

Large b behaviour and CGC/saturation approach: the BFKL equation with massive gluon.

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Papers: E.L., L. Lipatov and M. Siddikov:

- “*BFKL Pomeron with massive gluons*,” Phys. Rev. D **89** (2014) 074002 [arXiv:1401.4671 [hep-ph]];
- “*Semi-classical solution to the BFKL equation with massive gluons*” (in preparation);

The main questions to answer:

1. How the correct behaviour at large b will influence the high energy behaviour of the scattering amplitude in the BFKL dynamics.
2. What is the high energy asymptotic behaviour of the BFKL scattering amplitude that provides the high energy amplitude for the electro-weak theory which can be measured experimentally

Large b dependence of the BFKL Pomeron

- $$N(r_1, r_2; Y, b) = \int \frac{d\gamma}{2\pi i} \phi_{in}^{(0)}(\nu) e^{\omega(\gamma=\frac{1}{2}) + i\nu, 0} Y$$

$$\times \left\{ b_\nu (ww^*)^{\frac{1}{2}} + i\nu + b_{-\nu} (ww^*)^{\frac{1}{2}} - i\nu \right\} \xrightarrow{\nu \ll 1} \frac{r_1 r_2}{b^2} e^{\omega_0 Y}$$

- $$w w^* = \frac{r_1^2 r_2^2}{\left(\vec{b} - \frac{1}{2} (\vec{r}_1 - \vec{r}_2) \right)^2 \left(\vec{b} + \frac{1}{2} (\vec{r}_1 - \vec{r}_2) \right)^2}$$

$$N(r_1, r_2; Y, b) \leq 1 \quad \text{for} \quad b^2 \leq r_1 r_2 e^{\omega_0 Y}$$

Violation of Froissart theorem: (Kovner & Wiedemann)

$$\int d^2b N(r_1, r_2; Y, b) \propto s^{\omega_0} \gg Y^2 = \ln^2 s$$

Lessons from numerical solutions and theory considerations:

((Kovner & Wiedemann, McLerran and Iancu, Golec-Biernat & Stasto, Gotsman et al, Berger & Stasto, 2011)

- The confinement of quarks and gluon have to be included in the BFKL kernel (to include in the initial conditions is not enough);
- Suppressing large sizes of the produced dipoles in the decay *one dipole* → *two dipoles* we reproduce correct *b*-dependence;
- Since at large *b* the amplitude is small we do not need to take into account the non-linear corrections;

My talk at Low x WS'2013: modified BFKL kernel

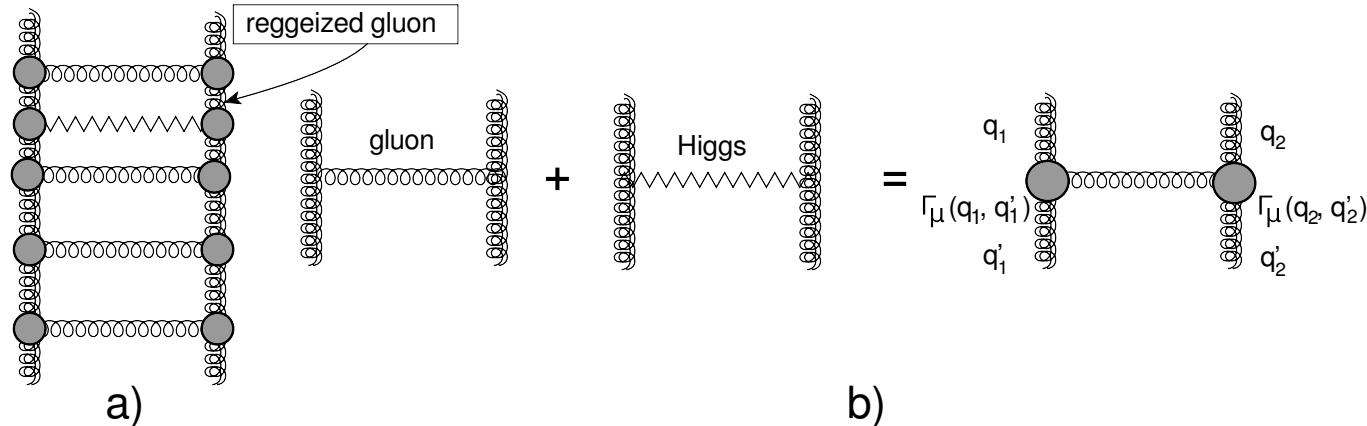
$$K(x_{12}, x_{20}|x_{10}) = \frac{x_{10}^2}{x_{12}^2 x_{02}^2} e^{-B(x_{12}^2 + x_{02}^2)}$$

