

# Updates of PDFs for the 2nd LHC run

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In collaboration with Alan Martin, Patrick Motylinski  
and Robert Thorne

and thanks to Ben Watt, Graeme Watt and James Stirling

# Outline

In this talk I will present results on continuing updates in PDFs within the **MSTW** framework. A new set is very close to being finalized, with no significant changes expected to the PDFs shown here. Updates:

- Changes in theoretical procedures (updated parameterisation, error treatment, nuclear corrections...).
- Inclusion of a variety of new data sets, including the most up-to-date **LHC** data:
  - $W^{\pm}$ ,  $Z$ ,  $t\bar{t}$
  - **HERA** updates
  - Jet data
  - DY@CMS double differential
  - Jet data @NNLO (not included in fit)

# Theory changes - extended parameterization

- Continue to use extended parameterisation with **Chebyshev polynomials** as in recent **MSTWCPdeut** study (**Eur.Phys.J. C73 (2013) 2318**).
- For valence and sea quarks, instead of taking

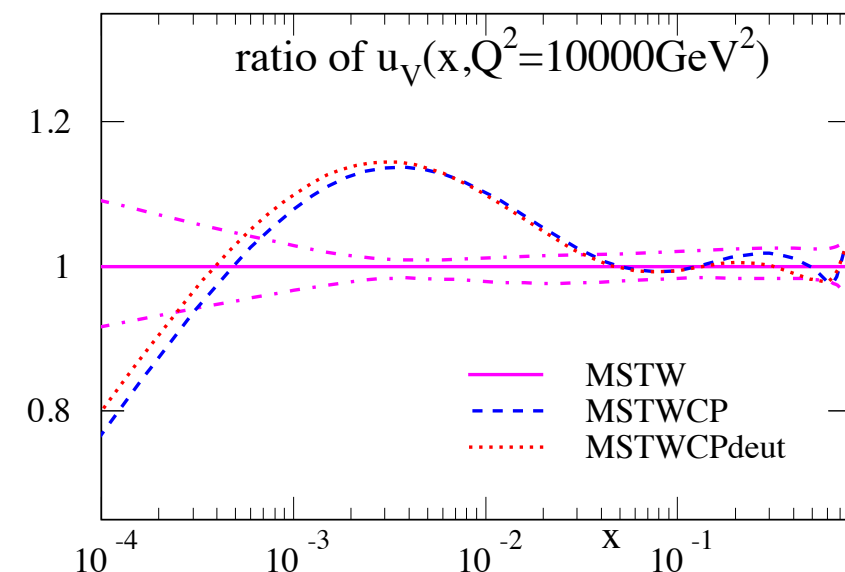
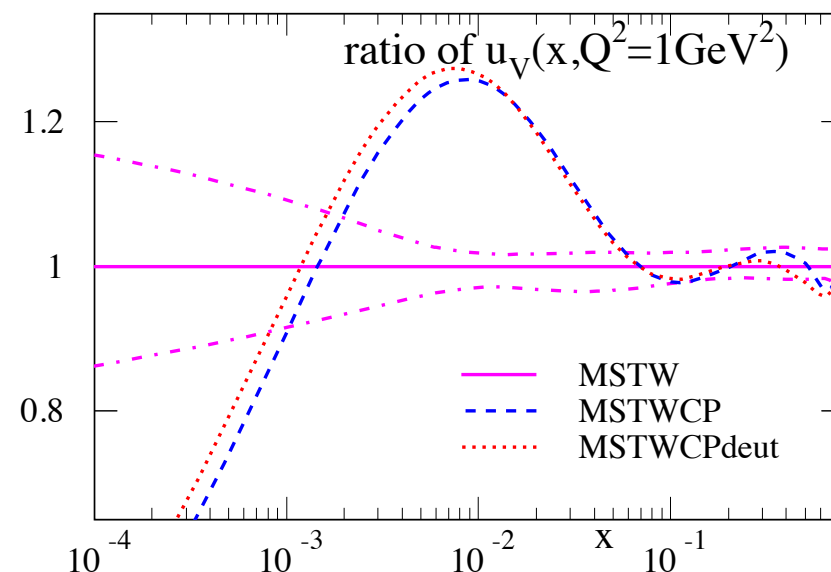
$$xf(x, Q_0^2) = A(1-x)^\eta x^\delta (1 + \epsilon x^{0.5} + \gamma x)$$

take

$$xf(x, Q_0^2) = A(1-x)^\eta x^\delta \left( 1 + \sum_{i=1}^n a_i T_i(y(x)) \right) \quad y(x) = 1 - 2\sqrt{x}$$

where  $T_i$  are **Chebyshev polynomials** - convenient choice of basis for interpolating polynomial. By considering different  $n$  can perform systematic study.

- Taking  $n = 4$  find significant improvement in global  $\Delta\chi^2$  and change in  $u_V$  for  $x \lesssim 0.03$ .



# Deuteron correction - fit

- In order to separate  $u$  and  $d$  at moderate to large  $x$  use DIS data on deuteron targets.

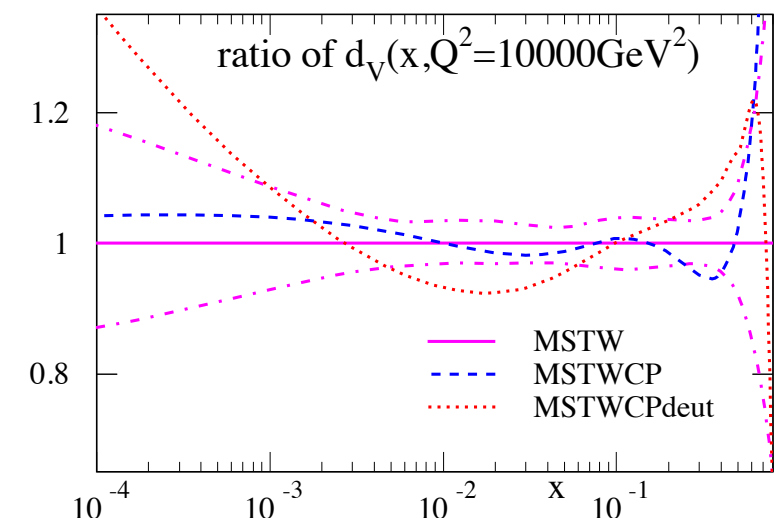
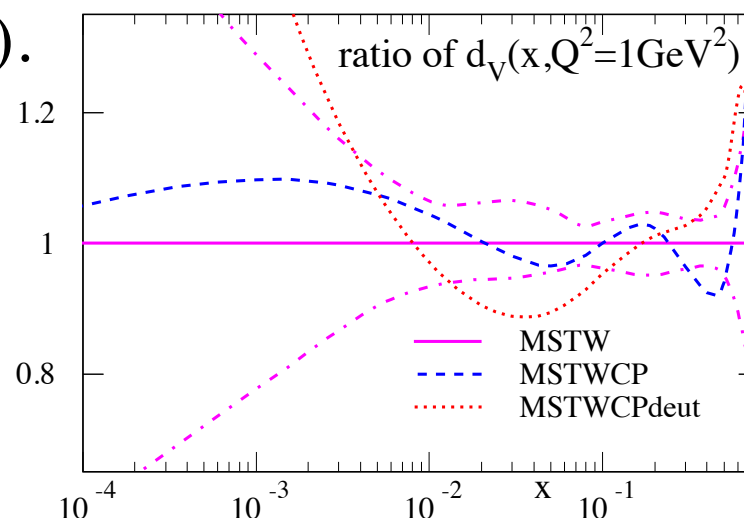
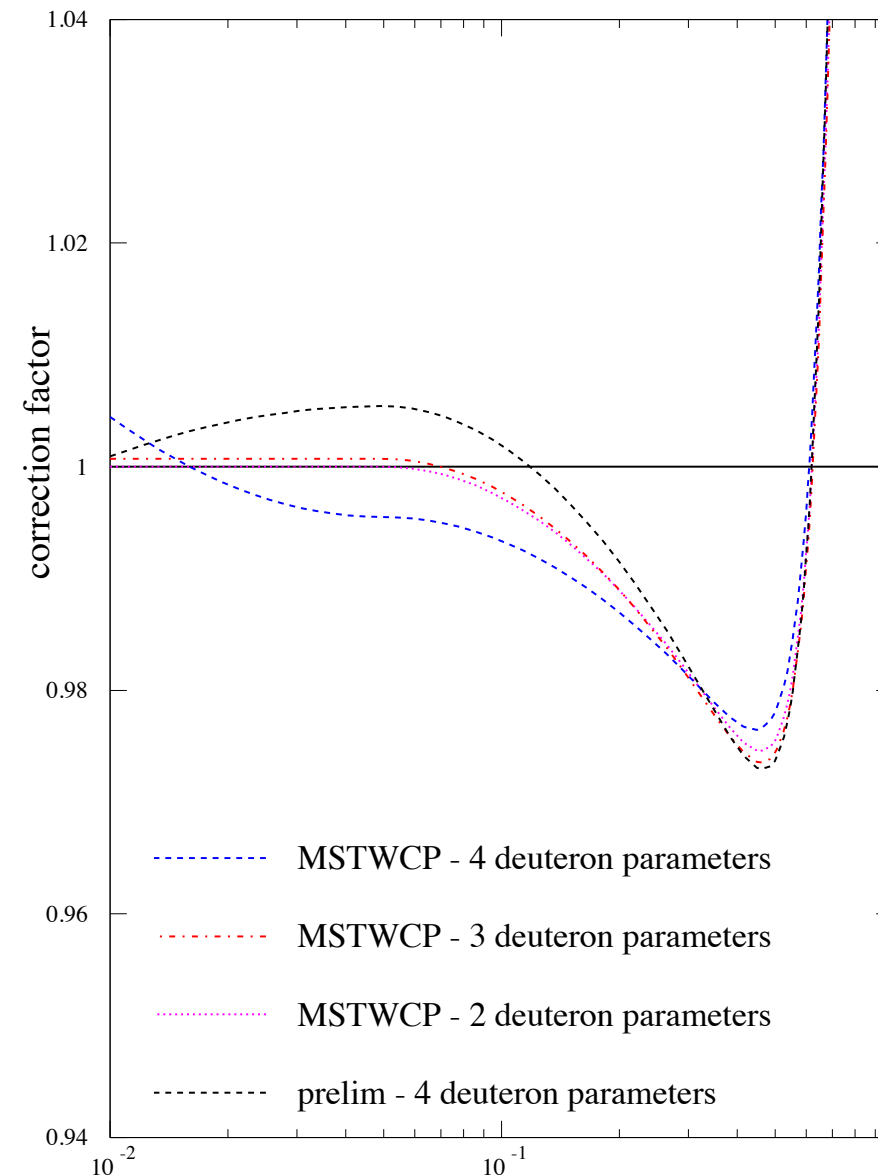
→ Need to include nuclear corrections.

$$F^d(x, Q^2) = c(x)(F^p(x, Q^2) + F^n(x, Q^2))/2$$

- Parameterise  $c(x)$  and allow to vary with no penalty in fit.

- Previously big improvement in fit for **MSTWCPdeut**, but not exactly as expected at lower  $x$ .

- Now behaves more like expectations, and 4 parameters are left free at **NLO** (and now **NNLO**). Uncertainty of about  $0.5 - 1\%$ . Feeds into PDF uncertainty.



# Treatment of errors

- Systematic errors generally multiplicative (uncertainty  $\propto$  measured value, as in e.g. overall normalization uncertainty  $\rightarrow$  percentage error).
- Using  $\chi^2$  definition:

$$\chi^2 = \sum_{i=1}^{N_{pts}} \left( \frac{D_i + \sum_{k=1}^{N_{corr}} r_k \sigma_{k,i}^{corr} - T_i}{\sigma_i^{uncorr}} \right)^2 + \sum_{k=1}^{N_{corr}} r_k^2,$$

where  $\sigma_{k,i}^{corr} = \beta_{k,i}^{corr} T_i$  and  $\beta_{k,i}^{corr}$  are the percentage error. Additive would use  $\sigma_{k,i}^{corr} = \beta_{k,i}^{corr} D_i$ . Previously did this for all but normalization uncertainty.

- Writing  $D_i + \sum_{k=1}^{N_{corr}} \beta_{k,i}^{corr} D_i \sim f * D_i$  then  $T_i - \sum_{k=1}^{N_{corr}} \beta_{k,i}^{corr} T_i \sim T_i / f$ ,

- And so:

both data and error scaled

$$\chi^2 \sim \left( \frac{D_i - T_i / f}{\sigma_i^{uncorr}} \right)^2 = \left( \frac{f * D_i - T_i}{f * \sigma_i^{uncorr}} \right)^2 \text{ rather than } \chi^2 \sim \left( \frac{f * D_i - T_i}{\sigma_i^{uncorr}} \right)^2.$$

incorrectly treating multiplicative errors as additive will bias results.

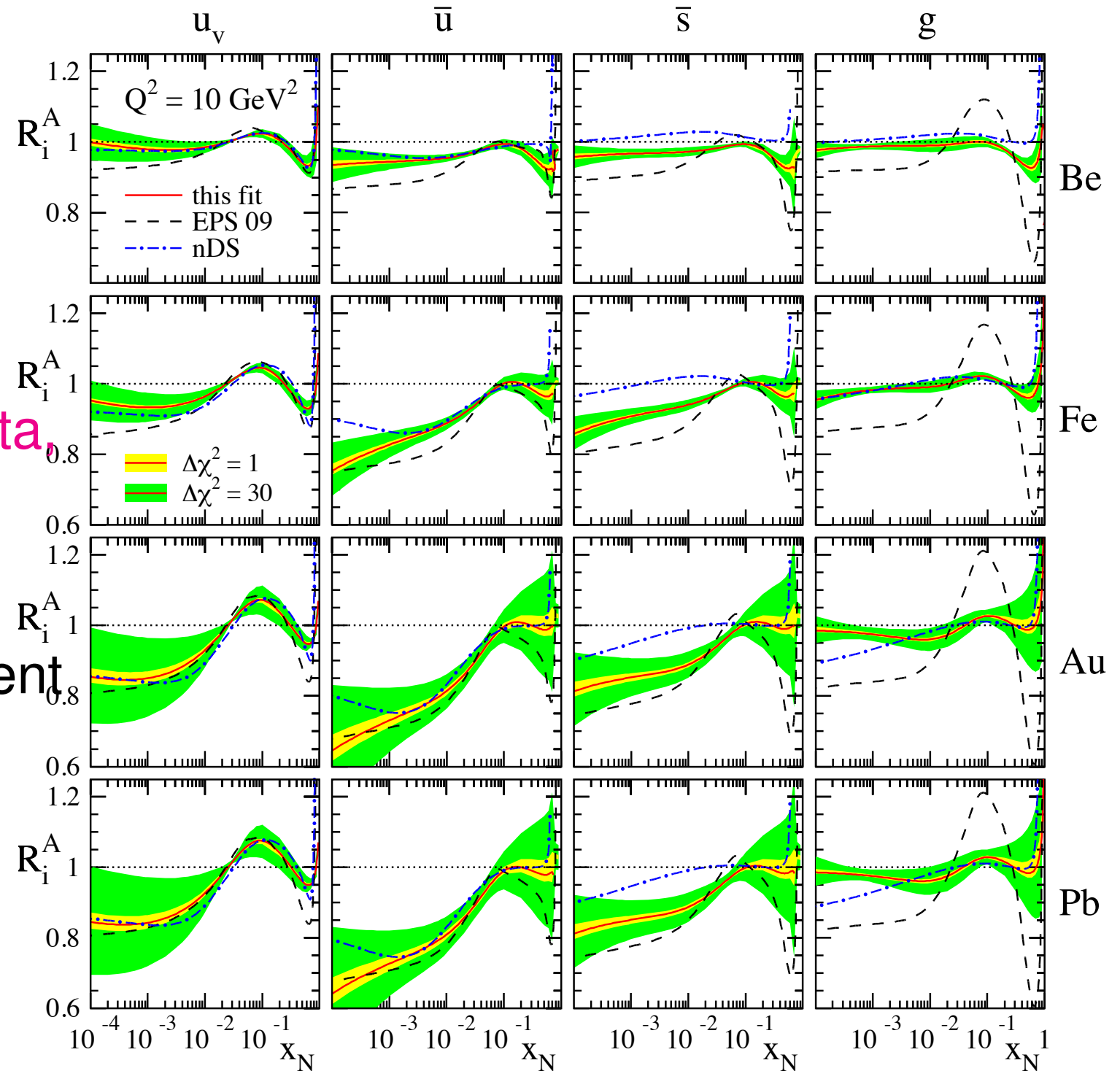
# Nuclear corrections

Have been using **de Florian, Sassot** nuclear corrections.

Update to more recent version, **de Florian, Sassot, Stratmann, Zurita**, Phys.Rev. D85 (2012) 074028.

Mainly similar, but different correction for small- $x$  strange.

Improves global fit by  $\sim 25$  units - **NuTeV  $F_2$ , HERA  $F_2$ , CMS jet**.



Only small change in strange quark, (no effect on **ATLAS, W,Z** fit).

# Other changes in theoretical procedures

- Now use “optimal” **GM-VFNS** choice (**Phys.Rev. D86 (2012) 074017**) which is smoother near to heavy flavour transition points (more so at **NLO**).
- Correct dimuon cross-sections for missing small contribution, i.e. where charm is produced away from the interaction point. Previously assumed this was accounted for by acceptance corrections. Previous checks showed correction is a small effect on strange distribution.
- Use **NMC** structure function data with  $F_L(x, Q^2)$  correction very close to theoretical  $F_L(x, Q^2)$  value. Very little effect.
- Branching ratio  $B_\mu \equiv B(D \rightarrow \mu)$  (for strangeness): now avoid those determined by fits to dimuon data relying on PDF input. Also apply error which feeds into PDFs. Use  $B_\mu = 0.092 \pm 10\%$  from **hep-ex/9708014**. Fits prefer  $B_\mu = 0.082 - 0.090 \pm 15\%$  with **NNLO** at lower end.

# Changes in data sets - HERA

- HERA Run-I neutral and charged current data from H1 and ZEUS replaced with combined data set with full treatment of correlated errors. Fit to data very good. Slightly better fit at NNLO.
- HERA combined data on  $F_2^c(x, Q^2)$  included. Fit quality  $\sim 60 - 65$  for 52 points.
- All direct published HERA  $F_L(x, Q^2)$  measurements included. Undershoot data a little at lower  $Q^2$ , but  $\chi^2$  not much more than one per point.
- Separate Run-II H1 and ZEUS data not included yet. Will wait for Run-II combination.

