Take a Walk to the Driplines

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Outline

- The Shell Model Realm; Where and How?
- SM calculations: many nucleons and many orbits: monopole anomalies and multipole universality
- Quadrupole dominance: The role of Monopole and Pairing
- From N=2Z, ^{60}Ca, to ^{68}Ni and N=Z, ^{80}Zr

Why Shell Model?

- To connect rigorously the free space nucleon nucleon interaction with the experimental spectroscopic data has been the Holy Grail of Nuclear Physics since its inceptions.
- Thanks to the breakthroughs of Brueckner, Kuo and Brown, and others, the goal seemed reachable.
- However, already at the end of the seventies, Pasquini and Zuker unveiled the monopole anomalies of the realistic effective interactions, which in particular did not produce the N=28 shell closures in ^{48}Ca and ^{56}Ni. But their origin was unclear.

The quest for the effective interaction today

- Evidence for the need of real three body forces came first from the GFMC and NCSM calculations of light nuclei.
- And χEFT has become the method of choice to obtain the NN and NNN interactions.
- New tools to improve the many body calculations are now available, like the use of V_{low-k} or the SRG techniques in conjunction with the NCSM or the CC approaches.

Many Particles and Orbits around the Fermi Level

- To solve the many body problem to spectroscopic accuracy, Large Scale Shell Model calculations have proven very successful when affordable.
- In other cases, approximations have to be made, either of physical (IBM) or mathematical (MCSM) nature
- Only recently, Beyond Mean Field calculations using Energy Density Functionals have been pushed to quantitative spectroscopy. However, many things that are trivial in LSSM, like the correct treatment of all the pairing channels or the inclusion of triaxial and higher multipolarity degrees of freedom, become extremely painful for BFM.

How?

- **LSSM calculations rely upon good effective interactions and physically sound valence space. But they need performant codes as well.**
- **The Antoine SM code computes 10^9 routinely and approaches the 10^{11} (M=0 Slater determinants) benchmark**

Where?

Physically sound and tractable SM valence spaces:

- **The classical 0$\hbar\omega$ spaces** *p*, *sd*, **and** *pf* **shells can be treated exactly.**
- **Nuclei (or states) at the** *p*-*sd* **and** *sd*-*pf* **borders, can be described to a very good approximation** *e. g.* **Low lying deformed and super deformed banda in ^{40}Ca.**
- **Neutron rich nuclei with protons in the** *sd*-**shell and neutrons in the** *pf*-**shell can be treated exactly as well** *e. g.* **^{32}Mg, ^{34}Si, ^{42}Si, ^{44}S.**
- **The space r3g ($1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$, $0g_{9/2}$), is also solved exactly. But its physical relevance is limited to a rather small part of its natural span,** *e. g.* **^{76}Ge, ^{82}Se.**

Where?

Physically sound and tractable SM valence spaces

- **The space r3gd ($1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$, $0g_{9/2}$ $1d_{5/2}$) for the neutrons and *pf* for the neutrons, for the very neutron rich isotopes from Calcium to Germanium.** *e. g.* **^{68}Ni, ^{64}Cr, ^{78}Ni, ^{80}Zn.**
- **The space *sdg* around ^{100}Sn**
- **The space r4h comprised between N=Z=50 and N=Z=82 for a small subset of the nuclei it encompasses; the Sn, Te , Xe and Ba isotopes up to N=82.**
- **Protons in r4h and neutrons in r5i; the very neutron rich Sn, Te , Xe and Ba isotopes, beyond N=82.**
- **Around ^{208}Pb**

Many Particles and Orbits around the Fermi Level

More is different:

Indeed, since you can treat more and heavier nuclei, but in addition, because some of these nuclei exhibit collective features which are better developed than in their *sd* shell Elliott's like precursors

And because their description was until now restricted to the mean field approaches

The Spherical Mean Field (Monopole Hamiltonian)

$$\mathcal{H}_m = \sum n_i \epsilon_i + \sum \frac{1}{(1+\delta_{ij})} \overline{V}_{ij}\, n_i (n_j - \delta_{ij})$$

the coefficients $\overline{V}$ are angular averages of the two body matrix elements, or centroïds of the two body interaction:

$$\overline{V}_{ij} = \frac{\sum_J V^J_{ijij}[J]}{\sum_J [J]}$$

the sums running over Pauli allowed values.