Gauge theory with massive gluon due to the Higgs mechanism

- It is not QCD but has the same colour structure;
- It has physics realization: electro-weak theory with zero Weinberg angle;
- It has the same problem with unitarity at high energies as QCD;
- It has correct exponential fall down at large b ;
- It is perfect training ground for answering the main question: how correct b behaviour will influens the high energy asymptotic behaviour;

Massive BFKL equation

(Fadin, Kuraev & Lipatov (1975 - 1977)



$$E = \omega / \bar{\alpha}_S$$

$$\kappa = p^2 / m^2$$

$$\begin{aligned}
 E\phi_E(\kappa) &= \underbrace{\frac{\kappa + 1}{\sqrt{\kappa}\sqrt{\kappa + 4}} \ln \frac{\sqrt{\kappa + 4} + \sqrt{\kappa}}{\sqrt{\kappa + 4} - \sqrt{\kappa}}}_{\text{kinetic energy (T)}} \phi_E(\kappa) \\
 &- \underbrace{\int_0^\infty \frac{d\kappa' \phi_E(\kappa')}{\sqrt{(\kappa - \kappa')^2 + 2(\kappa + \kappa') + 1}}}_{\text{potential energy (U)}} + \frac{N_c^2 + 1}{2N_c^2} \frac{1}{\kappa + 1} \int_0^\infty \frac{\phi_E(\kappa') d\kappa'}{\kappa' + 1}
 \end{aligned}$$

Qualitative features of the solution

- $E f_E(r) = \mathcal{H} f_E(r)$

$$\mathcal{H} = T(-\nabla^2) - 2K_0(|r|m) + \frac{N_c^2 + 1}{2N_c^2} K_0(|r|m) \int \frac{d^2 p'}{\pi} \frac{\phi_E(p')}{p'^2 + m^2}$$

$r m \ll 1$

$$E_{\text{BFKL}}(\gamma) = -2\psi(1) + \psi(\gamma) + \psi(1-\gamma)$$

$$f_{E=E^{\text{BFKL}}(\gamma)}(r) \xrightarrow{r \ll 1/m} f_{E=E(\gamma)}(r) \sim (r^2)^{\gamma-1}$$

$r m \gg 1$

$$E f_E(r) = T(-\nabla^2) f_E(r)$$

$$f_{E=E^{\text{BFKL}}(\gamma)}(r) \xrightarrow{r \gg 1/m} e^{-a r} \text{ with } T(-a^2) = E^{\text{BFKL}}(\gamma)$$

momentum rep.

