Unified BFKL & DGLAP evol. in terms of θ

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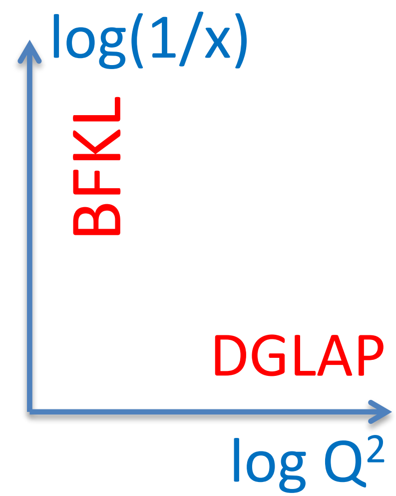
We present an evolution equation which simultaneously sums the leading BFKL and DGLAP logs for an integrated gluon distribution $g(x,\theta)$ in terms of a single variable ---- the emission angle of the gluon $\theta = k_t/xp$

Recall BFKL evolution is written in terms of a gluon distrib^n $f(x, k_t)$ unintegrated over its transverse momentum

Diffraction 2014, Primosten, Croatia, Sept.10-16

BFKL for unintegrated gluon $f(x, k_t)$

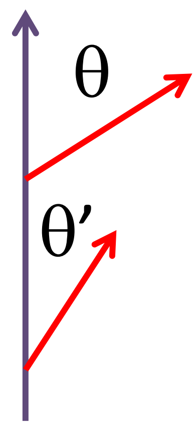
$$f(x, k_t) = f_0 + \frac{\alpha_s}{2\pi} \int d^2k_t' \frac{dx'}{x'} \mathcal{K}(k_t, k_t') f(x', k_t')$$



DGLAP for integrated gluon $g(x, Q^2)$

$$\frac{\partial g(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \sum_a \int dz\, P_{ga}(z)\, a\left(\frac{x}{z}, Q^2\right)$$

$$\boxed{xg(x, Q^2) = \int^{Q^2} \frac{dk_t^2}{k_t^2}\, f(x, k_t)}$$



Angular ordering of successive emissions due to coherence

$\theta = k_t / xp$ accounts for both DGLAP $\log k_t^2$ and BFKL $\log 1/x$

Possible unified evolution eq. for $\dfrac{\partial g(x, \theta)}{\partial \ln \theta}$??

unified eq. for unintegrated gluon   Kwiecinski, M, Stasto

$$f(x,k_t) = f_0(x,k_t) + \frac{\alpha_s}{2\pi}\left(\int d^2k_t' \frac{dx'}{x'}\mathcal{K}(k_t,k_t')f(x',k_t') + \int \frac{dk_t'^2}{k_t'^2} dz P(z) f\left(\frac{x}{z},k_t'\right) - DL\right)$$

BFKL   DGLAP

Need to subtract Double Log contrib^{n} $\int(dx'/x')(dk_t'^2/k_t'^2)$ hidden in both DGLAP and BFKL terms, so as to avoid double counting,

$$\overline{\mathcal{K}}(k_t,k_t')f(x',k_t') = \underbrace{2N_c\frac{k_t^2}{k_t'^2}\left[\frac{f(x',k_t')-f(x',k_t)}{|k_t'^2-k_t^2|} + \frac{f(x',k_t)}{\sqrt{4k_t'^4+k_t^4}}\right.}_{\mathcal{K}f}\left. - \frac{f(x',k_t')}{k_t^2}\right]$$

$$\boxed{\overline{\mathcal{K}} = \mathcal{K} - \frac{2N_c}{k_t'^2}}$$

unified eq. for unintegrated gluon Kwiecinski, M, Stasto

$$f(x,k_t) = f_0(x,k_t) +$$

$$\frac{\alpha_s}{2\pi}\left(\int d^2k_t'\frac{dx'}{x'}\mathcal{K}(k_t,k_t')f(x',k_t') + \int\frac{dk_t'^2}{k_t'^2}dzP(z)f\left(\frac{x}{z},k_t'\right) - DL\right)$$

BFKL DGLAP

$$\overline{\mathcal{K}} = \mathcal{K} - \frac{2N_c}{k_t'^2}$$

Double Log term needed to avoid double counting

kin. constraint

$$\overline{\mathcal{K}}(k_t,k_t')f(x',k_t') = 2N_c\frac{k_t^2}{k_t'^2}\left[\frac{\Theta(k_t^2/z-k_t'^2)f(x',k_t') - f(x',k_t)}{|k_t'^2-k_t^2|}\right.$$

$$\left. + \frac{f(x',k_t)}{\sqrt{4k_t'^4+k_t^4}} - \frac{\Theta(k_t^2-k_t'^2)f(x',k_t')}{k_t^2}\right].$$

