Spectral Characteristics of Radiation from Thomson and Compton Scattering of an Intense Laser Field by Relativistic Electrons

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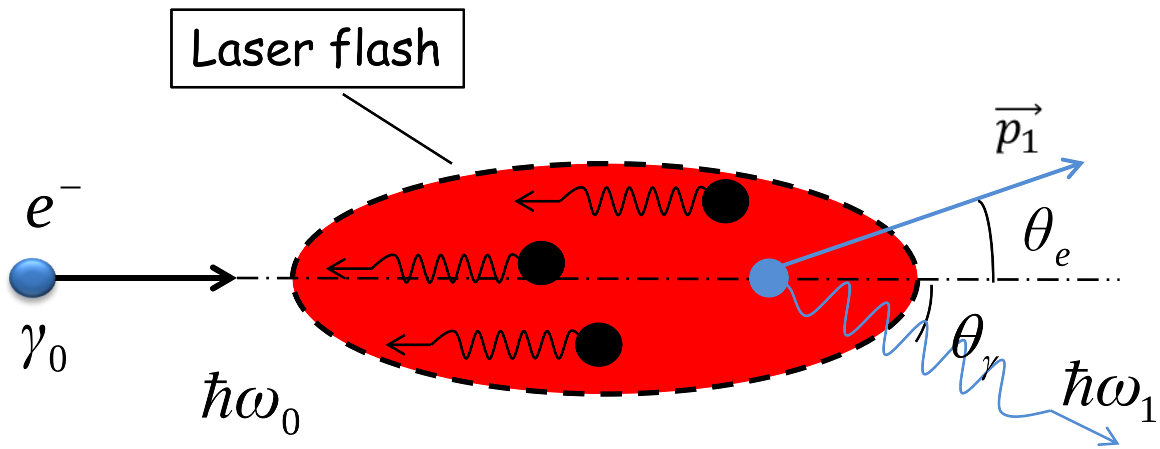
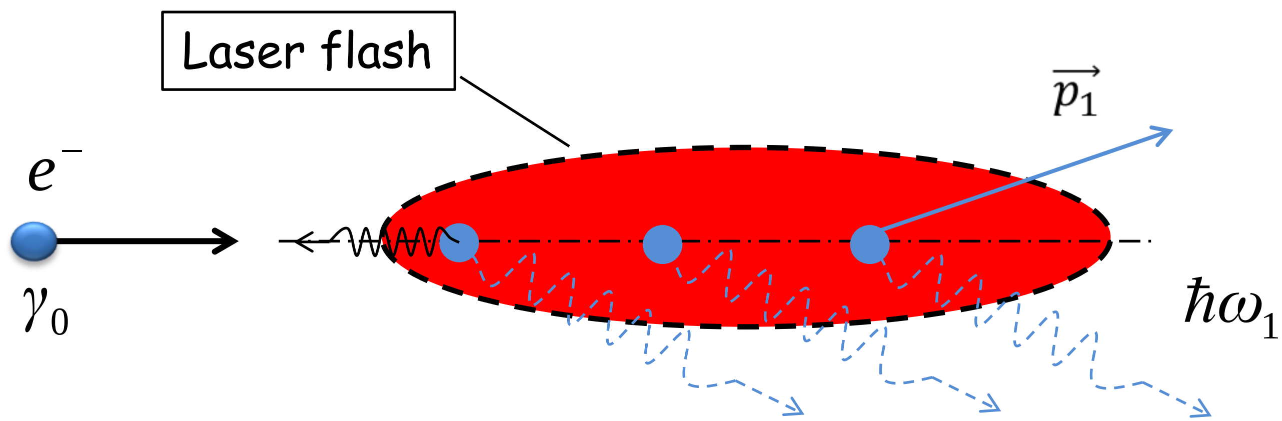
Compton backscattering (Linear Mode)



$$\theta_e \sim \frac{\hbar\omega_0}{mc^2} \leq 10^5$$

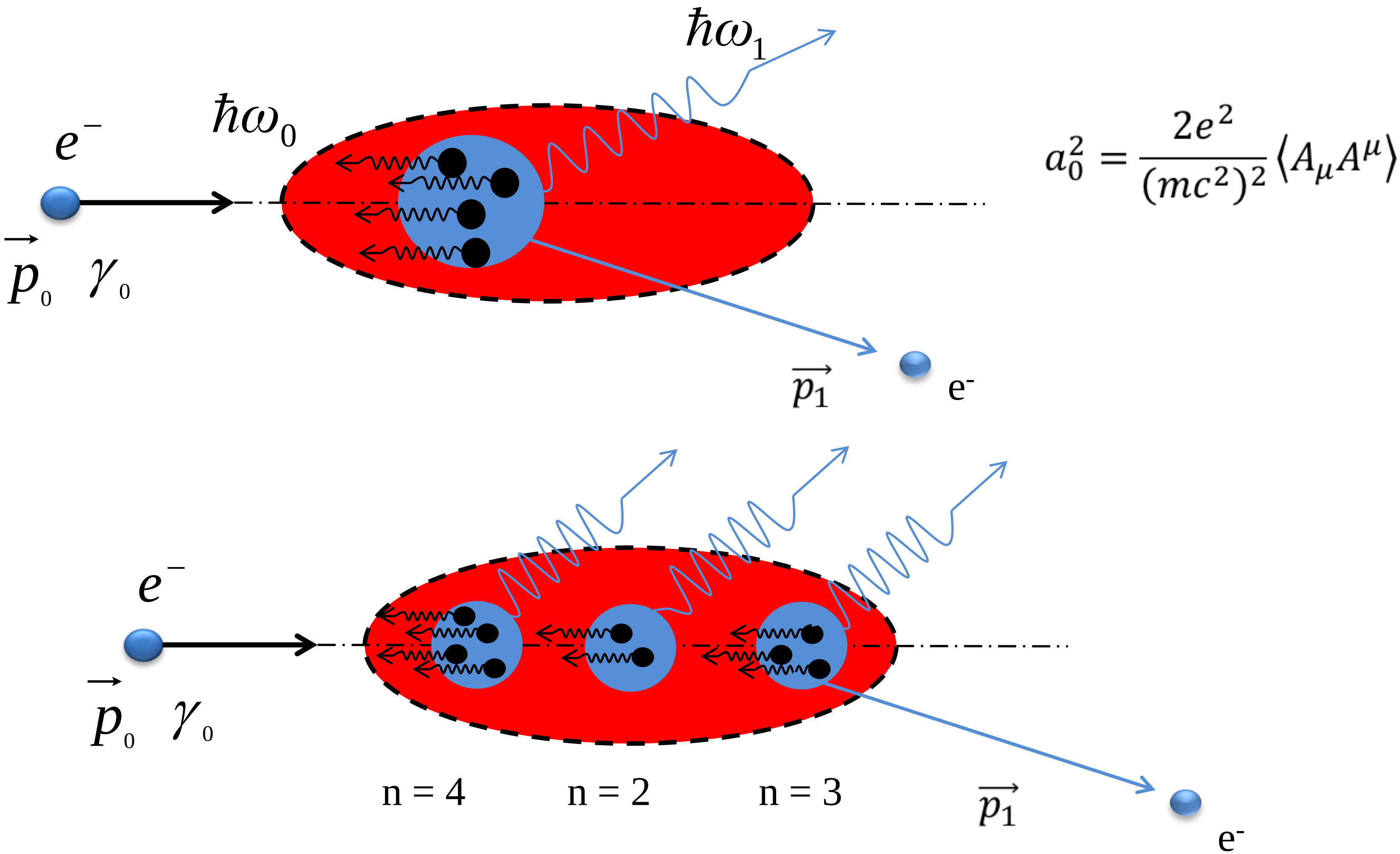
Multiple Compton Backscattering Process (MCBS)