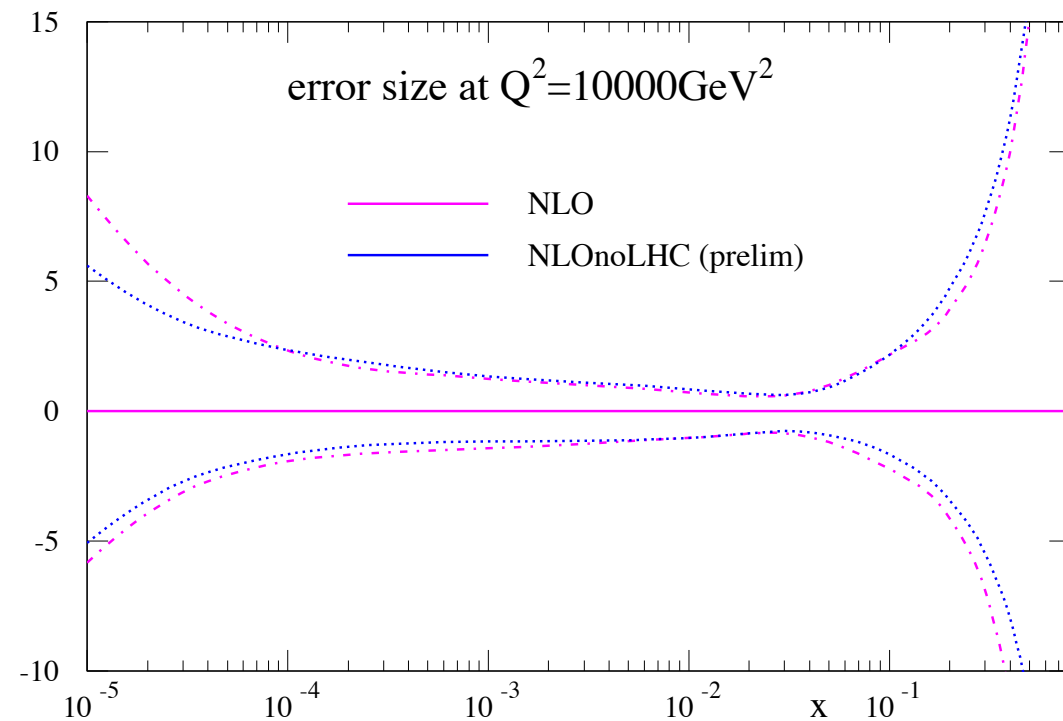
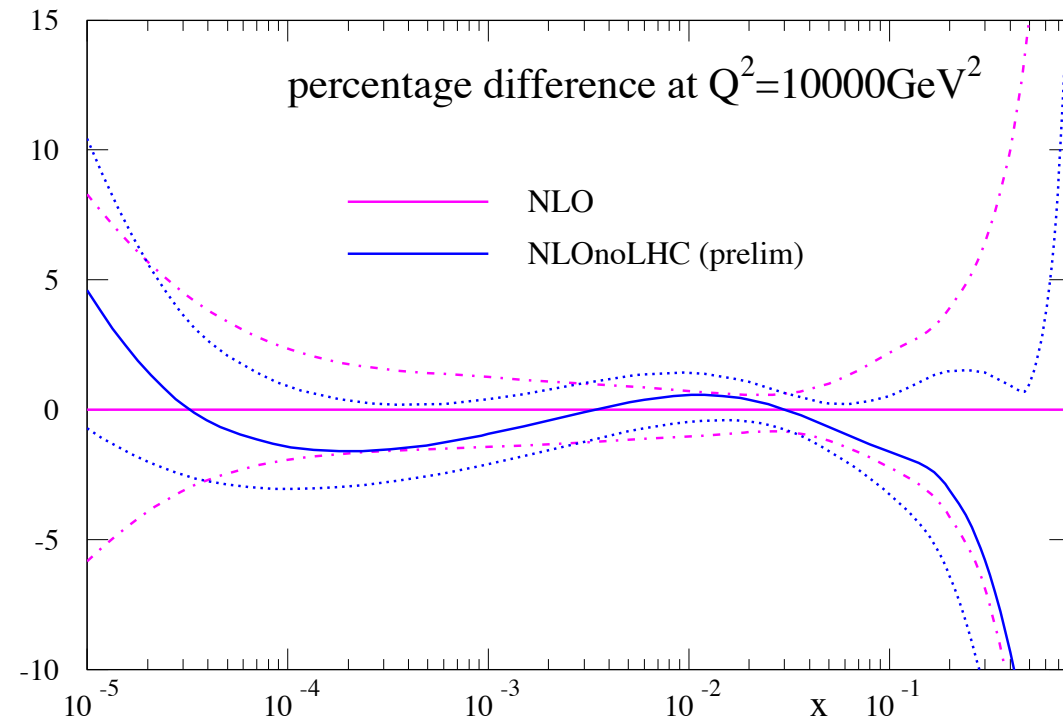


# Changes in data sets - Tevatron

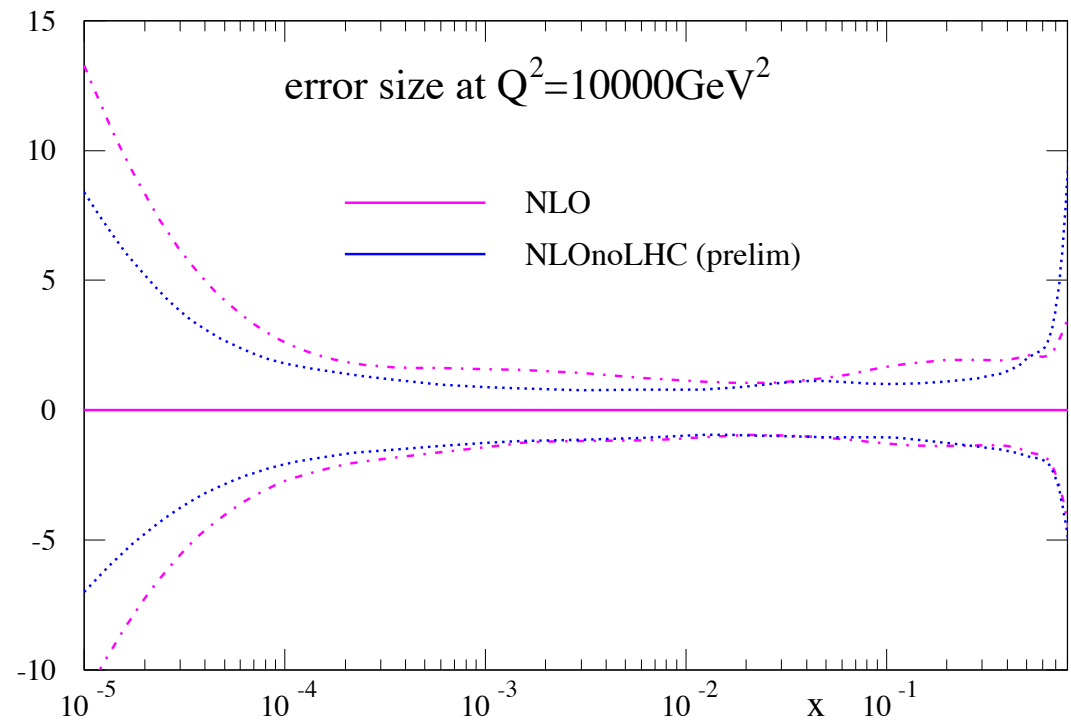
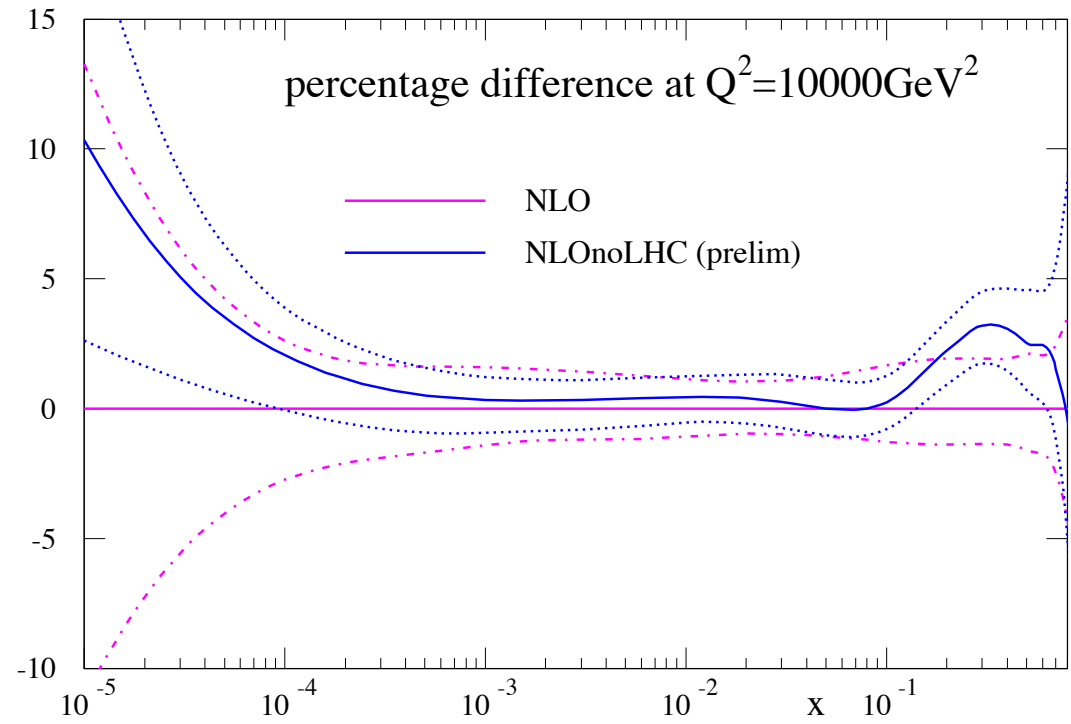
- **New(er)** Tevatron data sets included:
  - **CDF W**-asymmetry data
  - **D0** electron asymmetry data (  $p_{\perp} > 25 \text{ GeV}$  based on  $0.75 \text{ fb}^{-1}$  )
  - **New D0** muon asymmetry data (  $p_{\perp} > 25 \text{ GeV}$  based on  $7.3 \text{ fb}^{-1}$  )
- Include final numbers for **CDF Z**-rapidity data - final numbers changed after **MSTW2008** fit. (Also include very small photon contribution in theory.) Very little change.
- Not much change in PDFs (other than already seen in  $u_V - d_V$  ).
- At **NLO**  $\alpha_S(M_Z^2) = 0.1199$  from  $0.1202$  and at **NNLO**  $\alpha_S(M_Z^2) = 0.1180$  from  $0.1171$ .

# Changes in NLO PDFs - pre-LHC

Gluon at NLO



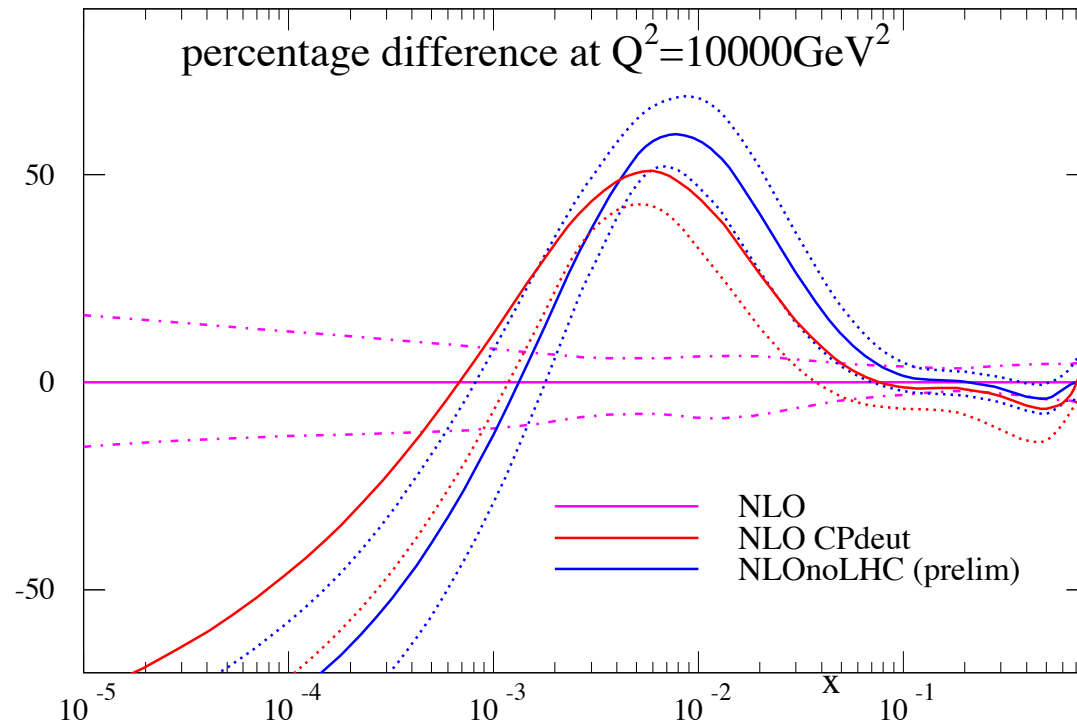
Light Quarks at NLO



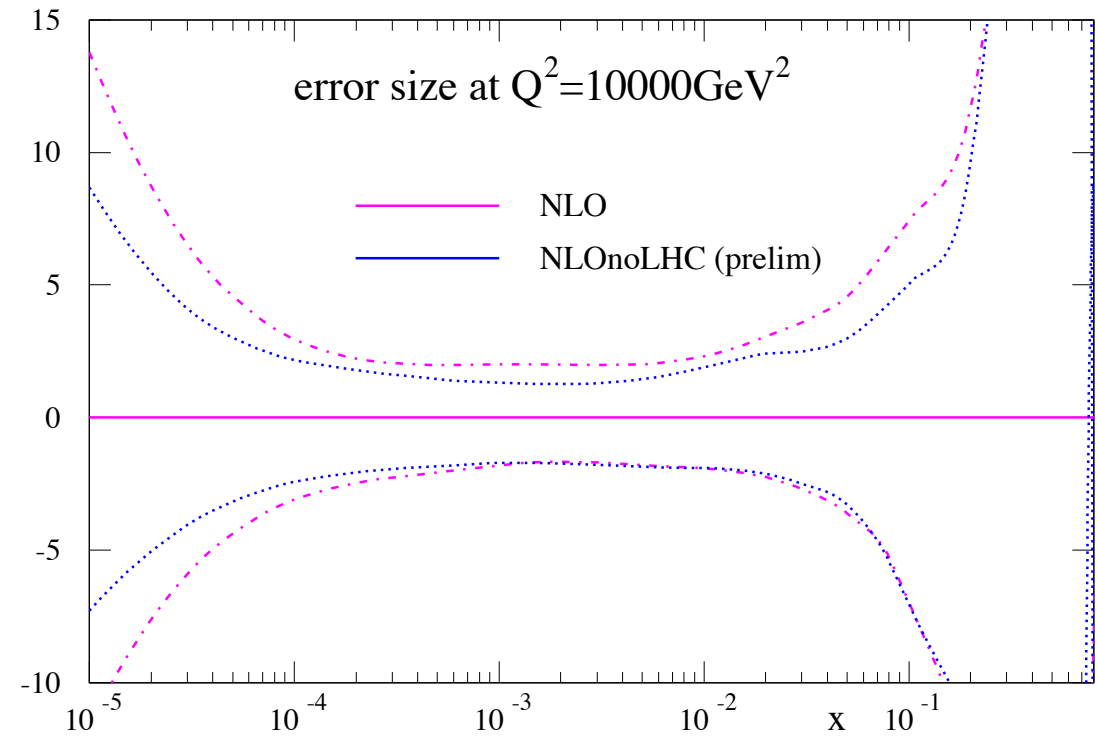
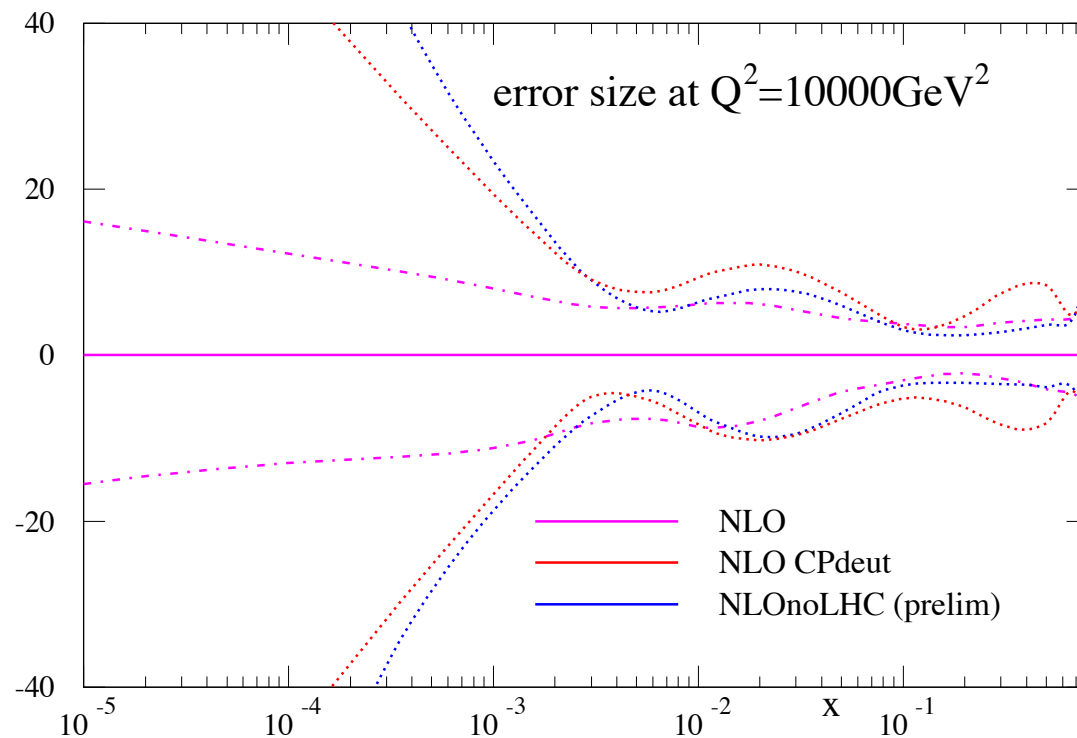
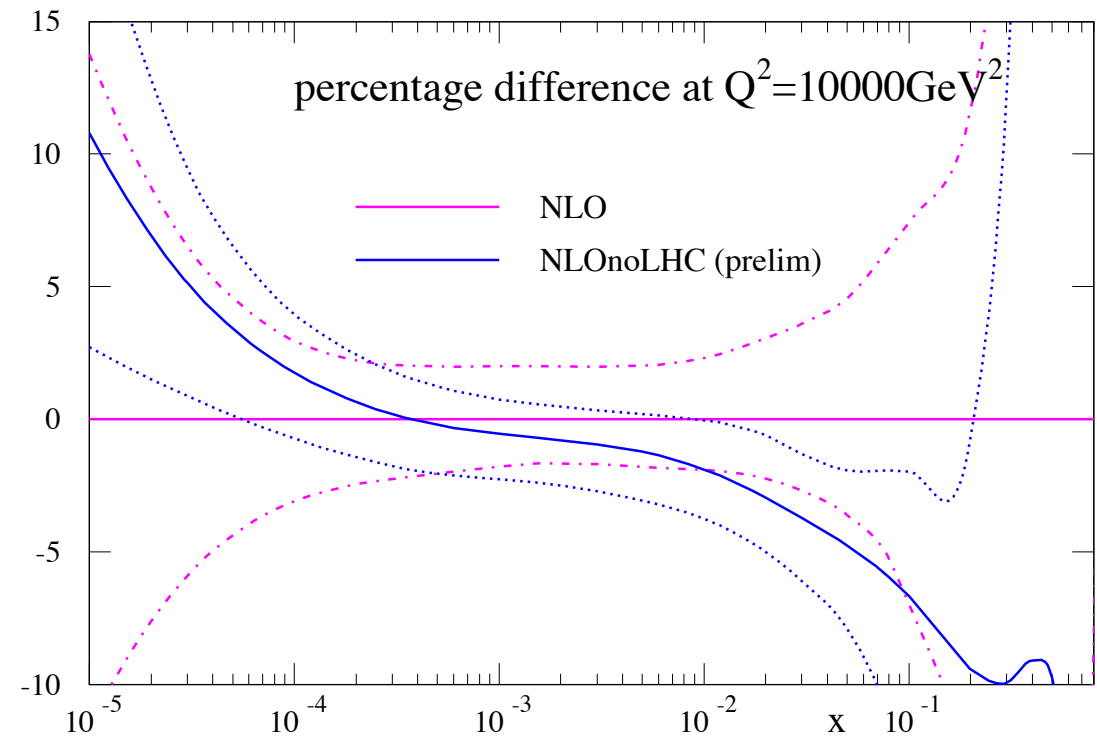
Increase in  $d$  at high  $x$  (mainly due to deuteron corr.). Overall small to moderate changes.

# Changes in NLO PDFs - pre-LHC

$x(u_V - d_V)$  at NLO



Strange at NLO



Note **large** changes in  $u_V - d_V$  and strange distributions.  
 → Due to extended parameterisation/deuteron corr.