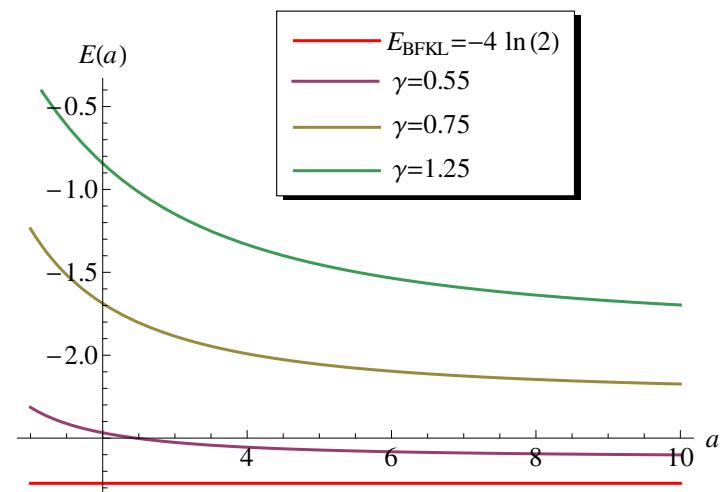
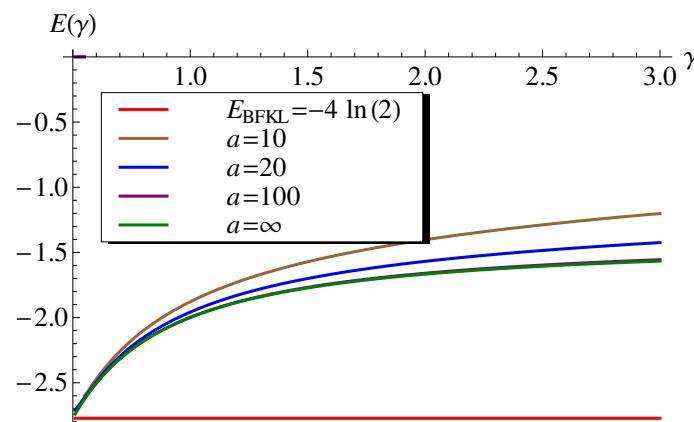
$$\phi_E(\kappa) = (\kappa^2 + a^2)^{-\gamma}$$

$$\phi_E(\kappa) \xrightarrow{\kappa \gg 1} \kappa^{-\gamma}; \quad \phi_E(\kappa) \xrightarrow{\kappa \ll 1} \text{Const}$$

Result # 1 : $E(\gamma; \text{BFKL } m \neq 0) = E(\gamma; \text{BFKL } m = 0)$

1. Variational method

- $E_{\text{ground}} \equiv -\omega_0 \leq F[\{\phi\}] = \frac{\langle \phi^*(p) | \mathcal{H} | \phi(p) \rangle}{\langle \phi^*(p) | \phi(p) \rangle}$
- Our choice: $\{\phi\} = (p^2 + m^2)^{-\gamma}$



Can be only states with $E < E^{\text{BFKL}} = -4 \ln 2$

2. The proof: $E \geq E^{\text{BFKL}} = -4 \ln 2$

The best trial function $\phi_{\text{trial}}^0 = \frac{1}{\sqrt{\kappa + a^2}} E_{\text{BFKL}} = -4 \ln 2$ at any a

- $\mathcal{H} = T(p) + U(r) = \{T(p) - T_0(p)\} + \mathcal{H}_0$
- $\mathcal{H}_0 \phi_{\text{trial}}^0 = (T_0(p) + U(r)) \phi_{\text{trial}}^0 = E_{\text{BFKL}} \phi_{\text{trial}}^0$

