The Role of Space Dispersion in Defining The Point Charge Imagination Near Surface of a Dielectric

Filippov G.M.
filippov38-gm@yandex.ru

This work devoted to analysis of influence of spatial dispersion on a point charge imagination near a surface of a dielectric or a metal.

The charge imagination sufficiently well describes the polarization field which occurs near a dielectric or metal surface in the presence of external point charge.

$$\varepsilon_{\omega} = 1 - \Theta(k_c - k)\omega_0^2 / \omega^2$$

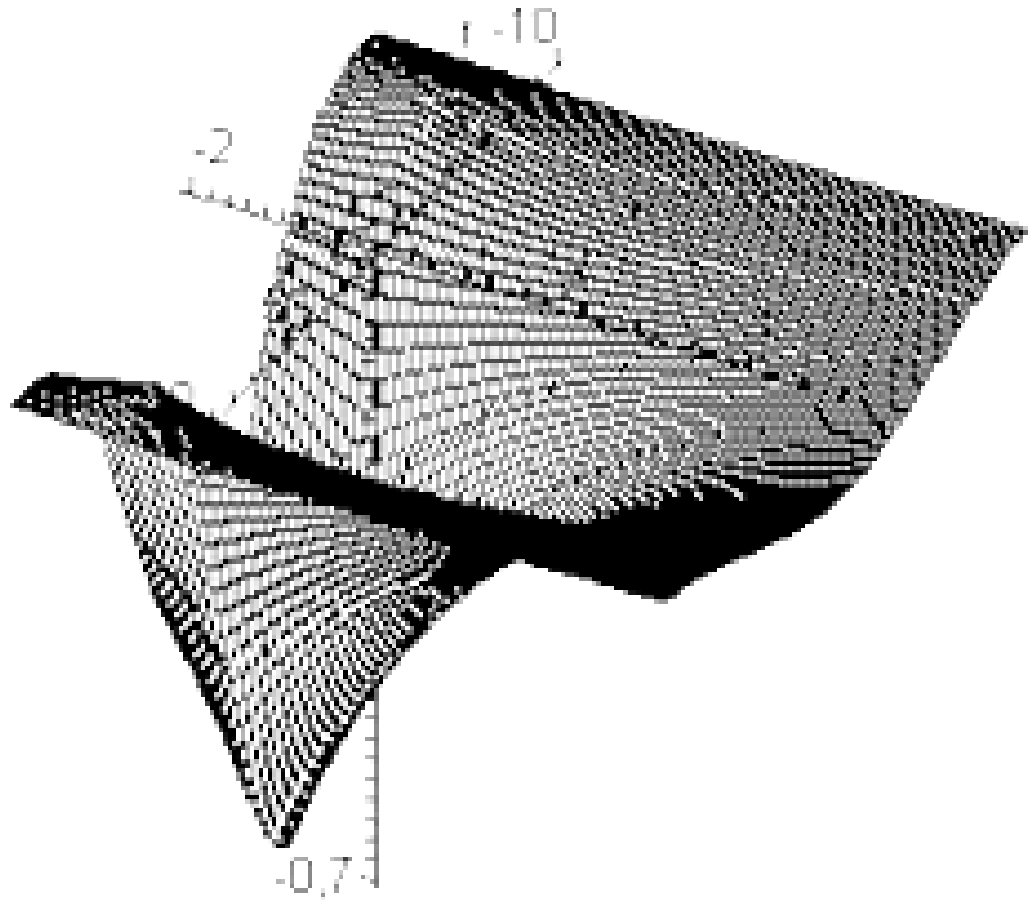
$$\Phi_s(z \geq 0) = -e \int_0^{k_c} dk_{\|} e^{-k_{\|}|b+z|} J_0(k_{\|} r)$$

