

Energy Density Functional Studies of Exotic Nuclear Structure

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University of Zagreb



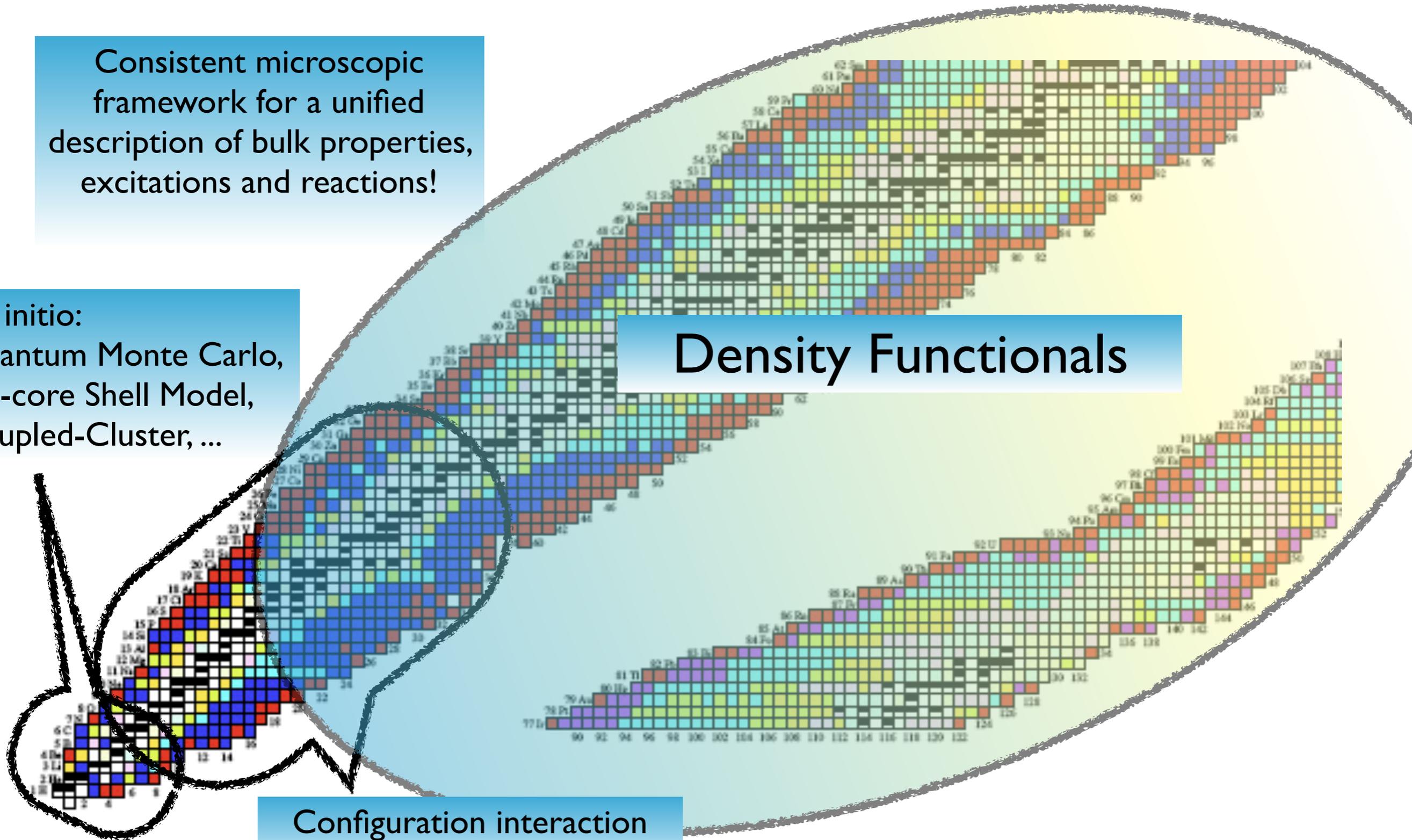
Energy Density Functionals

Consistent microscopic framework for a unified description of bulk properties, excitations and reactions!

Ab initio:
Quantum Monte Carlo,
No-core Shell Model,
Coupled-Cluster, ...

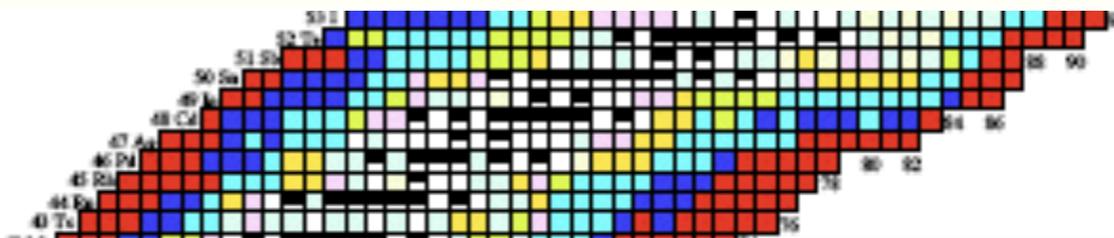
Density Functionals

Configuration interaction
(Interacting Shell-Model)

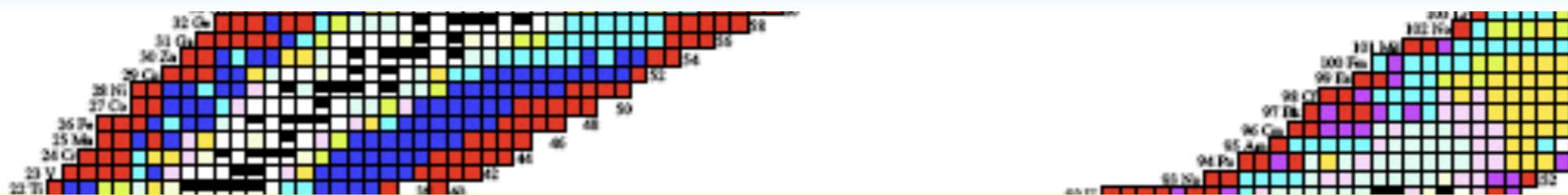


Energy Density Functionals

✓ the nuclear many-body problem is effectively mapped onto a *one-body problem* without explicitly involving inter-nucleon interactions!



✓ the exact density functional is approximated with *powers and gradients of ground-state densities and currents*.



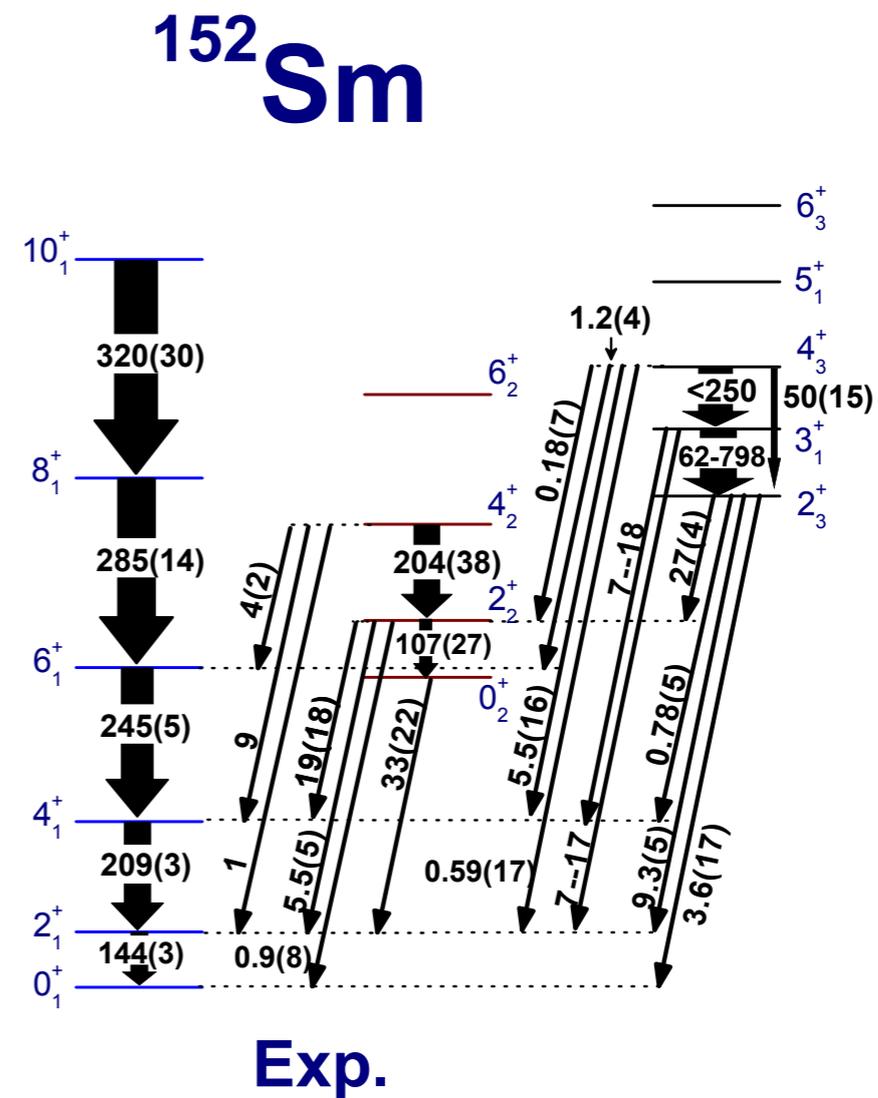
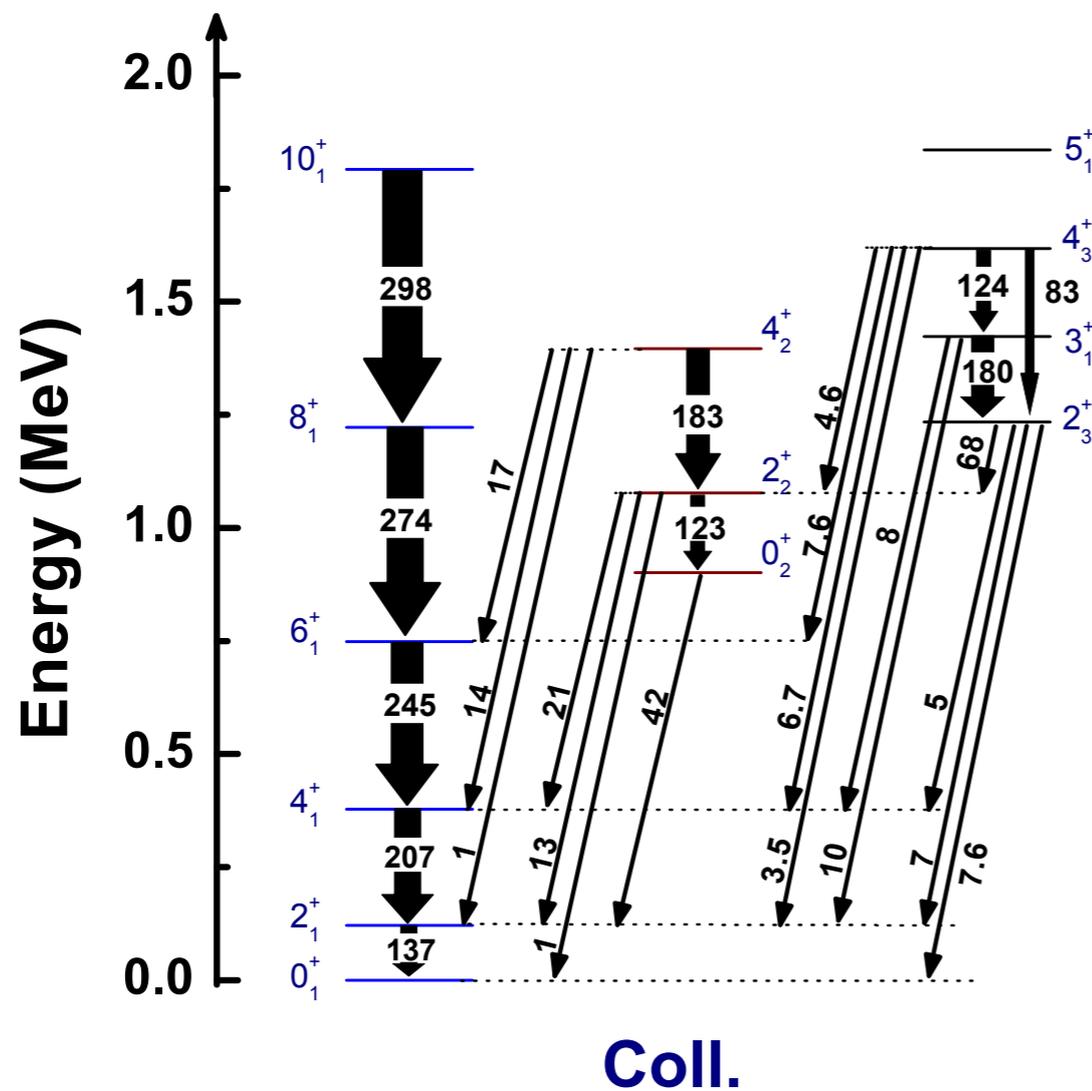
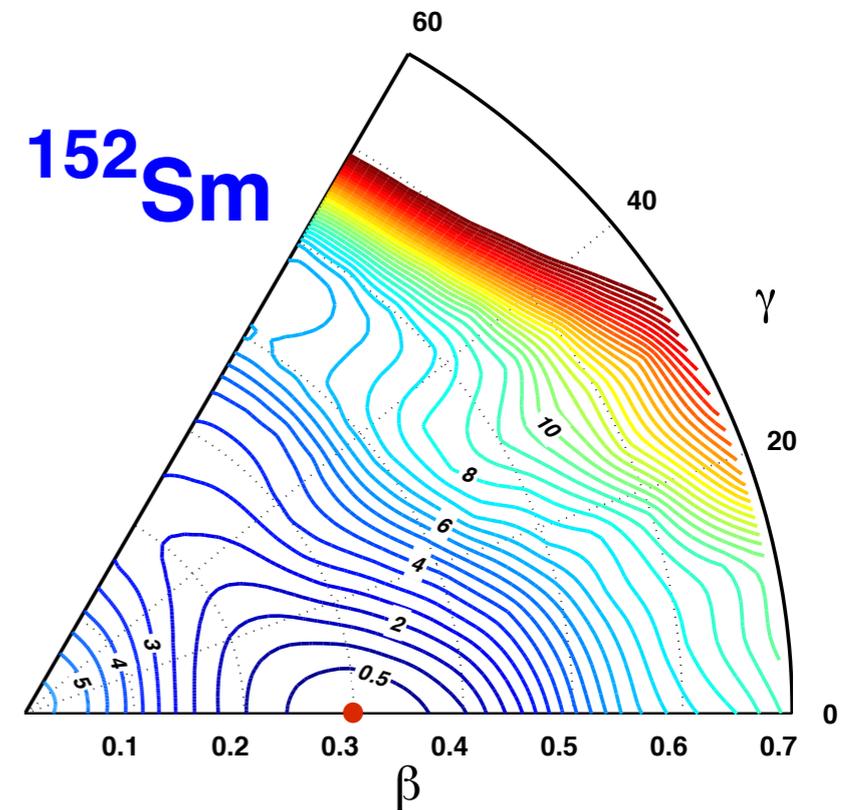
✓ *universal density functionals* can be applied to all nuclei throughout the chart of nuclides.



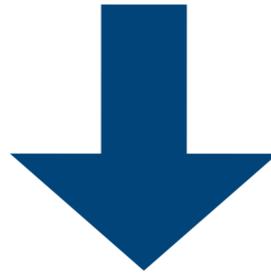
Important for extrapolations to regions far from stability!