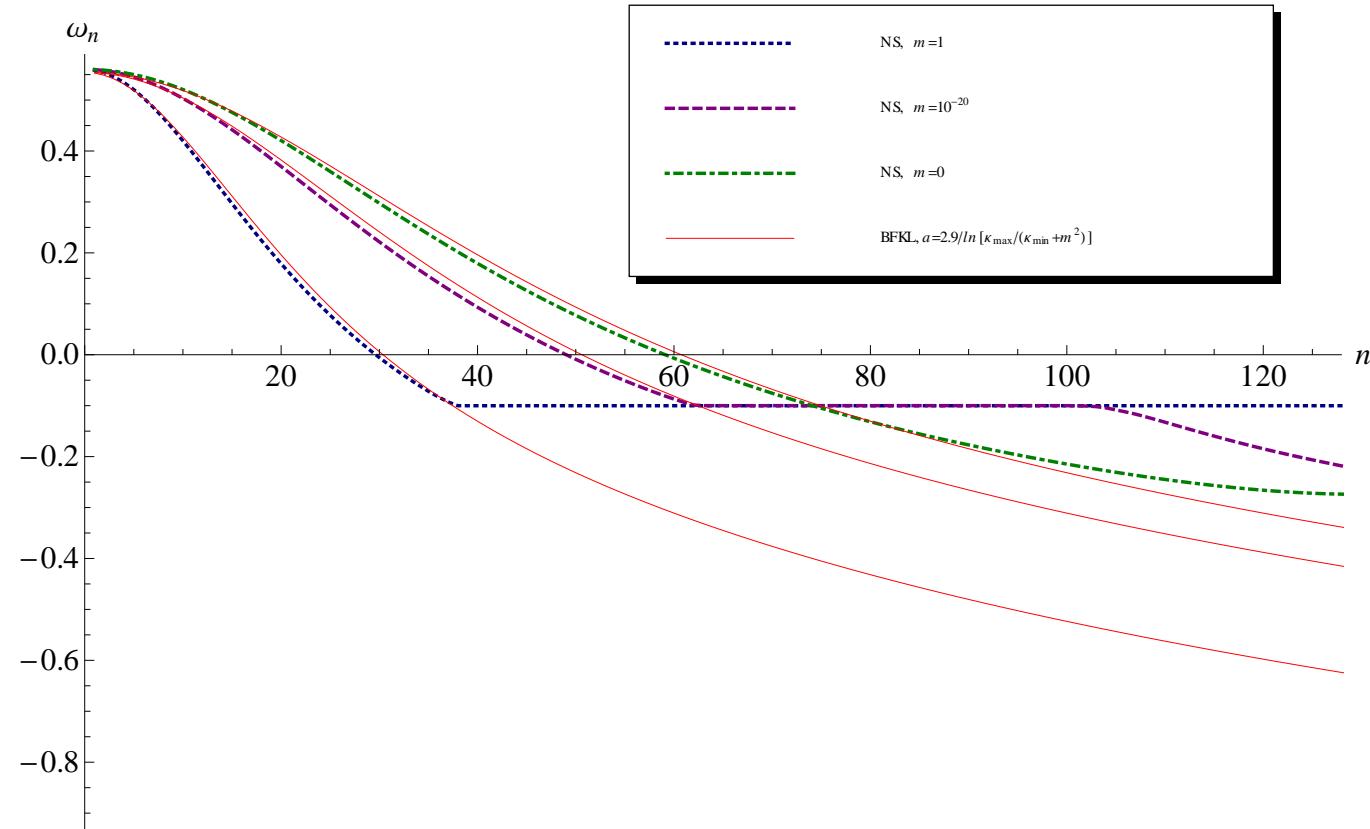
We prove $\{T(p) - T_0(p)\} \geq 0$

- $T_0(p) = E_{\text{BFKL}} - \frac{1}{\phi_{\text{trial}}^0(-i\nabla)} U(r) \phi_{\text{trial}}^0(r)$
- $\frac{1}{\phi_{\text{trial}}^0(p)} U(r) \phi_{\text{trial}}^0(r) = - \int \frac{d^2 p'}{\pi} \frac{\sqrt{p^2 + a^2}}{(|\vec{p} - \vec{p}'|^2 + 1) \sqrt{p'^2 + a^2}}$

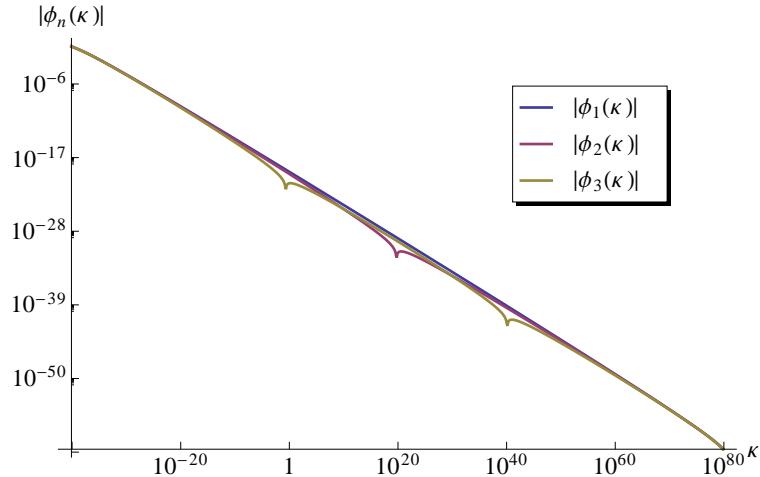
$$\{\dots\} \geq 0 \text{ for } 5/2 < a < a_0^2 = 5.26$$