$$x = \sqrt{z^2 + r^2} >> k_c^{-1}$$

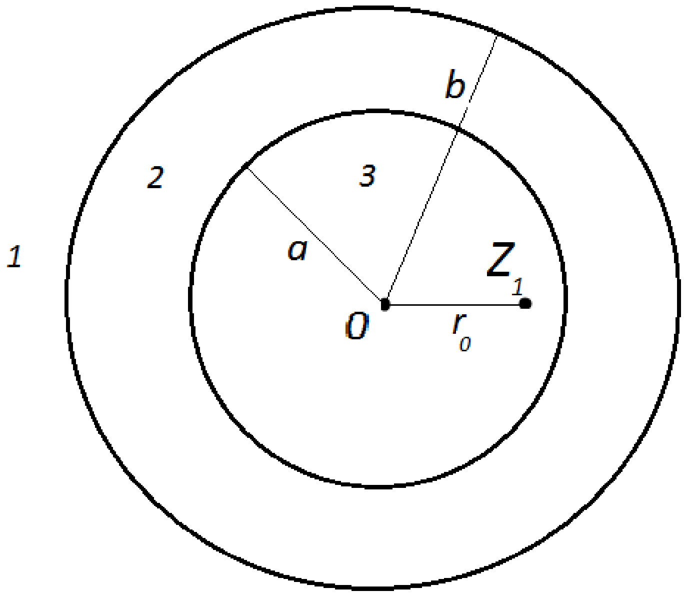
$$\Phi_s(z \geq 0) \approx -e \int_0^{\infty} dk_{\|} e^{-k_{\|}|b+z|} J_0(k_{\|} r) = -\frac{e}{\sqrt{(b+z)^2 + r^2}}$$

Image charge potential

The external charge is found in the point $z=1$, $r=0$. Minimum of the potential equal to $-Z_1k$.



Dielectric tube



$$\Phi_\omega = \sum_{k,m} e^{im\varphi}\left(R^{+}_{\omega km}(r)e^{ikz} + R^{-}_{\omega km}(r)e^{-ikz}\right).$$

$$\frac{d^2 R^{\pm}_{\omega km}}{dr^2} + \frac{1}{r}\frac{dR^{\pm}_{\omega km}}{dr} - \left(k^2 + \frac{m^2}{r^2}\right)R^{\pm}_{\omega km} = -4\pi\rho^{\pm}_{\omega km}(r)$$

$$R^{\pm}_{\omega km}(r) = R^{(ext)\pm}_{\omega km}(r) + \begin{cases} D^{\pm}K_m(|k|r) + G^{\pm}I_m(|k|r), & 0 \le r \le a \\ B^{\pm}K_m(|k|r) + C^{\pm}I_m(|k|r), & a < r \le b \end{cases}$$

$$R^{(ext)\pm}_{\omega km}(r) = -4\pi\int_0^r \left[I_m(|k|r)K_m(|k|r') - I_m(|k|r')K_m(|k|r)\right]\rho^{(ext)\pm}_{\omega km}(r')r'dr'$$

Two ways of calculation:

1. Free modes quantization and calculation of the Hamiltonian

2. Explicit solution to Maxwell's equations with subsequent quantization with the usage of singularities of analytical expressions.

Result of calculations

$$\Phi_m^{(pol)} = \frac{Z_1}{\pi}\int_0^{\infty} dk I_m(kr)\left(\frac{g_m^+(k)}{\Delta_m^+(k)}e^{ik(z-vt)} + \frac{g_m^-(k)}{\Delta_m^-(k)}e^{-ik(z-vt)}\right)e^{im(\varphi-\varphi_0)},\ \ r_0 < r < a$$