The Spherical Mean Field (Monopole Hamiltonian)

This can be written as well as:

$$\mathcal{H}_m = \sum_i n_i \left[\epsilon_i + \sum_j \frac{1}{(1+\delta_{ij})} \overline{V}_{ij} \left(n_j - \delta_{ij}\right) \right]$$

Thus

$$\mathcal{H}_m = \sum_i n_i \;\; \hat{\epsilon}_i(\left[n_j\right])$$

We call these $\hat{\epsilon}_i(\left[n_j\right])$ effective single particle energies (ESPE)

Effective Single Particle Energies

They give the evolution of the underlying (non observable) spherical mean field (aka, shell evolution) as we add particles in the valence space, as well the variations of the spherical mean field in a single nucleus for states which have different configurations.

They are the control parameter for the nuclear dynamics, given the universality of the nuclear correlators.

Monopole anomalies of the realistic NN interactions

They are the more blatant in the neutron-neutron interaction; for instance not producing a magic ^{48}Ca, or the location of the drip line in the Oxygen isotopes

On the contrary their monopole neutron proton tensor part is correct, and the spin orbit splittings well accounted for.

The blame probably rest in missing residual three body effects

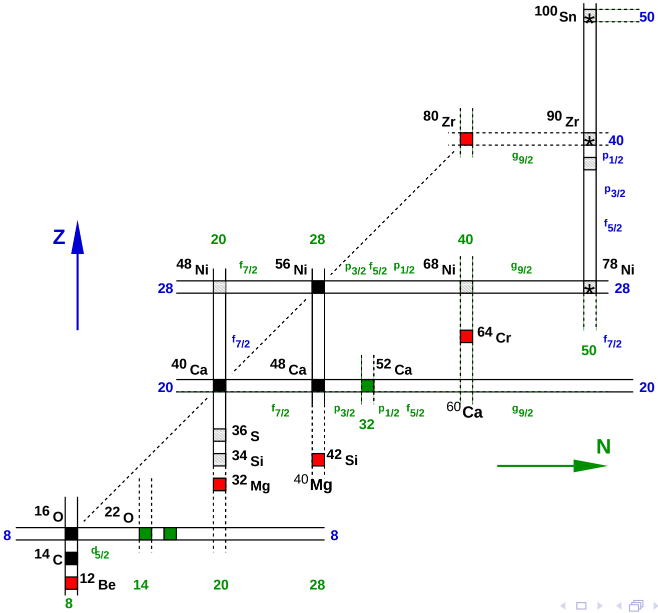
The Nuclear Correlators (Multipole Hamiltonian)

- The multipole hamiltonian is responsible for the collective nuclear behavior. It is universal and well given by the realistic NN interactions. Its main components are:
- BCS-like isovector and isoscalar pairing. When pairing dominates, as in the case of nuclei with only neutrons (or only protons) on top of a doubly magic nucleus, it produces nuclear superfluids.
- Quadrupole-Quadrupole and Octupole-Octupole terms of very simple nature ($r^\lambda Y_\lambda \cdot r^\lambda Y_\lambda$) which tend to make the nucleus deformed. In this limit, the pairing correlations mainly show up as responsible for the moment of inertia of the nuclear rotors.