Solution to $T(-a^2) = E(\gamma)$ gives a in this region

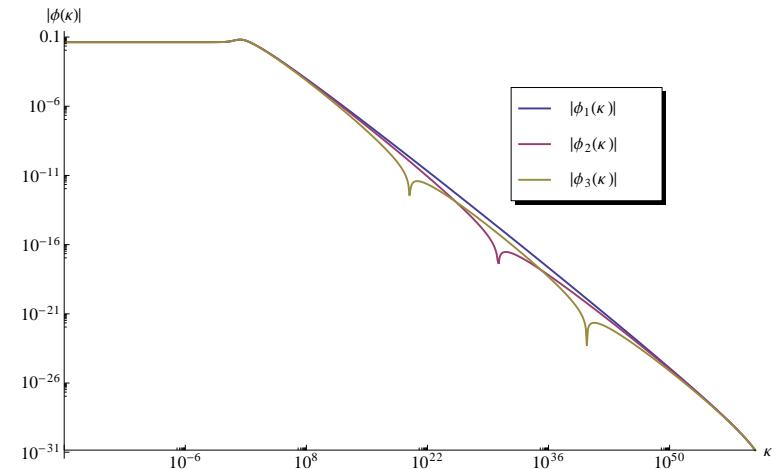
3. Numerical eigenvalues



Result # 2: Eigenfunctions



BFKL $m = 0$

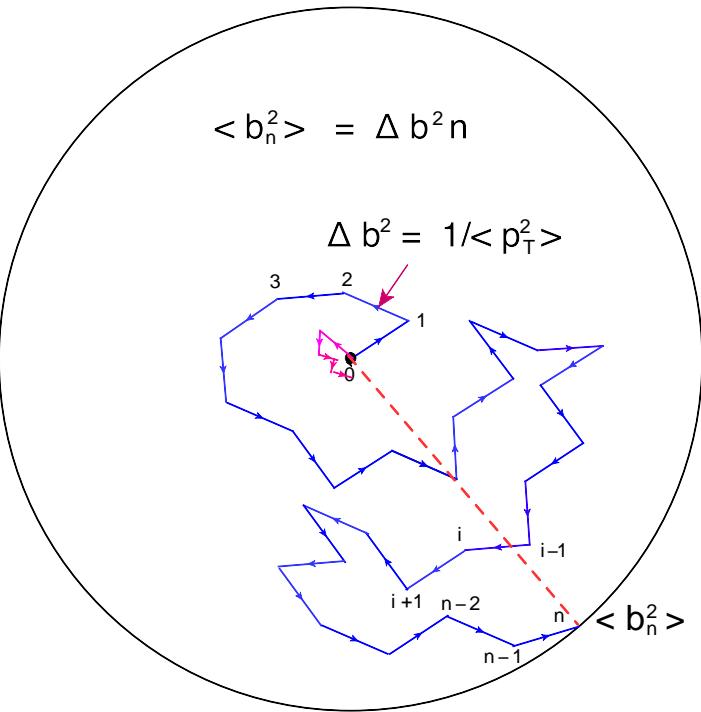


BFKL $m \neq 0$

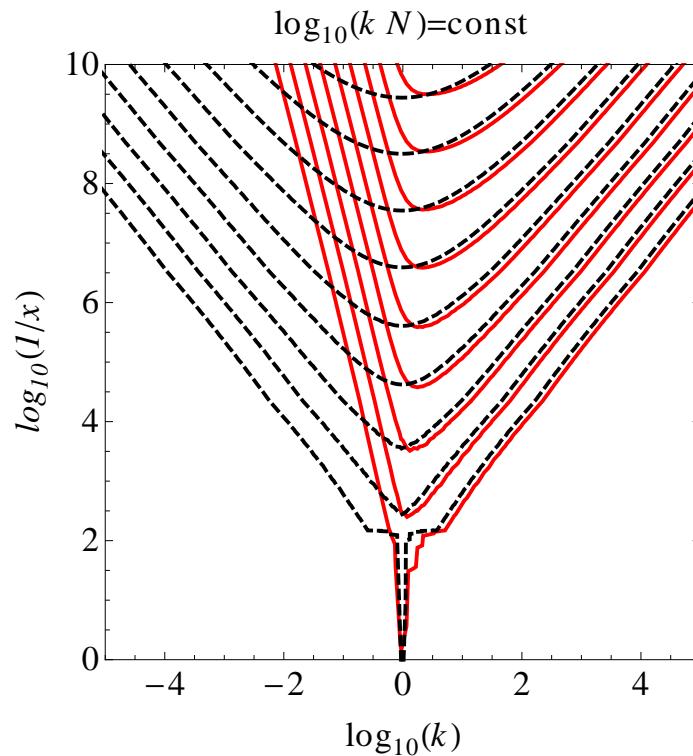
- $\phi^{(approx)}(\kappa, \nu) = \frac{\alpha(\nu)}{\sqrt{\kappa+4}} \sin(\nu \ln(\kappa) + b_\phi \nu)$
- $\gamma = \frac{1}{2} = i\nu$, $b_\phi = 1.865$ and $\ln(\kappa) = \ln\left(\frac{\sqrt{\kappa+4} + \sqrt{\kappa}}{\sqrt{\kappa+4} - \sqrt{\kappa}}\right)$