Change in branching ratio for  
 dimuon data not included yet

# Inclusion of LHC data

- Work done with R.S. Thorne and P. Motylinksi using FastNLO, APPLGrid, MCFM and DYNNLO/FEWZ. Allows direct inclusion of data at NLO into fit. At NNLO still rely on K-factor approximation:

$$K^{\text{NNLO}}(M, y) = 1 + \left( \frac{\alpha_s(M)}{\pi} \right) D(M, y) + \left( \frac{\alpha_s(M)}{\pi} \right)^2 E(M, y)$$

- ATLAS  $W^\pm$ ,  $Z$  rapidity data now included:
  - Before inclusion  $\chi^2 \sim 1.6$  per point at NLO,  $\chi^2 \sim 2$  per point at NNLO.
  - Inclusion results in some extra improvement at NLO,  $\chi^2 \sim 1.3$  with strongest pull on gluon PDF.
  - Also goes to  $\chi^2 \sim 1.3$  at NNLO. The most obvious change is in the strange quark (balance of W and Z production depends on strange).

# Inclusion of LHC data - $W, Z$

- **ATLAS** and **CMS**  $W^+ - W^-$  asymmetry data both included, and no longer an issue at all (c.f. extended parameterisation and deuteron corrections). Fit slightly better at **NLO**.
- **LHCb** data on  $W^+, W^-$  and  $Z \rightarrow e^+ e^-$  included. Both predicted/fit well at **NLO**. For the latter theory is a bit low at **NNLO** for  $y \sim 3.5$ . However, not evident in preliminary  $Z \rightarrow \mu^+ \mu^-$  data with higher precision.
- **CMS** data on  $Z \rightarrow e^+ e^-$ , and **ATLAS** high mass DY data included. Again both predicted/fit well.
- **CMS** Z double differential (rapidity distributions for  $20 < M_{ll} < 1500$  GeV divided into six bins) measurement included. Extends down to low mass: NNLO much better fit than NLO at lowest mass  $\sim 20 - 45$  GeV (more later).

# Inclusion of LHC data - $t\bar{t}$

- Include  $t\bar{t}$  data into fit:
  - $\sigma_{t\bar{t}}$  from the Tevatron (combined CDF and D0 cross section measurement).
  - All published data from ATLAS and CMS for 7 TeV and one point at 8 TeV .
  - Use  $m_t = 172.5 \text{ GeV}$  (value used in Tevatron combination) with an error of 1 GeV, with  $\chi^2$  penalty applied.
  - Predictions and fit good, with NLO preferring masses slightly below  $m_t = 172.5 \text{ GeV}$  and NNLO masses slightly above.

# Inclusion of LHC data - jets

- At NLO, LHC jet data is included in fit:
  - CMS together with ATLAS 7 TeV + 2.76 TeV data.
  - Before fitting, the ATLAS 7 TeV + 2.76 TeV  $\chi^2 = 112/114$  and CMS  $\chi^2 = 186/133$  - comparable to the best PDFs of other groups.
  - Simultaneous fit of CMS data together with ATLAS 7 TeV + 2.76 TeV gives some improvement for CMS, and a small amount for ATLAS. The experiments seem extremely compatible.
- At NLO, final extracted  $\alpha_S(M_Z^2) = 0.1193$
- LHC jets not included at NNLO - more later.

# Fit quality for LHC data - NLO

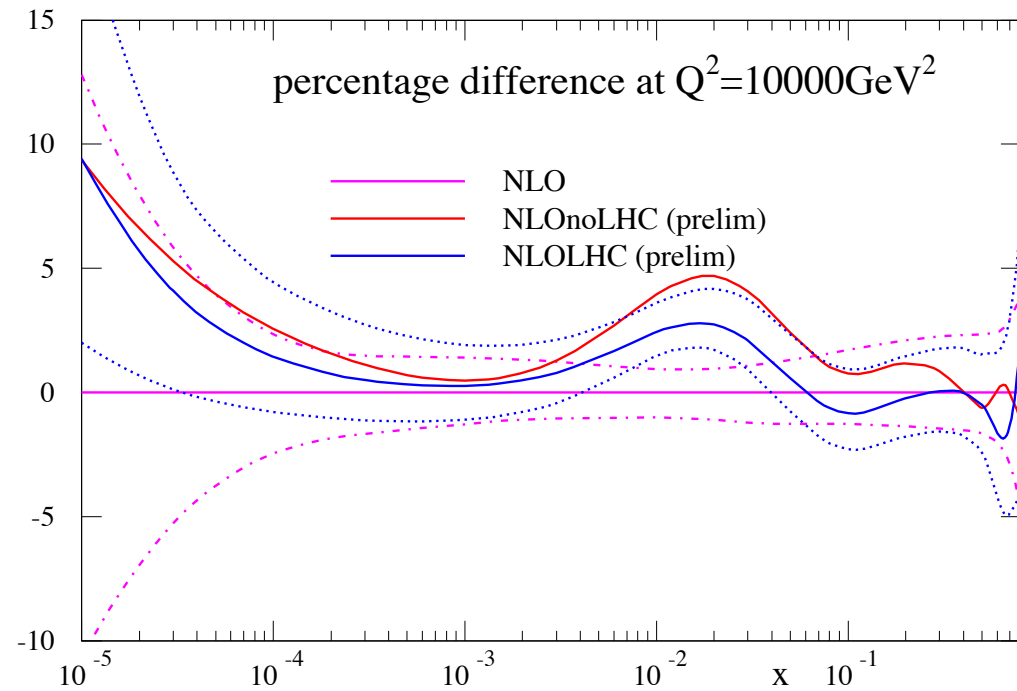
data set	$N_{pts}$	CPdeut	no LHC	prelim
ATLAS jets (2.76TeV+7TeV)	116	107	107	106
CMS jets (7TeV)	133	140	143	138
ATLAS $W^+, W^-, Z$	30	47	44	39
CMS $W$ asymm $p_T > 35\text{GeV}$	11	9	16	7
CMS asymm $p_T > 25\text{GeV}, 30\text{GeV}$	24	9	17	7
LHCb $Z \rightarrow e^+e^-$	9	13	13	13
LHCb $W$ asymm $p_T > 20\text{GeV}$	10	12	14	12
CMS $Z \rightarrow e^+e^-$	35	21	22	20
ATLAS High mass DY	13	20	20	21
TeV, ATLAS, CMS $\sigma_{t\bar{t}}$	13	8	10	7
CMS Low-high mass DY	132	385	396	373

- $W, Z$  data constrain gluon, as does  $\sigma_{t\bar{t}}$ .
  - CMS  $W$  asymmetry data constrains some flavour decomposition.
  - CMS double differential and ATLAS high mass DY have little impact on PDFs.
- However fit very poor at NLO in lowest mass bins (more later).

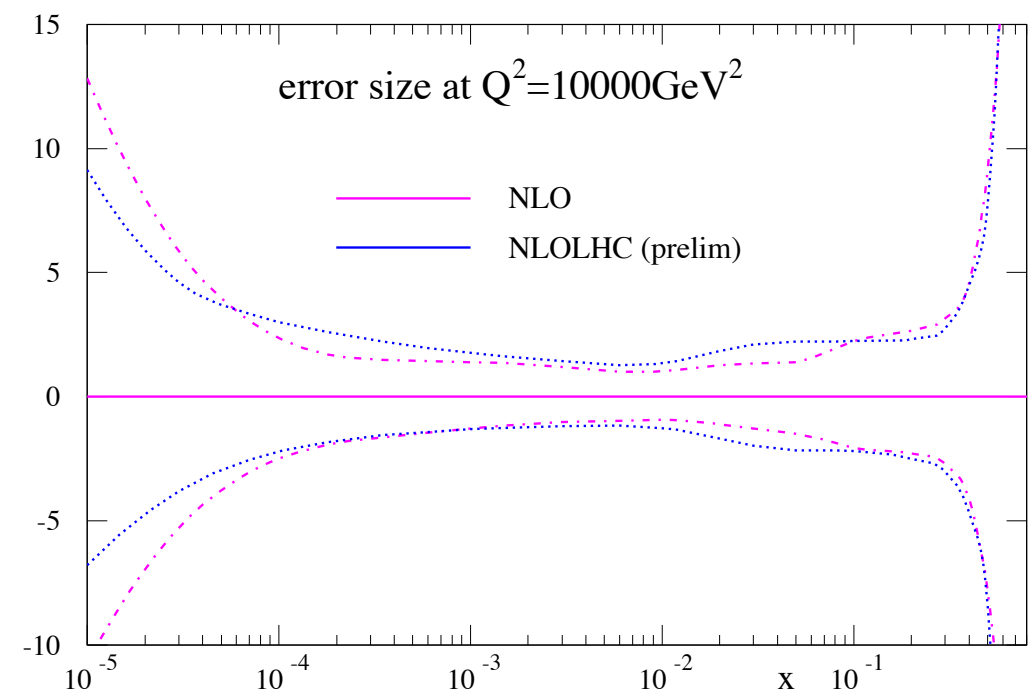
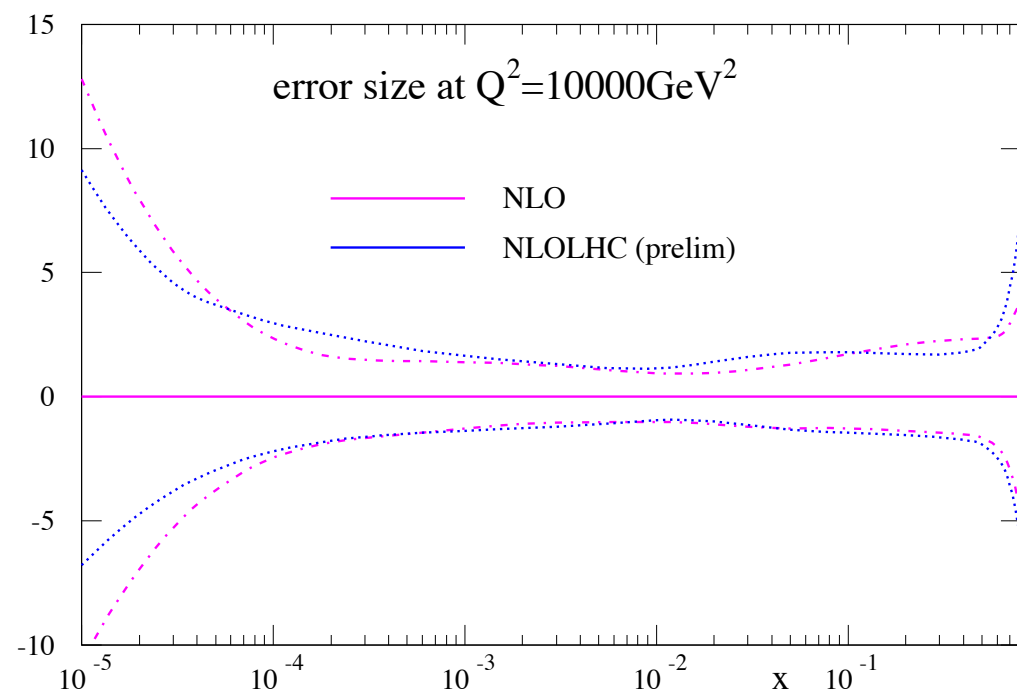
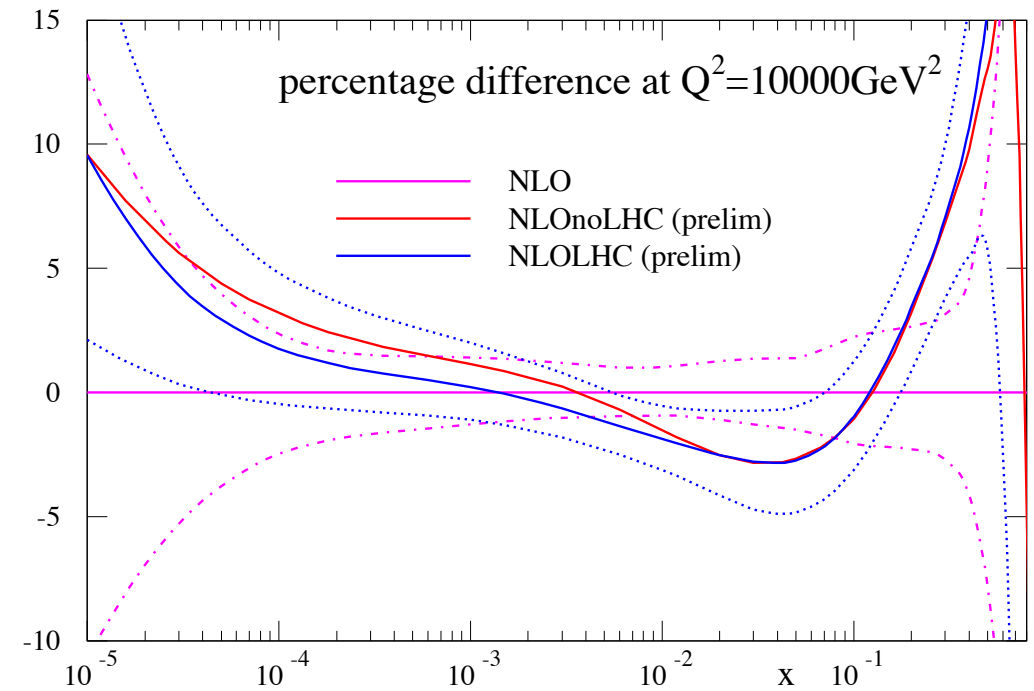


# Change in NLO PDFs

Up quark at NLO

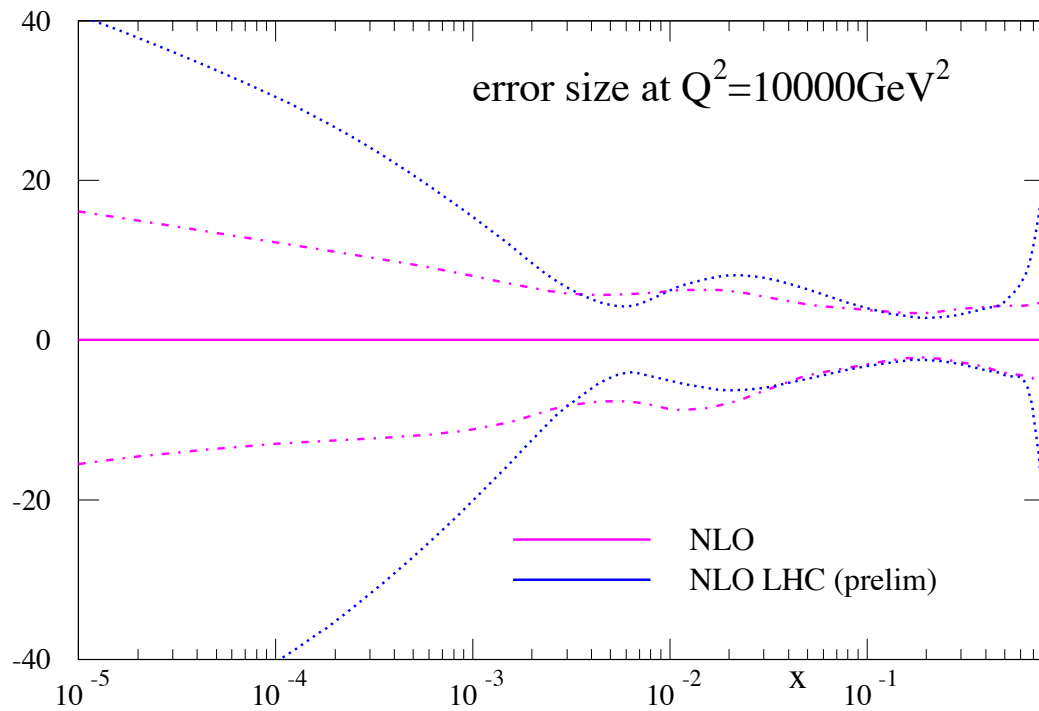
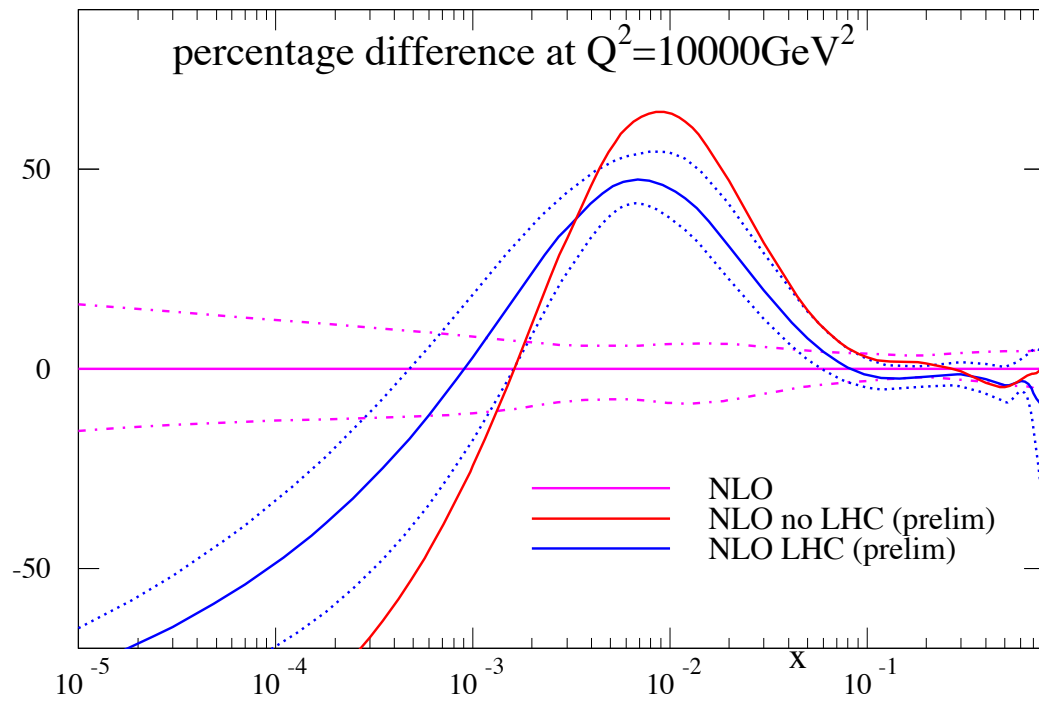