DGLAP ordering

$$\overbrace{k_t \to \theta = k_t/xp, \qquad k'_t \to \theta' = k'_t/x'p}$$

$$f(x,k_t) = f_0(x,k_t) + \frac{\alpha_s}{2\pi}\left(\int d^2k'_t \frac{dx'}{x'}\overline{\mathcal{K}}(k_t,k'_t) f(x',k'_t) + \int \frac{dk_t'^2}{k_t'^2} dz P(z) f\left(\frac{x}{z},k'_t\right)\right)$$

$d\ln(1/x') \to d\ln(\theta')$

after DL subt^n no logk_t dep.

$\int (dk_t'^2/k_t'^2) \to 2\int (d\theta'/\theta')$

$$f(x,\theta) = f_0(x,\theta_0) + \frac{\alpha_s}{2\pi}\int_{\theta_0}^{\theta}\left(\int d^2k'_t\, \overline{\mathcal{K}}(k_t,k'_t) \boxed{f(x',k'_t)} + 2\int_{z_{\min}}^{1} dz P(z) f\left(\frac{x}{z},\theta'\right)\right)\frac{d\theta'}{\theta'}$$

$$\frac{\partial(xg(x,\theta))}{\partial \ln\theta^2} = f(x,\theta) = f_0(x,\theta_0) + \frac{\alpha_s}{2\pi}\int_{\theta_0}^{\theta}\int d^2k'_t\, \overline{\mathcal{K}}(k_t,k'_t) \boxed{\frac{\partial[x'g(x',\theta')]}{\partial \ln\theta'^2}}\frac{d\theta'}{\theta'} + \int_x^1 dz P(z)\frac{x}{z} g\left(\frac{x}{z},z\theta\right)$$

see next slide

conventional
integrated PDF

unintegrated PDF

$$\boxed{xg(x,Q^2) = \int^{Q^2} \frac{dk_t^2}{k_t^2}\, f(x,k_t)}$$

express in terms of θ

$$k_t \to \theta = k_t/xp$$

$$xg(x,\theta) = \int^{\theta^2} f(x,\theta')\frac{d\theta'^2}{\theta'^2}$$

or

$$\frac{\partial(xg(x,\theta))}{\partial \ln\theta^2} = f(x,\theta)$$

$$\frac{\partial(xg(x,\theta))}{\partial \ln\theta^2} = f(x,\theta) = f_0(x,\theta_0) +$$

$$+ \frac{\alpha_s}{2\pi}\int_{\theta_0}^{\theta}\int d^2k'_t\,\overline{\mathcal{K}}(k_t,k'_t)\frac{\partial[x'g(x',\theta')]}{\partial \ln\theta'^2}\frac{d\theta'}{\theta'} + \int_x^1 dz P(z)\frac{x}{z}g\left(\frac{x}{z}, z\theta\right)$$

The value of the derivative over $\ln\theta'^2$ is calculated using the gluon PDF at angles θ' not equal to θ. This is a common property of BFKL (recall r.h.s. integrates over k'_t)

Not a problem --- g only enters at values $\theta' < \theta$ where the derivative is already known from previous evolution.