New scale: $\bar{p}_T^2 = 4 m^2$

Result # 3: $\langle |b^2| \rangle = \text{Const}(\Upsilon)$

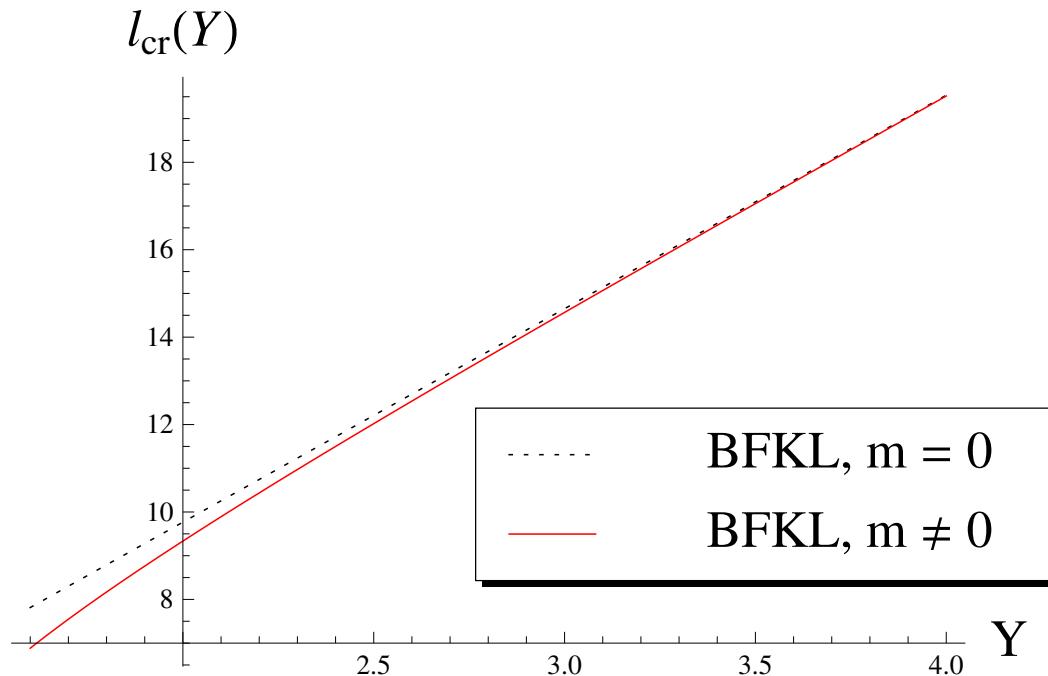


Gribov's diffusion



Contour plot

Result # 4: $Q_s^2(Y, b)$



$$N(Y, Q_s(Y, b)) = 0.1$$

Result # 5: Effective intercept in semi-classical approach

- $\Psi(Y, l) = \int \frac{d\gamma}{2\pi i} \phi_{in}(\gamma) \frac{e^{-E(\gamma, l)Y}}{(\kappa + a^2)^{1-\gamma}} = \int \frac{d\gamma}{2\pi i} \phi_{in}(\gamma) e^{-E(\gamma, l)Y + (\gamma-1)l}$