✓ an intuitive interpretation of mean-field results in terms of *intrinsic shapes* and *single-particle states*

✓ the *full model space* of occupied states can be used; no distinction between core and valence nucleons, *no need for effective charges!*



Nuclear Many-Body Correlations



short-range

(hard repulsive core of the NN-interaction)

long-range

nuclear resonance modes
(giant resonances)

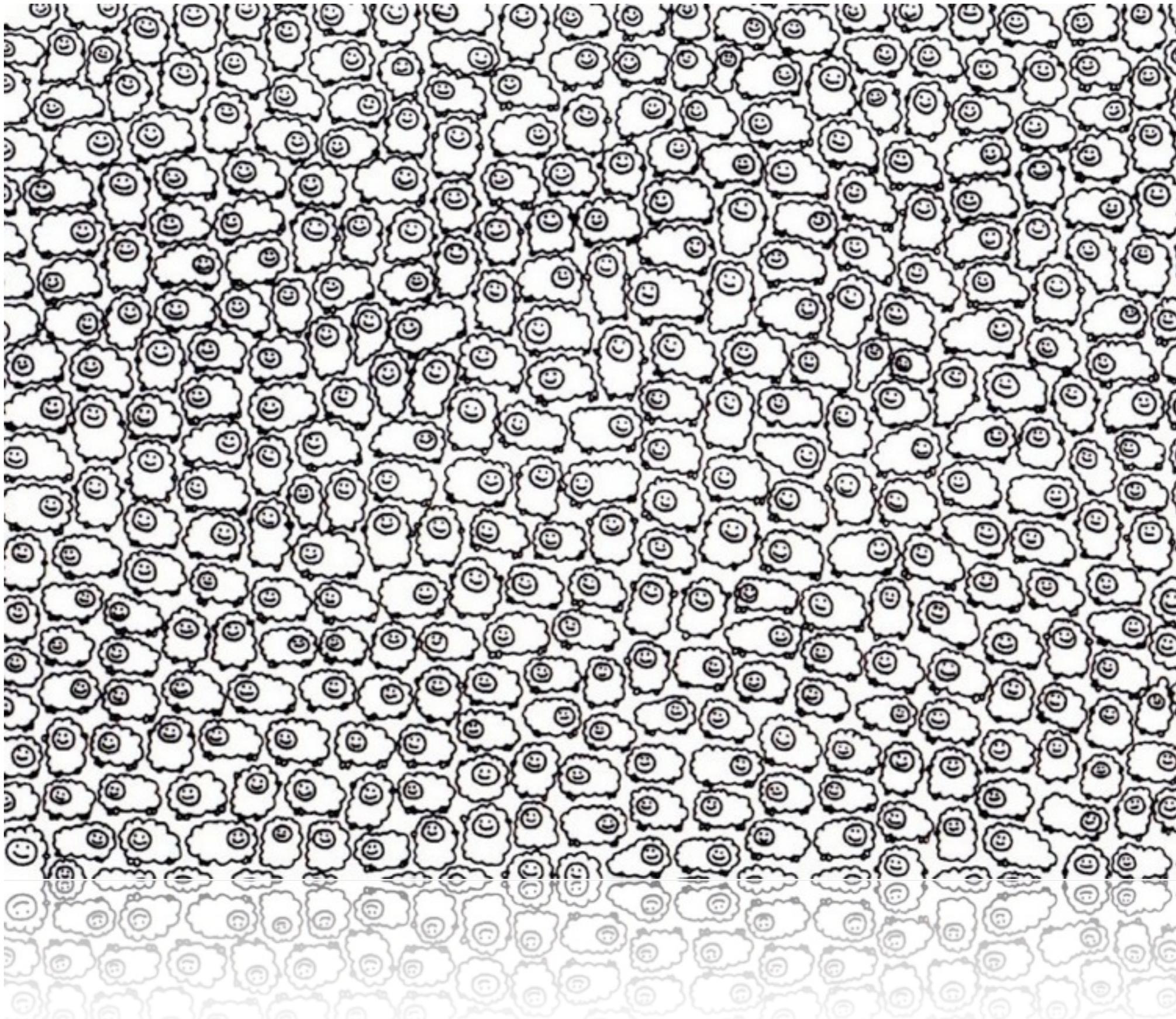
collective correlations

large-amplitude soft modes:
(center of mass motion, rotation,
low-energy quadrupole vibrations)

...vary smoothly with nucleon number!
Can be included implicitly in an effective Energy Density Functional.

...sensitive to shell-effects and strong variations with nucleon number!
Cannot be included in a simple EDF framework.

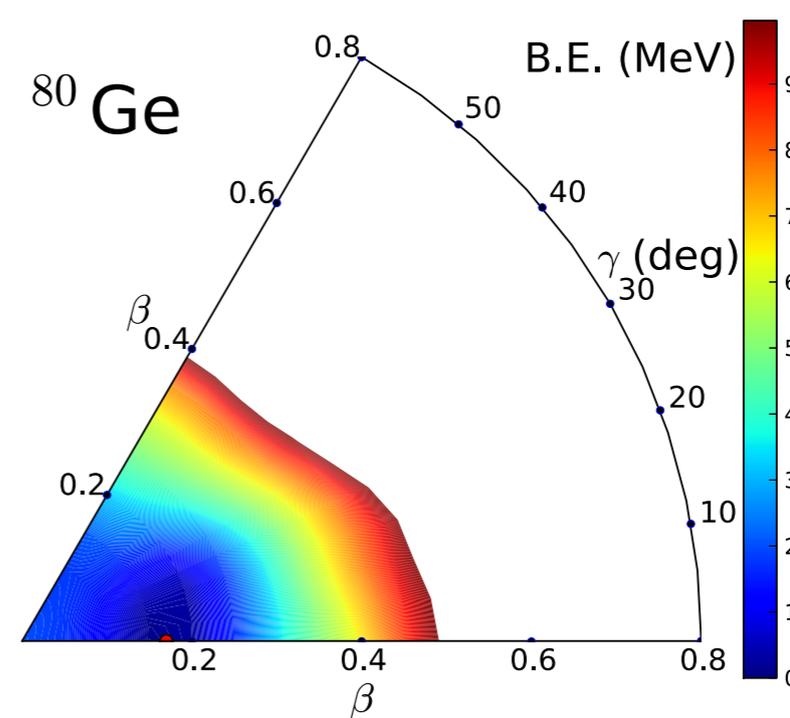
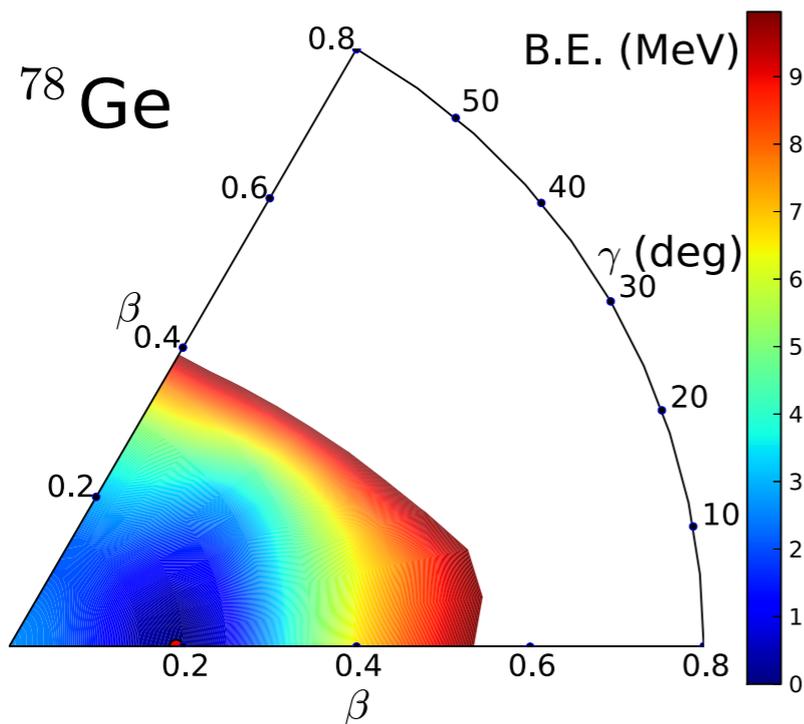
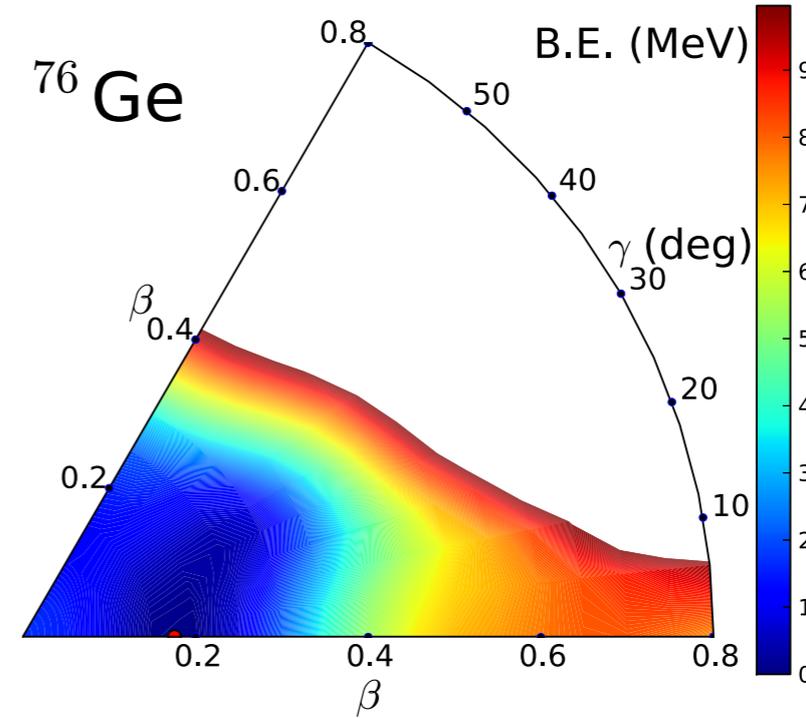
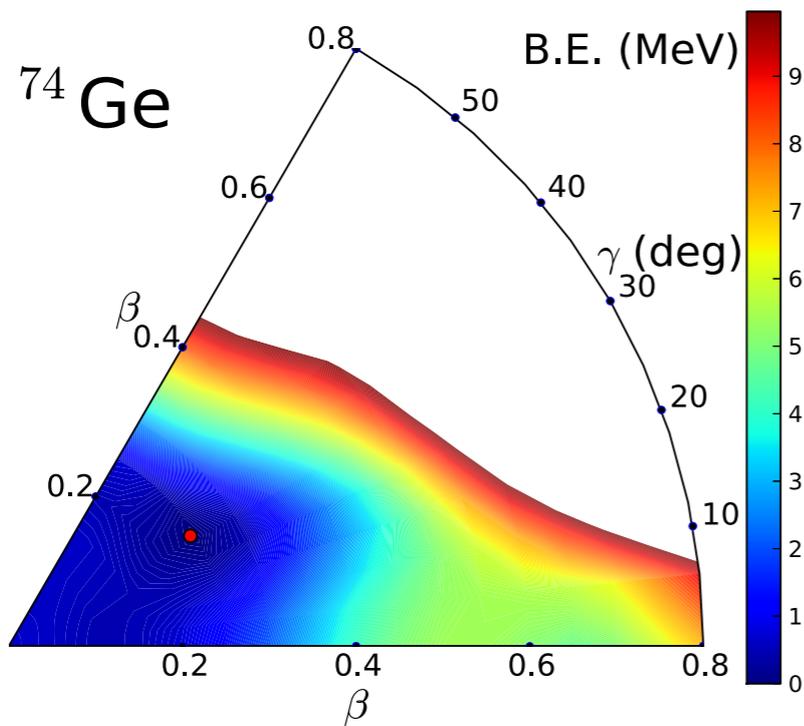
Collective correlations

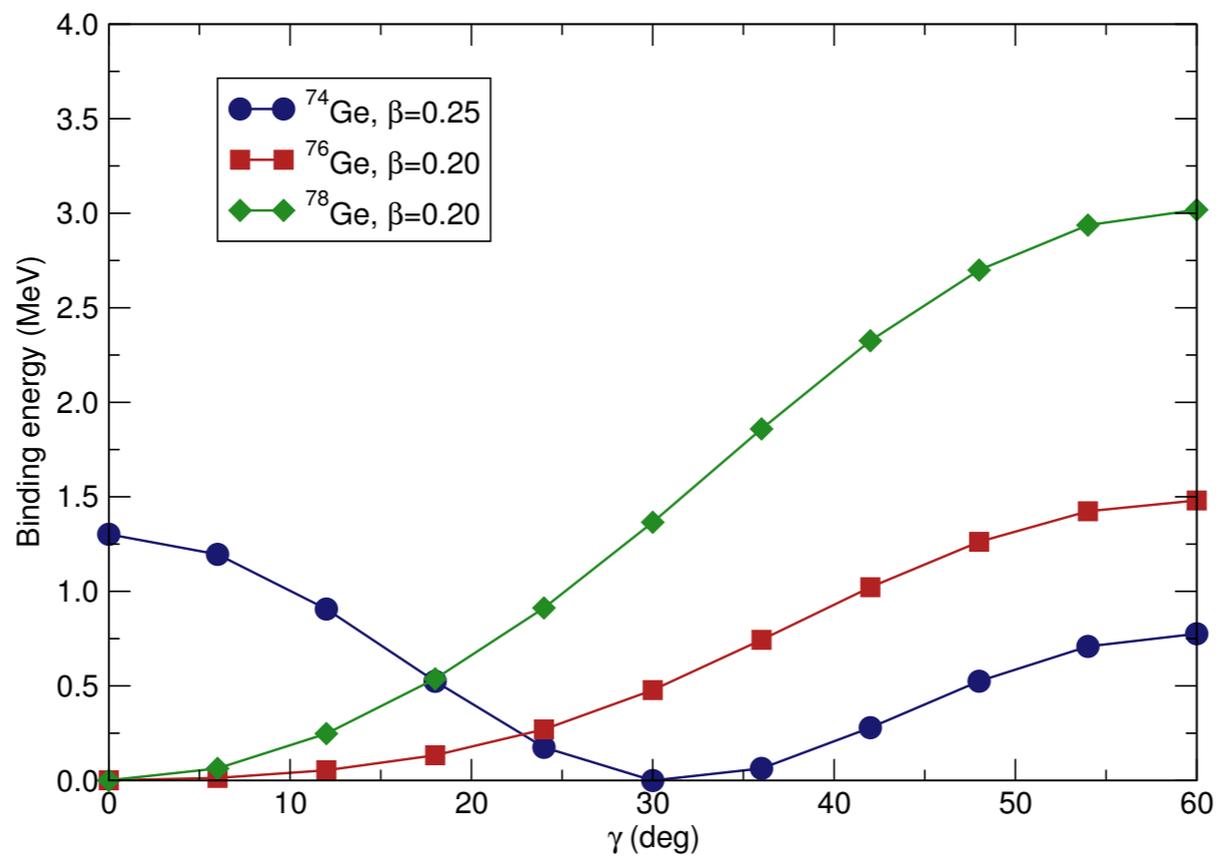
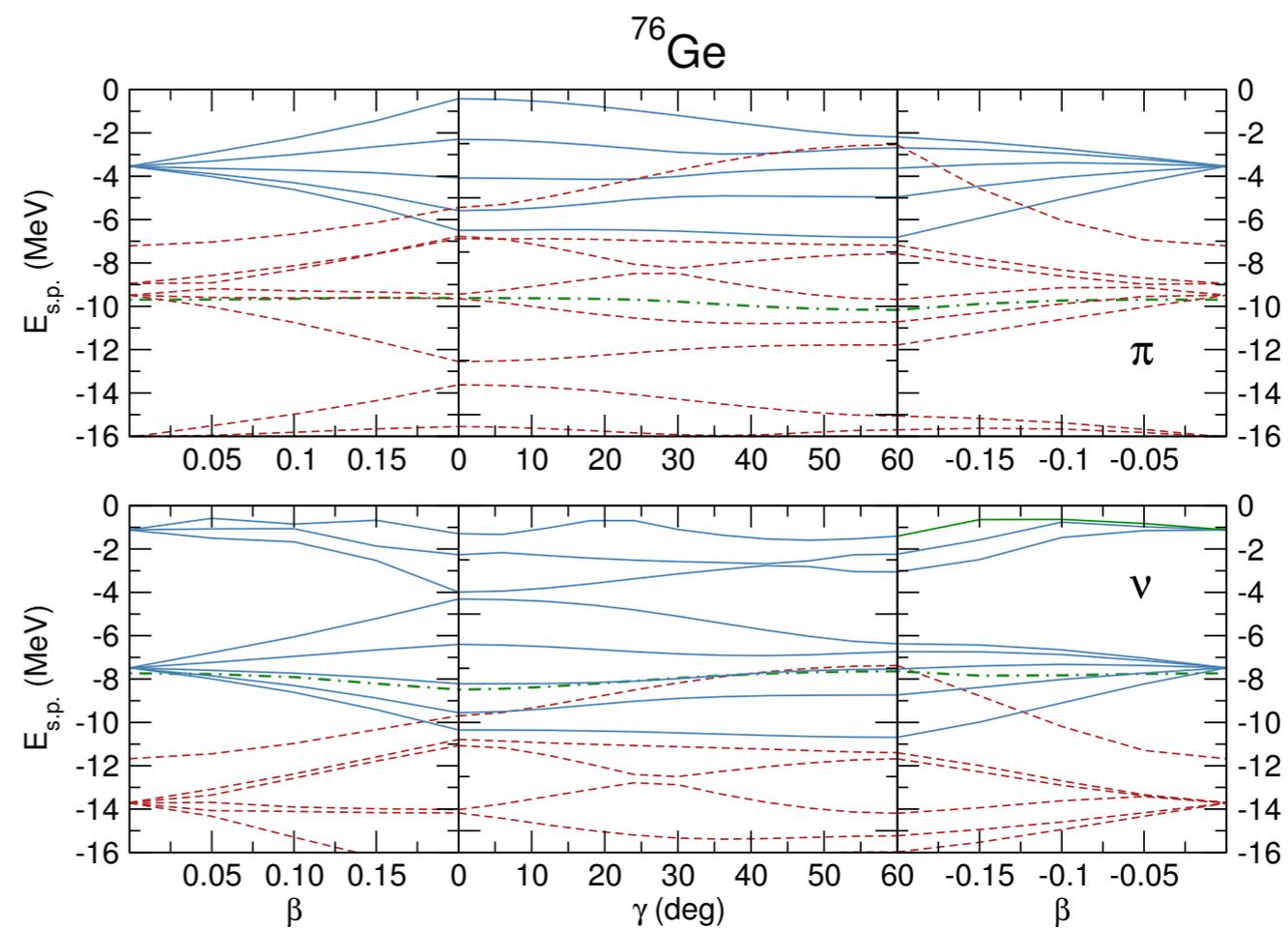
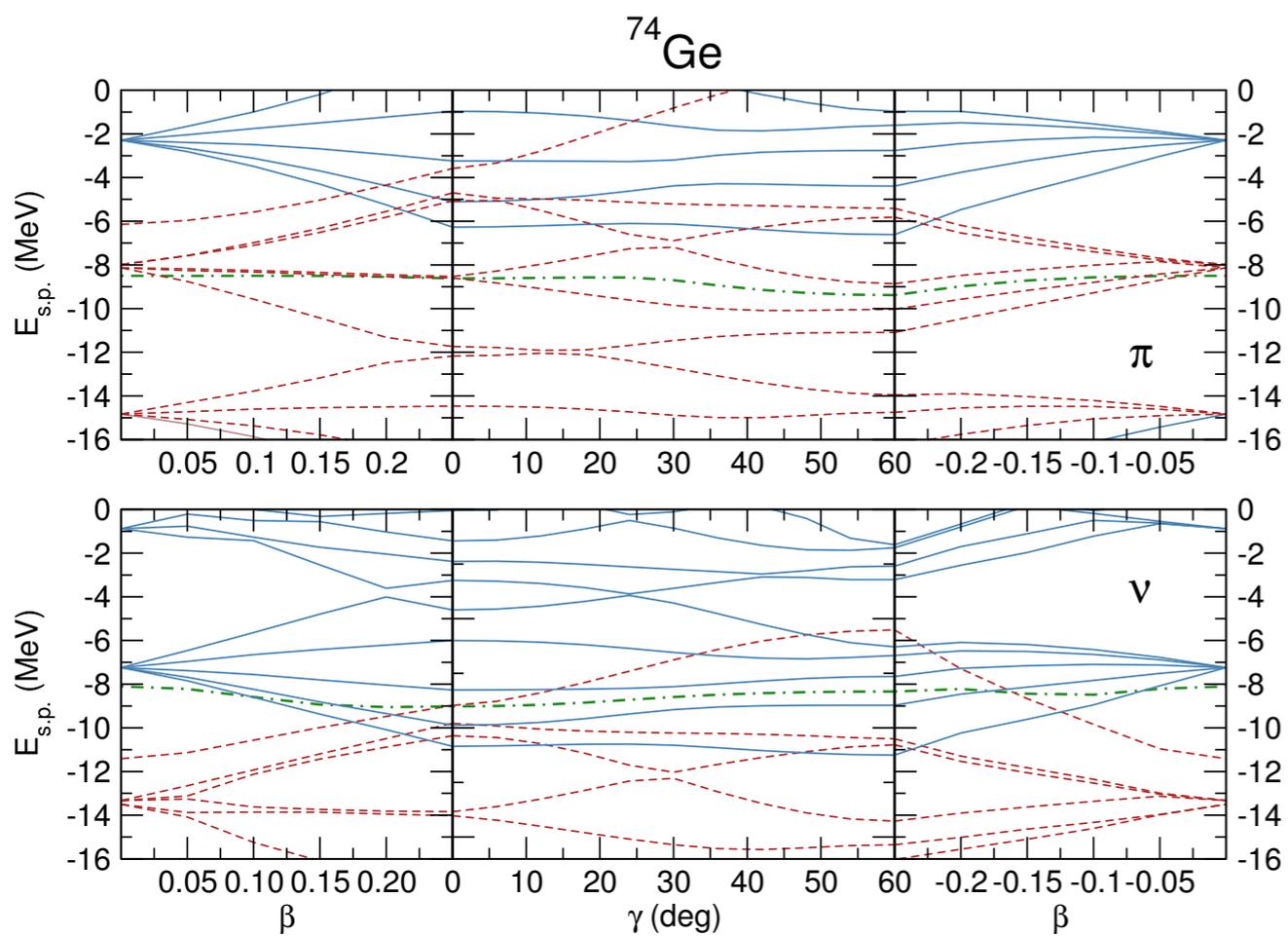


