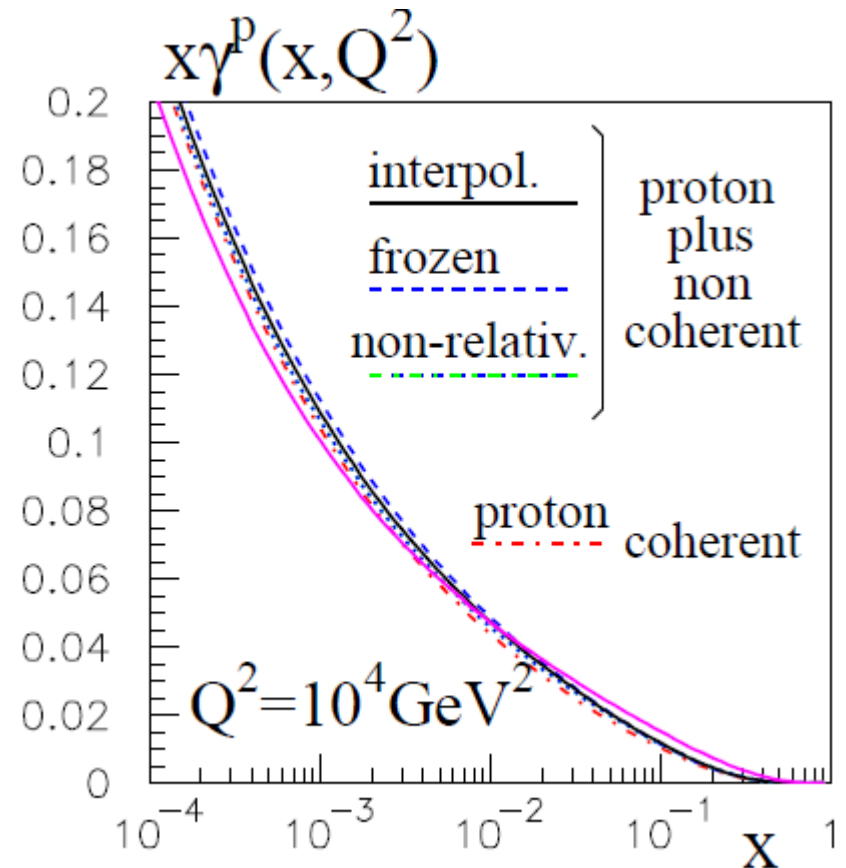
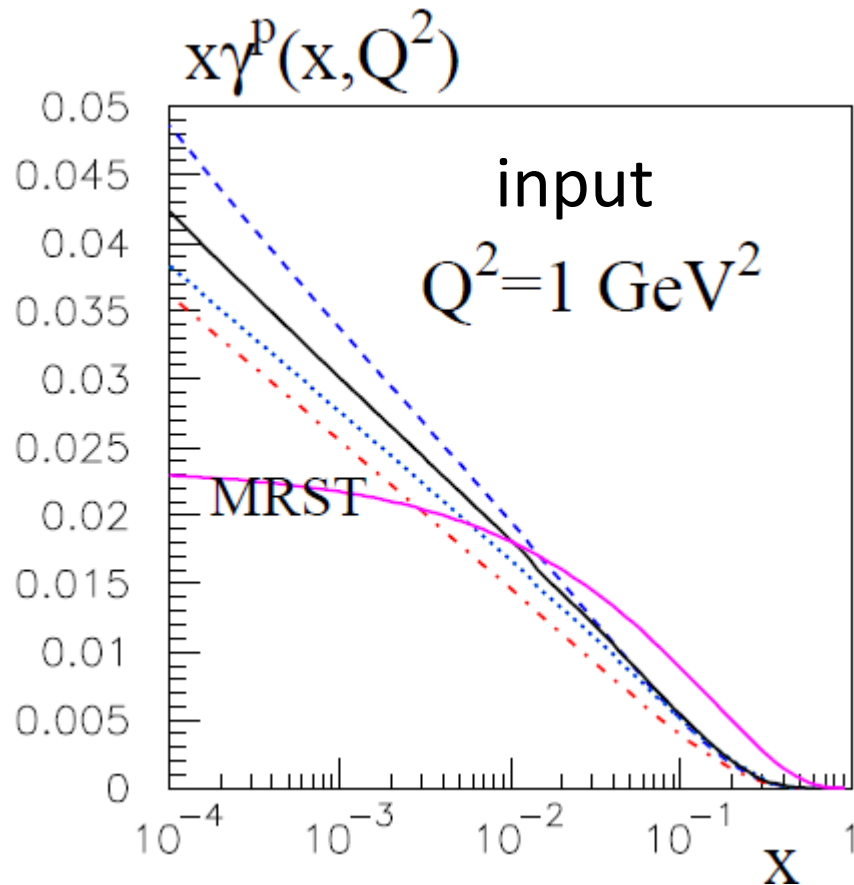
$$+ \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} \left( \sum_i \frac{e_i^2}{C_F} P_{qq} \otimes q_i + \sum_i e_i^2 P_{q\gamma} \otimes \gamma \right)$$

$$\frac{\partial g(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left( \sum_i P_{gq} \otimes q_i + P_{gg} \otimes g \right)$$

$$\frac{\partial \gamma(x, Q^2)}{\partial \log Q^2} = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} \left( P_{\gamma\gamma} \otimes \gamma + \sum_i e_i^2 P_{\gamma q} \otimes q_i \right)$$