k hard photons, k = 1,2,3,…

Compton backscattering (nonlinear mode)

Absorption of *n* laser photons



Luminosity

$$L = c(1+\beta_0)N_e N_L \int dV dt f_e(x,z,y,t) f_L(x,y,z,t),$$

N_e – number of electrons in bunches,
$N_L = U/\hbar\omega_0$ – number of photons in a laser pulse; U – energy of laser flash,
$F_{e(L)}(x,y,z,t)$ – distribution of each bunches in the space and time.

For 3D Gaussian distribution $F_{e(L)}$ with fixed parameters and head-on collisions:

$$L = \frac{N_e N_L}{2\pi\sqrt{\sigma_{Lx}^2+\sigma_{ex}^2}\sqrt{\sigma_{Ly}^2+\sigma_{ey}^2}},$$

$\sigma_{e(L)}$ – transverse dimensions of electron (laser) bunches.

As a rule, $\sigma_{Lx} > \sigma_{ex,ey}$; for $\sigma_{Lx}^2 = \sigma_{Ly}^2 = \rho_L^2/2$

$$L = \frac{N_e N_L}{\pi\rho_L^2},$$

no dependence on bunch length

Mean number of emitted photons per an electron (mean number of collisions)

$$\bar{k} = \frac{L\sigma}{N_e},$$

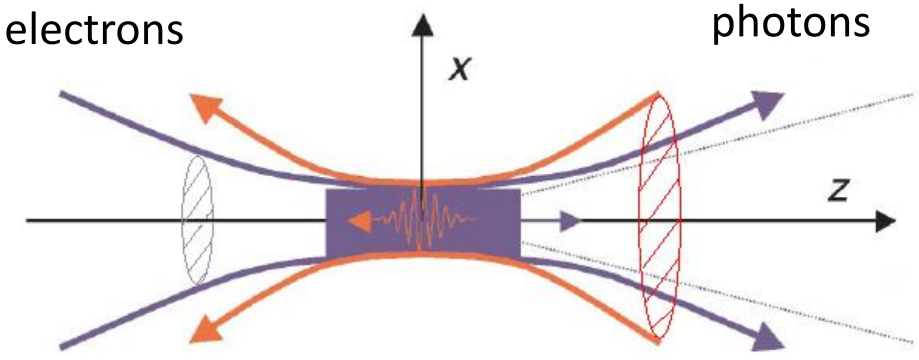
σ – cross-section of Compton scattering

More strict model F.Hartemann et al. PRST-AB, (2005)1007 02

$$f_L \sim \exp(-\frac{\rho^2}{\rho_L^2}), \qquad \rho_L^2 = \rho_0^2\left(1+\frac{z^2}{z_R^2}\right), \quad \rho_e^2 = \rho_b^2(1+\frac{z^2}{\beta_f^2}),$$

$$f_e \sim \exp\left(-\frac{\rho^2}{\rho_e^2}\right),$$

z_R is Rayleigh length, β_f is the beta function



Longitudinad distributions of beams were approximated by Gaussians with parameters l_L and l_e.

In the Hartemann model the mean number of scattered photons per an electron :

$$\bar{k} = \frac{16\sqrt{\pi}}{3}\frac{r_e^2}{\rho_0^2} N_L f(l_e, l_L, r), \text{ where}$$

$$f(l_e,l_L,r) = \exp[(1+r^2)/(\mu^2+r^2\eta^2)]/[(\mu^2+r^2\eta^2)(1+r^2)]^{1/2} \times \{1-\Phi[(1+r^2)^{1/2}/(\mu^2+r^2\eta^2)^{1/2}]\},$$

$$\mu = \frac{l_L}{2\sqrt{2}z_R}, \ \eta = \frac{l_e}{2\sqrt{2}\beta_f}, \ r = \frac{\rho_b}{\rho_0}, \ \Phi(x) \text{ is the error function.}$$

Simplest case

$$\rho_{\mathbf{L}} = const,\ \rho_{\mathbf{e}} = const,\ x_0 << 1$$

In the limit $z_R \to \infty, \beta_f \to \infty$

$$1-\Phi(x) \approx \frac{1}{\sqrt{\pi}} \frac{\exp(-x^2)}{x},$$

$$\bar{k} \approx \frac{16}{3} N_L \frac{r_e^2}{\rho_L^2 + \rho_e^2} \approx 2 N_L \frac{(8/3)\pi\, r_e^2}{\pi\, \rho_L^2} \approx 2\, n_0\, \sigma_T\, l_L$$

n_0 – laser photon concentration

or using luminosity L: $\bar{k} = \frac{L\sigma_T}{N_e}, \quad \sigma_T = \frac{8\pi}{3} r_e^2$ - Thomson cross-section

Cross-section of Compton scattering is determined by kinematics (parameter x_0) and laser field strength parameter a_0.