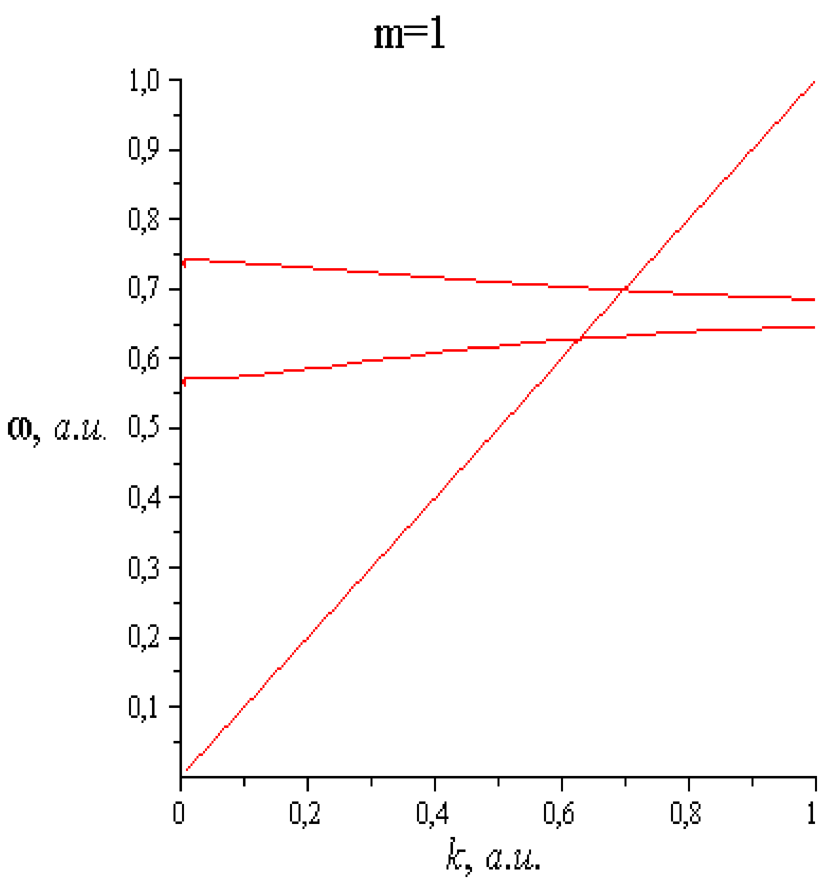
$$g_m^{\pm} = (\varepsilon_\omega^{\pm} - 1) I_m(kr_0)\Big\{kaK_m(ka)K'_m(ka) +$$

$$+(\varepsilon_\omega^{\pm} - 1)kbK_m(kb)\Big(kaK_m(ka)[I'_m(kb)K'_m(ka) - K'_m(kb)I'_m(ka)] - K'_m(kb)\Big)\Big\}_{\omega=kv}$$

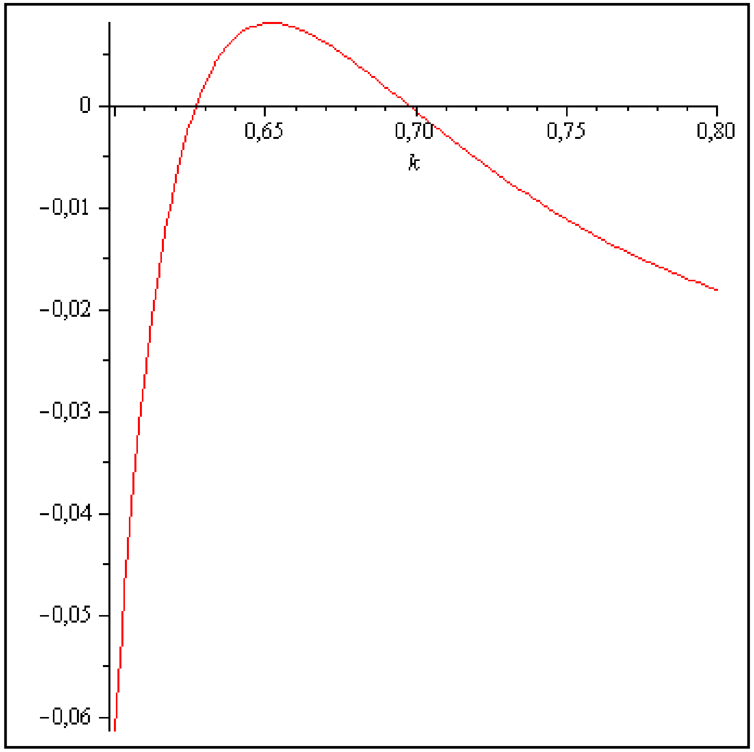
$$\Delta_m^{\pm} = 1 - ka\left\{(\varepsilon_\omega^{\pm}(k) - 1)\left[I_m(ka)K'_m(|k|a) - \frac{b}{a}K_m(kb)I'_m(kb)\right] + \right.$$