At large  $Q^2$  the behaviour of  $\gamma^p(x, Q^2)$  becomes stable and relatively insensitive to the form taken for the input.

The evolution  $d\gamma^p/d\log Q^2$  is driven by  $P_{\gamma q} \otimes q$  and the quark PDFs are well determined



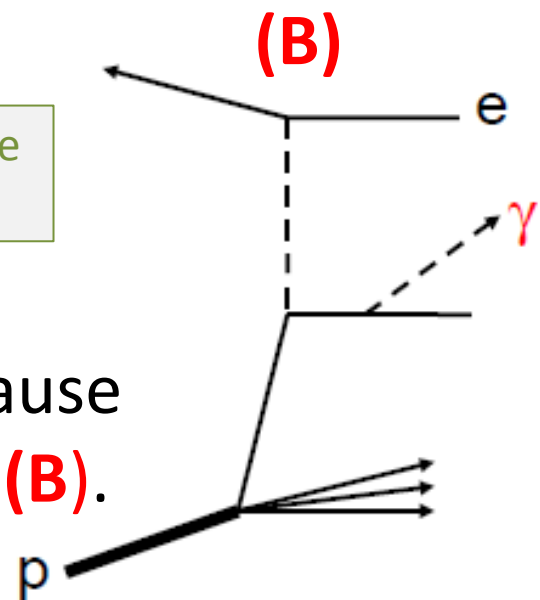
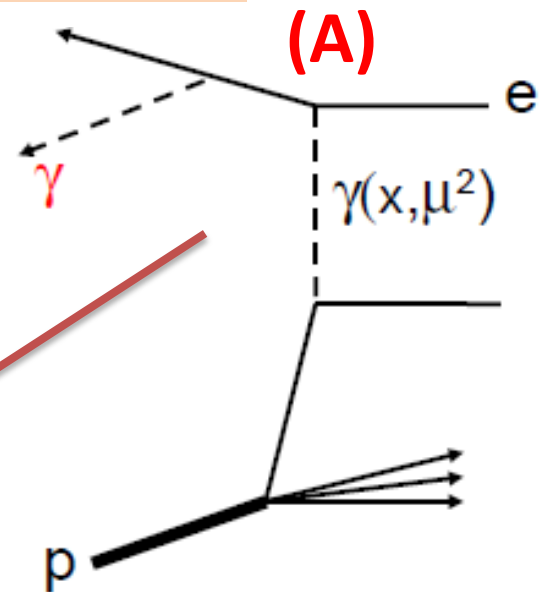
Expt<sup>al</sup> probe of photon PDF:  $ep \rightarrow e\gamma X$  data

Photons observed in direction of incoming  $e$  are mediated by  $e\gamma \rightarrow e\gamma$ , **diag.(A)** --- so data at high negative rapidity,  $\eta^\gamma$ , at angles close to  $e$  beam, are driven by photon PDF.

$\eta^\gamma$ range	$d\sigma/d\eta^\gamma$ (pb)	
	ZEUS	(A)
$-0.7 - -0.3$	$17.4 \pm 1.1$	16.4
$-0.3 - 0.1$	$13.0 \pm 0.9$	7.7
$0.1 - 0.5$	$10.7 \pm 1.0$	2.7
$0.5 - 0.9$	$8.7 \pm 1.3$	0.8

+/-20% scale uncertainty

The discrepancy as  $\eta^\gamma$  increases is because then emission is mainly from  $q$ 's, **diag (B)**.



**Conclusions** We determine the input  $\gamma$  PDF. The uncertainty is relatively small, since the major part,  $\gamma_{\text{coh}}^p$ , of the distrib<sup>n</sup> (which is produced by the coherent emission of the photon from a proton that remains intact) is well known.

Compare with other determinations

Group	input photon PDF	data
MRST 2004	model for $\gamma_{\text{incoh}}^p$	predict $ep \rightarrow e\gamma X$
NNPDF 2013	freely parametrised	fit to LHC D-Yan
CTEQ prelim.	parametrise with $p_0(\gamma)$	fit to $ep \rightarrow e\gamma X$
this work	calc. $\gamma_{\text{coh}}^p$ (dominates) + model for $\gamma_{\text{incoh}}^p$	predict $ep \rightarrow e\gamma X$