Classical consideration: $a_0^2 = 2 I_0 \lambda_0^2 \frac{2e}{\pi m c^2}, \quad I_0 = \frac{P}{\pi \rho_L^2} = \frac{U}{\pi \rho_L^2 \tau_L},$

U – energy of laser pulse

QED: $a_0^2 = 4\alpha \lambdabar_e^2 \lambda_0 n_0, \quad n_0 = \frac{U}{\hbar\omega_0} \frac{1}{\pi \rho_L^2 c \tau_L}, \quad \lambdabar_e = \frac{\hbar}{mc}.$

The mean number of emitted photons $\bar{k}$ depends on a_0^2:

$$\bar{k} = 2 n_0 \sigma_T l_L = \frac{\alpha}{2} a_0^2 \frac{\sigma}{r_e^2} \frac{l_L}{\lambda_0} \approx \frac{4}{3} \pi \alpha a_0^2 \frac{l_L}{\lambda_0}$$

CROSS – SECTION OF THE COMPTON SCATTERING

depends on a number of absorbed photons and a_0^2

Invariant spectral variable:

$$y^{(n)} = \frac{k_1^{(n)} k_0}{p_0 k_0} = \frac{\hbar\omega^{(n)}}{\gamma_0 m c^2},$$

The "partial" differential cross-section

$$\frac{d\sigma^{(n)}}{dy^{(n)}} = \frac{4\pi r_0^2}{x_0 a_0^2} \times \left\{-4J_n^2 + \frac{a_0^2}{2}\left(1 - y^{(n)} + \frac{1}{1-y^{(n)}}\right)\left(J_{n-1}^2 + J_{n+1}^2 - 2J_n^2\right)\right\},$$

J_m^2 is the Bessel function of order m=n-1,n,n+1 depending on argument:

$$z_n = \sqrt{2} n a_0 \sqrt{\frac{y^{(n)}}{(1-y^{(n)}) n x_0}} \times \left[1 - \frac{y^{(n)}\left(1+\frac{a_0^2}{2}\right)}{1-y^{(n)} n x_0}\right].$$

The cross-section of the process with absorption of n photons:

$$\sigma^{(n)} = \int_0^{y_{max}^{(n)}} \frac{d\sigma^{(n)}}{dy^{(n)}} dy^{(n)}, \qquad y_{max}^{(n)} = \frac{n x_0}{1 + n x_0 + \frac{a_0^2}{2}}$$

The total cross-section: $\sigma_{non\ lin} = \sigma_{tot} = \sum_{n=1}^{\infty} \sigma^{(n)} = \sum_{n=1}^{n_{max}} \sigma^{(n)}$; $P_{non\ lin}(n) = \frac{\sigma^{(n)}}{\sigma_{tot}}$.

The maximal number n_{max} → from condition $\sigma^{(n_{max}+1)}/\sigma_{tot} < 0.005$.