Down quark at NLO

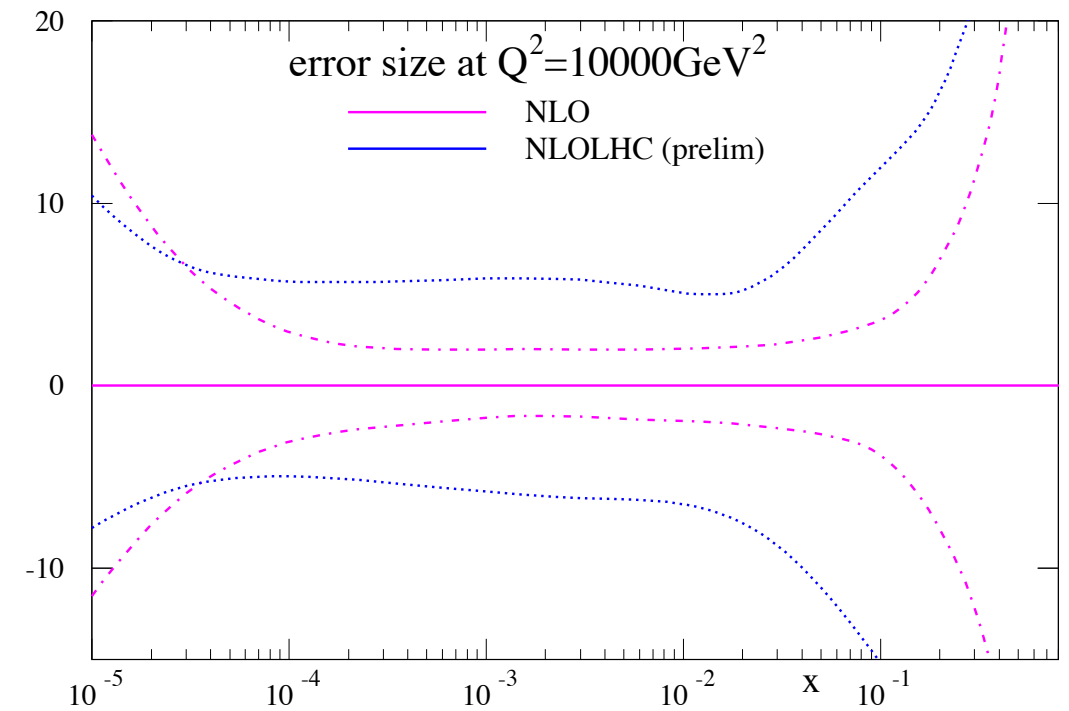
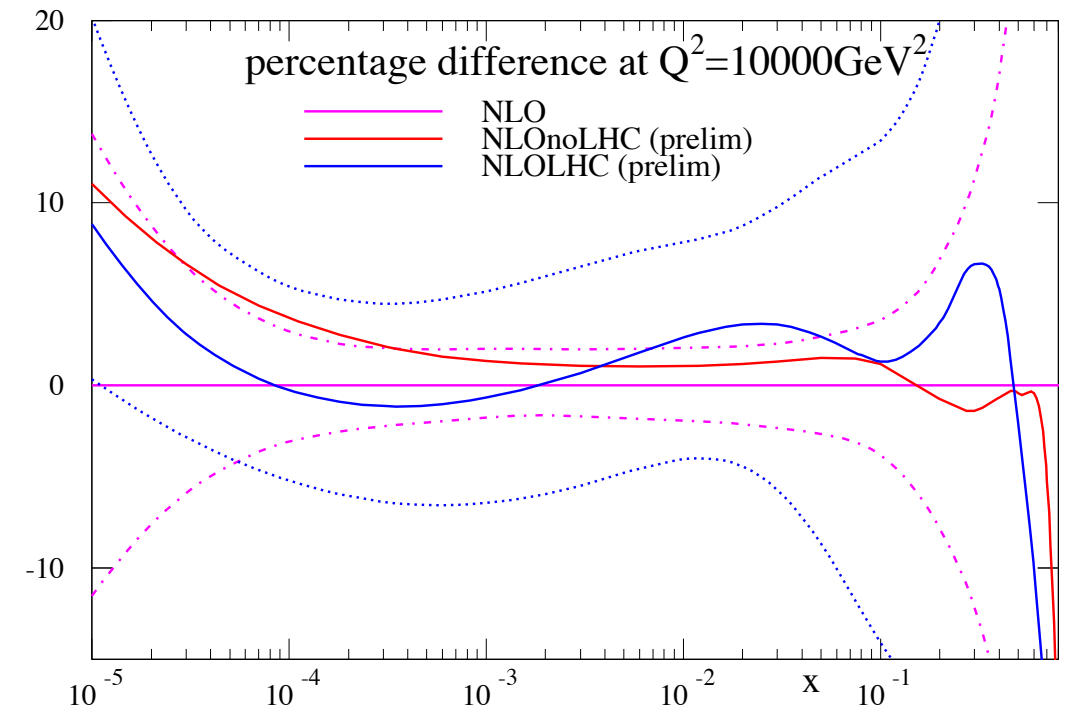


Includes theoretical updates and LHC data (not completely final version)

$x(u_V - d_V)$  at NLO



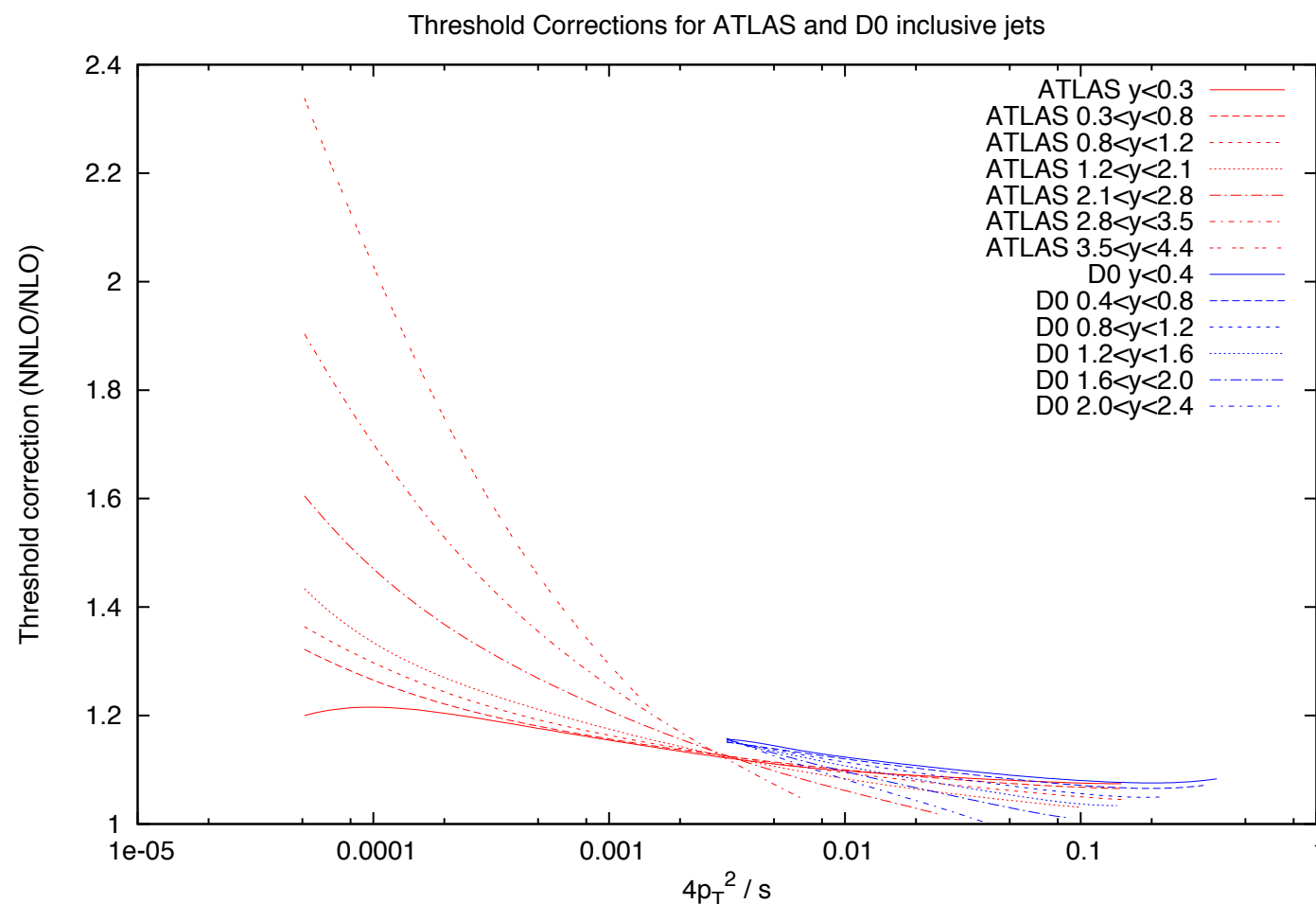
Strange+antistrange quark at NLO



Much expanded  $s + \bar{s}$  uncertainty is clear (error on  $B_\mu$ ).

# NNLO: LHC jet data?

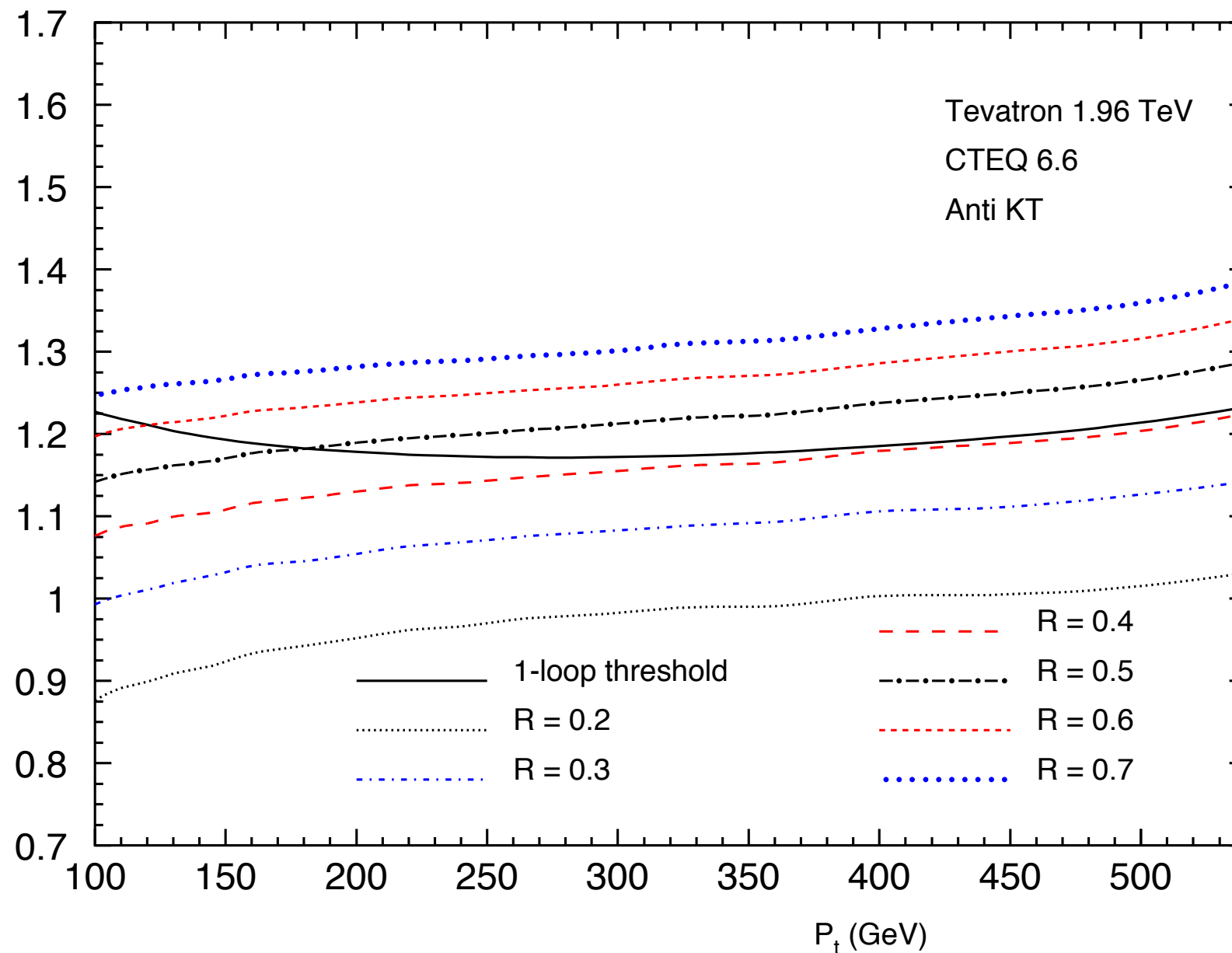
- For **Tevatron** data use approximate “threshold” corrections (**Kidonakis and Owens**),  $\sim 10\%$  positive correction.  $\frac{2p_{\perp}}{\sqrt{s}} \sim 1$
- **LHC** corrections very similar in the highish  $x$  region (as probed at the **Tevatron**), however these blow up for low  $x$ , i.e. far from threshold, which is probed at the **LHC**.



B.J.A. Watt, P. Motylinski  
and R.S. Thorne, arXiv:  
1311.5703

→ need full **NNLO** calculation. Enormous project for full **NNLO** calculation (**Gehrmann-de-Ridder, Gehrmann, Glover and Pires**) nearing completion. Some channels calculated, and some indications of the full form of the correction.

# NNLO threshold corrections- recent results

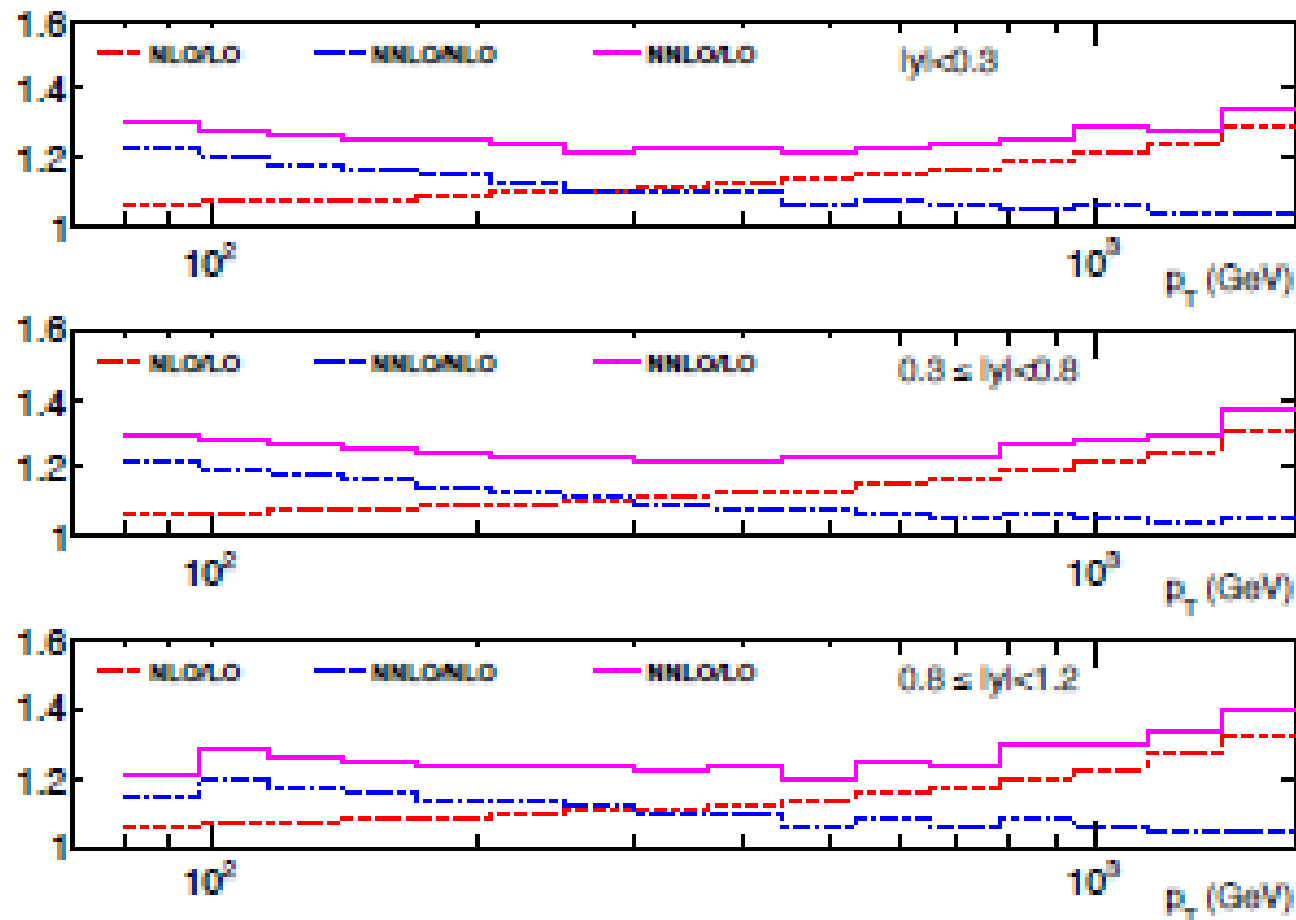


- Recent repeat of threshold calculation by [Kumar, Moch \(arXiv:1309.5311\)](#) and comparison to exact NLO results for different jet radii  $R$ .
- Big variation with  $R$  at NLO and threshold calculation which has no  $R$  dependence matches best with  $R \sim 0.3 - 0.4$ . Bit lower than typical Tevatron value, however adjusting leads to little change.

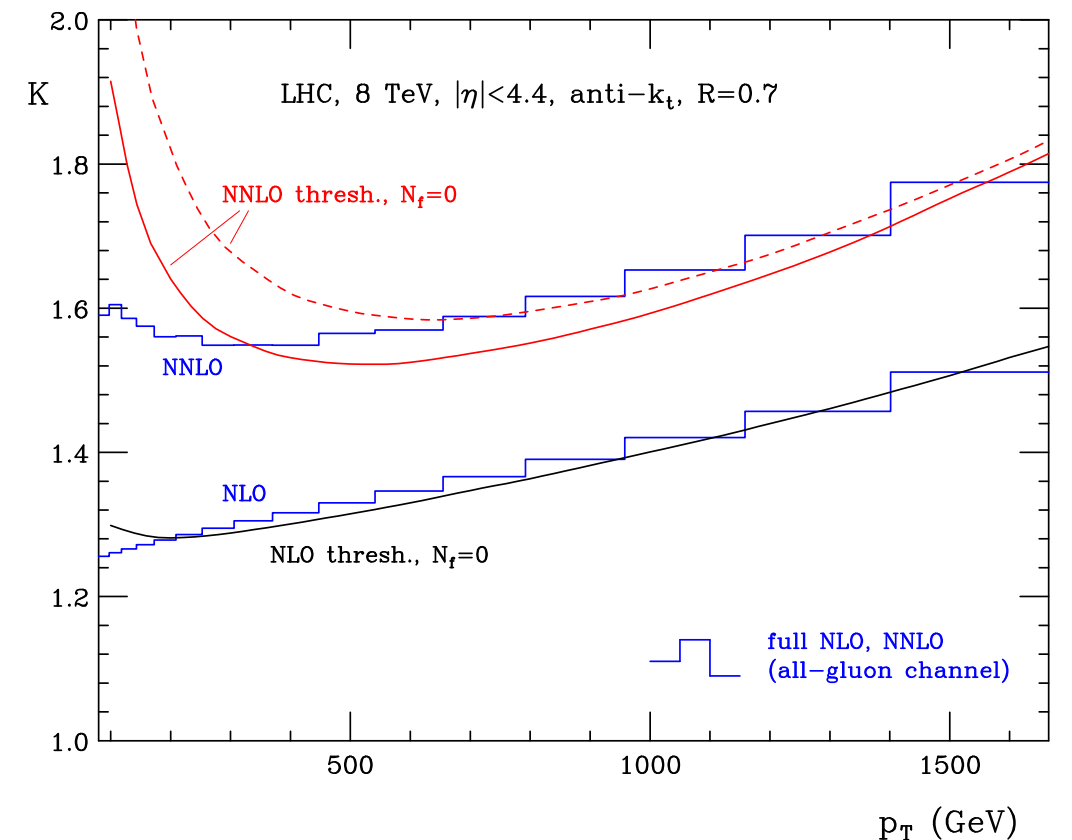
# NNLO correction- existing results

Inclusive jet production:  
double differential distributions

$R = 0.4$  Gehrmann-de-Ridder et al.



de Florian et al.



- Calculation of all-gluon contribution to jet production performed by Gehrmann-de-Ridder et al.
- Result appears to be fairly similar to threshold correction near threshold by de Florian et al (arXiv:1310.7192). Overall  $\sim 5 - 10\%$  positive correction which increases at lower  $p_{\perp}$ .

# NNLO jets - PDF updates

- As default NNLO set still fits Tevatron data. Seems safe as data are always relatively near to threshold, and corrections do not obviously break down at lowest probed  $p_{\perp}$ .
- In order to test robustness: have repeated MSTW2008 fits with extreme modified K-factors for NNLO jets, i.e. multiply standard corrections by 0 or 2 and use constant  $K = 1.15$ . All within one sigma, even for extreme changes.
- Different story for LHC data. In general much farther away from threshold, lowest not stable in threshold corrections, and large uncertainty at highest rapidity. Therefore do not include in fit.
- Test: try putting in very approximate NNLO correction of  $\sim 5 - 20\%$  which grows at lower  $p_{\perp}$ . “Smaller” and “larger” K-factor with corrections of  $\sim 10\%$  and  $\sim 20\%$  at  $p_{\perp} = 100 \text{ GeV}$  - rapidity independent. Prediction is good: fit quality a small amount worse than at NLO, though deteriorates slowly with larger K-factor.

# Fit quality for LHC data - NNLO

- Jet data not fitted but quality checked using “smaller” K-factor,  $\sim 10\%$ .

data set	$N_{pts}$	CPdeut	no LHC	prelim
ATLAS jets (2.76TeV+7TeV)	116	(107)	(123)	(119)
CMS jets (7TeV)	133	(142)	(137)	(135)
ATLAS $W^+, W^-, Z$	30	72	53	39
CMS $W$ asymm $p_T > 35\text{GeV}$	11	18	15	9
CMS asymm $p_T > 25\text{GeV}, 30\text{GeV}$	24	18	17	10
LHCb $Z \rightarrow e^+e^-$	9	23	22	20
LHCb $W$ asymm $p_T > 20\text{GeV}$	10	24	21	13
CMS $Z \rightarrow e^+e^-$	35	30	24	22
ATLAS High mass DY	13	18	16	17
TeV, ATLAS, CMS $\sigma_{t\bar{t}}$	13	8	11	8
CMS Low-high mass DY	132	159	151	149

- Large improvement after fit in ATLAS  $W, Z$  data, mainly from strange quark, and in CMS  $Z \rightarrow e^+e^-$  data, CMS  $W$  asymmetry and LHCb  $W^+, W^-$  data.
- CMS  $W$  asymmetry data constrains some flavour decomposition.