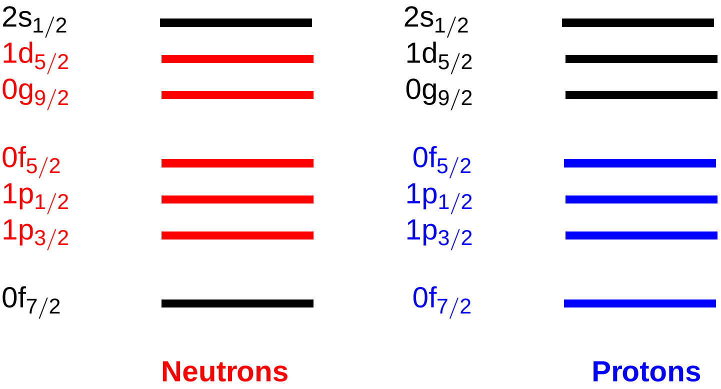
Why do the quadrupole correlations thrive in the nucleus?

- When valence protons an neutrons occupy the degenerate orbits a major oscillator shell, and for an attractive Q·Q interaction, the many body problem has an analytical solution in which the ground state of the nucleus is maximally deformed (Elliott's model)
- In cases when both valence neutrons and protons occupy quasi-degenerate orbits with Δj= 2 and Δj=2, including j=p+1/2 (Quasi-SU3), or quasi-spin multiplets (Pseudo-SU3)
- For example, $0f_{7/2}$ and $1p_{3/2}$, or $0g_{9/2}$ $1d_{5/2}$ and $2s_{1/2}$ form Quasi-SU3 multiplets and $0f_{5/2}$, $1p_{3/2}$ and $1p_{1/2}$ a Pseudo-SU3 triplet

Landscape of medium mass exotica

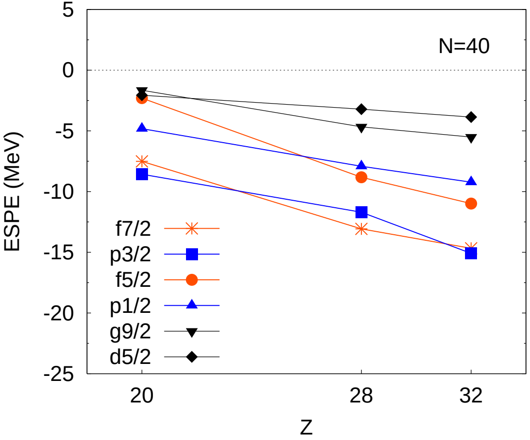


The valence space adequate for the N=40 isotones contains all these ingredients



For N=Z, ^{56}Ni provides a good core. Approaching N=2Z, one should rather switch to ^{48}Ca

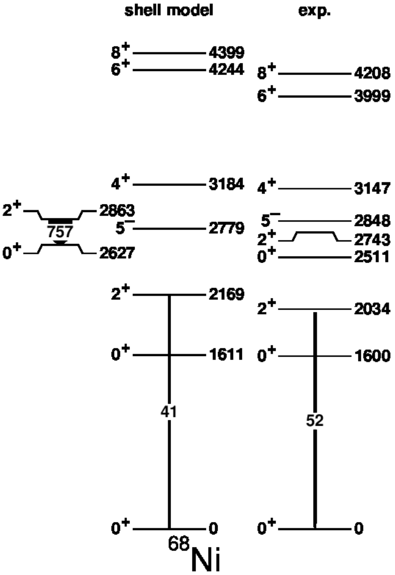
The neutron ESPES at N=40. Anchoring in ^{60}Ca.



These ESPES are enforced into the LSSM calculations by the experiment. They should provide a prime meeting point with the "ab initio" calculations. Notice the unorthodox spherical mean field at Z=20. A similar situation is found for the N=20 isotopes at Z=8.

Triple coexistence in ^{68}Ni

- In a first approximation, ^{68}Ni has a double shell closure in its *gs*
- But its low energy spectrum is much more complex
- Three coexisting 0^+ states appear between 0 and $\sim$ 2.5 MeV
- The 0^+_2 state excitation energy has been remeasured to be 1.6 MeV (F. Recchia et al. 2013)
- And the 0^+_3 (A. Dijon et al. 2012) is predicted by the LSSM calculations to be the band head of a very low-lying superdeformed band of $6p - 6h$ nature!



