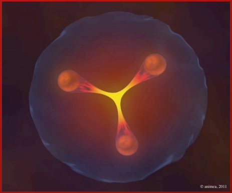
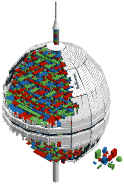
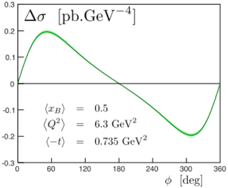
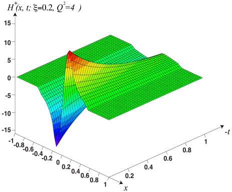
DE LA RECHERCHE À L'INDUSTRIE



Towards a Model of Pion Generalized Parton Distributions From Dyson-Schwinger Equations







Diffraction 2014 | Hervé MOUTARDE

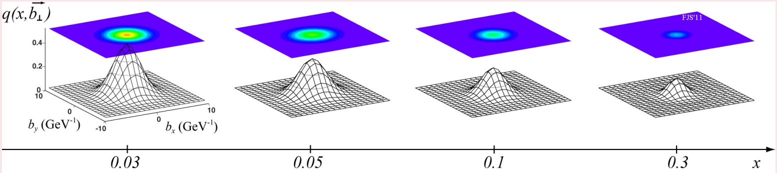
Sept 13rd, 2014

Motivations.

3D imaging of nucleon's partonic content but also. . .

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in the nucleon.
- Insights on:
  - **Spin** structure,
  - **Energy-momentum** structure.
- **Probabilistic interpretation** of Fourier transform of GPD$(x, \xi = 0, t)$ in **transverse plane**.

Transverse plane density (Goloskokov and Kroll model)



- Important topic for several **past, existing and future** experiments: H1, ZEUS, HERMES, CLAS, CLAS12, JLab Hall A, COMPASS, EIC, . . .
- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- **Recent applications** of the Dyson-Schwinger and Bethe-Salpeter framework to **hadron structure studies**.

Overview.

Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.

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- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- **Recent applications** of the Dyson-Schwinger and Bethe-Salpeter framework to **hadron structure studies**.
- Here develop **pion GPD model** for simplicity.
- No planned experiment on pion GPDs but existing proposal of DVCS on a virtual pion.

Amrath *et al.*, Eur. Phys. J. **C58**, 179 (2008)

Overview.

Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.

- Important topic for several **past, existing and future** experiments: H1, ZEUS, HERMES, CLAS, CLAS12, JLab Hall A, COMPASS, EIC, . . .
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- **Recent applications** of the Dyson-Schwinger and Bethe-Salpeter framework to **hadron structure studies**.

Steps towards a **pion GPD** model:

1. GPDs: Theoretical Framework
2. GPDs in the Dyson-Schwinger and Bethe-Salpeter Approach
3. Results: Theoretical Constraints and Phenomenology

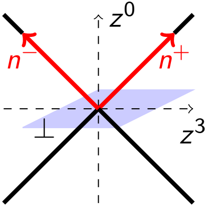
GPDs: Theoretical Framework

Pion Generalized Parton Distribution.

Definition and symmetry relations.

$$H^q_\pi(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- From **isospin symmetry**, all the information about pion GPD is encoded in $H^u_{\pi^+}$ and $H^d_{\pi^+}$.
- Further constraint from **charge conjugation**: $H^u_{\pi^+}(x,\xi,t) = -H^d_{\pi^+}(-x,\xi,t)$.

- Introduce **isovector** and **isoscalar** GPDs:

$$
\begin{aligned}
H^{I=0}(x,\xi,t) &= H^u_{\pi^+}(x,\xi,t) + H^d_{\pi^+}(x,\xi,t) \\
H^{I=1}(x,\xi,t) &= H^u_{\pi^+}(x,\xi,t) - H^d_{\pi^+}(x,\xi,t)
\end{aligned}
$$

- Compute Mellin moments of GPDs:

$$
\begin{aligned}
\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=0}(x,\xi) &= 0 \ (m \text{ even}) \\
\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=0}(x,\xi) &= \sum_{\substack{i=0 \\ \text{even}}}^{m} (2\xi)^i C^{I=0}_{mi} + (2\xi)^{m+1} C^{I=0}_{m\,m+1} \ (m \text{ odd}) \\
\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=1}(x,\xi) &= \sum_{\substack{i=0 \\ \text{even}}}^{m} (2\xi)^i C^{I=1}_{mi} \ (m \text{ even}) \\
\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=1}(x,\xi) &= 0 \ (m \text{ odd})
\end{aligned}
$$

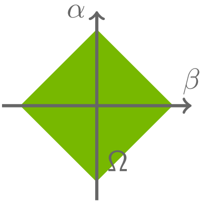
Double Distributions.