Shape evolution and triaxiality in germanium isotopes

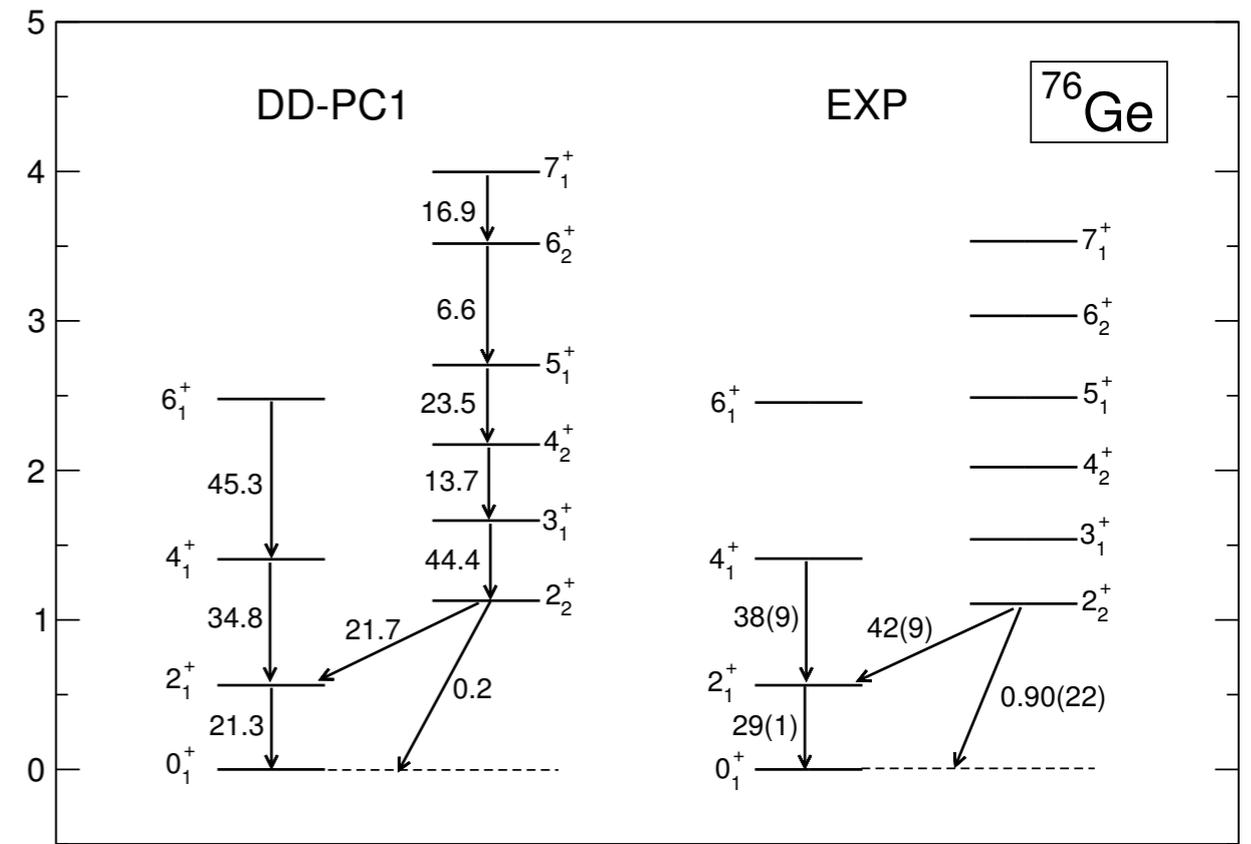
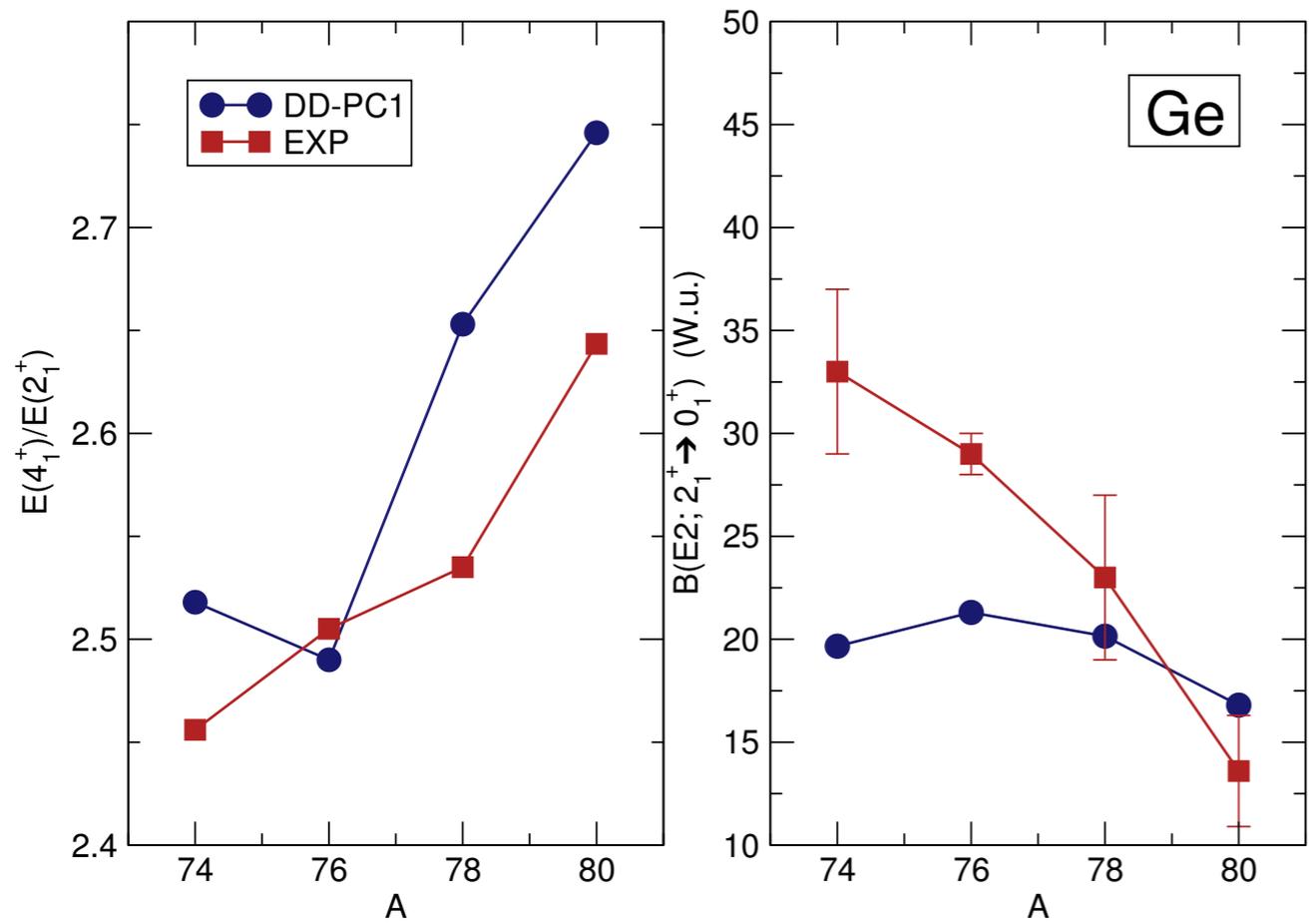
T. Nikšić, P. Marević, and D. Vretenar

PHYSICAL REVIEW C **89**, 044325 (2014)

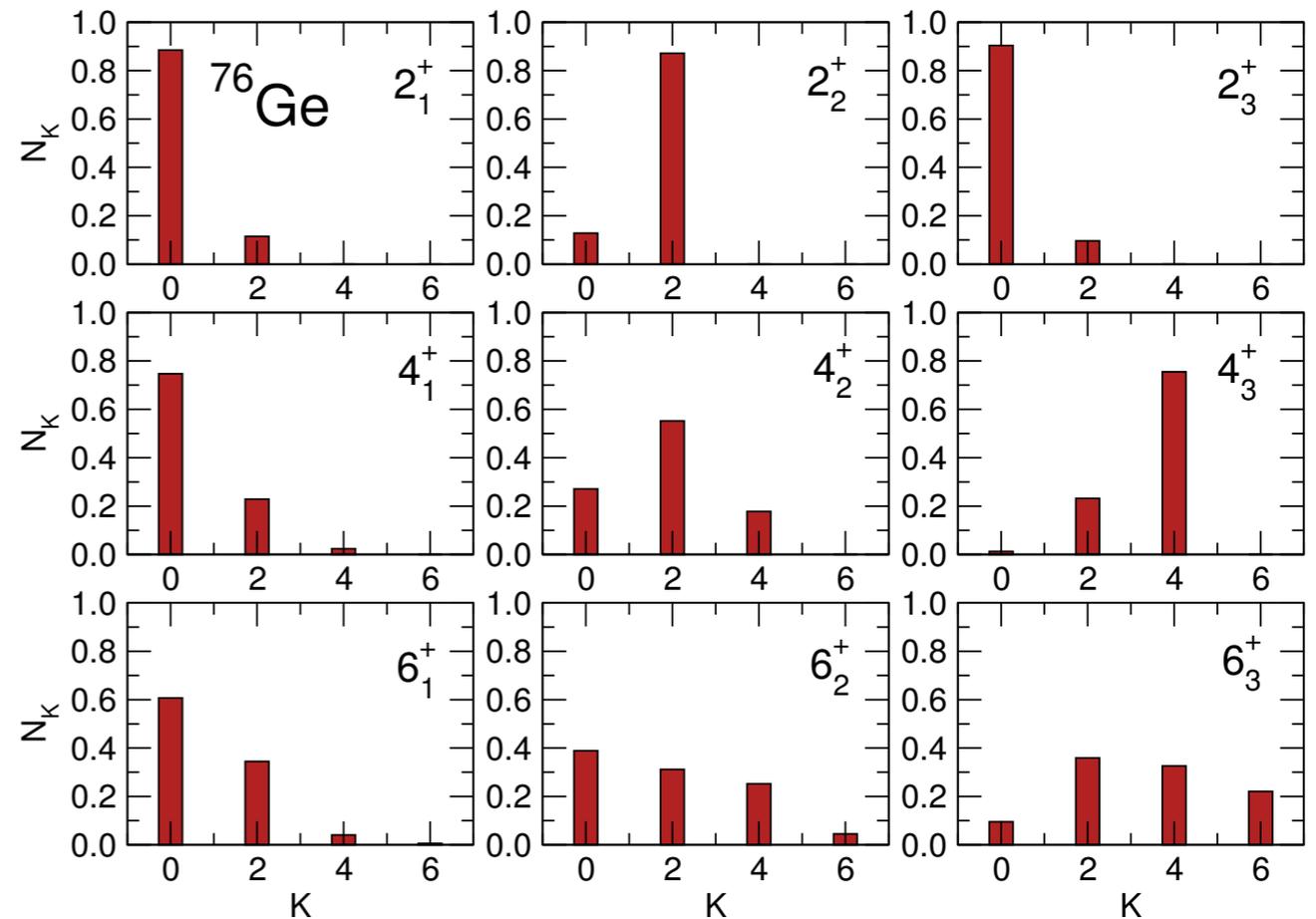




Quadrupole collective Hamiltonian based on the functional DD-PCI



Distribution of K components (projection of the angular momentum on the body-fixed symmetry axis) in the collective wave functions of the nucleus ^{76}Ge .

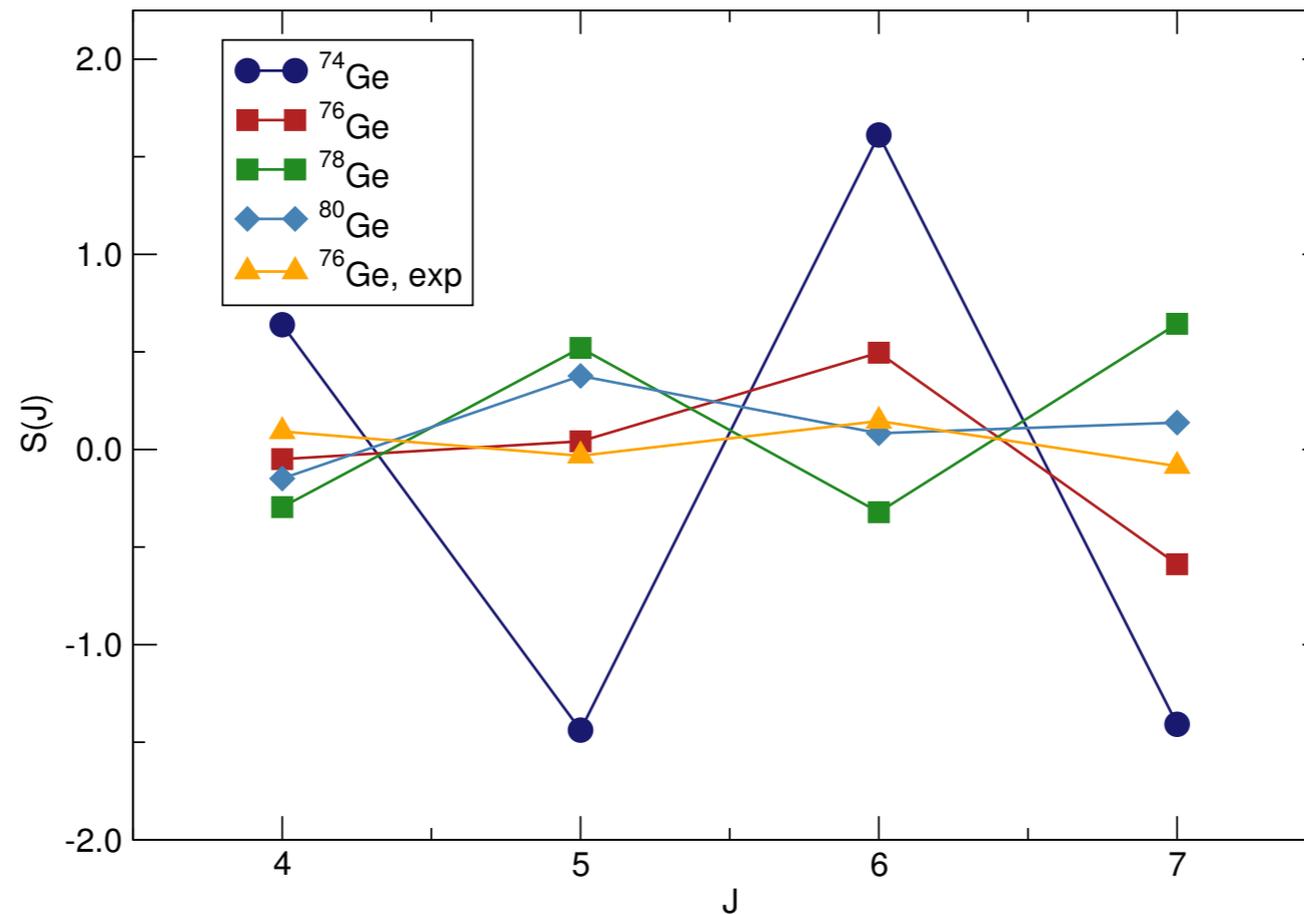


The level of K-mixing is reflected in the staggering in energy between odd- and even-spin states in the γ band:

$$S(J) = \frac{E[J_\gamma^+] - 2E[(J-1)_\gamma^+] + E[(J-2)_\gamma^+]}{E[2_1^+]}$$

Deformed γ -soft potential $\Rightarrow S(J)$ oscillates between negative values for even-spin states and positive values for odd-spin states.