$$\left.\left(\varepsilon_\omega^{\pm}(k) - 1\right)^2 kbK_m(kb)I_m(ka)\left[-K'_m(kb)I'_m(ka) + K'_m(ka)I'_m(kb)\right]\right\}.$$

Equation Δ=0 gives us a possibility to calculate the frequencies of a free electric oscillations of a tube. In this calculation we consider the frequency ω as the independent variable and set only after calculating the eigenfrequencies.



Surface modes for the Al tube (a=4, b=6).
The sraigh line corresponds to w=kv



Calculation of potential and forces

$$\frac{I_m(kr)g_m^{\pm}(k)}{\Delta_m^{\pm}(k)}e^{ik(z-vt)} = \wp\left[\frac{I_m(kr)g_m^{\pm}(k)}{\Delta_m^{\pm}(k)}e^{ik(z-vt)}\right] \mp i\pi\sum_a \frac{I_m(k_a r)g_m^{\pm}(k_a)}{\left(\partial\Delta_m^{\pm}(k)/\partial k\right)_{k=k_a}}e^{ik_a(z-vt)},$$

After summing over m with different signs the potential takes a form

$$\Phi_m^{(pol)} = \frac{4Z_1}{\pi}\cos(m(\varphi-\varphi_0))\wp\int_0^\infty dk I_m(kr)\frac{g_m(k)}{\Delta_m(k)}\cos\left(k(z-vt)\right) -$$

$$4Z_1\cos(m(\varphi-\varphi_0))\sum_a \frac{I_m(k_a r)g_m(k_a)}{\left(\partial\Delta_m(k)/\partial k\right)_{k=k_a}}\sin\left(k_a(z-vt)\right), \quad r_0 < r < a$$