A convenient tool to encode GPD properties.

- Define Double Distributions F^q and G^q as matrix elements of **twist-2 quark operators**:

$$\left\langle P+\frac{\Delta}{2}\middle|\bar{q}(0)\gamma^{\{\mu}i\overset{\leftrightarrow}{D}^{\mu_1}\ldots i\overset{\leftrightarrow}{D}^{\mu_m\}}q(0)\middle|P-\frac{\Delta}{2}\right\rangle = \sum_{k=0}^{m}\binom{m}{k}$$

$$\left[F^q_{mk}(t)2P^{\{\mu}-G^q_{mk}(t)\Delta^{\{\mu}\right]P^{\mu_1}\ldots P^{\mu_{m-k}}\left(-\frac{\Delta}{2}\right)^{\mu_{m-k+1}}\ldots\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$



with

$$F^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}F^q(\beta,\alpha)$$

$$G^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}G^q(\beta,\alpha)$$

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Radyushkin, Phys. Rev. **D59**, 014030 (1999)
Radysuhkin, Phys. Lett. **B449**, 81 (1999)

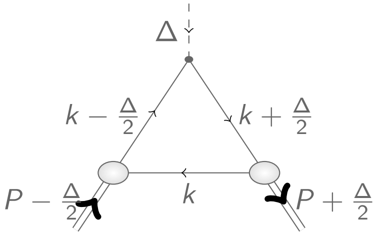
- Representation of GPD:

$$H^q(x,\xi,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha,t) + \xi G^q(\beta,\alpha,t)\big)$$

- Support property: $x \in [-1,+1]$.
- Discrete symmetries: F^q is α-even and G^q is α-odd.

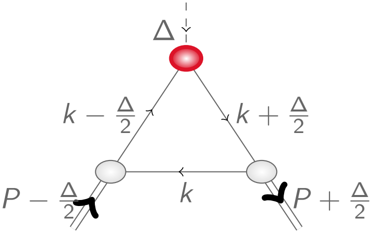
GPDs in the Dyson-Schwinger and Bethe-Salpeter Approach

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



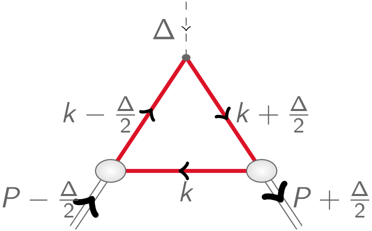
- Compute **Mellin moments** of the pion GPD H.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0)\gamma^+ (i \overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$



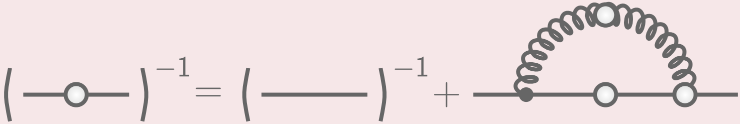
- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.

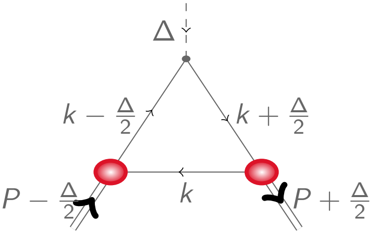
Dyson - Schwinger equation



GPDs in the rainbow ladder approximation.

Evaluation of triangle diagrams.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
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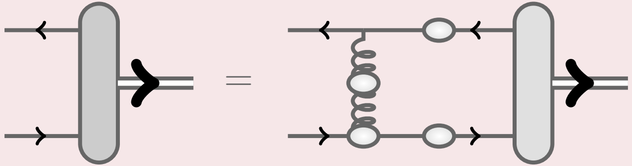
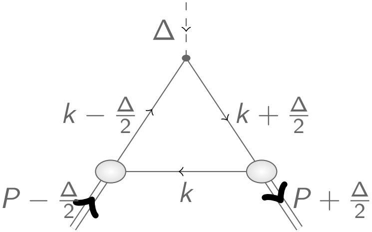
Bethe - Salpeter equation



$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

- Most GPD properties **satisfied by construction**.