γ -rigid triaxial potential $\Rightarrow S(J)$ oscillates between positive values for even-spin states and negative values for odd-spin states.



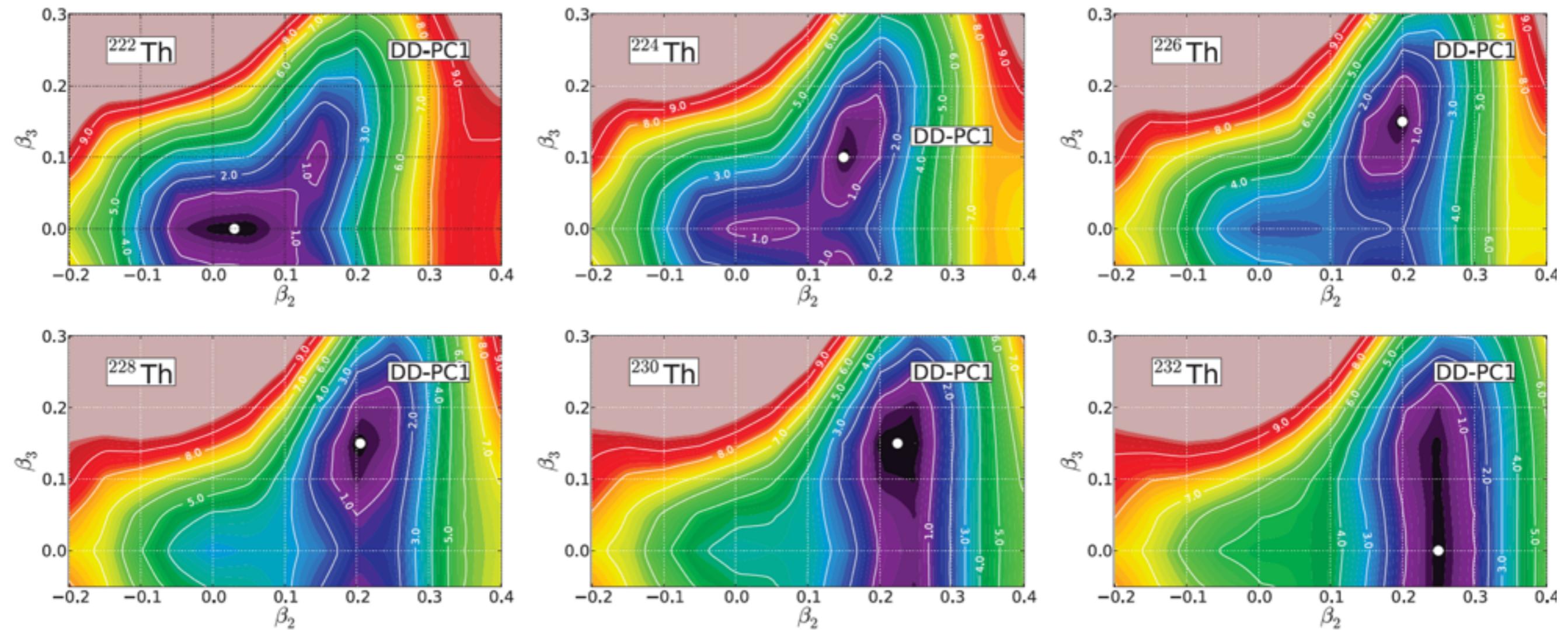
The mean-field potential of ^{76}Ge is γ soft. The inclusion of collective correlations (symmetry restoration and quantum fluctuations) drives the nucleus toward triaxiality, but they are not strong enough to stabilize a $\gamma \approx 30^\circ$ triaxial shape.

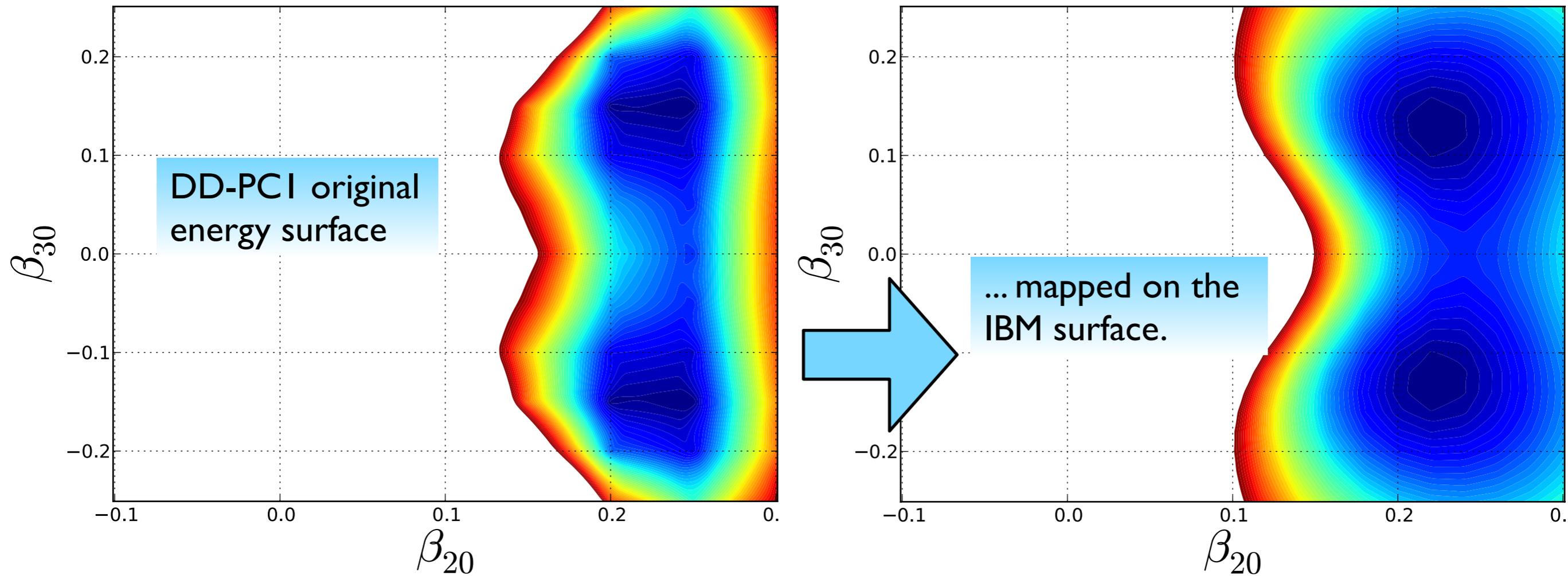
Octupole shape-phase transitions in light actinide and rare-earth nuclei

NOMURA, VRETENAR, NIKŠIĆ, AND LU

PHYSICAL REVIEW C **89**, 024312 (2014)

Axially symmetric deformation energy surfaces of $^{222-232}\text{Th}$ in the (β_2, β_3) plane:



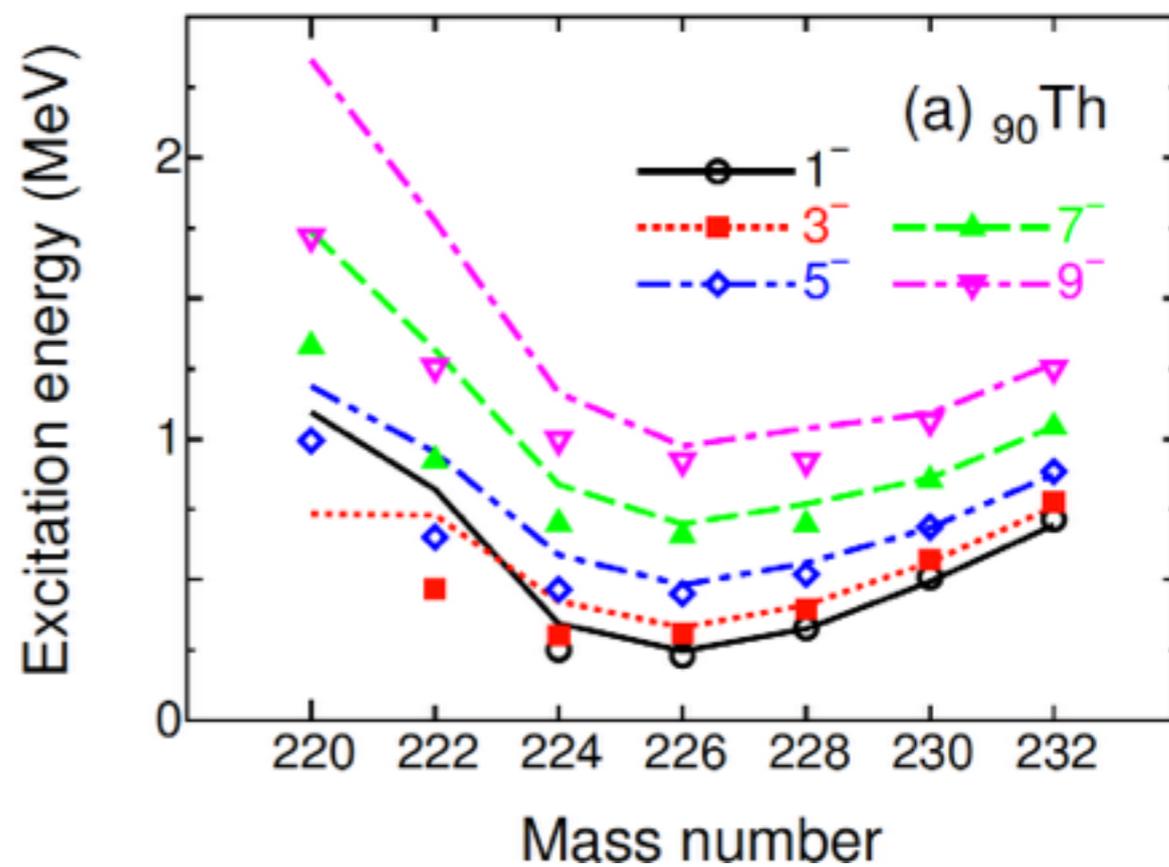
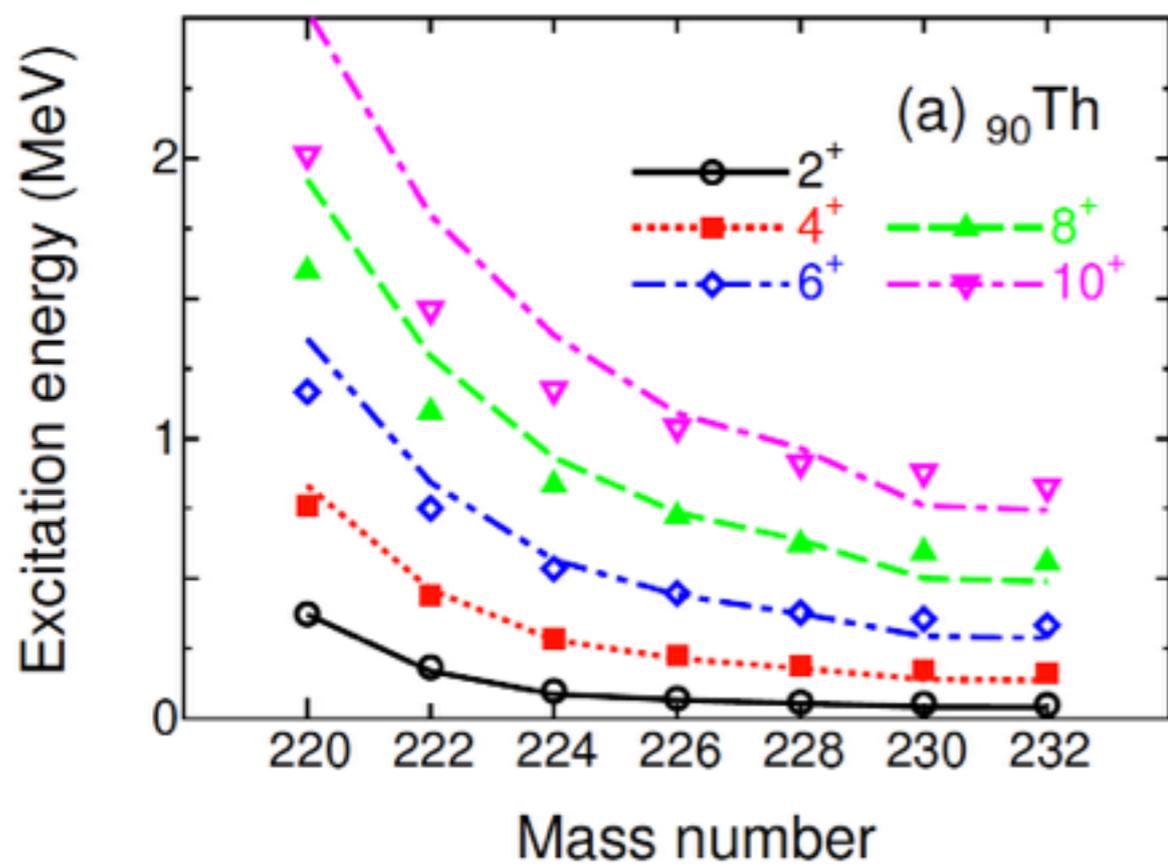
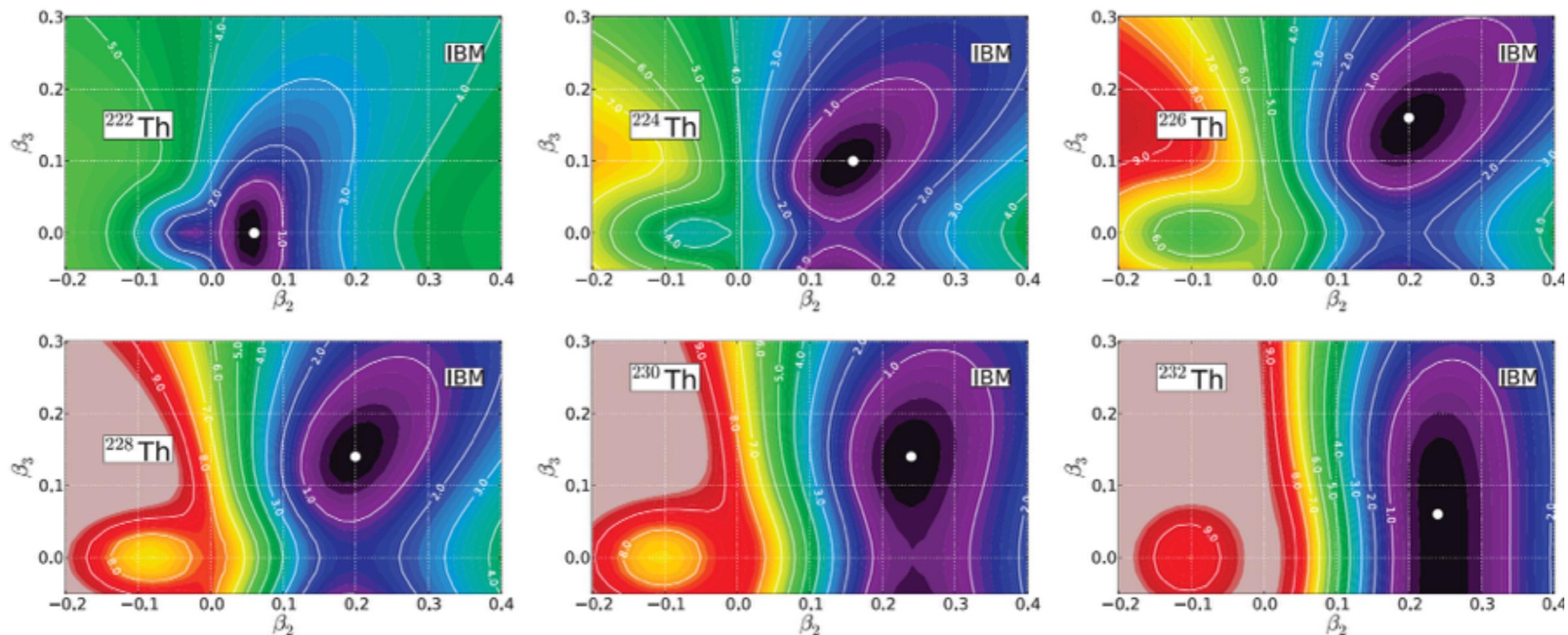