# Fit quality for LHC data - NNLO

- Jet data not fitted but quality checked using “larger” K-factor,  $\sim 20\%$ .

data set	$N_{pts}$	CPdeut	no LHC	prelim
ATLAS jets (2.76TeV+7TeV)	116	(117)	(132)	(128)
CMS jets (7TeV)	133	(145)	(137)	(139)

and “smaller” K-factor,  $\sim 10\%$

data set	$N_{pts}$	CPdeut	no LHC	prelim
ATLAS jets (2.76TeV+7TeV)	116	(107)	(123)	(119)
CMS jets (7TeV)	133	(142)	(137)	(135)

- **ATLAS** jet data deteriorates more than **CMS**, which with increase in systematics is largely insensitive to **K**-factor, though even prefers smaller one. Difficult to guess relative size of **K**-factor at two different energies.



# NNLO fit with jets - change in gluon

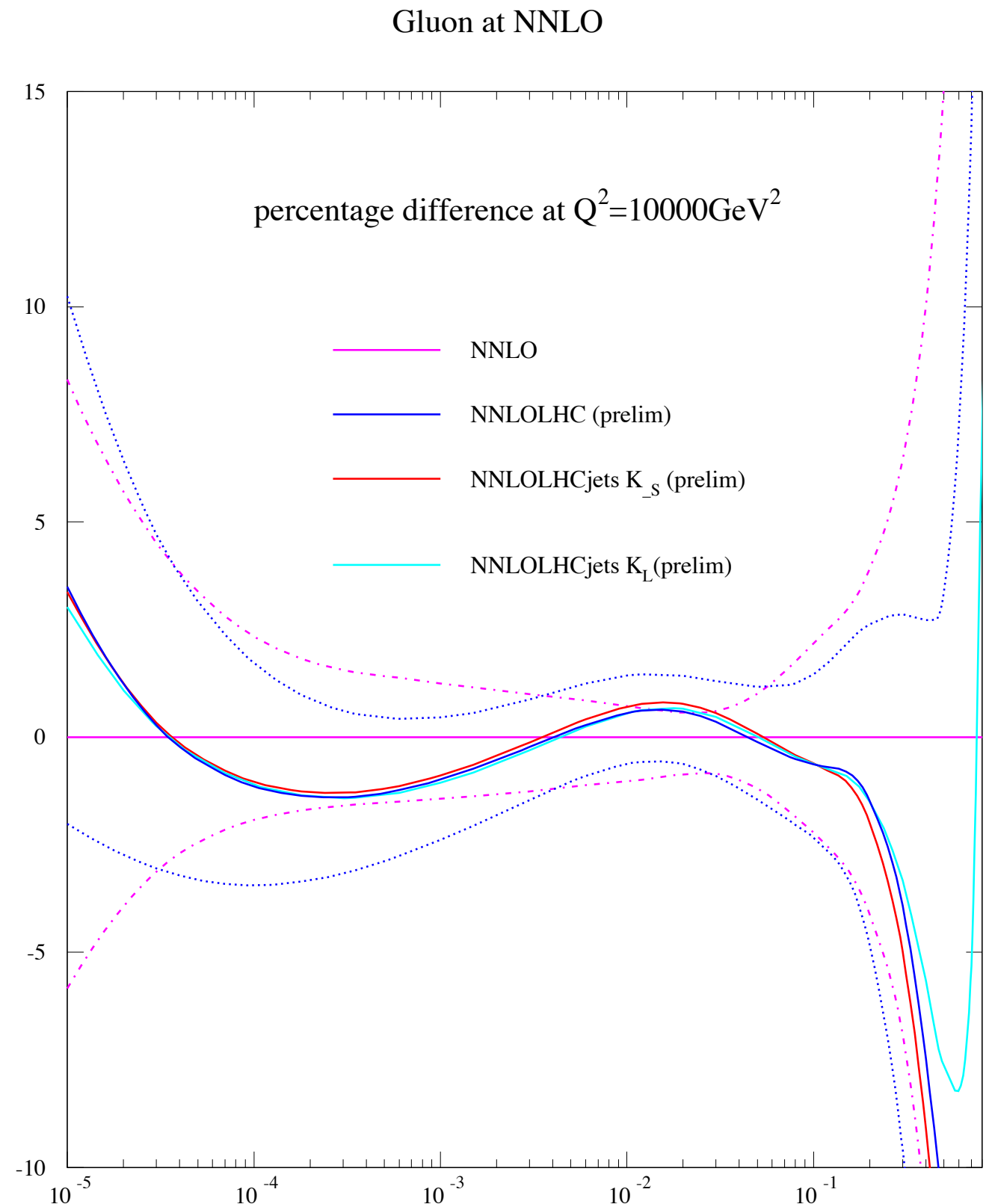
- Ratio of  $g(x, Q^2)$  for the default NNLO fit to that in MSTW2008, and also fits where jet data are included with “smaller” and “larger” K-factor.
- In both cases changes in gluon,  $\alpha_S(M_Z^2)$  and fit to other data are extremely small.

“Smaller” K-factor (  $\sim 10\%$  )

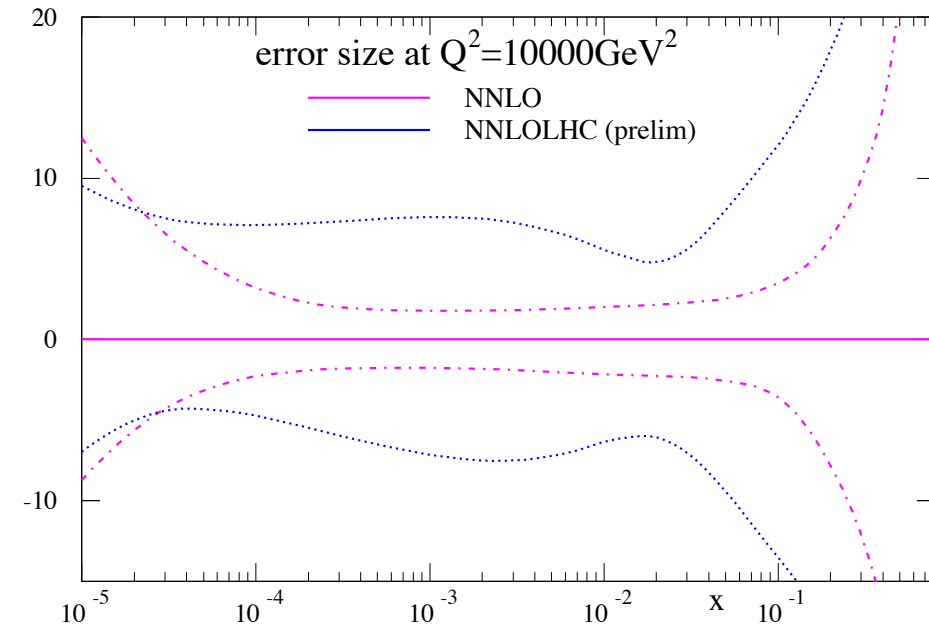
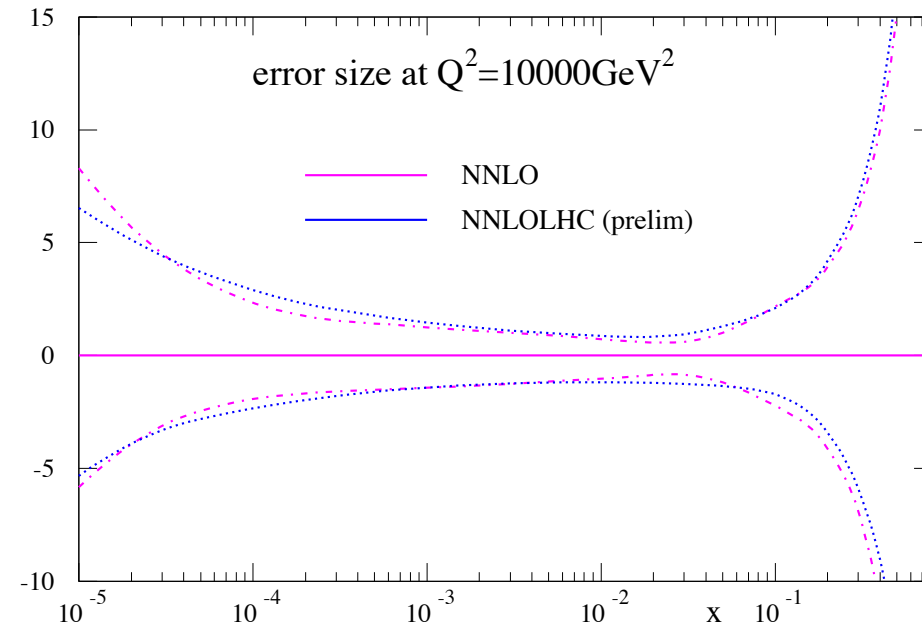
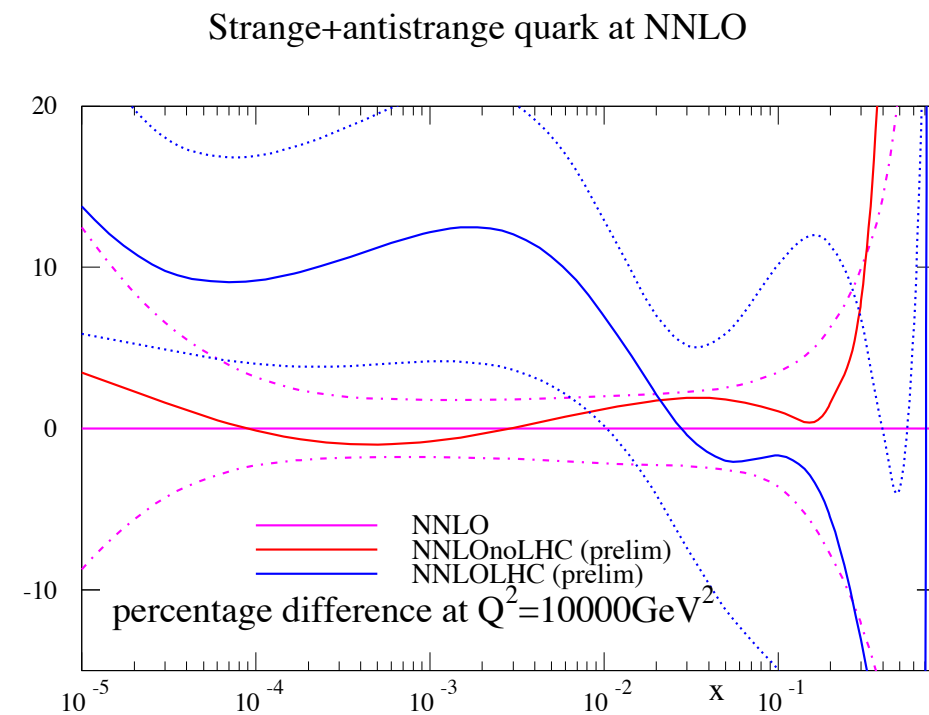
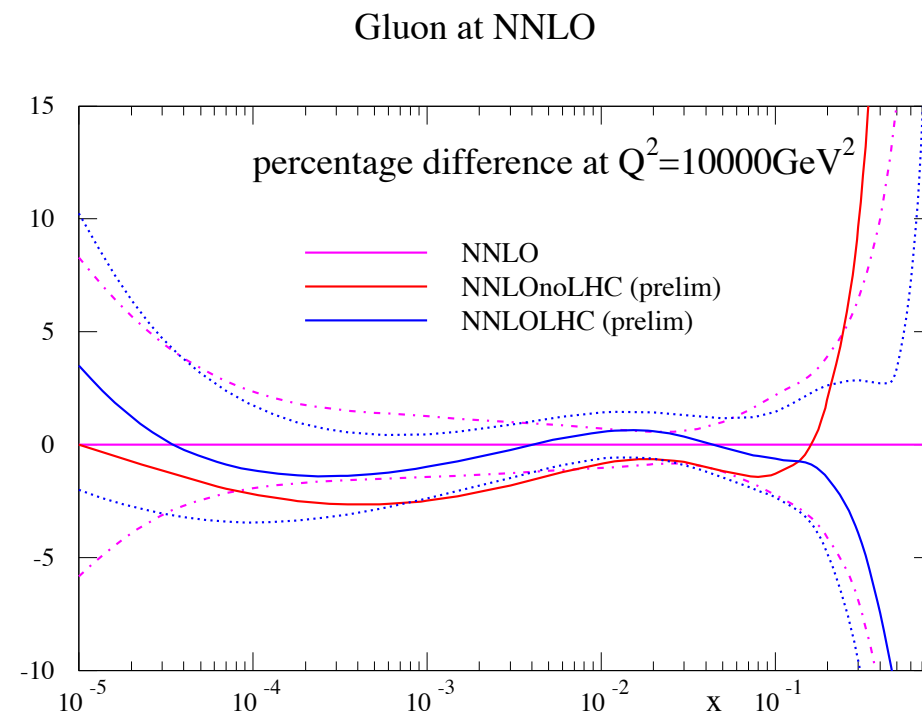
ATLAS  $\chi^2 = 119/116 \rightarrow 106/116$   
and CMS  $\chi^2 = 138/133 \rightarrow 139/133$

“Larger” K-factor (  $\sim 20\%$  )

ATLAS  $\chi^2 = 128/116 \rightarrow 118/116$   
and CMS  $\chi^2 = 139/133 \rightarrow 141/133$



# Change in NNLO PDFs



Includes theoretical updates and LHC data (not absolutely final). Gluon uncertainty slightly larger at high  $x$  - no jet data in fit.

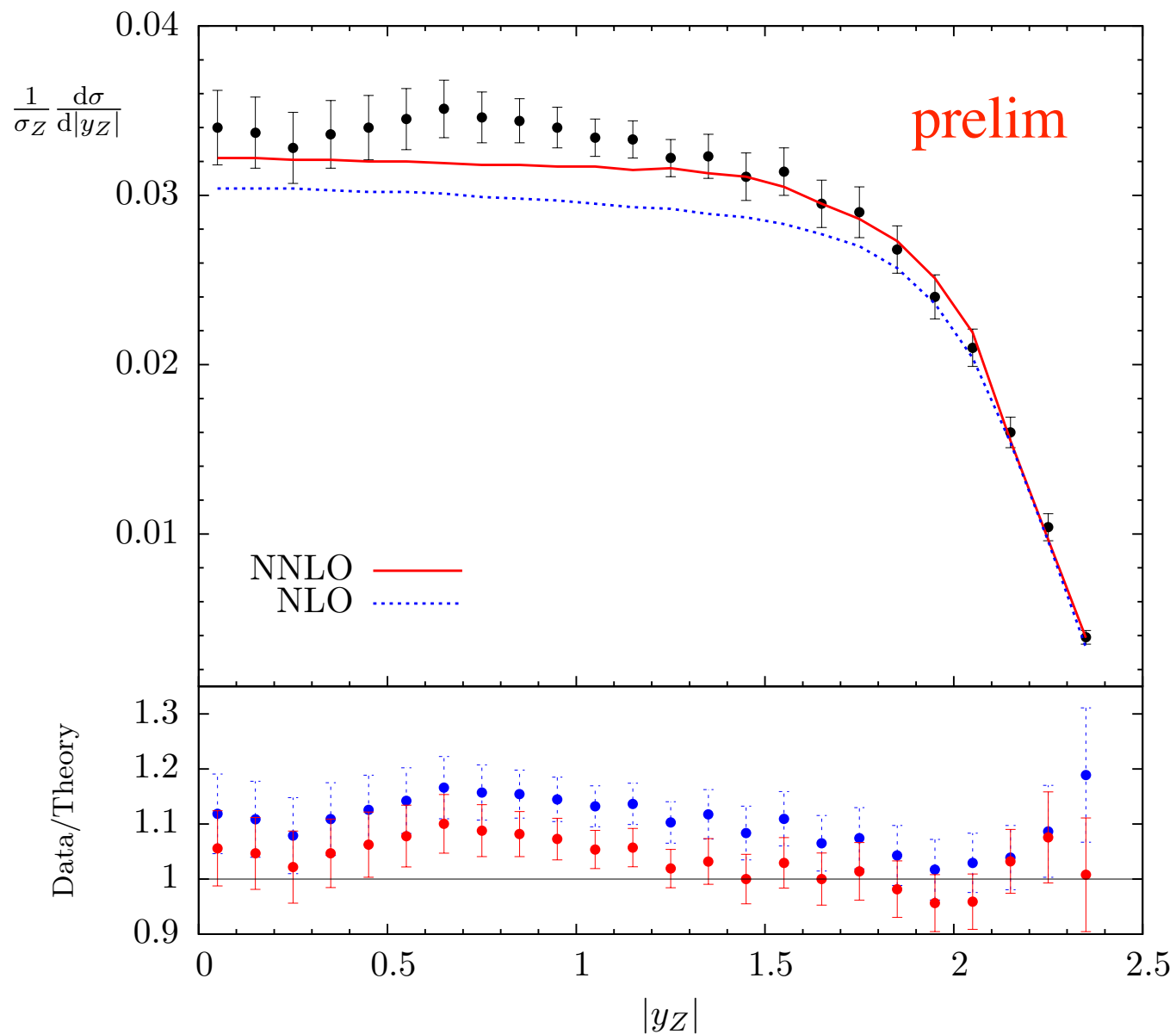
# CMS Drell-Yan data

- Fit very poor at **NLO**:

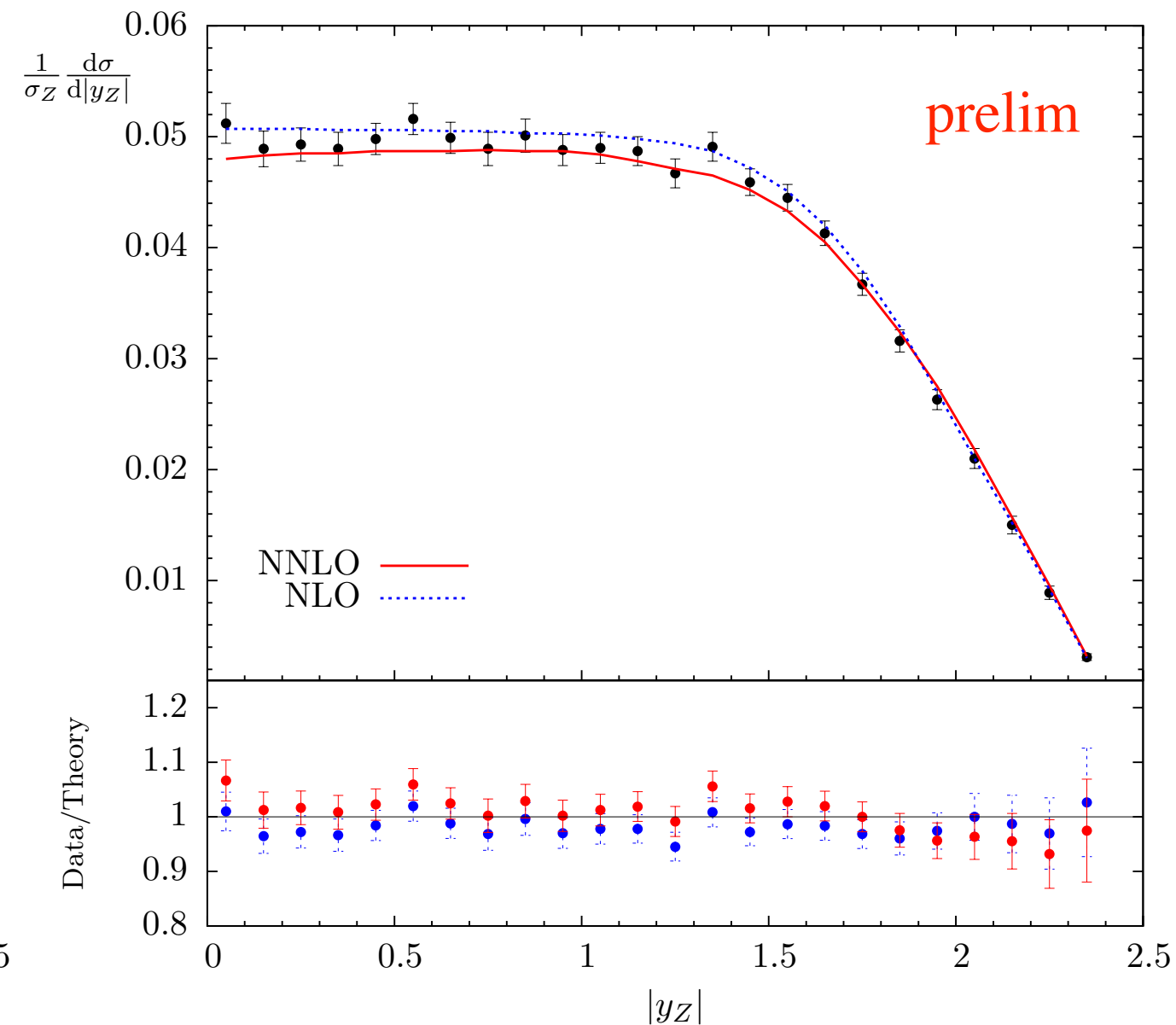
data set		$N_{pts}$	CPdeut	no LHC	prelim
CMS Low-high mass DY	NNLO	132	159	151	149
CMS Low-high mass DY	NLO	132	385	396	373