GPDs in the rainbow ladder approximation.

Evaluation of triangle diagrams.

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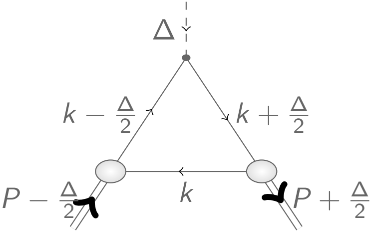

- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

- Most GPD properties **satisfied by construction**.
- Also compute crossed triangle diagram.

Algebraic model.

Intermediate step before using numerical solutions of Dyson-Schwinger and Bethe-Salpeter equations.

- Expression for GPD Mellin moments:

$$2(P^+)^{m+1}\langle x^m\rangle^u = \mathrm{tr}_{CFD}\int\frac{\mathrm{d}^4k}{(2\pi)^4}(k^+)^m\, i\Gamma_\pi\left(k-\frac{\Delta}{2},P-\frac{\Delta}{2}\right)$$
$$\times S(k-\frac{\Delta}{2})i\gamma^+\; S(k+\frac{\Delta}{2})i\bar{\Gamma}_\pi\left(k+\frac{\Delta}{2},P+\frac{\Delta}{2}\right)S(k-P)$$

- Expressions for vertices and propagators:

$$S(p) = \left[-i\gamma\cdot p+M\right]\Delta_M(p^2)$$
$$\Delta_M(s) = \frac{1}{s+M^2}$$
$$\Gamma_\pi(k,p) = i\gamma_5\frac{M}{f_\pi}M^{2\nu}\int_{-1}^{+1}\mathrm{d}z\,\rho_\nu(z)\;\left[\Delta_M(k_{+z}^2)\right]^\nu$$
$$\rho_\nu(z) = R_\nu(1-z^2)^\nu$$

with R_ν a normalization factor and $k_{+z}=k-p(1-z)/2$.

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

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Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

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  - Dimensionful parameter M.

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Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:
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  - Dimensionless parameter ν

Algebraic model.

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$$

with R_ν a normalization factor and $k_{+z} = k - p(1-z)/2$.

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:
  - Dimensionful parameter M.
  - Dimensionless parameter ν. **Fixed to 1** to recover asymptotic pion DA.

Realistic model.

Implementing vertices and propagators coming from the numerical resolution of the Dyson-Schwinger and Bethe-Salpeter equations.

- Numerical solutions of equations are taken into account by a fit with the following Ansätze:
  - Ansatz for quark propagator:

$$S(p) = \sum_{j=1}^{j_m} \left(\frac{z_j}{i\not{p} + m_j} + \frac{z_j^*}{i\not{p} + m_j^*} \right)$$

  - Ansatz for scalar functions in Bethe Salpeter amplitude:

$$F(k; P) = c \int_{-1}^{+1} \mathrm{d}z \, \rho_\nu(z) \Lambda k^2 \Delta_\Lambda^2(k_z^2) + \text{other similar terms}$$

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Use experience from algebraic model.
- In principle slightly more complex. In practice many more terms. *Work in progress*.

Results: Theoretical Constraints and Phenomenology

Analytic expression in the DGLAP region.

$$H^u_{x\geq\xi}(x,\xi,0) = \frac{48}{5}\left\{ \frac{3\left(-2(x-1)^4\left(2x^2-5\xi^2+3\right)\log(1-x)\right)}{20\left(\xi^2-1\right)^3} \right.$$

$$\frac{3\left(+4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\tanh^{-1}\left(\frac{(x-1)\xi}{x-\xi^2}\right)\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(x^3(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^4\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(-5x(x(x(x+2)+36)+18)\xi^2-15\xi^6\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(2(x-1)\left((23x+58)\xi^4+(x(x(x+67)+112)+6)\xi^2+x(x((5-2x)x+15)+3)\right)\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(\left(15(2x(x+5)+5)\xi^4+10x(3x(x+5)+11)\xi^2\right)\log\left(1-\xi^2\right)\right)}{20\left(\xi^2-1\right)^3}$$

$$\left.+\frac{3\left(2x(5x(x+2)-6)+15\xi^6-5\xi^2+3\right)\log\left(1-\xi^2\right)}{20\left(\xi^2-1\right)^3}\right\}$$

- **Analytic expression** in the DGLAP region.
- Similar expression in the ERBL region.