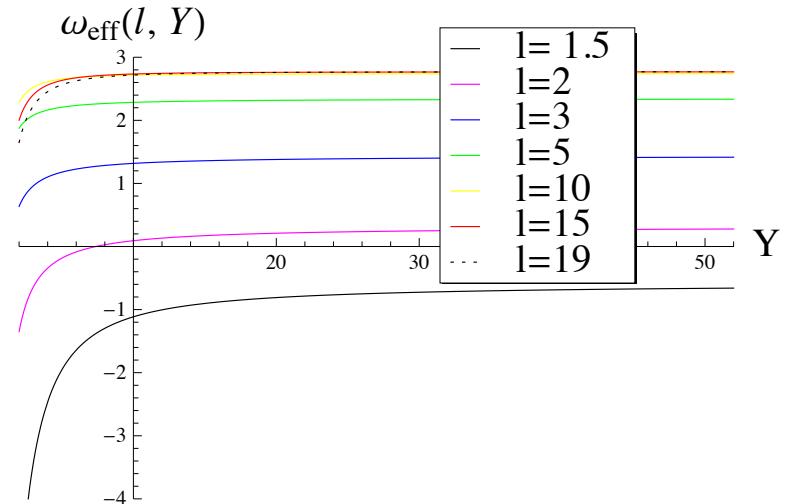
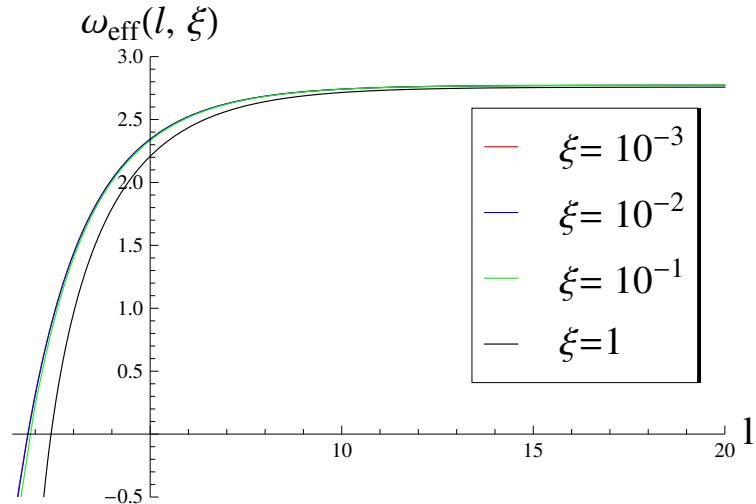
Steepest decent method (saddle point equation):

- $-\frac{\partial E(\gamma_{SP}(Y, l, a), l)}{\partial \gamma} Y + l = 0$

- $\Psi(Y, l = \ln(\kappa + a^2)) = \phi_{in}(\gamma_{SP}(Y, l))$

$$\times e^{-E(\gamma_{SP}(Y, l), l)Y + (\gamma_{SP}(Y, l) - 1)l} \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\gamma}{2\pi i} e^{-\frac{1}{2} \frac{\partial^2 E(\gamma_{SP}(Y, l), l)}{\partial \gamma_{SP}^2} Y (\gamma - \gamma_{SP}(Y, l))^2}$$