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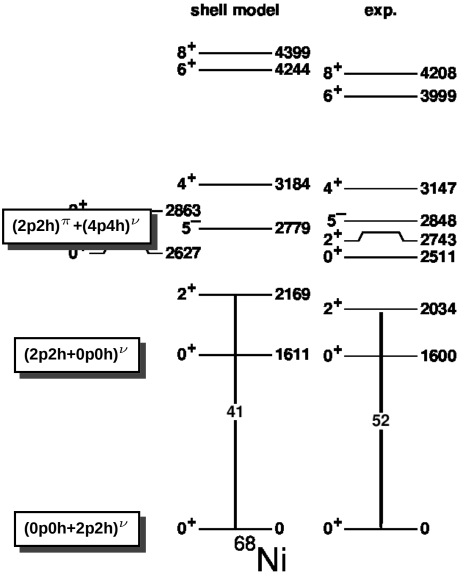

Triple coexistence in ^{68}Ni

- **In a first approximation, ^{68}Ni has a double shell closure in its *gs***
- **But its low energy spectrum is much more complex**
- **Three coexisting 0^+ states**

The calculations describe transition rates ranging over more than 2 orders of magnitude **Configuration mixing and relative transition rates between low-spin states in ^{68}Ni:**

F. Recchia et al., Phys. Rev. C88, 041302(R) (2013)

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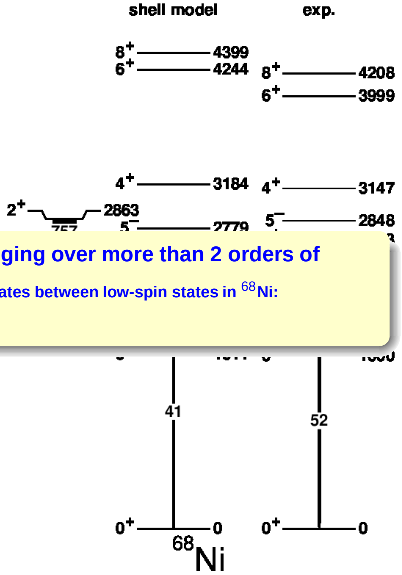
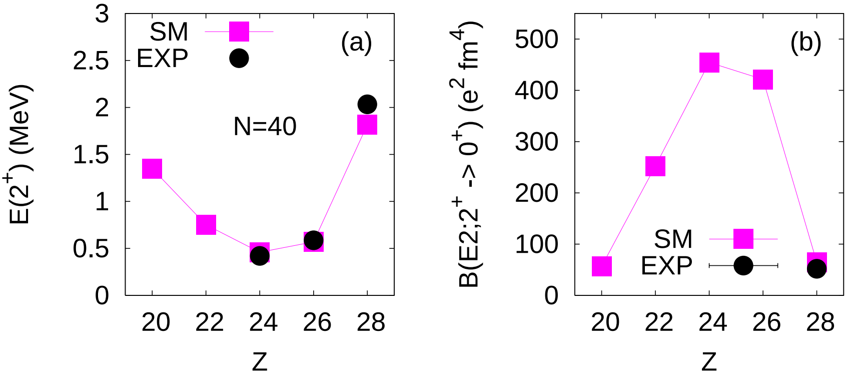


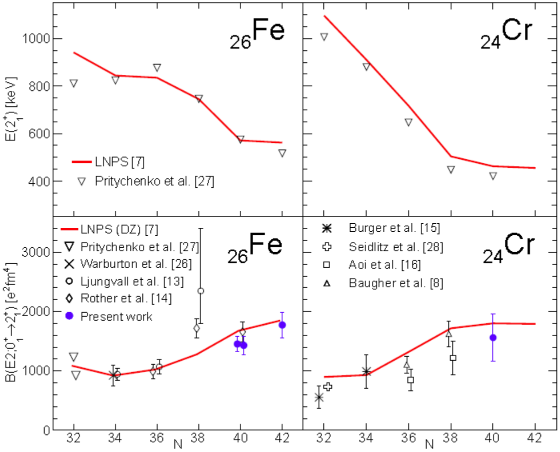
The island of deformation south of ^{68}Ni