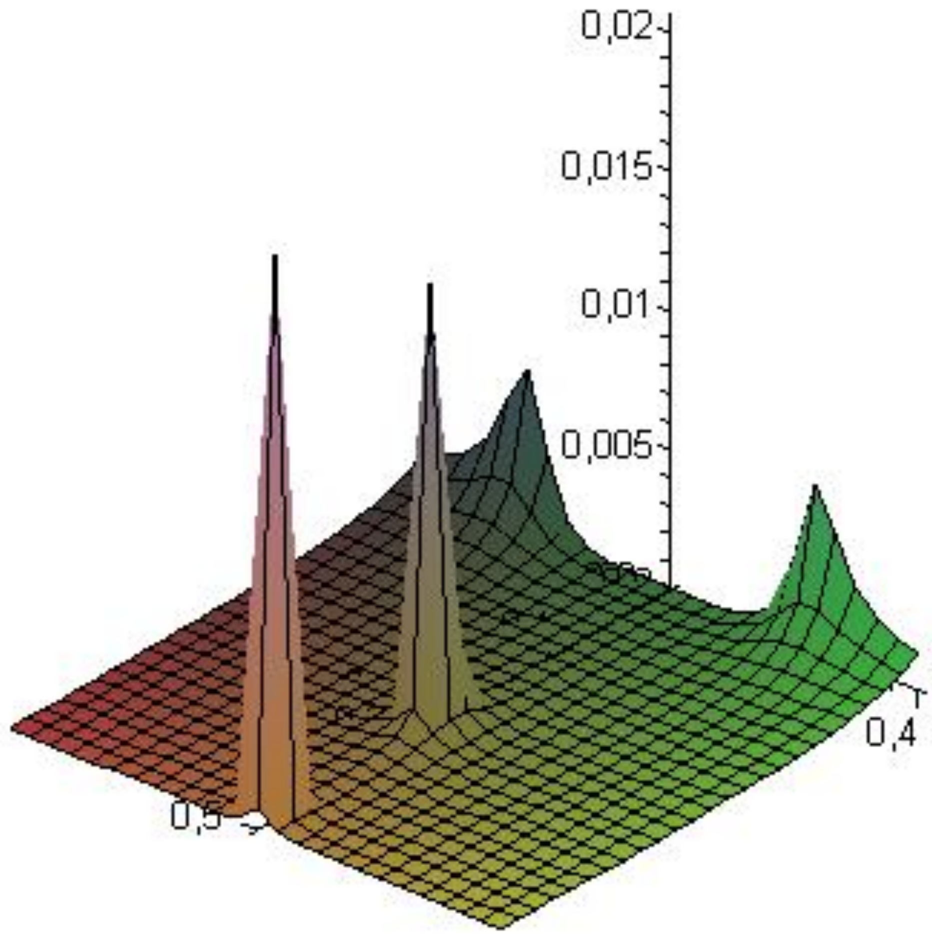
Forces

$$F_z = -Z_1 \left[\frac{\partial \Phi^{(pol)}}{\partial z} \right]_{z=vt} = -4Z_1^2 \sum_m \sum_a k_a \frac{I_m(k_a r_0) g_m(k_a)}{\left(\partial \Delta_m(k) / \partial k \right)_{k=k_a}}$$

The stopping is defined by the free excitations only

$$F_r = -Z_1 \left[\frac{\partial \Phi^{(pol)}}{\partial r} \right]_{z=vt} = -\frac{4Z_1^2}{\pi} \sum_m \wp \int_0^\infty \frac{k dk}{\Delta_m(k)} I_m'(kr) g_m(k)$$

Contribution of different eigenexcitations. The integrand in the complex k-plane.



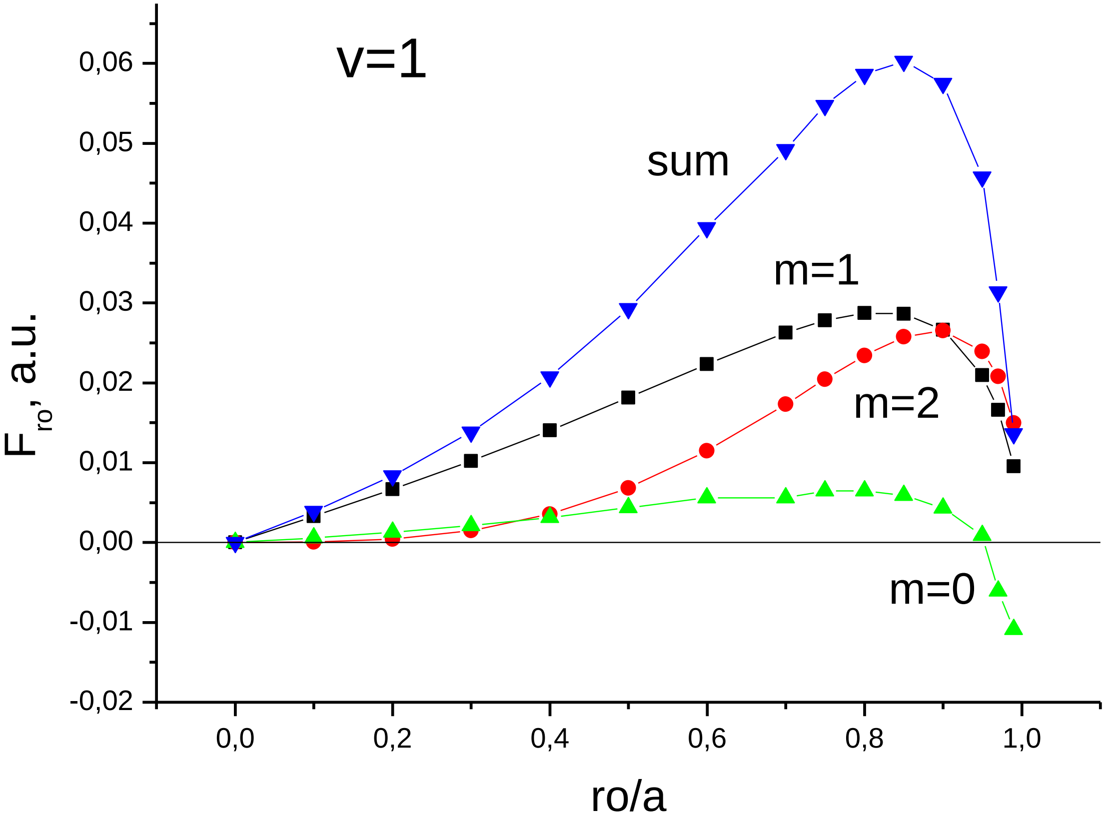
Space dispersion restrictions:

$$l=[r_x p]$$

$$l_c=rp_c$$

For Al p_c=0.68

The radial force in the Al tube (a=4, b=6).



References

1. King-Yien Ng. PR D. **42, No.5,** 1819-1828, 1990.
2. U. Schroeter, Dereux A. Phys. Rev., **6 4**, 125420, 2001.
3. O. Ste´phan,1,* D. Taverna,1 M. Kociak,1 K. Suenaga,2 L. Henrard,3 and C. Colliex. Phys. Rev.B 66, 155422 (2002).
4. Que W. J. Phys.: Condens. Matter **14** (2002) 5239–5253.
5. Tu Y.H., Kwei C.M., Li Y.C., Tung C.J. Phys. Rev. B **74**, 045403, 2006.
6. S. Segui, J.L. Gervasoni, N.R. Arista. Plasmon excitation in nanotubes. Comparison with capillaries and wires. Radiation Physics and Chemistry 76 (2007) 582–586.
7. A. S. Sabirov . Journal of Surface Investigation. X-ray, Synchrotron and Neutron Techniques, 2014, Vol. 8, No. 2, pp. 297–300. © Pleiades Publishing, Ltd., 2014.
8. Cohen-Tannoudgi et al.
9. Filippov G.M.
10. J. Jackson, Classical Electrodynamics, J. Wiley & Sons, New York, London, Sydney, 1962.

The Role of Space Dispersion in Defining The Point Charge Imagination Near Surface of a Dielectric

Filippov G.M.[1,a]
[1]Cheboksary Polytechnic Institute, Cheboksary,
P.Lumumba –str. 8, Russian Federation
[a]filippov38-gm@yandex.ru

Keywords: electric field, mirror imagination, space dispersion, dielectric tube, polarization field

Abstract. This paper devoted to analysis of influence of spatial dispersion on a point charge imagination near a surface of a dielectric or a metal.

1. Introduction

The charge imagination sufficiently well describes the polarization field which occurs near a dielectric or metal surface in the presence of external point charge. Usually, in classical electrodynamics (see, e.g. [1-6]), this problem is solved without taking into account the spatial dispersion of polarization properties of the solid near which surface the external point charge occurs. In this paper we try to investigate which significant corrections should be made if the medium obey the spatial dispersion. This problem attains a new importance in connection to the channeling phenomena in a thin channels like nanotubes or thin capillaries and cylindrical holes in solids.