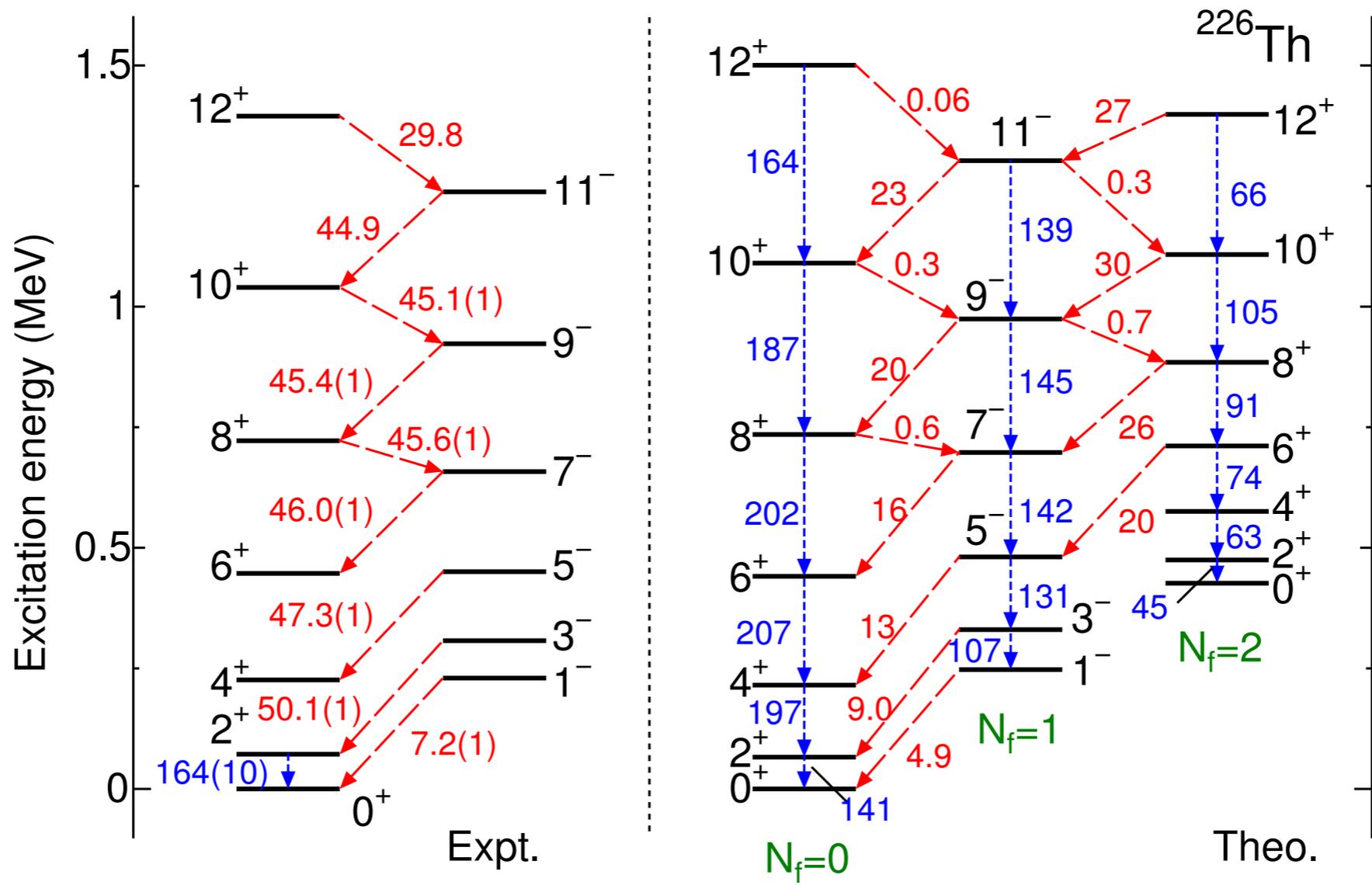
Mapping the microscopic PES on the expectation value of the IBM Hamiltonian in the *sdf*-boson condensate state:

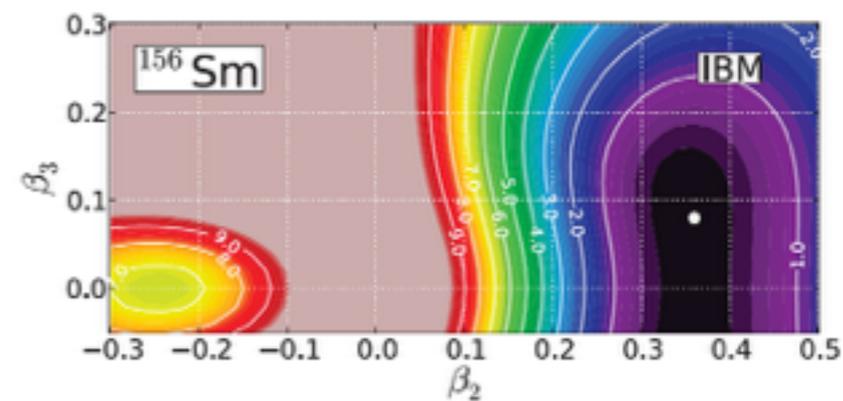
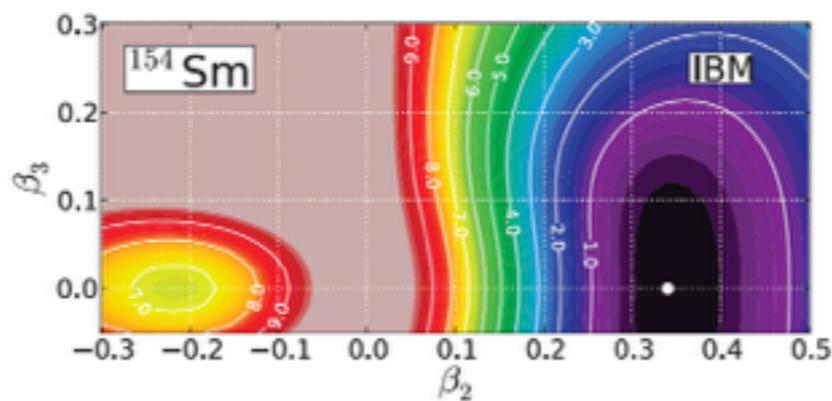
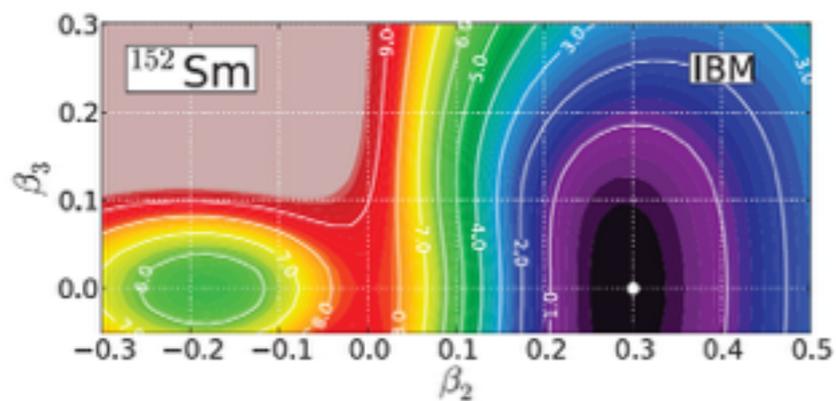
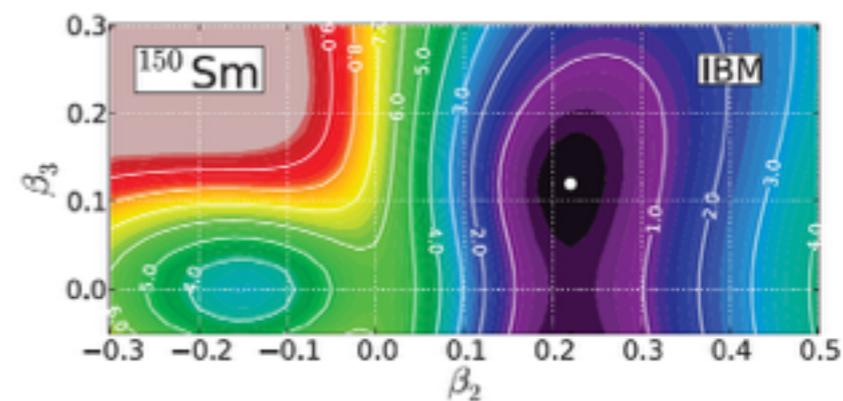
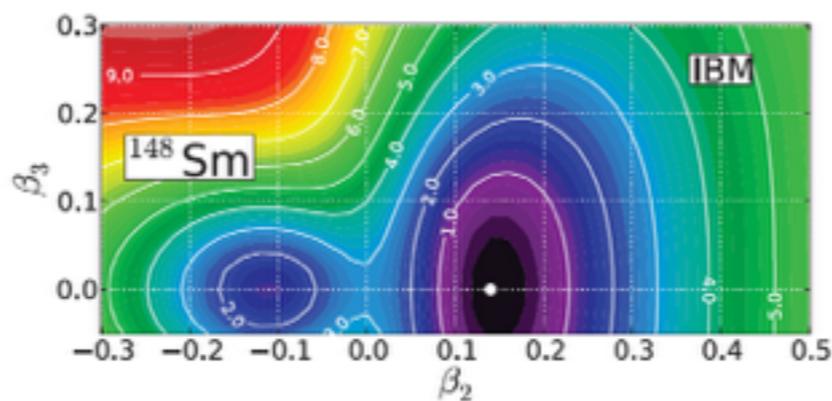
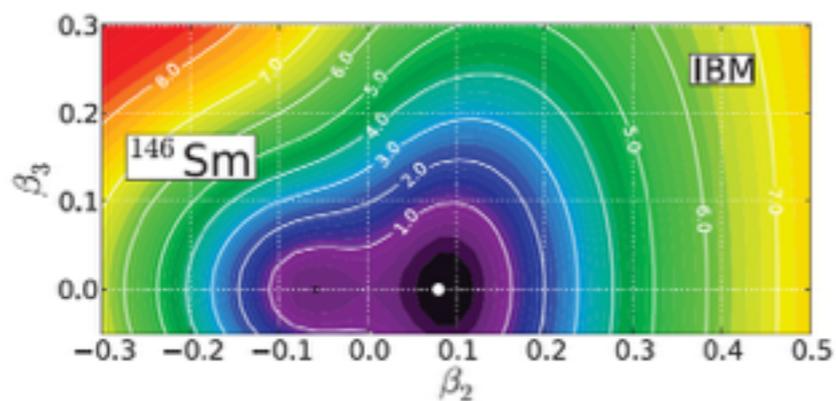
$$\hat{H} = \epsilon_d \hat{n}_d + \epsilon_f \hat{n}_f + \kappa_2 \hat{Q} \cdot \hat{Q} + \alpha \hat{L}_d \cdot \hat{L}_d + \kappa_3 : \hat{V}_3^\dagger \cdot \hat{V}_3 :$$

$$|\phi\rangle = \frac{1}{\sqrt{N!}} (\lambda^\dagger)^N |-\rangle$$

$$\lambda^\dagger = s^\dagger + \beta_2 d_0^\dagger + \beta_3 f_0^\dagger$$

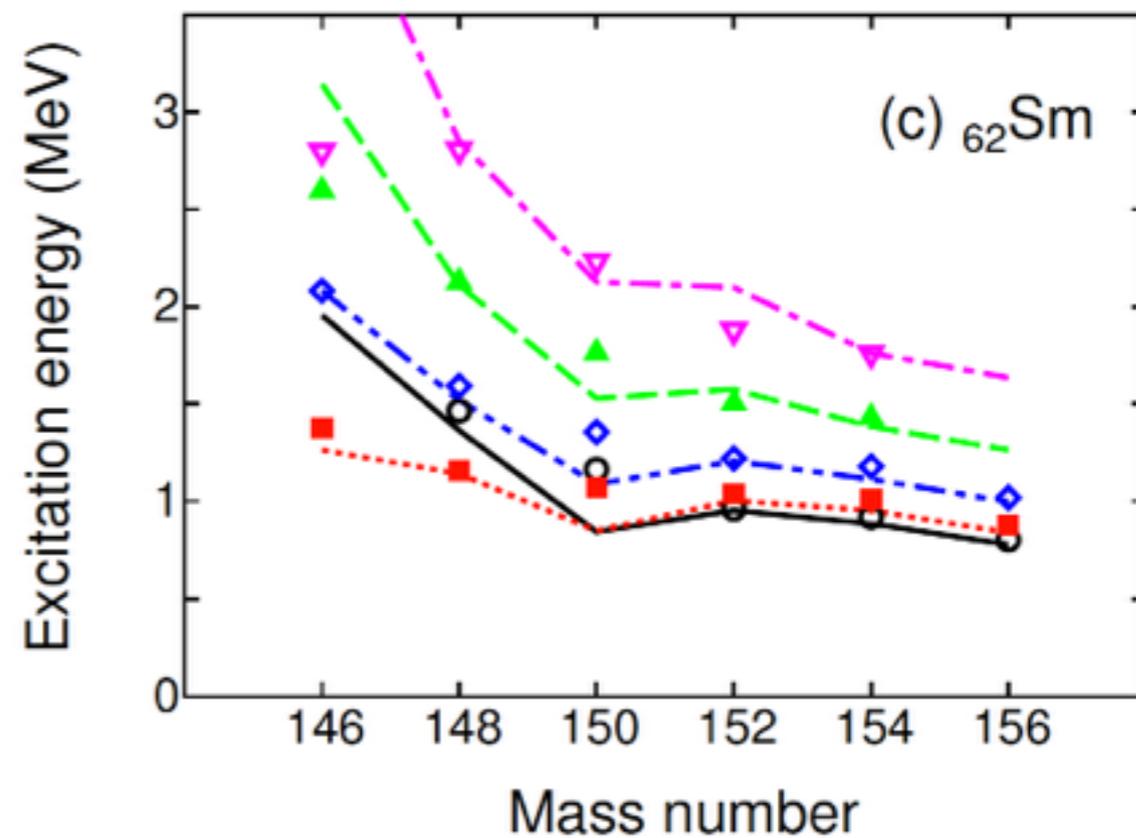
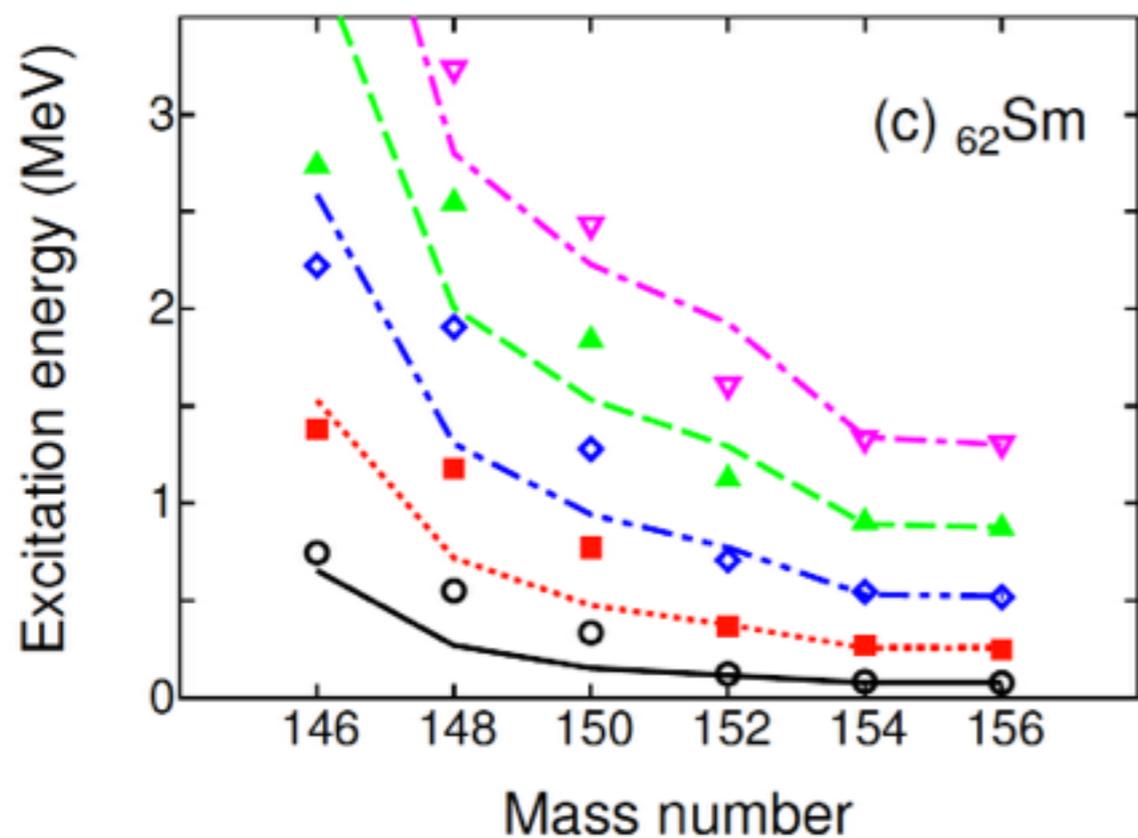