- In lowest mass bins,  $20 < M_{ll} < 45$  GeV, cuts on leptons ( $p_{\perp} > 9, 14$  GeV)  
mean **NLO** is effectively **LO**, and fit is very poor:

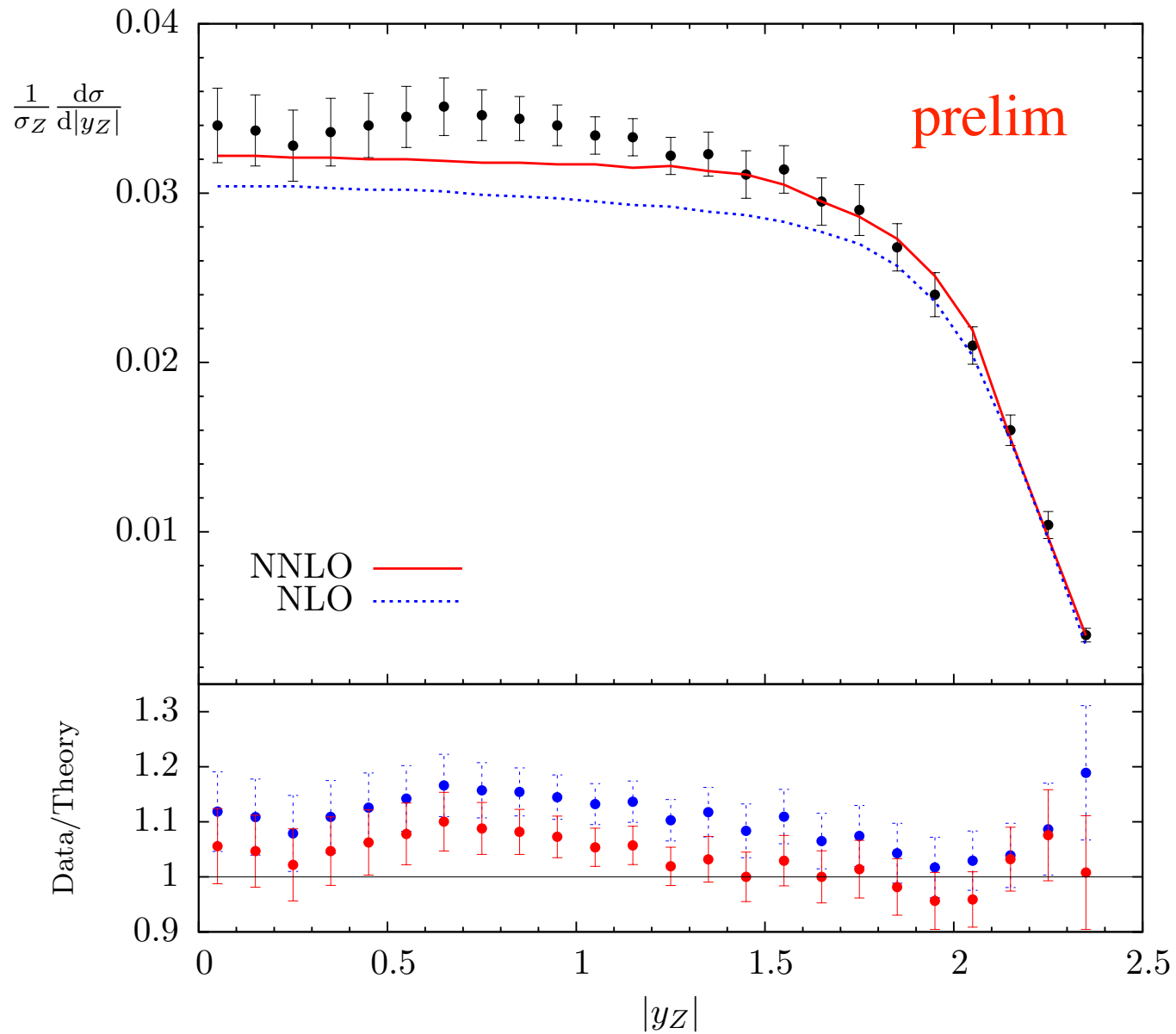
$20 < M_{ll} < 30$  GeV



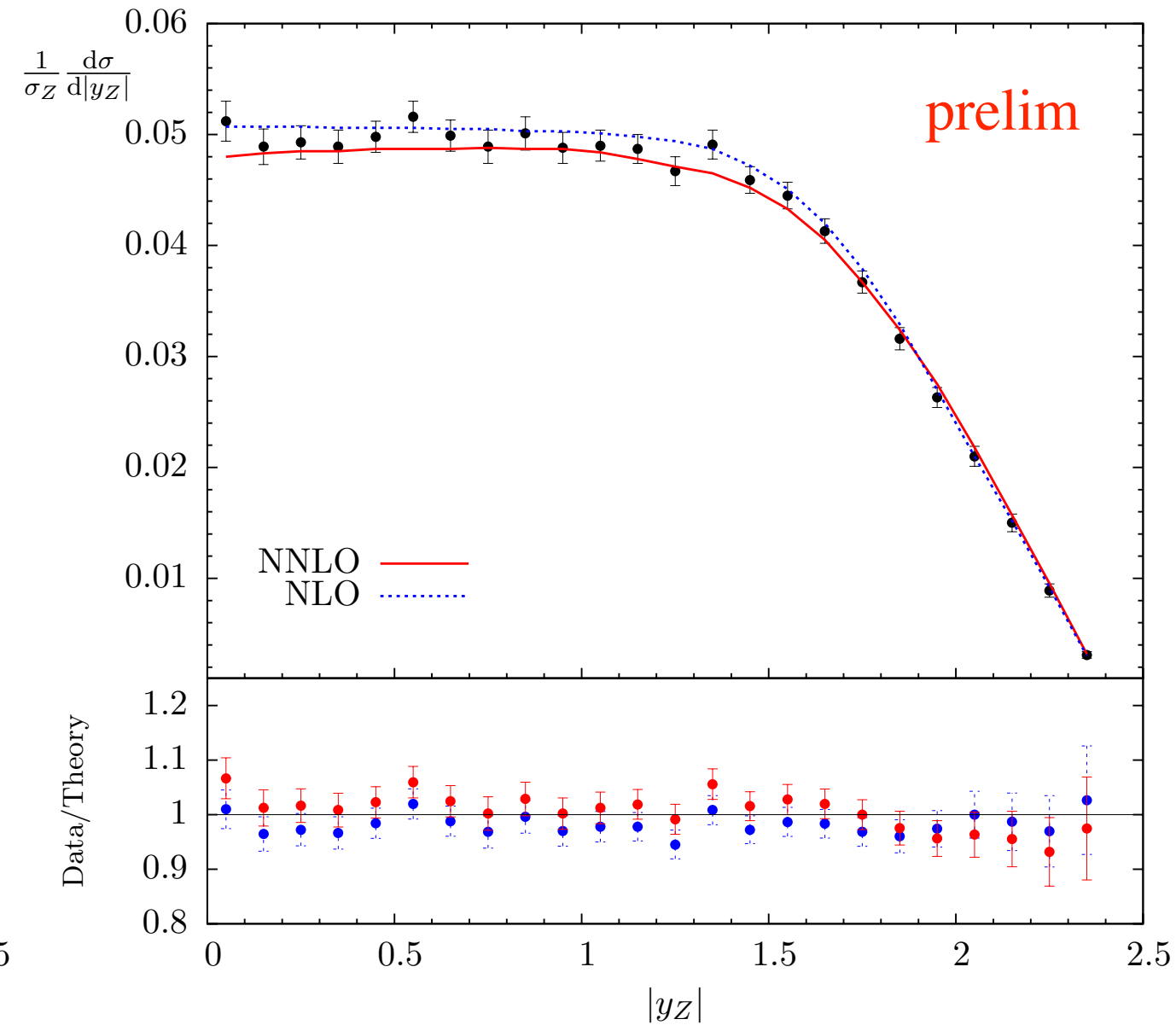
$30 < M_{ll} < 45$  GeV



$20 < M_{ll} < 30 \text{ GeV}$



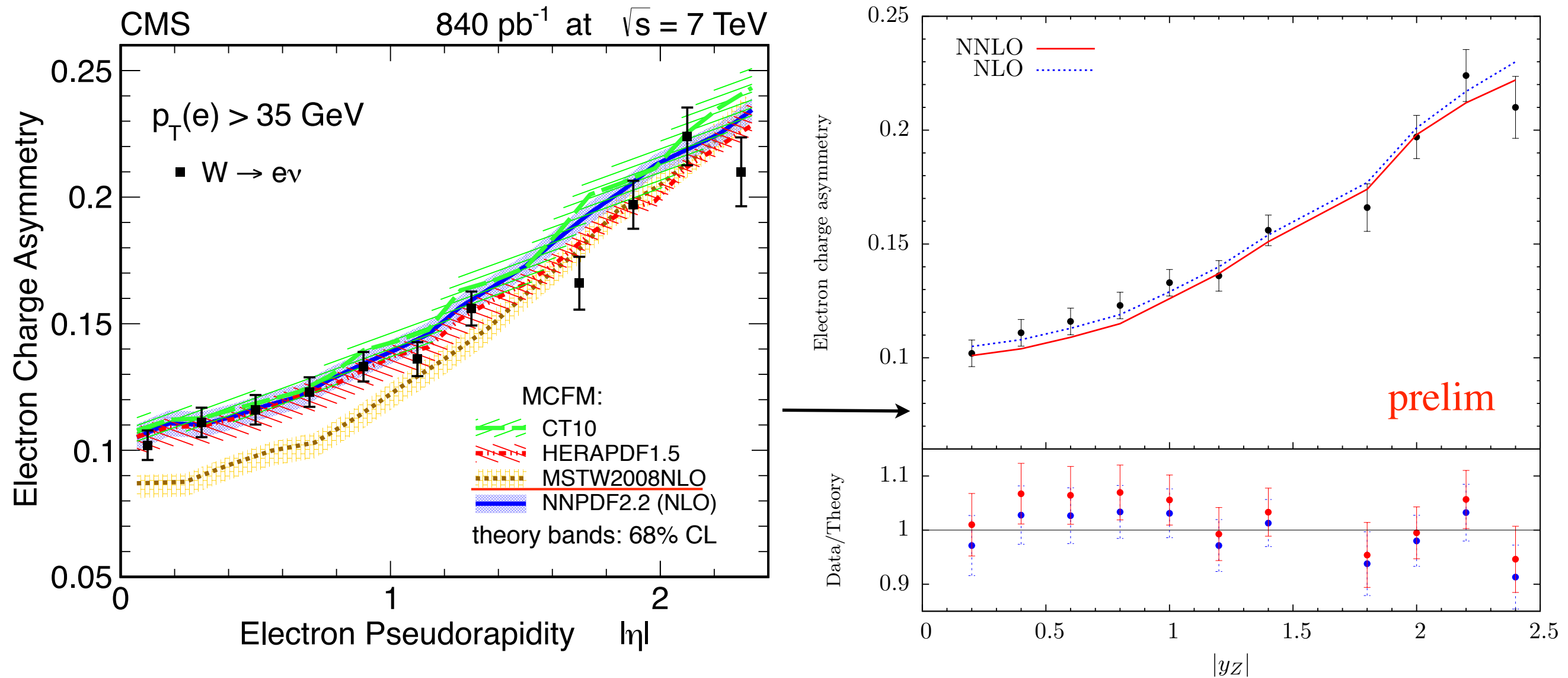
$30 < M_{ll} < 45 \text{ GeV}$



- Enormously improved fit quality at **NNLO** due to improvement in theory.
- Sensitivity to strange fraction in quarks, but differs at **NLO** and **NNLO** and weak compared to direct constraint from dimuon data.

# CMS W asymmetry - fit

- CMS W asymmetry ([arXiv:1206.2598](https://arxiv.org/abs/1206.2598)) - no longer an issue (extended parametrization and deuteron corrections).



NNLO

data set	$N_{pts}$	CPdeut	no LHC	prelim
CMS $W$ asymm $p_T > 35\text{GeV}$	11	18	15	9

## $\alpha_S(M_Z^2)$ as a data point

- $\alpha_S(M_Z^2)$  coming out similar to 2008 fit. Still a NLO/NNLO difference. Both fairly compatible with global average  $\rightarrow$  try inputting this as a data point.
- Try world average (minus DIS data) of  $\alpha_S(M_Z^2) = 0.1187 \pm 0.0007$  (rather small uncertainty, mainly from lattice):

NLO : already within one sigma, essentially no change -  $\alpha_S(M_Z^2) = 0.1199 \rightarrow 0.1195$  with  $\Delta\chi^2 < 2$ .

NNLO : best fit gives  $\alpha_S(M_Z^2) = 0.1172 \rightarrow 0.1177$ , i.e. very close to 0.118.  $\Delta\chi^2 < 2$

- Also force  $\alpha_S(M_Z^2) = 0.118$ :

NLO :  $\Delta\chi^2 \sim 16$ , but not single set deteriorates significantly.

NNLO : basically no further change.

# Conclusions

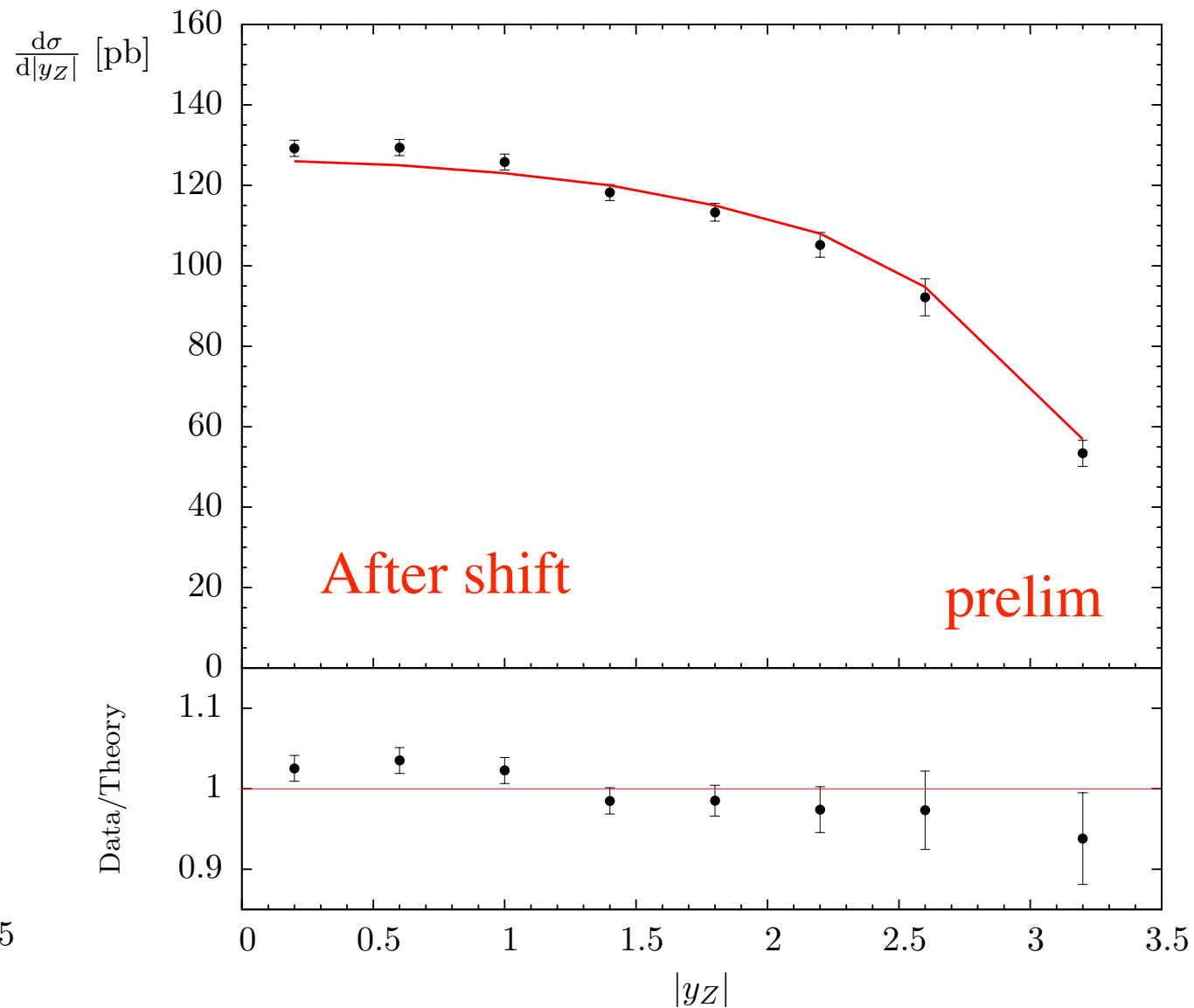
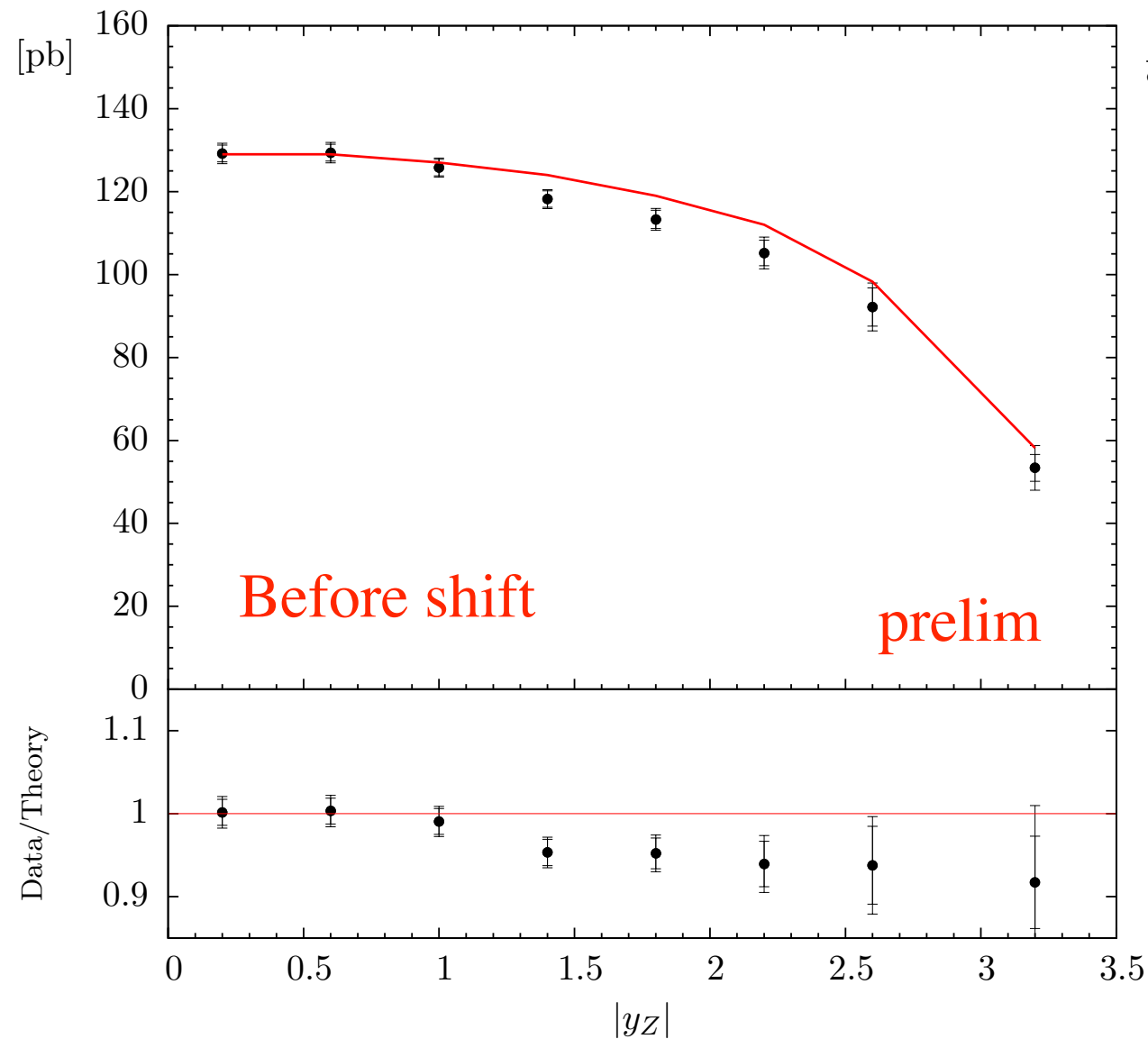
- Ongoing, but very near final, updates on PDFs - soon to be released.
- Various theoretical improvements described: parameterisation, deuteron corrections, heavy flavour treatments, nuclear corrections, branching ratio for dimuon data.
- Inclusion of up-to-date [HERA](#) and [Tevatron](#) data
- Directly included most relevant published [LHC](#) data: [ATLAS](#), [CMS](#), [LHC](#) W,Z rapidity data, top cross sections and all published [ATLAS](#) and [CMS](#) inclusive jet data (but not at [NNLO](#)).
- Fit good (except for [CMS](#) double differential at [NLO](#) - but clear reason for this). No PDF conflicts.
- So far few dramatic effects on PDFs. Mainly strange quark and low  $x$  valence quarks, largely due to change in methodology, but also newer data. Larger strange uncertainty from branching ratio error.
- Some uncertainty in [NNLO](#) effect on jets. Have decided at present to wait for full [NNLO](#) calculation. However, comparison now suggests [NNLO](#) fit is happy with moderate guesses for K-factors, with no real change in PDFs or coupling.