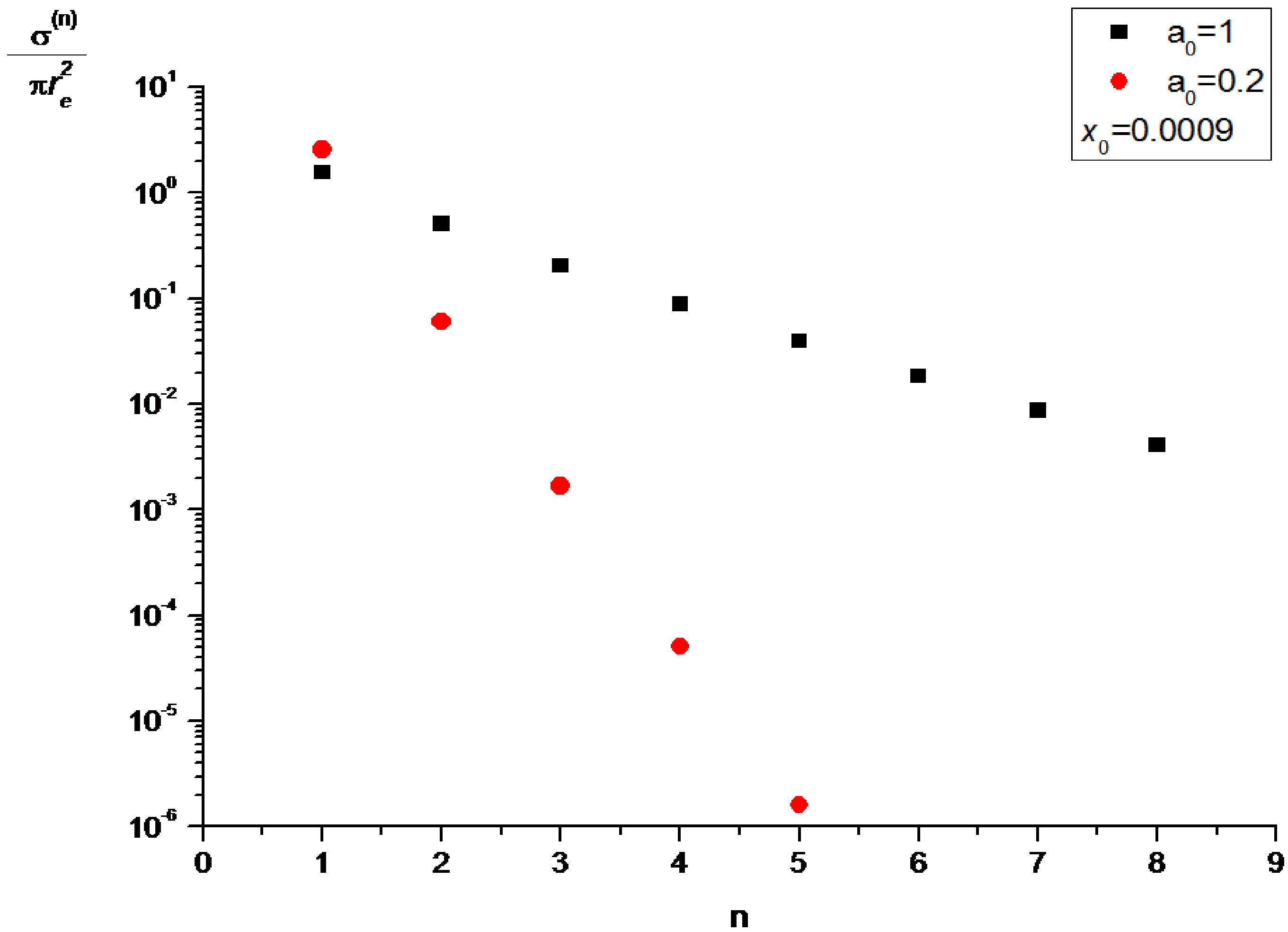


Fig. 1

Fig. 1. Dependence of nonlinear Compton partial cross-section on the number of "absorbed" photons *n* for different parameters a_0 (E_0 = 50 MeV; $\hbar\omega_0$ = 1.17 eV; x_0 = 0.0009)

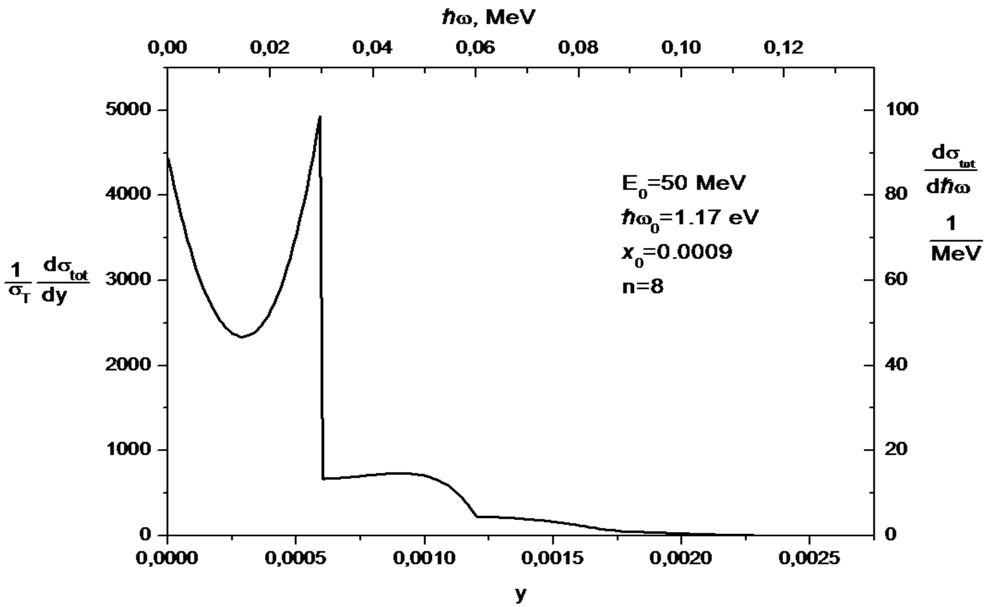


Fig. 2 a

Fig. 2 a) The cross-section energy dependence of nonlinear Compton backscattering for the same parameters for $a_0 = 1$ (per unit energy interval – right axis).

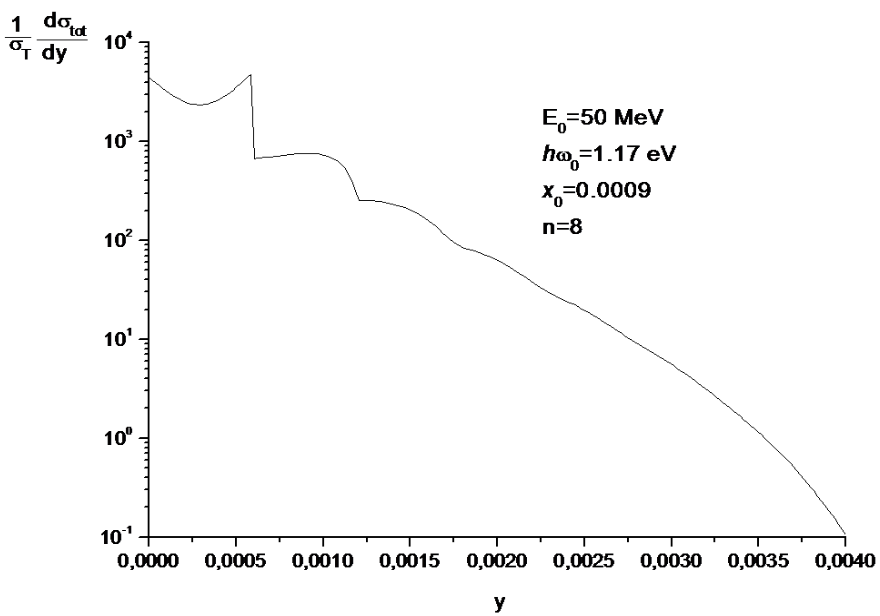


Fig. 2 b

Fig. 2 b) The same dependence on the logarithmic scale

Thomson scattering

Classical consideration of electromagnetic wave scattering by a charge (no recoil, formally $x_0 \to 0$)

Frequency of n-th harmonics: $$\omega_n = \frac{4\gamma_0^2\omega_0}{1+(\gamma_0\theta_\gamma)^2+a_0^2/2}$$

The dimensionless spectral variable:

$$S_n = \frac{\omega_n}{4\gamma_0^2\omega_0} = \frac{n}{1+(\gamma_0\theta)^2+a_0^2/2}, \qquad 0 \le S_n \le \frac{n}{1+a_0^2/2}$$