POSITIVE PARITY

NEGATIVE PARITY

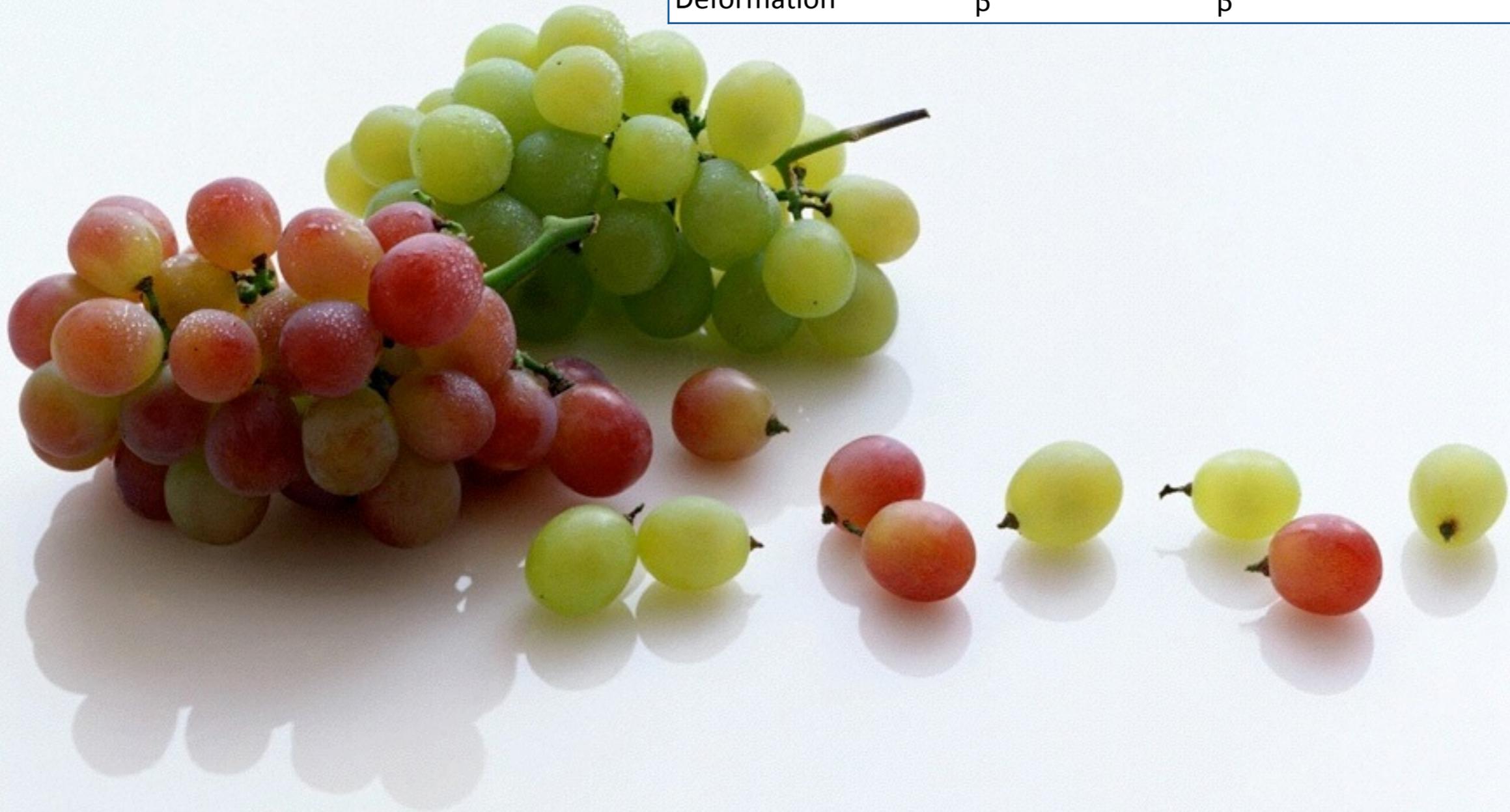


Localization and clustering in the nuclear Fermi liquid

^{20}Ne

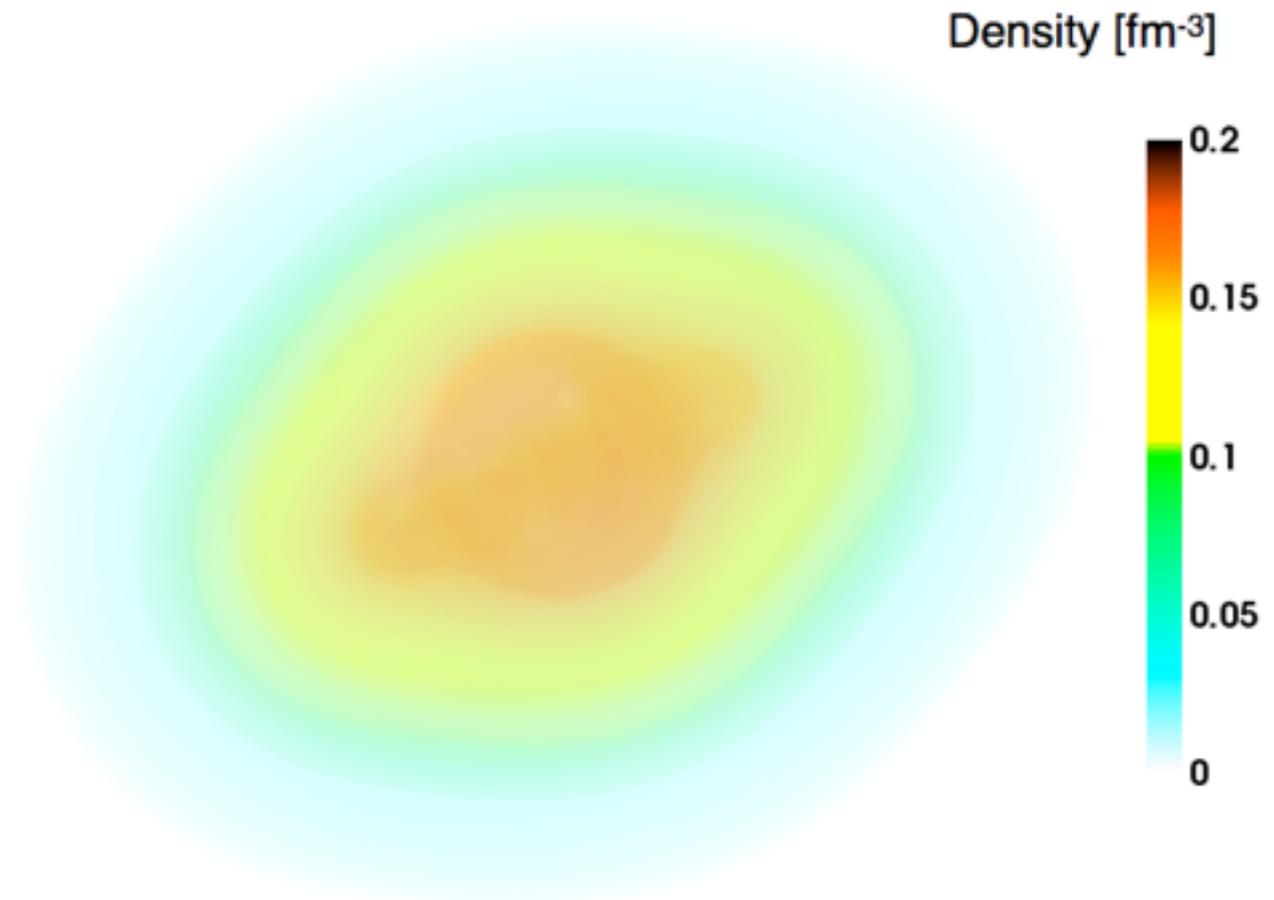
Equilibrium properties calculated using two different density functionals:

	Skyrme SLy4	Rel. DD-ME2	EXP
Binding energy	157.2 MeV	156.4 MeV	160.6 MeV
Charge radius	3.04 fm	2.98 fm	3.0 fm
Matter radius	2.92 fm	2.86 fm	2.85 fm
Deformation	β	β	