Verification of theoretical constraints.

Analytic expressions for the GPD H.

- **Analytic expression** in the DGLAP region.
- Similar expression in the ERBL region.
- **Explicit check** of **support property** and **polynomiality** with correct powers of ξ.

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- Also direct verification using Mellin moments of H.

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- Similar expression in the ERBL region.
- **Explicit check** of **support property** and **polynomiality** with correct powers of ξ.
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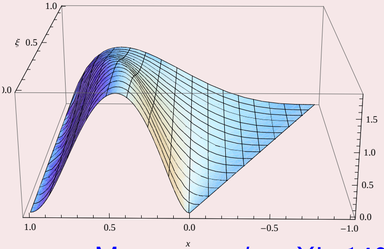
Valence $H^u(x, \xi, t)$ as a function of x and ξ at vanishing t.



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

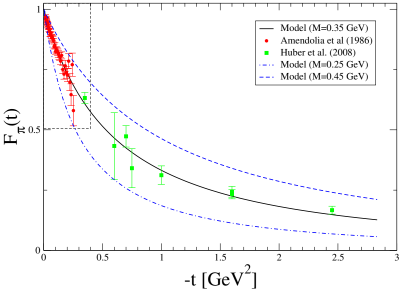
Pion form factor.

Determination of the model dimensionful parameter M.

- Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{I=1}(x, \xi, t) = 2F_\pi(t)$$

- Single dimensionful parameter $M \simeq 350$ MeV.



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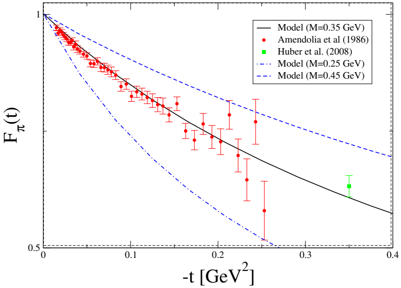

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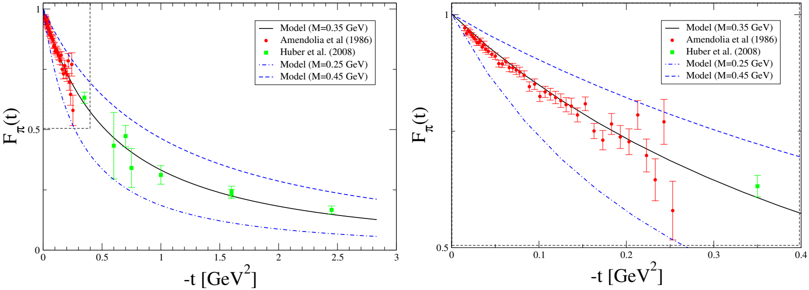


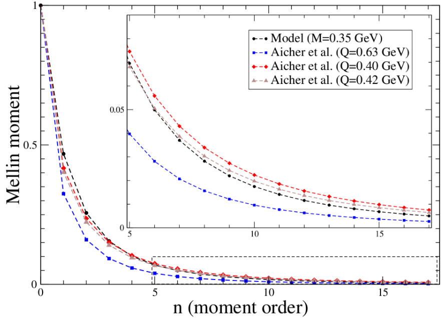
Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Pion PDF obtained from forward limit of GPD:

$$q(x) = H^q(x, 0, 0)$$

- Use LO DGLAP equation and compare to PDF extraction.

Aicher *et al.*, Phys. Rev. Lett. **105**, 252003 (2010)



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Find model initial scale $\mu \simeq 400$ MeV.

Conclusions

- Computation of GPDs and DDs in the **nonperturbative framework** of Dyson-Schwinger and Bethe-Salpeter equations.
- **Explicit check** of several theoretical constraints, including polynomiality and support property.
- Simple algebraic model exhibits **most features of the numerical solutions** of the Dyson-Schwinger and Bethe-Salpeter equations.
- **Very good agreement** with existing pion form factor and PDF data. Extension to realistic (numerical) model looks promising.

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