The spectral distribution of emitted photons from an electron trajectory with $N_{osc} \gg 1$ oscillations in the laser field for the fixed harmonic number n:

$$\frac{dN^{(n)}}{dS_n} = 2\pi\alpha a_0^2 N_{osc}\left\{\frac{\left[n-S_n(2+a_0^2)\right]^2}{2S_n a_0^2\left[n-S_n\left(1+\frac{a_0^2}{2}\right)\right]}J_n^2(nz)+J_n^{`2}(nz)\right\}, \;\; nz=\sqrt{2}a_0\sqrt{S_n n-S_n^2(1+a_0^2/2)}$$

The resulting spectrum: $$\frac{dN}{dS} = \sum_{n=1}^{n_{max}}\frac{dN^{(n)}}{dS_n}, \qquad 0 \le S \le \frac{n_{max}}{1+a_0^2/2}$$

Using the relation:

$$dN = \frac{N_{ph}}{S_{ph}} d\sigma_{Thom}, \qquad where \;\; \frac{N_{ph}}{S_{ph}} = \frac{U}{\hbar\omega_0 \pi \rho_L^2} = \frac{\alpha}{4}\frac{a_0^2}{r_e^2}\frac{l_L}{\lambda_0}$$

It is possible to get the differential Thomson “partial” cross-section:

$$\frac{d\sigma_{Thom}^{(n)}}{dS_n} = \frac{4r_e^2}{\alpha a_0^2 N_0}\frac{dN^{(n)}}{dS_n} = 8\pi r_e^2 \times \left\{ \frac{[n - S_n(2 + a_0^2)]^2}{2S_n a_0^2 \left[n - S_n\left(1 + \frac{a_0^2}{2}\right)\right]} J_n^2(nz) + J_n^{\text{`}2}(nz) \right\}$$

And total cross-section:

$$\frac{d\sigma_{Thom}}{dS} = \sum_{n=1}^{n_{max}} \frac{d\sigma_{Thom}^{(n)}}{dS_n}$$

The multiple nonlinear Compton/Thomson scattering process
is connected with two stochastic processes:

- Random collision of an electron with laser photons (k = 0,1,2,…);
- Random number of photons absorbed in each collision (n = 1,2,3,…). (Monte-Carlo simulation only)

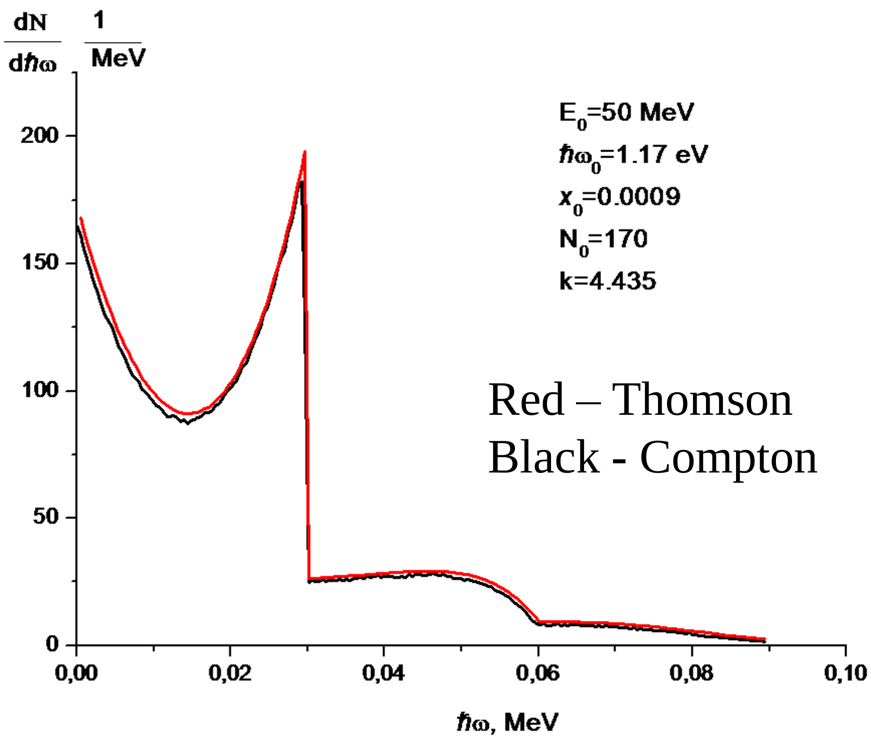


Fig. 3

Fig. 3. Comparison of cross-sections for Thomson and Compton spectra generated by an electron with energy of 50 MeV travelling through a laser pulse consisting of N_0 = 170 cycles (L = 600 fs). The mean number of emitted photons per electron is $\bar{k}$ = 4.4.