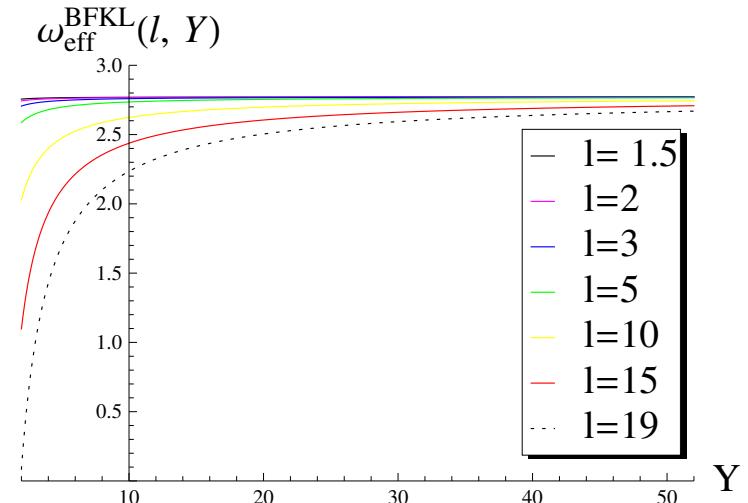
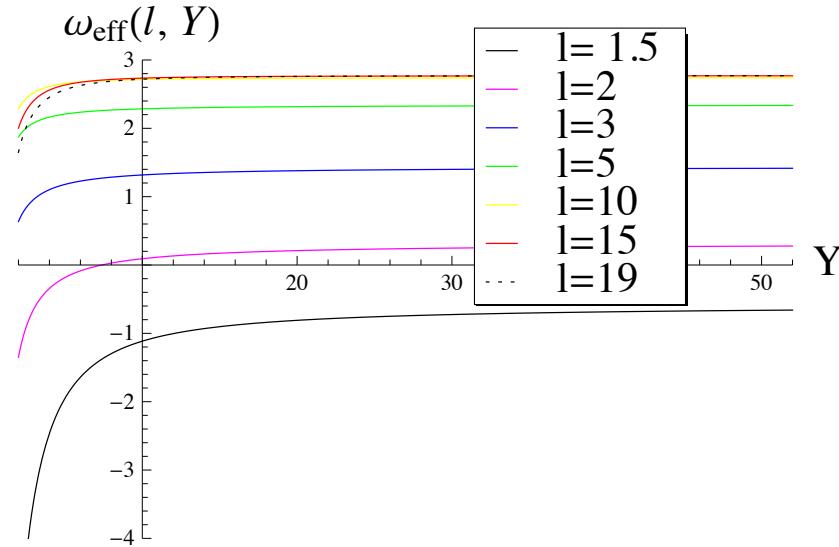
$$= \frac{\phi_{in}(\gamma_{SP}(Y, l))}{\sqrt{2\pi \left| \frac{\partial^2 E(\gamma_{SP}(Y, l), l)}{\partial \gamma_{SP}^2} \right| Y}} e^{-\frac{1}{2}l} e^{\underbrace{-E(\gamma_{SP}(Y, l), l)Y + (\gamma_{SP}(Y, l) - \frac{1}{2})l}_{\omega_{eff} Y}}$$



New scale: $l_{\text{soft}} = 1.85$ $p_{\text{soft}}^2 = m^2(e^{l_{\text{soft}}} - a^2) = 2.92m^2$

- For $l \geq l_{\text{soft}}$ $\omega_{\text{eff}} \geq 0 \rightarrow$ problems with unitarity;
- For $l \leq l_{\text{soft}}$ $\omega_{\text{eff}} \leq 0 \rightarrow$ no problems with unitarity;

Effective intercept of the massless BFKL Pomeron



Conclusions

1. $E(\text{BFKL } m \neq 0) = E(\text{BFKL } m = 0)$
2. The convenient parametrization of the wave functions is found with the new scale $\approx 4m^2$;
3. Average b^2 does not depend on Y at high energies;
4. $Q_s(\text{BFKL } m \neq 0) = Q_s(\text{BFKL } m = 0)$ at $Y \gg 1$;
5. New scale for scattering $p_{soft}^2 \approx 2.92 m^2$. For $p < p_{soft}$ - no problems with unitarity;