# Backup



# Correlated errors - data shift

ATLAS Z rapidity (arXiv:1109.5141)



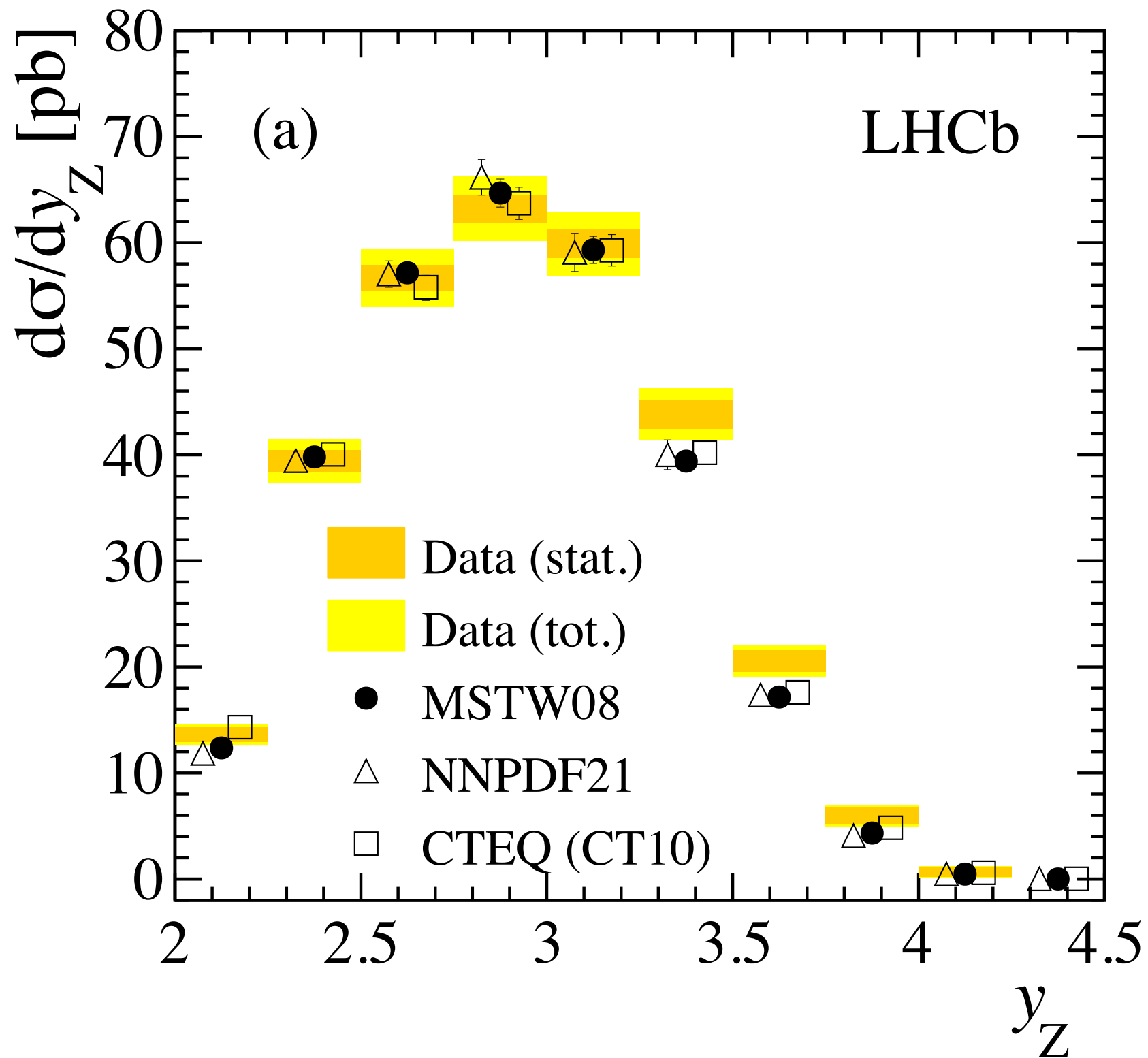
Recall treatment of correlated errors:  $\chi^2 \sim \left( \frac{D_i - T_i/f}{\sigma_i^{uncorr}} \right)^2 = \left( \frac{f * D_i - T_i}{f * \sigma_i^{uncorr}} \right)^2$

→ Data (and error) allowed to shift by fraction  $f$  to give best fit.

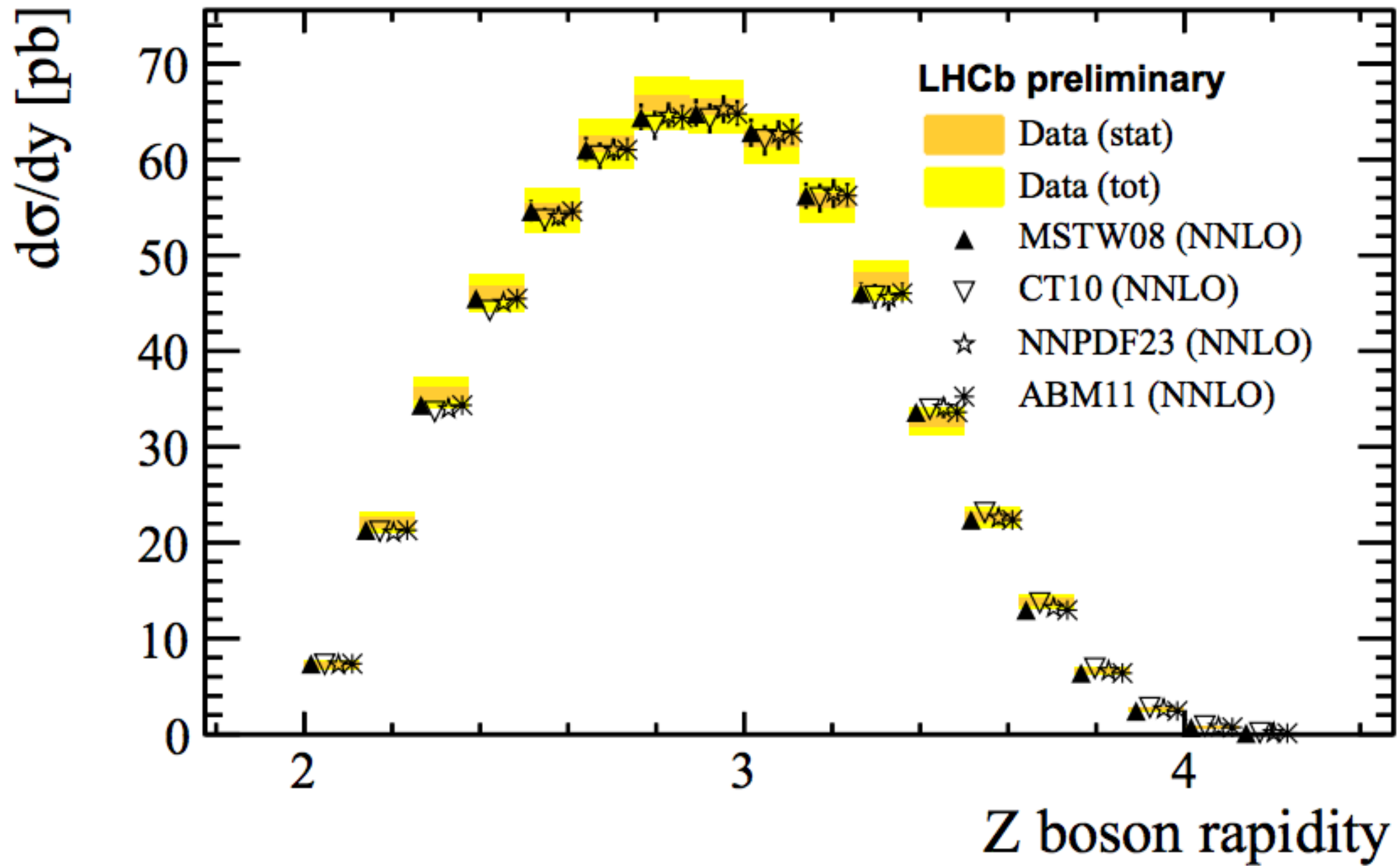
Change in various cross section predictions compared to uncertainty for **MSTW2008**.

	no LHC NLO	no LHC NNLO	LHC NLO	LHC NNLO	unc.
$W$ Tevatron (1.96 TeV)	+1.0	+2.1	-0.5	+0.2	1.8
$Z$ Tevatron (1.96 TeV)	+2.4	+2.6	+0.5	+0.1	1.9
$W^+$ LHC (7 TeV)	+2.5	+0.9	+0.3	-1.1	2.2
$W^-$ LHC (7 TeV)	-0.3	+1.1	-0.8	-1.9	2.2
$Z$ LHC (7 TeV)	+1.1	+1.1	+0.2	-1.5	2.2
$W^+$ LHC (14 TeV)	+3.0	+0.8	+0.7	-0.9	2.4
$W^-$ LHC (14 TeV)	+0.6	+0.6	-0.3	-1.6	2.4
$Z$ LHC (14 TeV)	+1.7	+0.6	+0.2	-0.6	2.4
Higgs Tevatron	-3.5	+2.8	-3.1	-3.2	5.1
Higgs LHC (7 TeV)	-1.2	+0.9	-1.4	-2.1	3.3
Higgs LHC (14 TeV)	-2.0	+0.1	-1.2	-2.3	3.1
$t\bar{t}$ Tevatron	+0.5	+4.9	-1.6	-0.7	3.2
$t\bar{t}$ LHC (7 TeV)	-3.1	+3.3	-2.9	-2.5	3.9
$t\bar{t}$ LHC (14 TeV)	-2.0	+1.7	-2.0	-2.0	3.1

Some changes of order size of uncertainty - none dramatic.



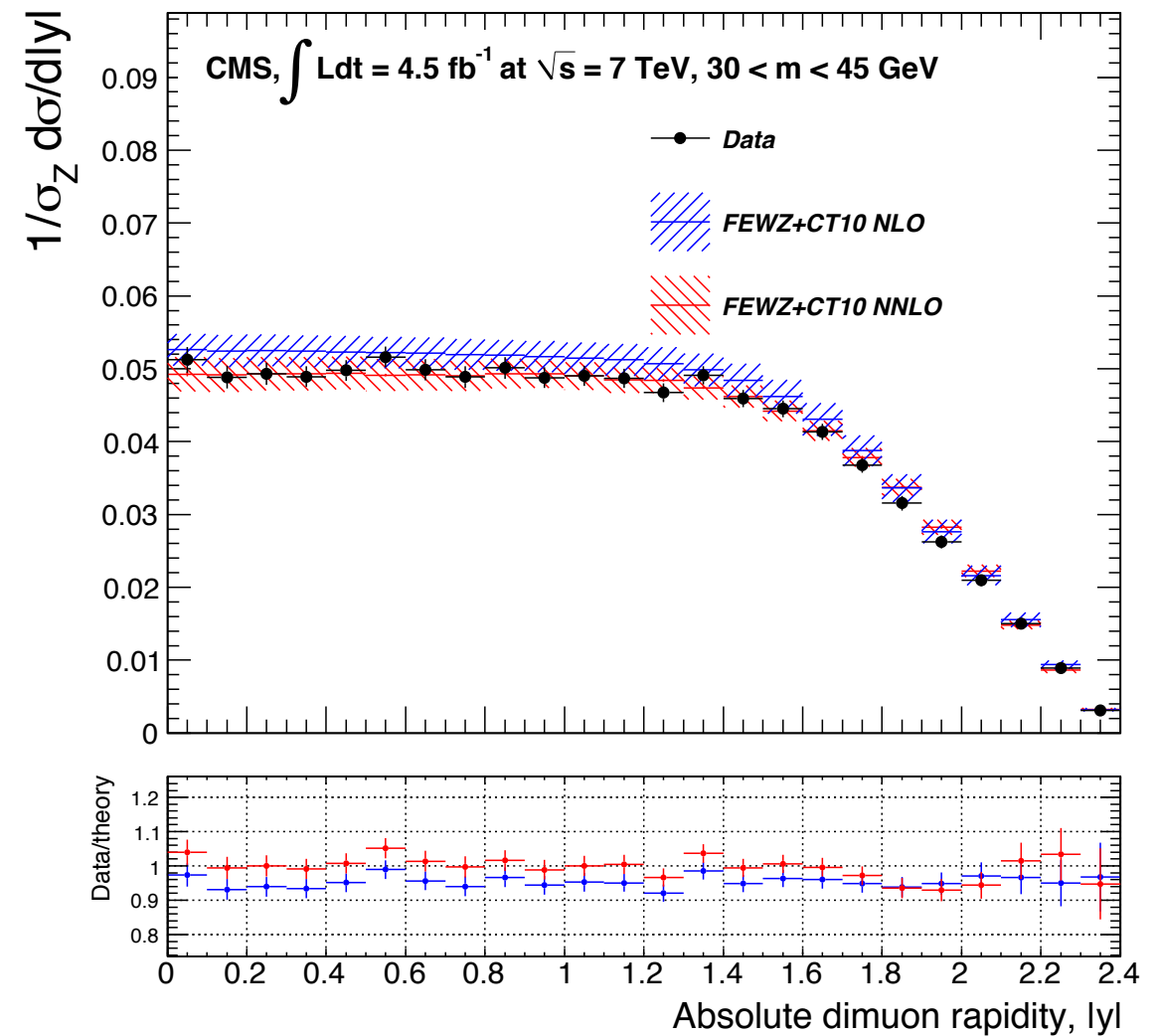
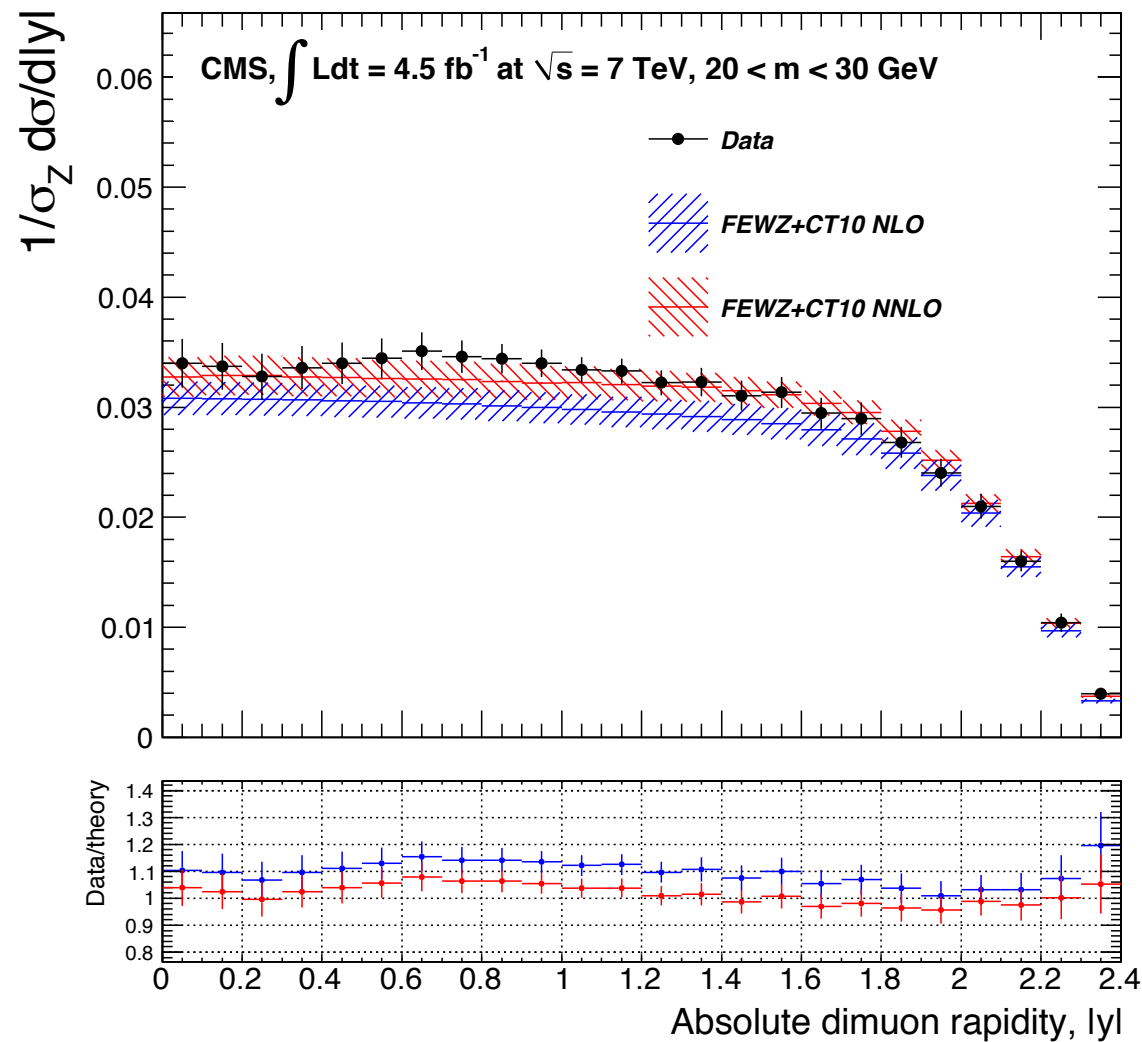
Points near to  $y = 3.5$  overshoot predictions in general. Feature not present in prelim. higher luminosity  $Z \rightarrow \mu^+ \mu^-$  data.