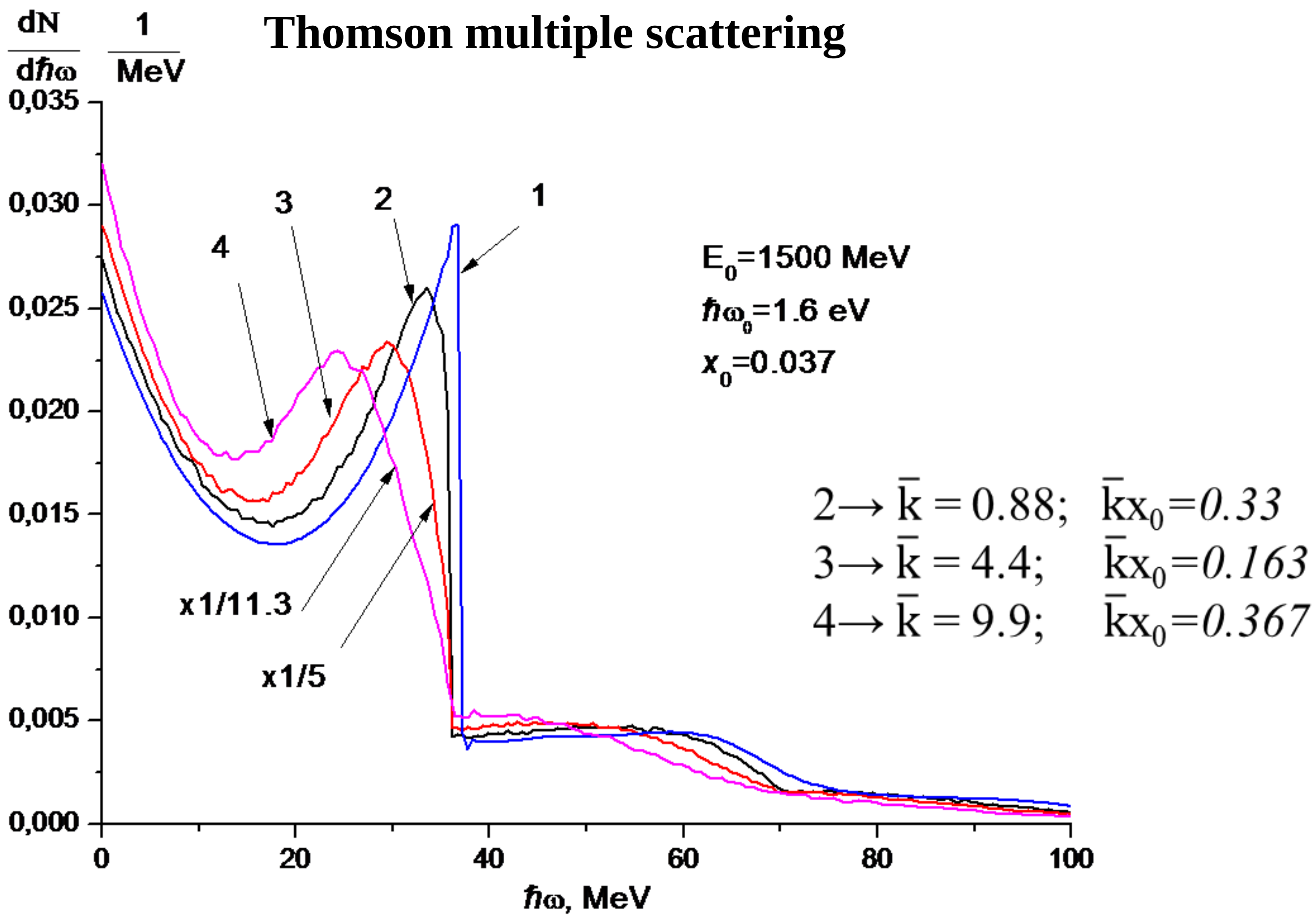


Fig. 4

Fig. 4. The same as in the previous figure for an electron $E_0 = 1500$ MeV travelling through laser pulses with different duration (curves 2 - $\bar{k} = 0.88$; curve 3 - $\bar{k} = 4.4$; curve 4 - $\bar{k} = 9.9$). The areas under the curves 1 and 2 are equal to the mean number of photons ($\bar{k} = 0.88$), but there are the normalizing factors for the curve 3 (4.4/0.88) and for curve 4 (9.9/0.88).

Monte-Carlo simulation

of nonlinear MCBS process

- Calculation of $\sigma^{(n)}, \sigma_{tot}, P_{nonlin}(n)$;
- Simulation of the path length between collisions along laser pulse;
- Simulation of the number of absorbed photons n in each collision;
- Simulation of the energy loss in a collision;
- Calculation of the emitted photon angle θ and the check of the criterion $\theta \leq \theta_c$.

ISSN 1063-7796, Physics of Particles and Nuclei, 2014, Vol. 45, No. 5, pp. 1000– 1012. © Pleiades Publishing, Ltd., 2014.
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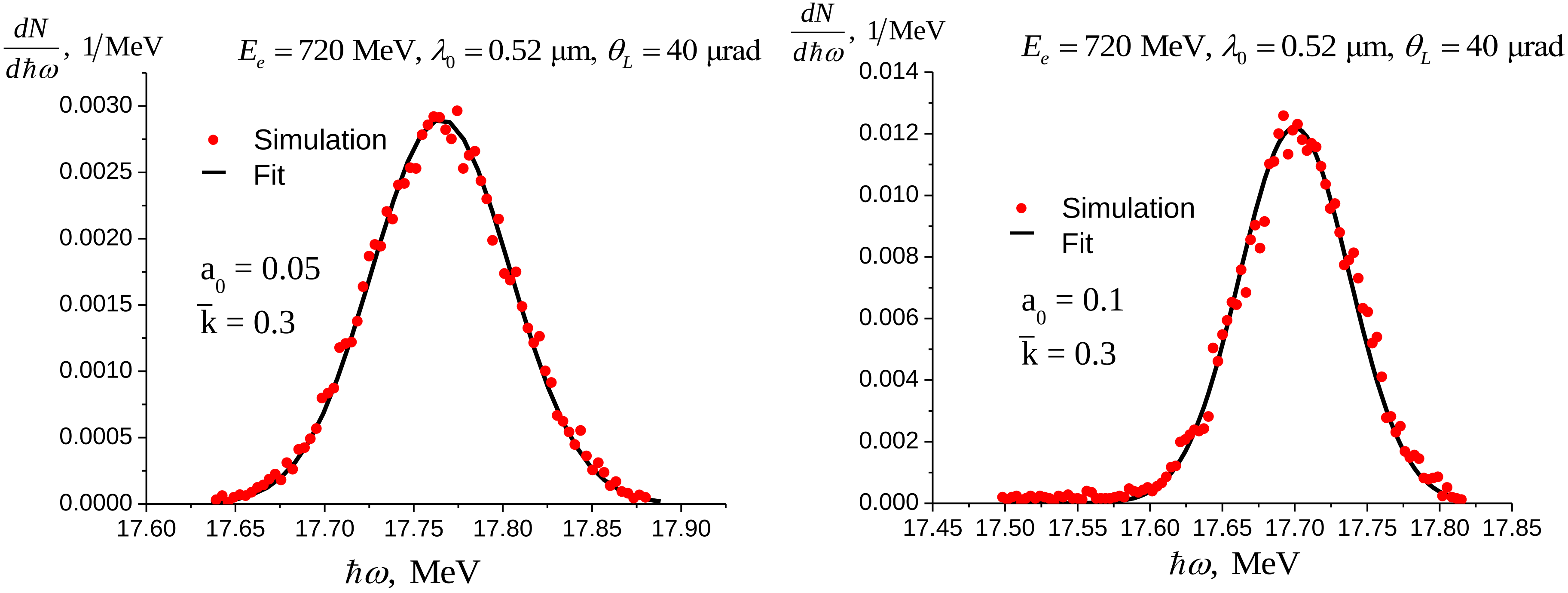
Statistical Simulation of Multiple Compton Backscattering Process