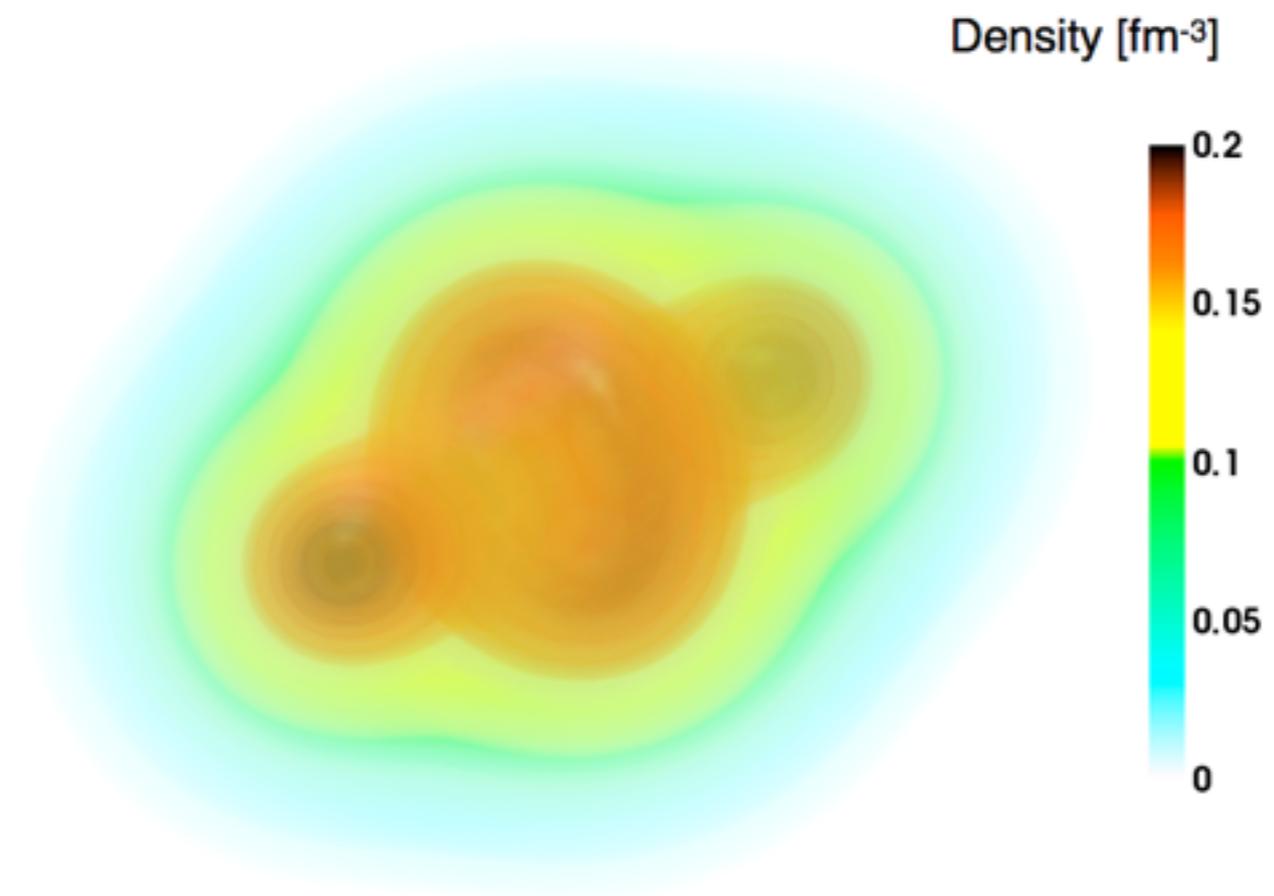


^{20}Ne : intrinsic equilibrium density distributions

Skyrme Ly4

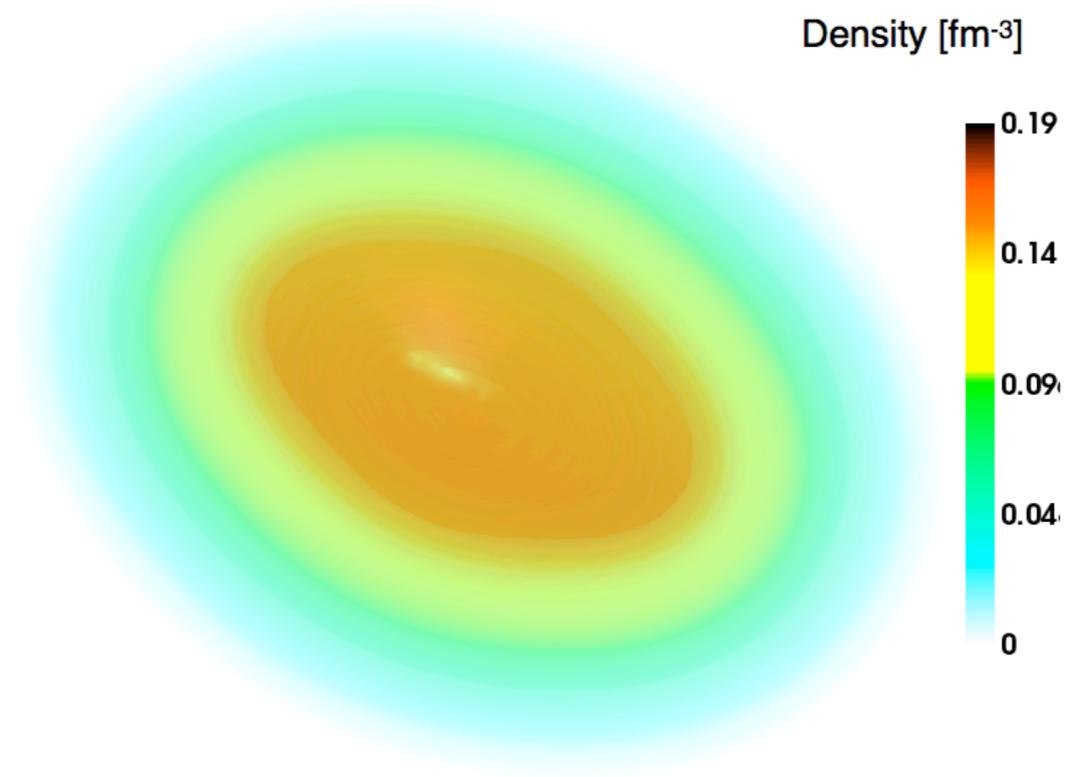


DD-ME2

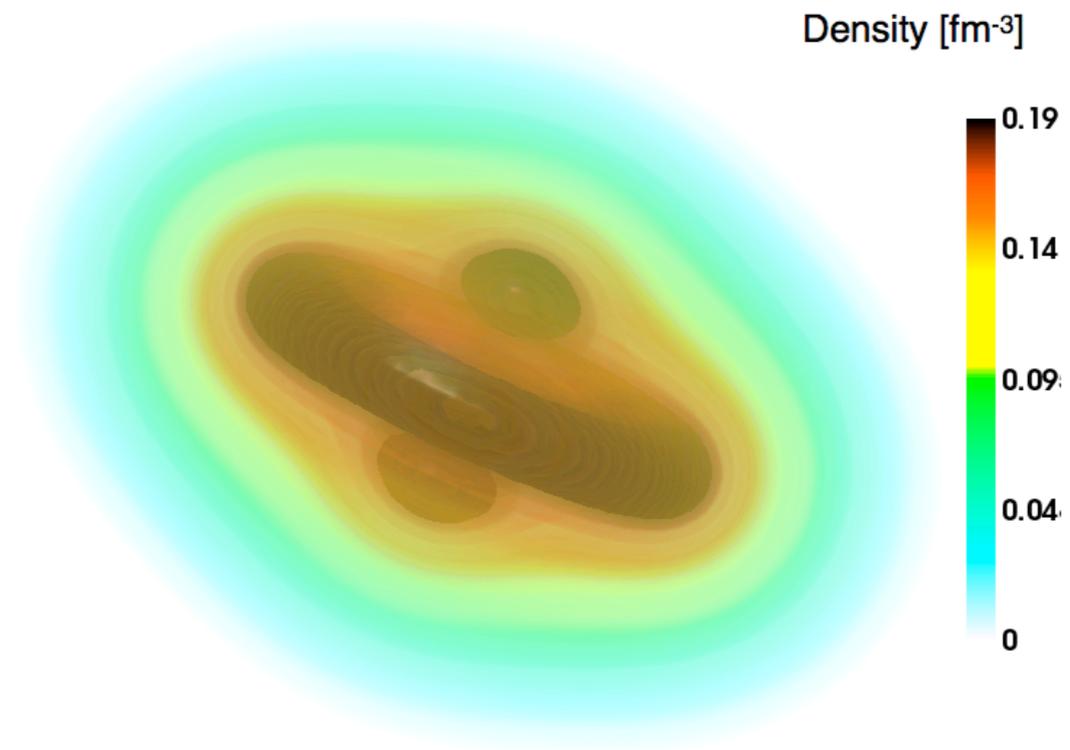


^{28}Si : intrinsic equilibrium density distributions

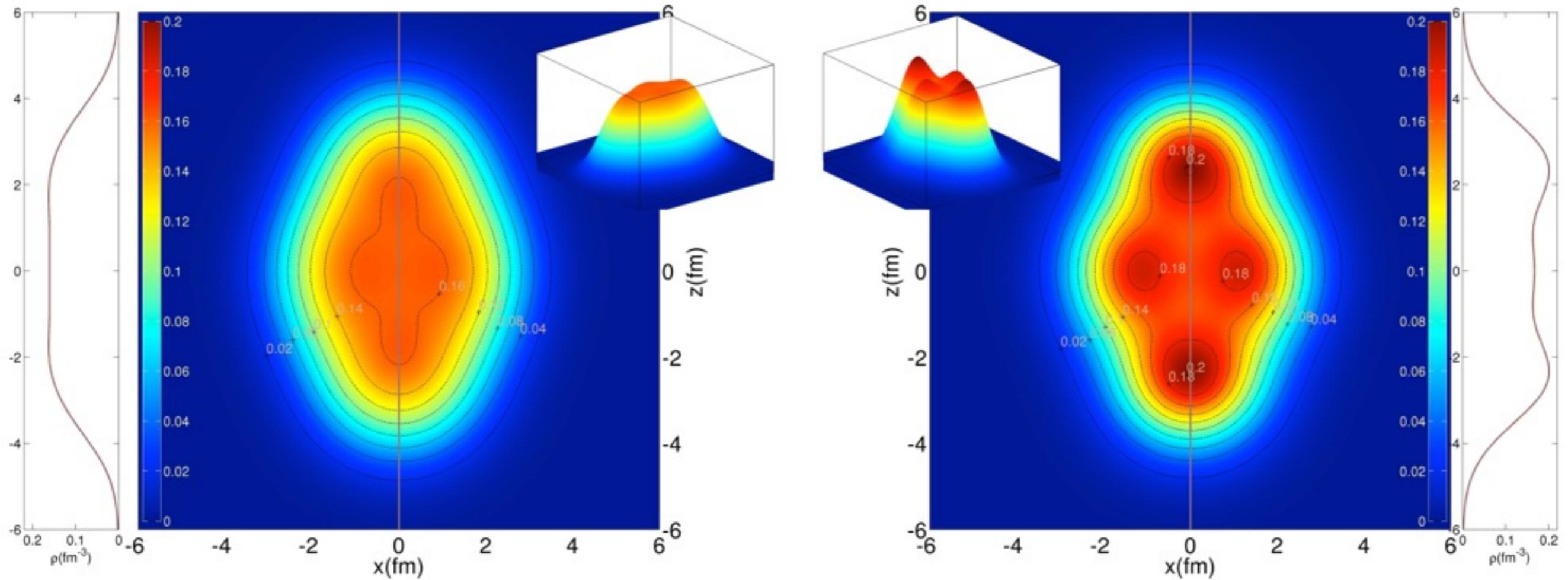
Skyrme Ly4



DD-ME2



^{20}Ne : intrinsic equilibrium density distributions

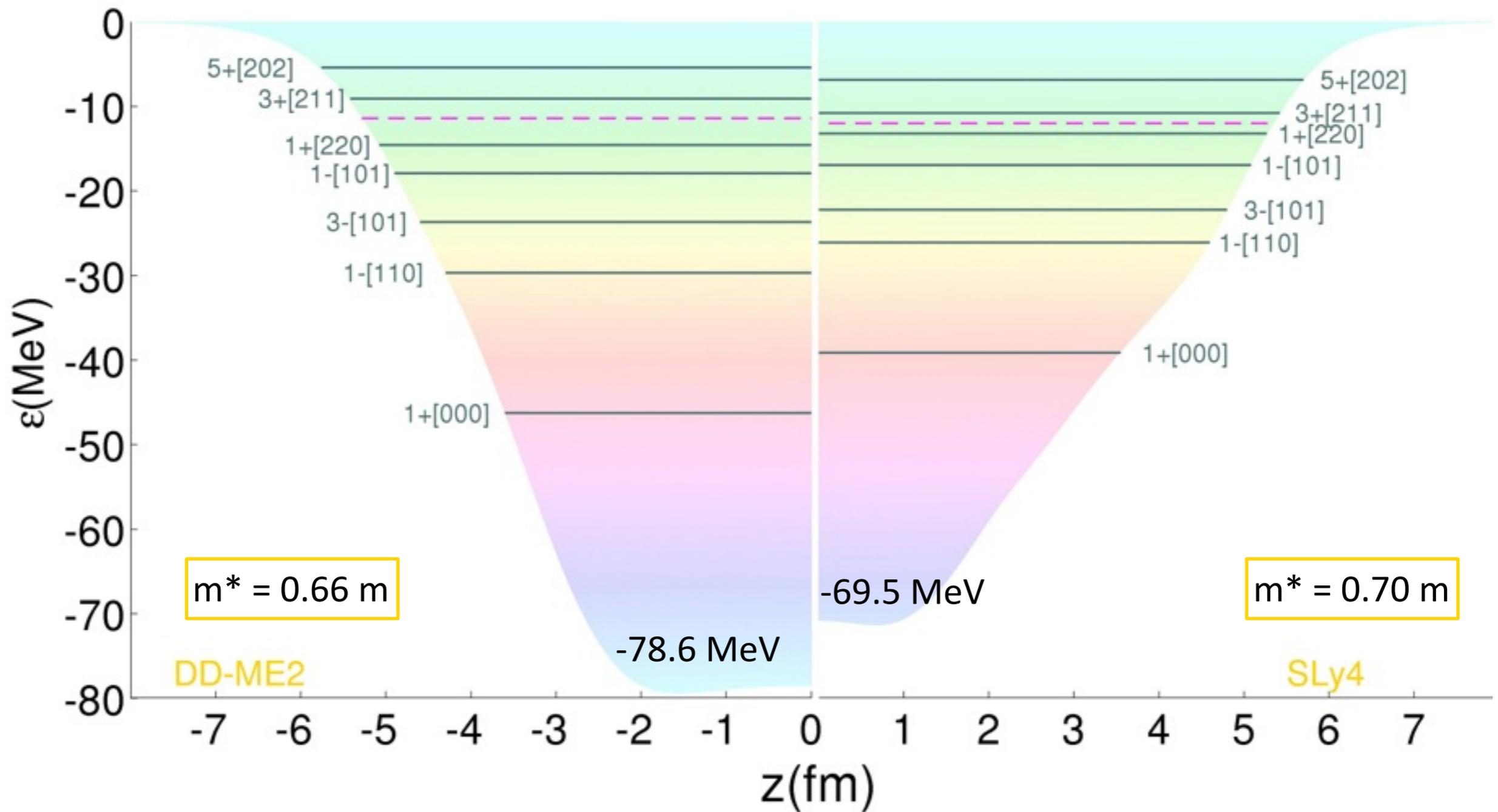


How atomic nuclei cluster

RESEARCH LETTER

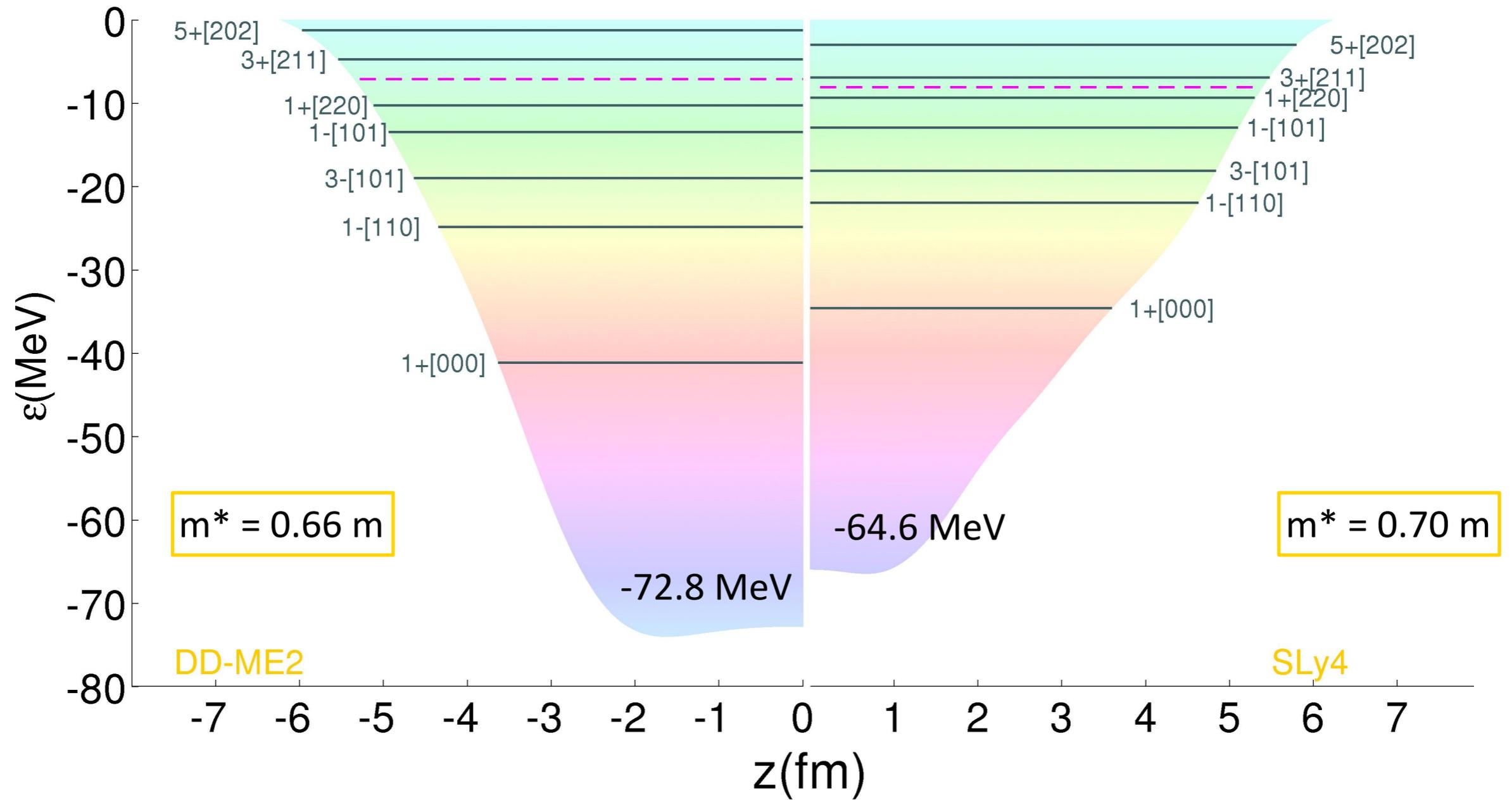
J.-P. Ebran¹, E. Khan², T. Nikšić³ & D. Vretenar³

Effect of the depth of the confining potential:



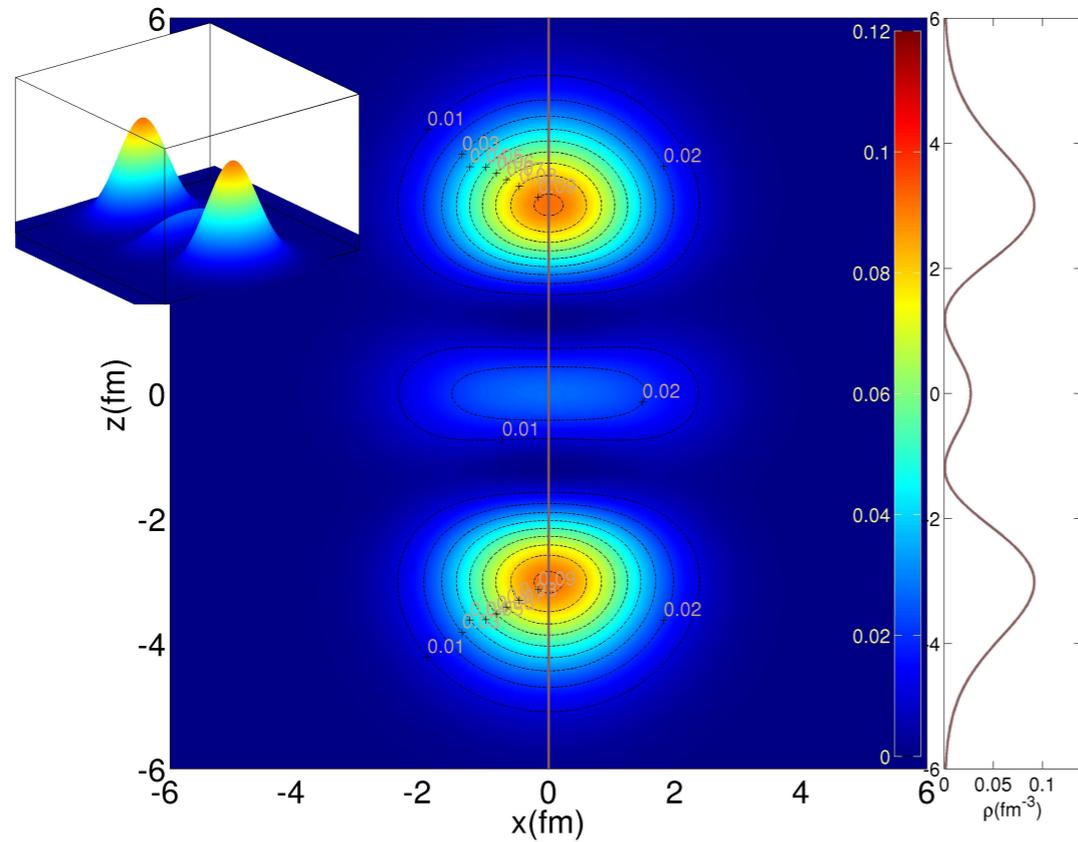
^{20}Ne – single-neutron Nilsson levels at equilibrium deformation

Effect of the depth of the confining potential:

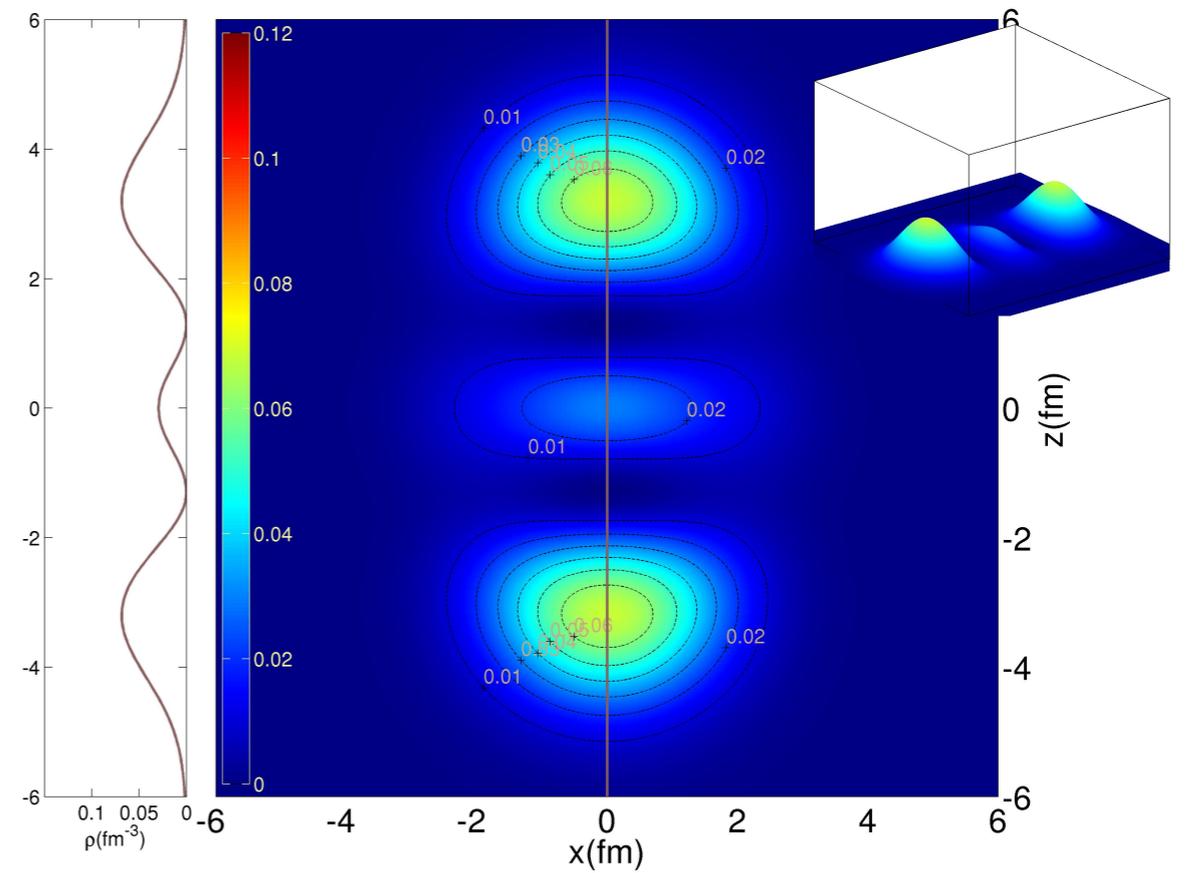


^{20}Ne – single-proton Nilsson levels at equilibrium deformation

Partial nucleon density distributions: highest occupied level $1/2^+[220]$

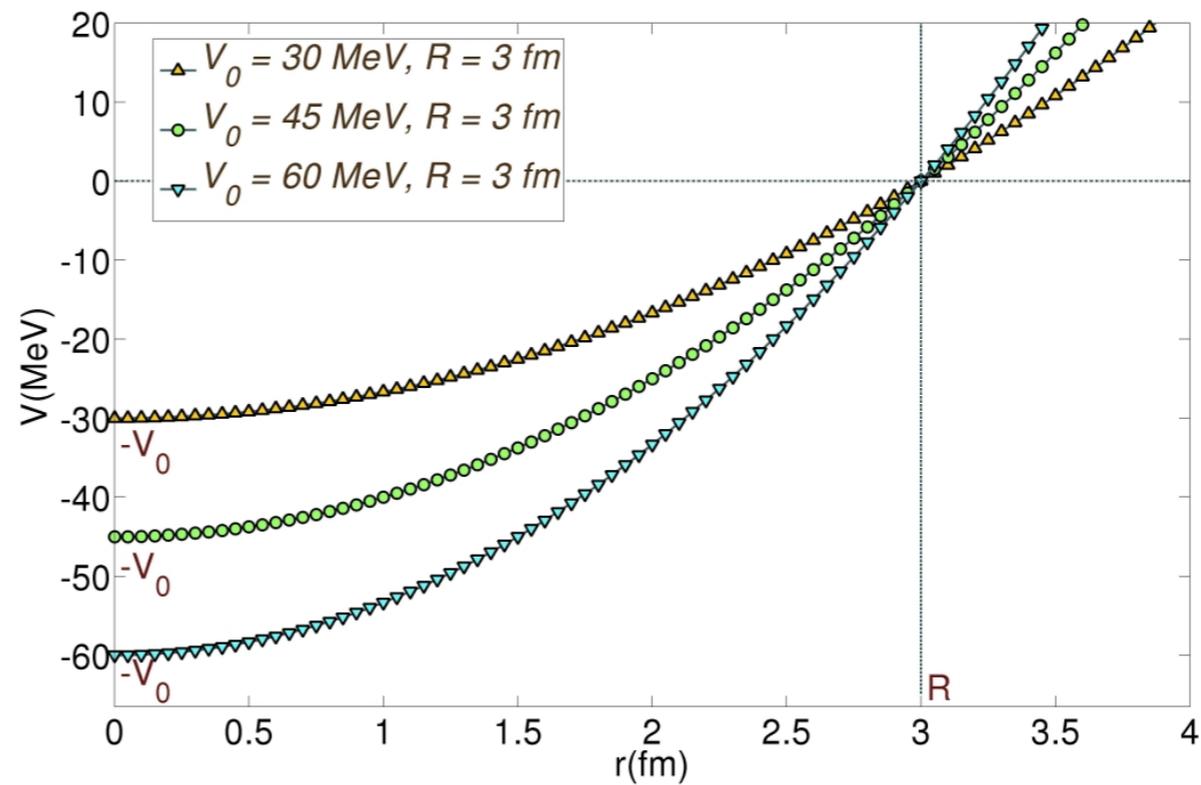


DD-ME2

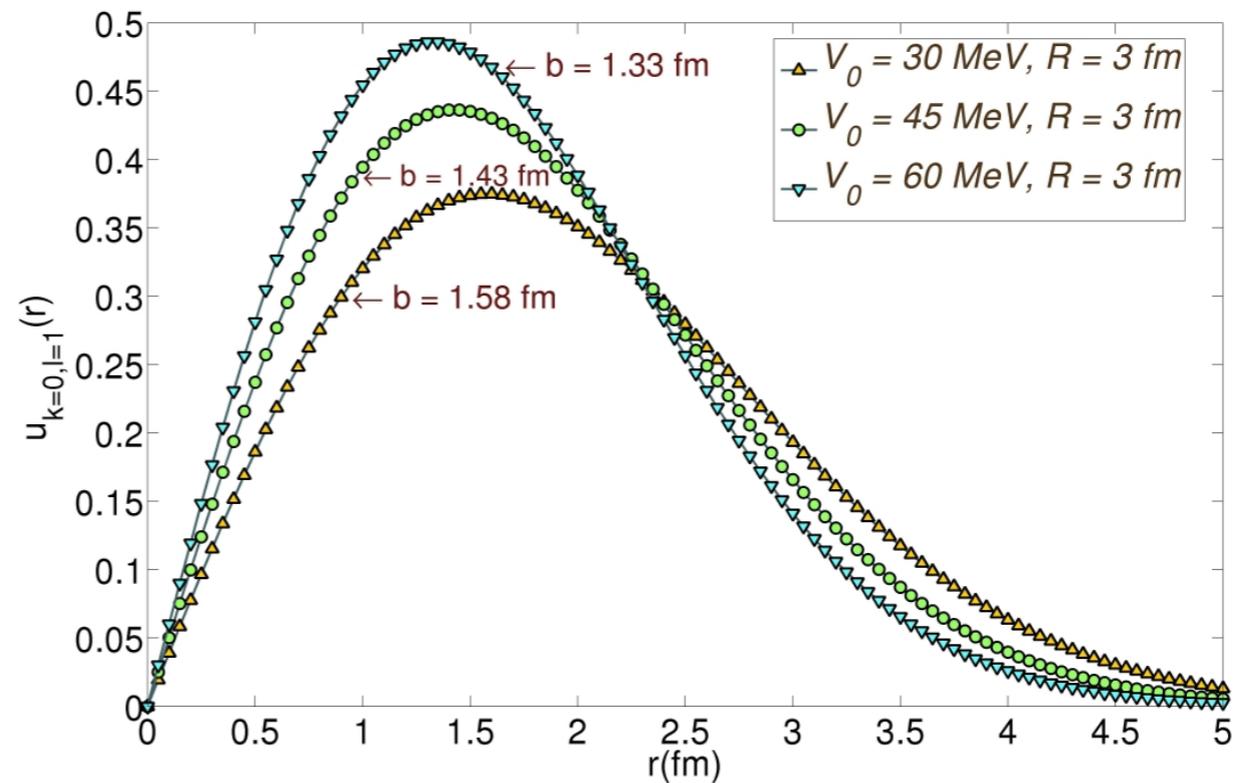


Skyrme Ly4

Spherical harmonic oscillator potentials of different depths but with the same radius $\rightarrow 3$ fm.



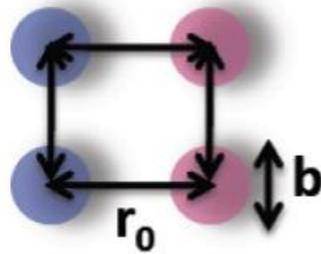
Radial wave functions of the corresponding first p-state.



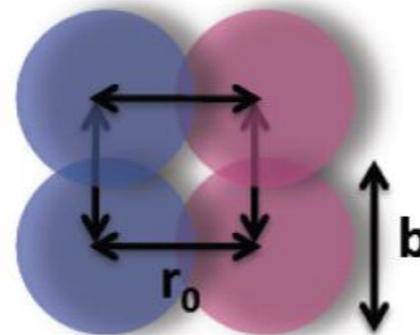
Transition from a crystalline to a quantum liquid phase:

$b \rightarrow$ dispersion of the single-fermion wave function
 $r_0 \rightarrow$ typical inter-nucleon distance

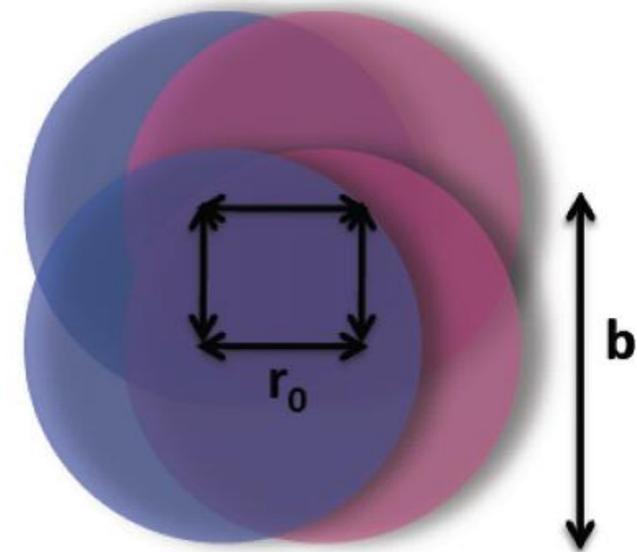
$$\alpha = b/r_0$$



Crystal
 $\alpha < 1$

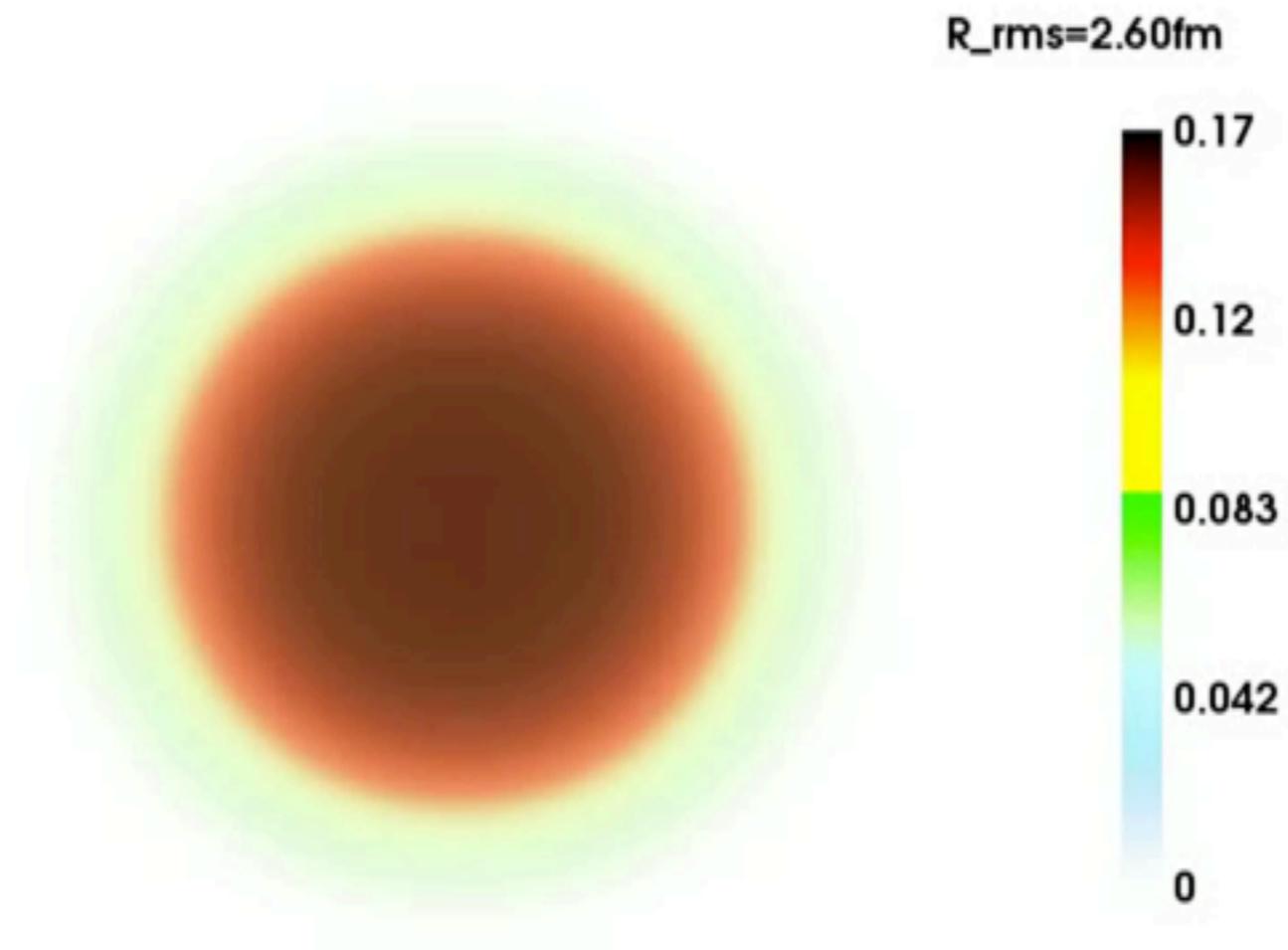


Molecule
 $\alpha \sim 1$



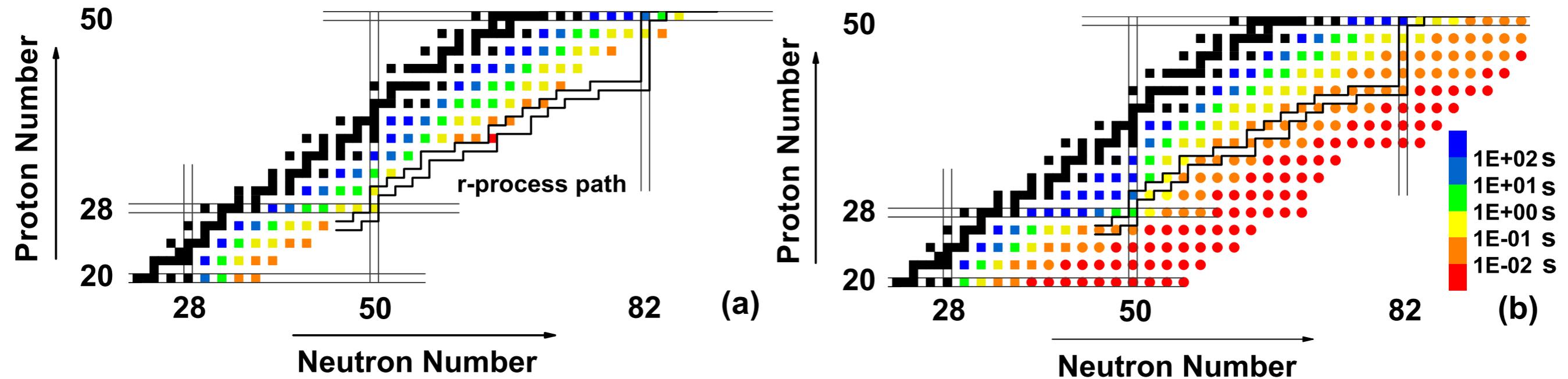
Quantum
liquid
 $\alpha > 1$

Role of nuclear saturation \Rightarrow spontaneous α -clustering at low density



β -decay half-lives of neutron-rich nuclei and matter flow in the r-process

Niu, Niu, Liang, Long, Nikšić, Vretenar, Meng, Phys. Lett. B **723**, 172 (2013).



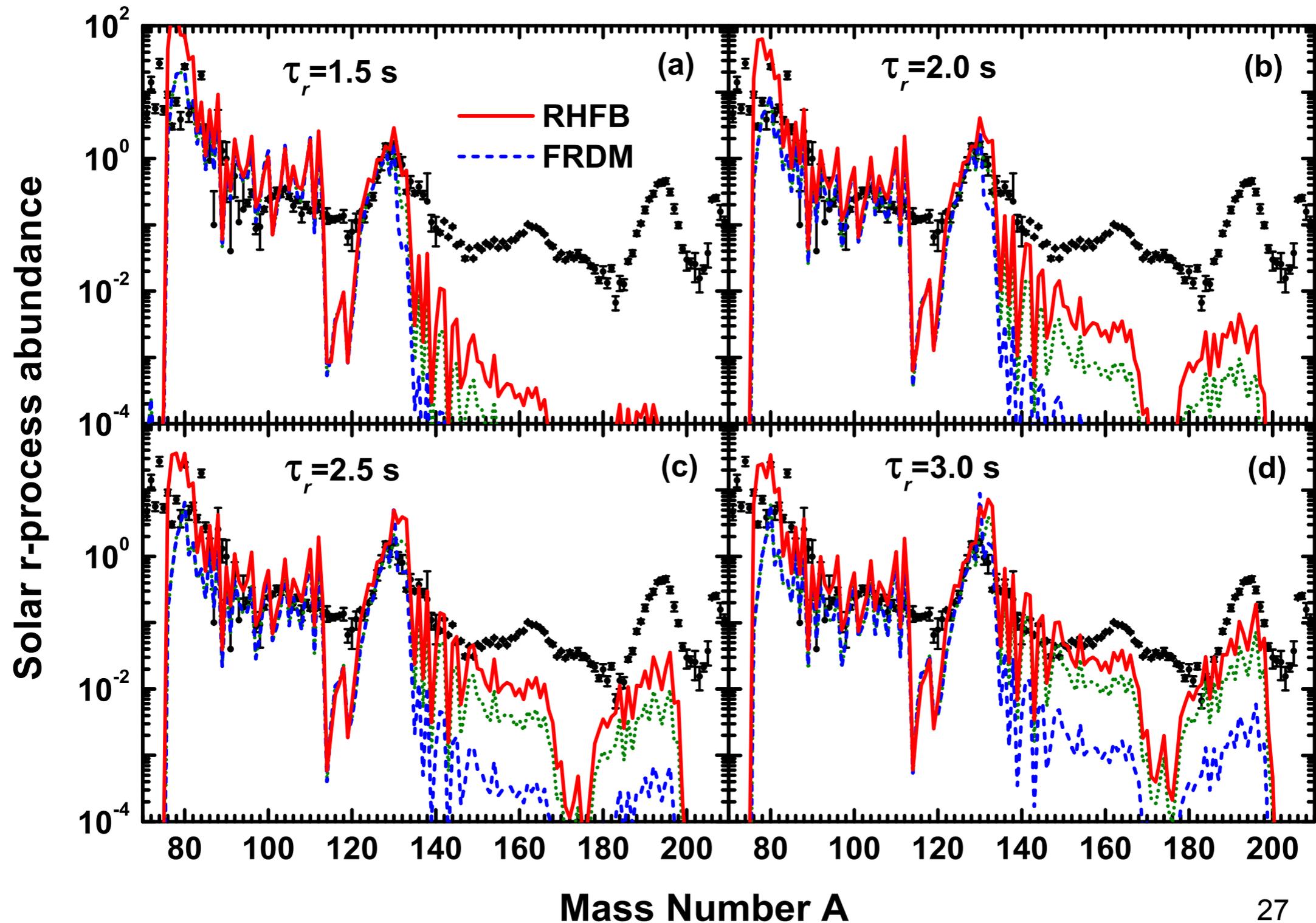
Contour maps of experimental and theoretical β -decay half-lives for the $Z = 20-50$ even-even nuclei.

Half-lives of r-process bottleneck nuclei near $N = 82$

Nucleus	Half-life (s)		
	RHFB + QRPA	FRDM + QRPA	Exp.
^{124}Mo	0.0108	0.0106	–
^{126}Ru	0.0205	0.0342	–
^{128}Pd	0.0486	0.1251	–
^{130}Cd	0.1685	1.1232	0.162 ± 0.007
^{134}Sn	0.7530	3.5410	1.050 ± 0.011

→ impact of the predicted β -decay half-lives on r - process abundances:

The impact of nuclear β -decay half-lives on the r-matter flow. The curves correspond to calculated r-process abundances in comparison to data denoted by the points. Panels (a) - (d) correspond to neutron irradiation times $\tau_r = 1.5, 2.0, 2.5,$ and 3.0 s.



Nuclear Energy Density Functional Framework

✓ unified microscopic description of the structure of stable and nuclei far from stability, and reliable extrapolations toward the drip lines.

✓ when extended to take into account collective correlations, EDFs describe deformations, shape-coexistence and shape-phase transition phenomena associated with shell evolution.

✓ Time-dependent NDFT → fully self-consistent (Q)RPA analysis of giant resonances, low-energy multipole response in weakly-bound nuclei, astrophysical applications.