- **Removing protons from the $0f_{7/2}$ orbit, activates the quadrupole collectivity, which, in turn, favors the np-nh neutron configurations across N=40, that take advantage of the quasi-SU3 coherence of the doublet $0g_{9/2}$- $1d_{5/2}$.**
- **Large scale SM calculations in the valence space of the full *pf*-shell for the protons and the $0f_{5/2}$ $1p_{3/2}$ $1p_{1/2}$ $0g_{9/2}$ and $1d_{5/2}$ orbits for the neutrons, predict a new region of deformation centered at ^{64}Cr.**

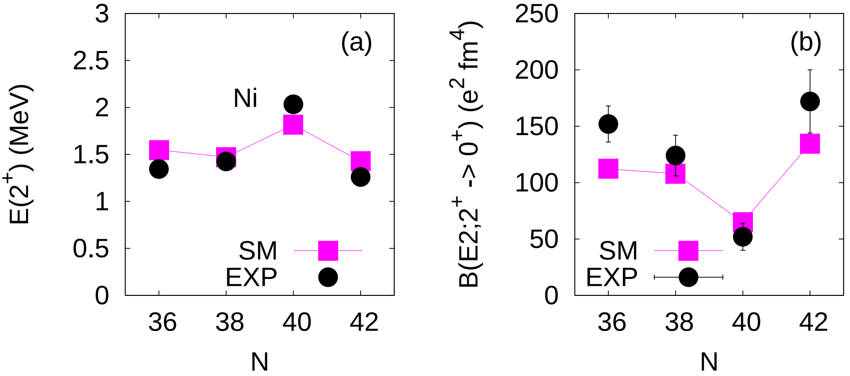
The N=40 isotones



The Iron and Chromium Isotopes



The Nickel Isotopes



Quadrupole dominance in heavy nuclei in the Quasi + Pseudo SU3 frame

- **The Quasi + Pseudo SU3 or Quasi + Quasi SU3 scheme plus the monopole field provide a SM toolkit to locate deformed structures in the (N, Z, E^*) landscape.**
- **The ground states of the N=Z nuclei between ^{68}Se and ^{92}Pd, (and their neighbors) are dominated by configurations with np-nh jumps across N=Z=40.**
- **Why? Because the n particles sit in Quasi-SU3 orbits and the n holes in Pseudo-SU3, thus maximizing their quadrupole moments and, a fortiori, their quadrupole correlation energy, which suffices to beat by large the monopole energy cost of crossing the N=Z=40 gap.**

Intrinsic Quadrupole moments for Pseudo (r3g) and Quasi-SU3 (gds): prolate states

The most favorable configurations from the quadrupole point of view are:

^{72}**Kr**

4p-4h; Q_0= 60 b^2 8p-8h; Q_0= 73 b^2 12p-12h; Q_0= 74 b^2

^{76}**Sr**

4p-4h; Q_0= 51 b^2 8p-8h; Q_0= 77 b^2 12p-12h; Q_0= 79 b^2

^{80}**Zr**

8p-8h; Q_0= 72 b^2 12p-12h; Q_0= 83 b^2 16p-16h; Q_0= 85 b^2

Monopole *vs* Quadrupole

- **We have recently shown that these quadrupole moments are resilient to departures of the SPE from their degenerated limit (Zuker *et al.* arXiv:1404.0224)**
- **The key point here is that the quadrupole energy gains grow with the square of the quadrupole moment whereas the monopole losses are at most proportional to the number of particle-hole jumps**
- **In ^{76}Sr and ^{80}Zr the deformed configurations, 8p-8h and 12p-12h win comfortably. In ^{72}Kr the 4p-4h prolate and the oblate solutions (oblate meaning $(0g_{9/2})^4$ instead of $(gds)^4$) are degenerated as we shall discuss next.**
- **From Q_0 one can deduce the B(E2)'s. The $2^+ \rightarrow 0^+$ are equal to $Q_0{}^2/50.3$ and the $4^+ \rightarrow 2^+$ a factor 1.43 larger.**