A. P. Potylitsyn[a, b] and A. M. Kolchuzhkin[c]

Collimated photon spectra

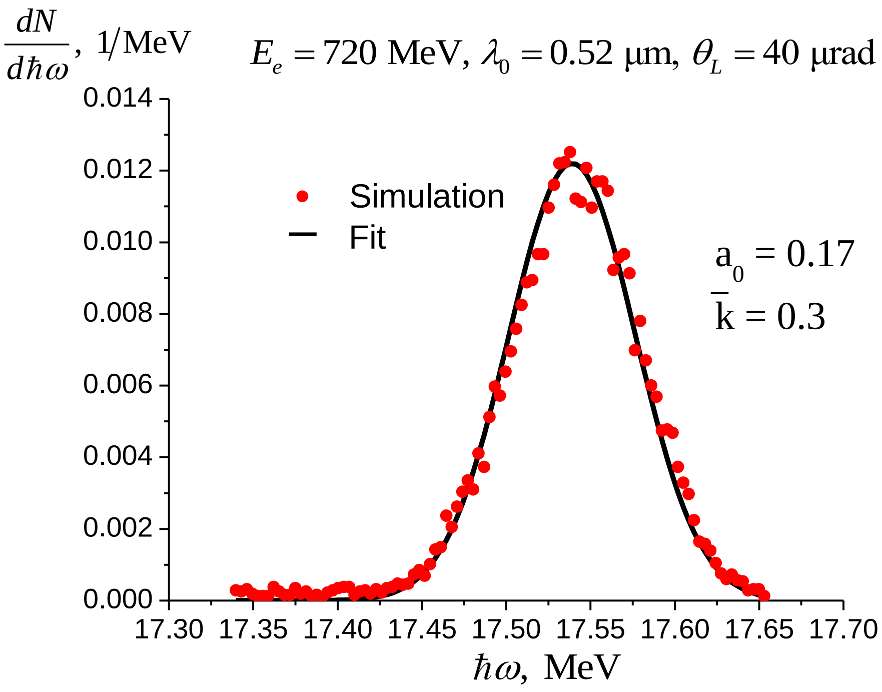
$$\Delta E_e / E_e = 0.1\%$$

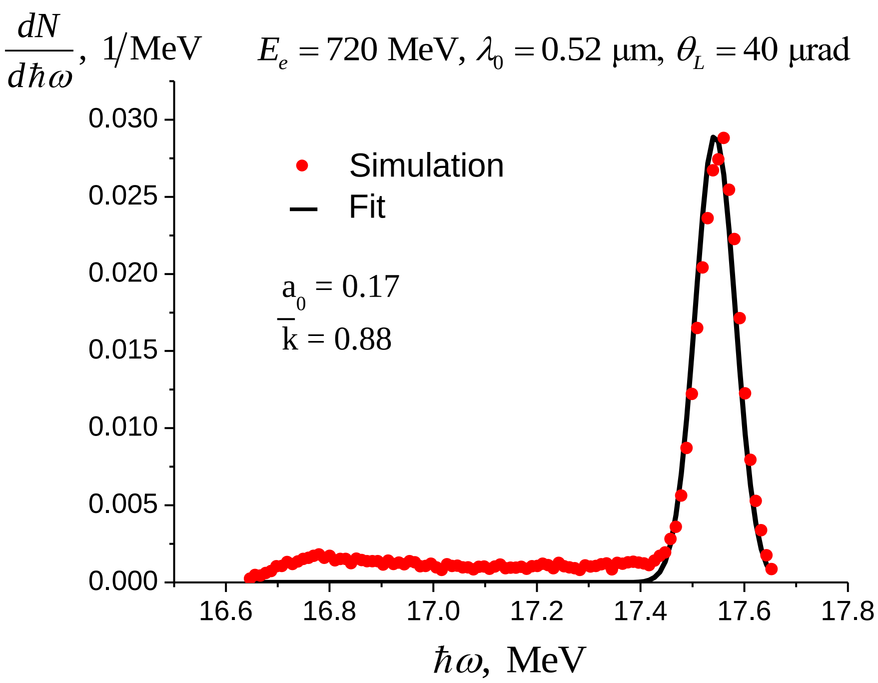


FWHM = 89 keV; σ = 38 keV

Collimated photon spectra

$$\Delta E_e / E_e = 0.1\%$$





For multiplicity $\bar{k} \geq 1$ soft photons pass through a collimator worsening the monochromaticity of the resulting beam

L. Serafini (2011)

Energy-angular Spectral distribution

For a bunch the energy spread of the collected photons depends on

- Collecting angle θ_M
- Bunch energy spread
- Transverse emittance

$$\frac{\Delta E_X}{E_X} \cong 2\frac{\Delta\gamma}{\gamma} + (\gamma\vartheta_M)^2 + 2\left(\frac{\varepsilon_n}{\sigma_\perp}\right)^2 + \frac{a_0^2}{1+a_0^2}$$