Higher luminosity LHCb  $Z \rightarrow \mu^+ \mu^-$  data.

## CMS Drell Yan data.

Fit very poor at **NLO** in lowest mass bins (where it is effectively **LO**), even when data highly weighted.



Repeat **MSTW2008** fits with modified K-factors for **NNLO** jets, i.e. multiply standard correction by **0** or **2** and use constant  **$K = 1.15$** .

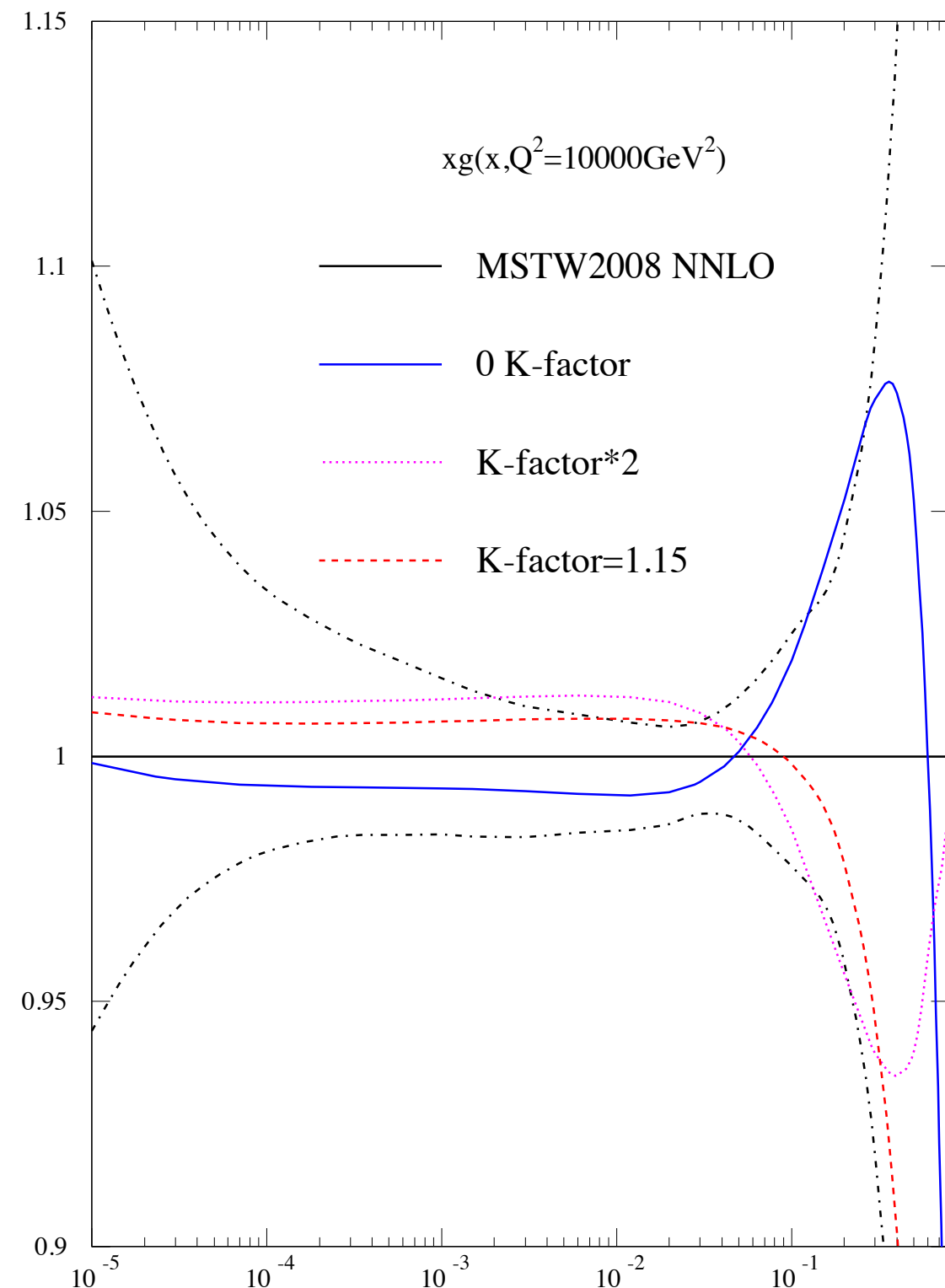
Extreme variations.

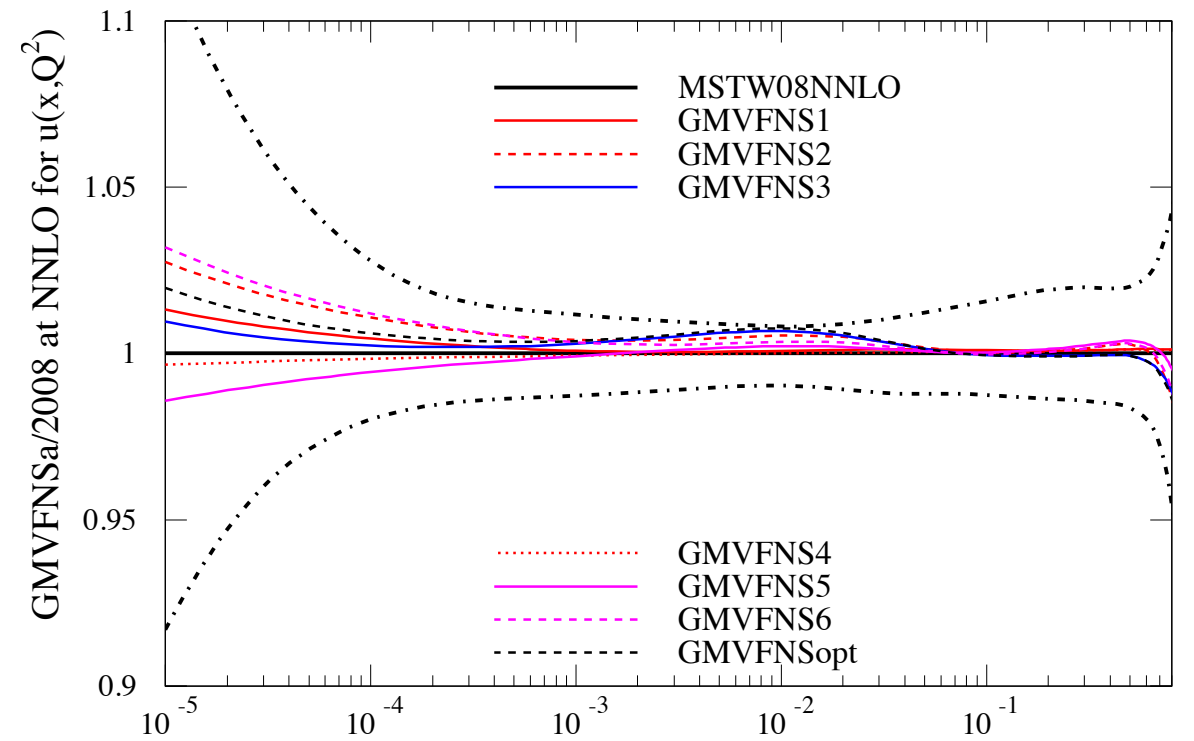
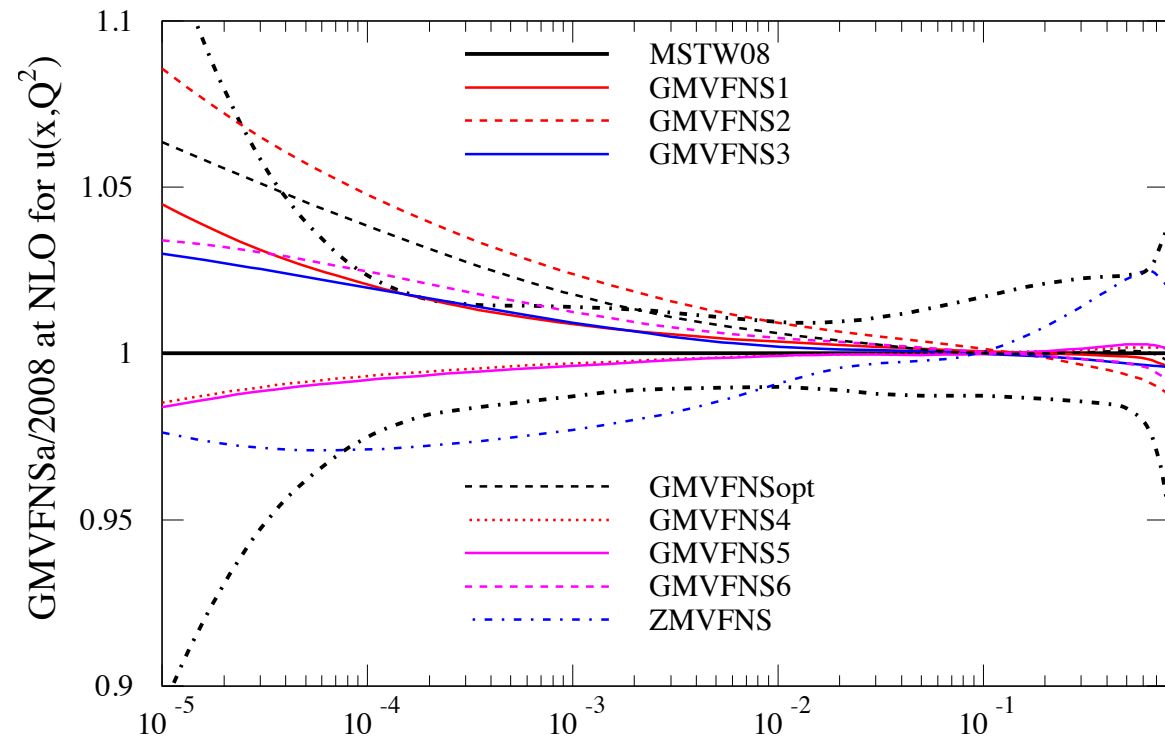
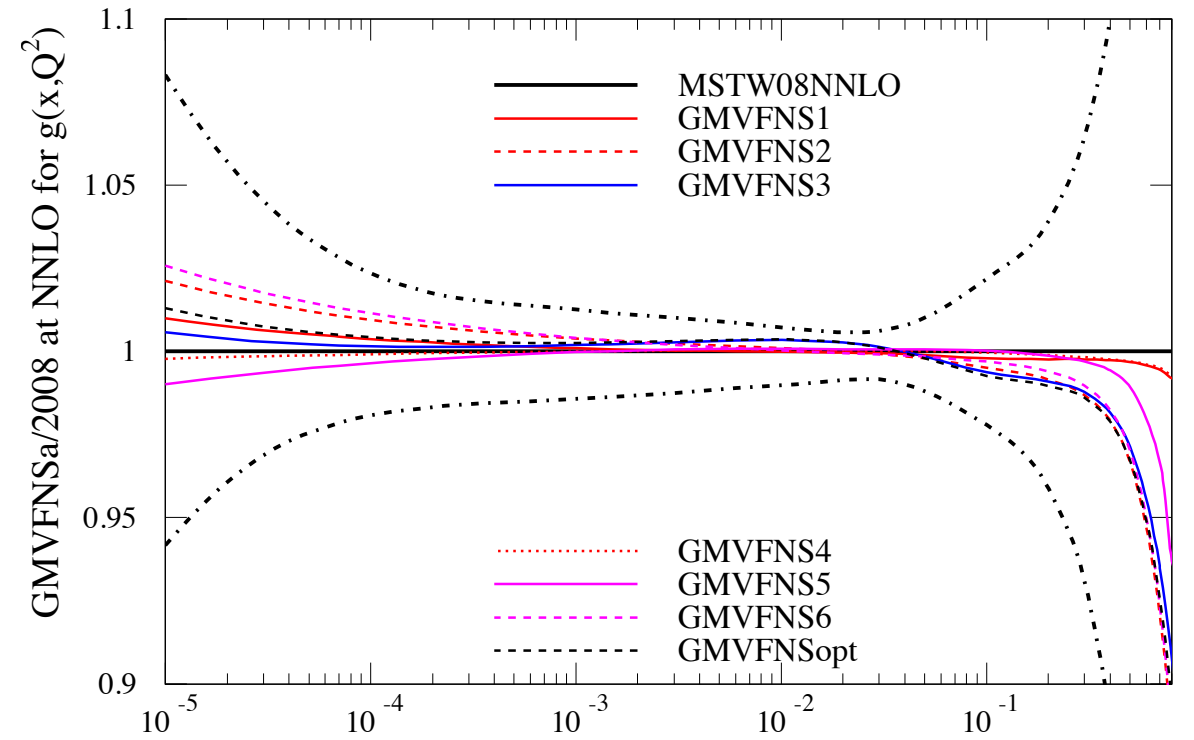
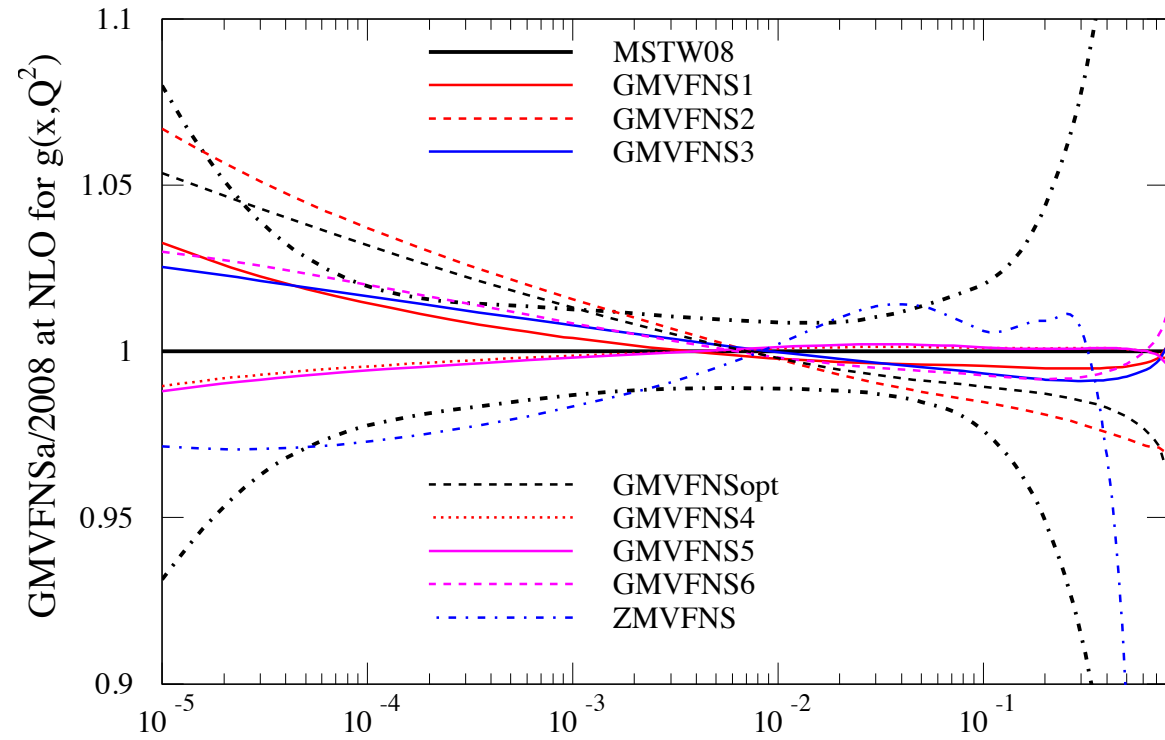
Changes in gluon relatively small. Larger K-factor slightly worse  $\chi^2$ . Zero K-factor slightly better  $\chi^2$ ,  **$K = 1.15$**  almost no change.

$$K = 0 \quad \alpha_S(M_Z^2) = 0.1181$$

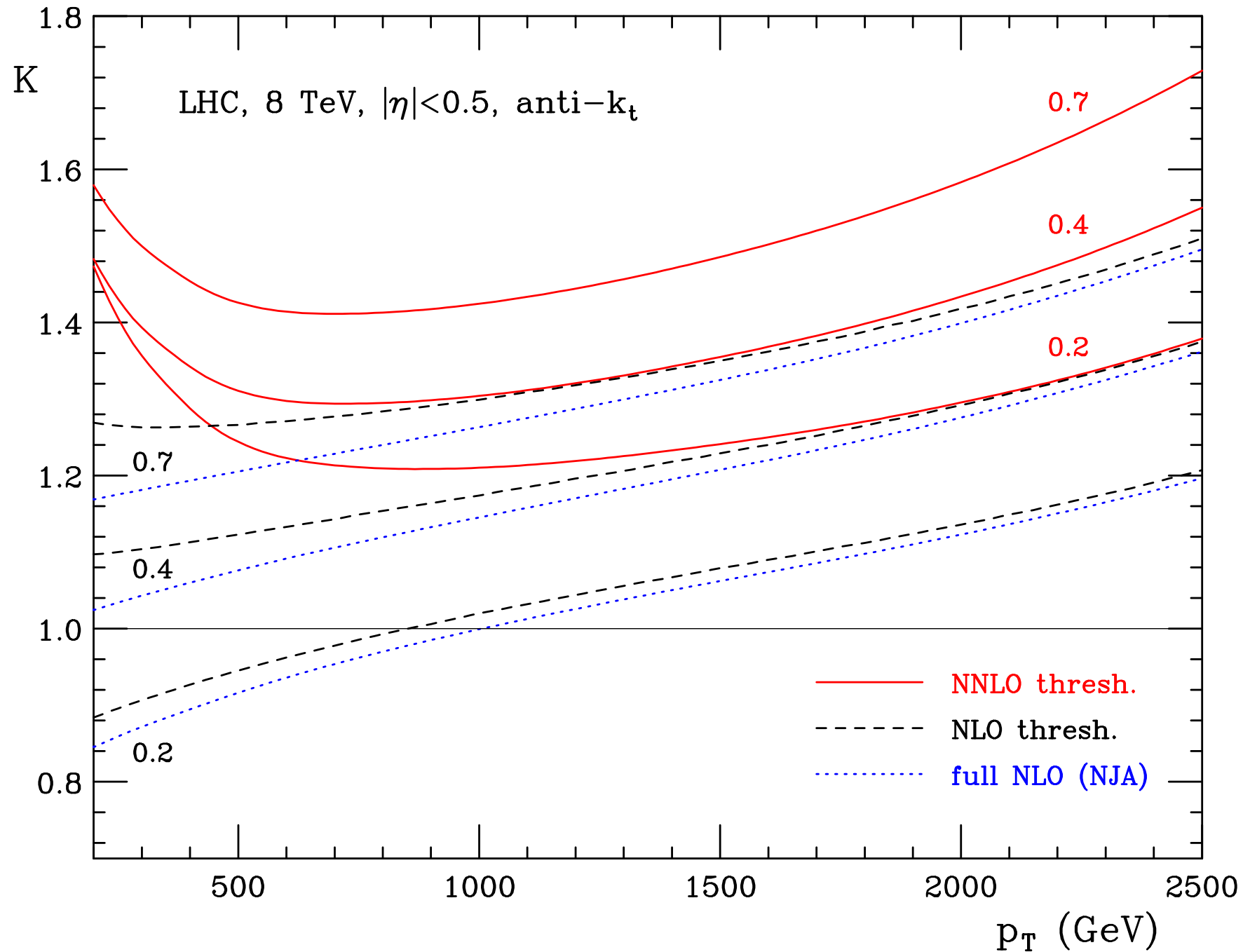
$$K * 2 \quad \alpha_S(M_Z^2) = 0.1159$$

$$K = 1.15 \quad \alpha_S(M_Z^2) = 0.1167$$





Using smoother schemes leads to some change in PDFs, with tendency for slight increase at small  $x$  and slight decrease at high  $x$  for gluon. Much smaller at **NNLO** than **NLO**. No real change in  $\alpha_S(M_z^2)$ .



- Very recent improved calculation by [de Florian et al. \(arXiv:1310.7192\)](#) has built in **R** dependence. Shows correct variation at **NLO** but little extra **R** dependence at **NNLO**. Still has problems at low  $p_\perp$ .