Comparing with experiment

Using b^2=4.5 fm^2 we obtain the following B(E2) values:

- ^{72}Kr; $2^+ \rightarrow 0^+$; 1470 e^2fm^4; $4^+ \rightarrow 2^+$; 2100 e^2fm^4
- ^{76}Sr; $2^+ \rightarrow 0^+$; 2380 e^2fm^4; $4^+ \rightarrow 2^+$; 3410 e^2fm^4
- ^{80}Zr; $2^+ \rightarrow 0^+$; 2800 e^2fm^4; $4^+ \rightarrow 2^+$; 4000 e^2fm^4

To compare with the available experimental results:

- ^{72}Kr; $2^+ \rightarrow 0^+$; 810(150) e^2fm^4; $4^+ \rightarrow 2^+$; 2720(550) e^2fm^4
- ^{76}Sr; $2^+ \rightarrow 0^+$; 2200(270) e^2fm^4
- ^{80}Zr; no data yet

Excellent agreement except for the $2^+ \rightarrow 0^+$ of ^{72}Kr. But this is a blessing in disguise because it led us to understand better the prolate oblate coexistence in this nucleus.

^{72}Kr, a case of full prolate oblate mixing

- **It is common lore to speak of prolate-oblate or prolate-spherical coexistence when an excited 0^+ state appears at very low energy. This is the case in ^{72}Kr, whose first excited state is a 0^+ at 671 keV followed by a 2^+ at 710 keV. The very large B(E2) of the transition $4^+ \rightarrow 2^+$ strongly suggest that the 2^+ belongs to a prolate band which extends up to J=16^+. But, if so, where is the band head?**
- **If we follow down the J(J+1) sequence from the upper part of the band we should expect it 250 keV below the 2^+ , which is very close to the experimental excitation energies of the 2^+ in ^{76}Sr and ^{80}Kr. Obviously the distortion must be due to the mixing of the prolate prolate band-head with a near lying oblate state.**

^{72}Kr, prolate oblate mixing, a (very) simple model

- **The first element to take into account is that the oblate and prolate 4p-4h states do not mix directly;** *i.e.* $\langle p|H|o\rangle$ **= 0**
- **The mixing should then proceed through 2p-2h or 6p-6h states. Lets take these to be represented by an auxiliary state $|I\rangle$, and further assume that it lies at about ΔE=4 MeV (as our calculations show) and that its coupling to both prolate and oblate states is equal to δ. Taking them degenerated for simplicity, the mixing matrix reads:**
- $$\begin{pmatrix} 0 & 0 & \delta \\ 0 & 0 & \delta \\ \delta & \delta & \Delta E \end{pmatrix}$$

^{72}Kr, prolate oblate mixing, a (very) simple model

- For $\delta \sim$ 1 MeV, which is a sensible choice, the eigenvalues are: –0.5 MeV, 0.0 MeV and +4.5 MeV. They fit nicely the experimental energies.
  The eigenstates corresponding to the two lower eigenvalues are:
  $|0^+_1\rangle$ = 43% $|p\rangle$ + 43% $|o\rangle$ + 16% $|l\rangle$ and
  $|0^+_2\rangle$ = 50% $|p\rangle$ + 50% $|o\rangle$
- Therefore, the B(E2)($2^+ \rightarrow 0^+_1$) will be approximately one half of the expected value for the prolate band in full accord with the experimental data
- What is the shape of an object which is an even mixture of prolate and oblate? What is the nature of this mixing of shapes? Or should we speak of a shape entangled state?

Conclusions

- The onset of the different modes of quadrupole collectivity depend on the structure of the spherical mean field
- In the valence space comprising the *pf* shell and the $0g_{9/2}$ and $1d_{5/2}$ we have explained the appearance of large prolate deformation at N=Z in terms of different realizations of SU3.
- Similar arguments explain the onset of deformation in the new island of inversion around ^{64}Cr and its precursor, the superdeformed band of ^{68}Ni