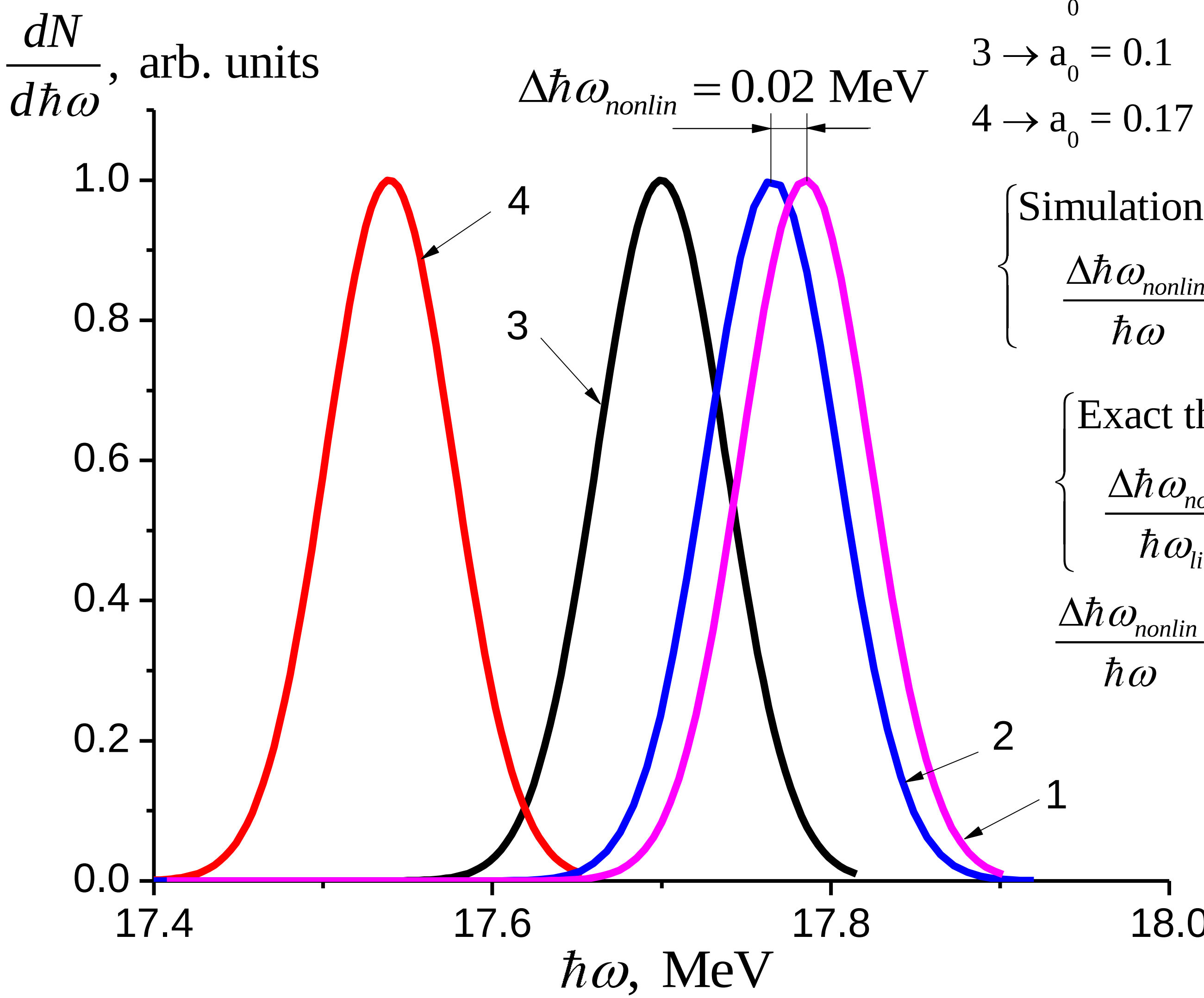
$$\Psi = \bar{\gamma}\bar{\vartheta} \cong (\bar{p}_\perp / mc) \cong (\varepsilon_n / \sigma_\perp)$$

$$Optimized\ \ Bandwidth \cong 2(\varepsilon_n / \sigma_\perp)^2$$

$$Optimized\ \ Spectral\ \ Density \propto$$
$$Luminosity / (\varepsilon_n / \sigma_\perp)^2 \propto Q / \varepsilon_n^2$$

“Red-shift” of spectral lines due to nonlinearity of the MCBS process

$E_e = 720$ MeV, $\lambda_0 = 0.52$ μm, $\theta_L = 40$ μrad



$1 \rightarrow a_0 = 0$

$2 \rightarrow a_0 = 0.05$ (U= 0.5 J; ρ_L = 25 μm)

$3 \rightarrow a_0 = 0.1$ (U= 0.5 J; ρ_L = 15 μm)

$4 \rightarrow a_0 = 0.17$ (U= 1 J; ρ_L = 15 μm)

Simulation (a_0 =0.05; x_0 =0.025)

$$\frac{\Delta\hbar\omega_{nonlin}}{\hbar\omega} = 0.0011 = 0.11\%$$

Exact theory ($a_0, x_0 << 1$)

$$\frac{\Delta\hbar\omega_{nonlin}}{\hbar\omega_{lin}} = \frac{a_0^2}{2}(1 - x_0)$$

$$\frac{\Delta\hbar\omega_{nonlin}}{\hbar\omega} = 0.0012 = 0.12\%$$

Monochromaticity of the collimated γ-beam

$$\frac{\Delta\hbar\omega}{\hbar\omega} = 2\frac{\Delta\gamma_0}{\gamma_0} + (\gamma_0\theta_c)^2 + 2\left(\frac{\varepsilon_n}{\sigma_\perp}\right)^2 + \frac{a_0^2}{1+a_0^2},$$

ε_n – normalized emittance
$\sigma_\perp$ - transverse e^- beam size

The last term is connected with "red shift" with transition from the linear regime to the nonlinear one.
A broadening of the spectral line is caused by the variable value of the nonlinearity parameter a_0 along electron trajectory:

$$a_0(r) = 2\sqrt{\alpha n_0(r)\bar{\lambda}_e^2\lambda_0}$$

$$\Delta a_0^2 = \langle a_0^2 \rangle - \langle a_0 \rangle^2$$

$$\left(\frac{\Delta\hbar\omega}{\hbar\omega}\right)_{nonlinear} \sim \frac{1}{2}\Delta a_0^2$$

Summary

1. For kinematic range $x_0 << 1$ photon spectra calculated in semi classical (Thomson regime) and quantum consideration give identical results if the more strong condition $\bar{k}x_0 << 1$ is fulfilled ($\bar{k}$ - the mean number of emitted photons).

2. Even for laser fields with $a_0 \sim 0.05$ (for instance, as at the ELI-NP project) simulation of the spectral line shape should be performed with taking into account a nonlinearity of the CBS process.

3. There is the additional reason for broadening of the spectral line connected with "jitter" of the nonlinearity parameter a_0 varying from a shot to shot Δa_0^2

Thanks for your attention