Check that $\theta' < \theta$: in the DGLAP term $\theta' = z\theta < \theta$

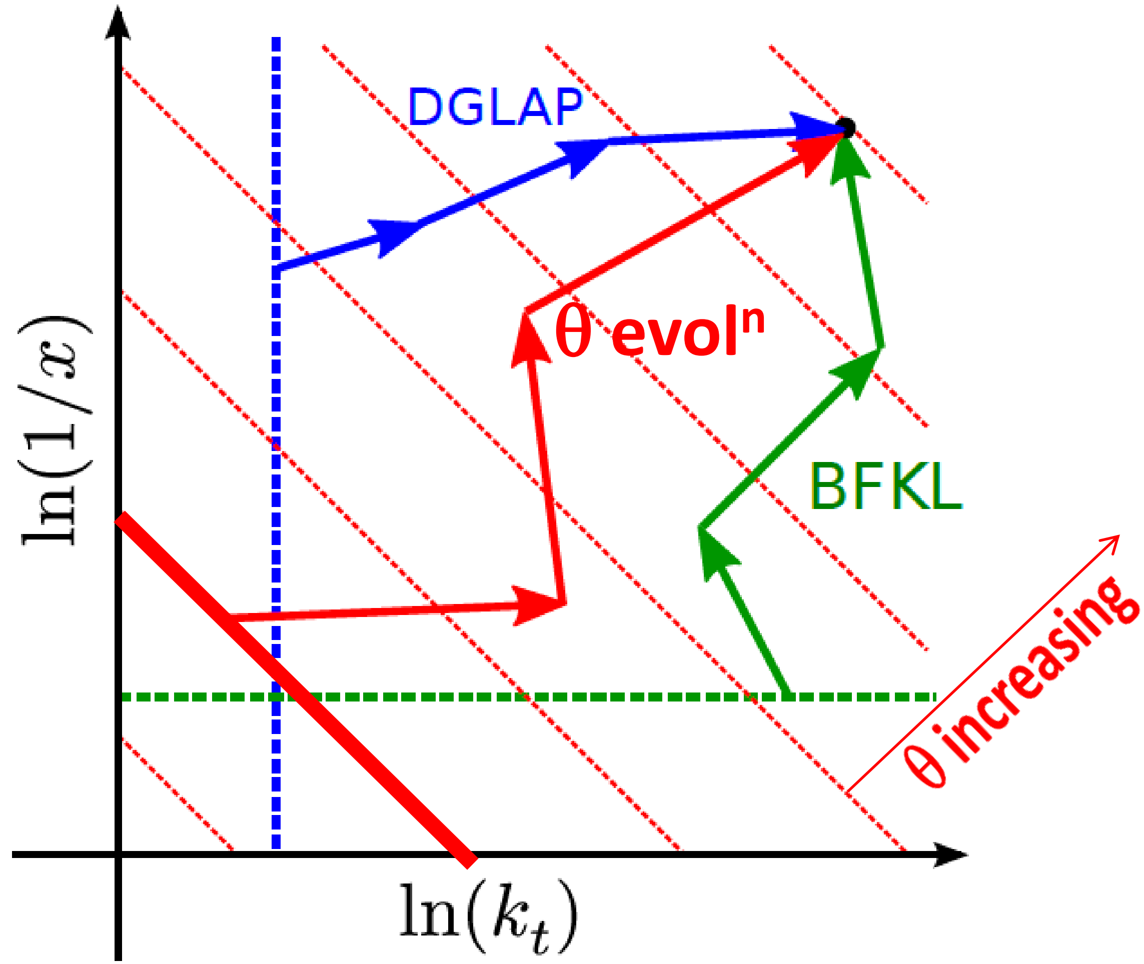
in the BFKL term we have kin. constraint

$$k_t'^2 < k_t^2/z \longrightarrow \theta' = \frac{zk'_t}{xp} < \sqrt{z}\frac{k_t}{xp} < \sqrt{z}\theta$$

Advantage of using θ

$$\frac{\partial(xg(x,\theta))}{\partial \ln\theta^2} = f_0(x,\theta_0) + \frac{\alpha_s}{2\pi}\int_{\theta_0}^{\theta}\int d^2k'_t\,\overline{\mathcal{K}}(k_t,k'_t)\frac{\partial[x'g(x',\theta')]}{\partial \ln\theta'^2}\frac{d\theta'}{\theta'} + \int_x^1 dz P(z)\frac{x}{z}g\left(\frac{x}{z}, z\theta\right)$$

After the double-log subtr^n, k'_t integral converges as $k'_t \to 0$. So, use simple form for $k'_t < k_0$



Evol^n from θ_0 (domain of rel. large x and low k_t)
-- natural input large-dist. confinement (~0.5fm)

Energy-momentum conservation

Unlike the DGLAP part, the leading log BFKL part gives additional energy to new partons, which leads to a small violation of energy-momentum conservation. Can add NLL contribution which restores conservation. Contribution extends into $\theta' > \theta$ region, so need one or two iterations to ensure conservation

see also 1406.2910

Straightforward to include the quark contribution in evolution

Straightforward to extend to NLO, but not so simple

Conclusions

There is an evolution eq., which sums both the leading BFKL and DGLAP logs, for an **integrated** distribution $g(x,\theta)$, where θ = angle of emitted gluon. Coherence → $\theta' < \theta$

It is not a problem that r.h.s. contains $g(x',\theta')$, since for $\theta' < \theta$, we know $g(x',\theta')$ from previous evolution step. [Unlike conventional BFKL for unintegrated $f(x,k_t)$, where there is diffusion into $k'_t > k_t$ (as well as $k'_t < k_t$)].

$\theta' < \theta$ is provided by the kin. constraint $k'^2_t > k_t/z$, which also gives for a major part of higher-order BFKL contrib^n.

Unified BFKL/DGLAP evolution in terms of a **single** variable, θ, should be convenient for Monte Carlo simulations.