Consider the calculation of the interaction of a point charge with a uniform semi-infinite dielectric medium with a flat surface, based on the concept of the field of surface elementary excitations of the electric type (field of surface plasmons) . In this case, we go beyond the classical electrodynamics using partly quantum mechanical notions. Chose the dielectric function of a semi-infinite conducting medium with a plane boundary $z=0$ in a simplest form

$$\varepsilon_\omega = 1 - \Theta(k_c - k)\omega_0^2/\omega^2 \ , \tag{1}$$

where the spatial dispersion takes in that the wave length of the plasma oscillations assumes to be not less the minimal value $\lambda_c = 2\pi/k_c$. Here the maximal wave vector k_c has the order of the Fermi momentum divided by the Plank constant. The potential of the polarization field takes to be

$$\Phi_s(z \ge 0) = -Z\int_0^{k_c} J_0(k_{||}r)e^{-k_{||}|b+z|}dk_{||} \quad . \tag{2}$$

The point charge Z is assumed to be placed on a distance b in vacuum over the plane boundary on the z-axis at $z>0$. As it follows from (1), at the distances $x=\sqrt{z^2+r^2} >> k_c^{-1}$ we have

$$\Phi_s(z \ge 0) \approx -Z\int_0^{\infty} e^{-k_{||}|b+z|}J_0(k_{||}r)dk_{||} = -\frac{Z}{\sqrt{(b+z)^2+r^2}}$$

and therefore, at the great distances the polarization field approximately coincides with the field of a point image charge which displaced on the other side of the boundary plane and obeying the

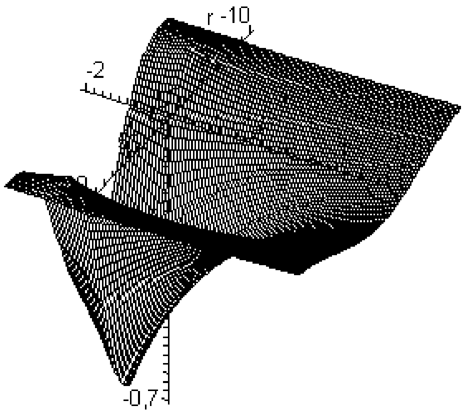


Fig.1.

negative sign. But at the small distances $x \sim k_c^{-1}$ the image charge doesn't point (in opposite to the external charge). It is distributed in the volume with the characteristic size $l \sim k_c^{-1}$. In particular, the minimal potential energy of interaction between the external charge and the image charge isn't infinity and has the value $U_{\min} = -Zk_c$. In the Fig.1 the image potential is shown in coordinates r and z where for the illustration purposes the coordinate r admitted to negative values. The external unit point charge is placed in the point $z = b = 1, r = 0$. Minimum of the image potential is found in the point $z = -b, r = 0$. In the neighborhood of the minimum the potential has a more complex behavior than it can be anticipated. In particular, its first derivatives in the minimum does not equal to zero.

2. Dielectric tube

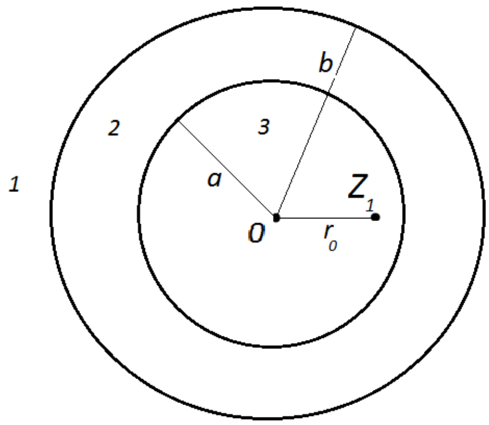


Fig.2

Consider now a case of a dielectric/metal tube. Some important electromagnetic properties of such a specimen was described in the work [5]. In this case (see the cross section in the fig.1) we assume the external point charge Z is moving with the constant velocity v parallel to the tube's axis at the distance $r_0 < a$ from the axis. We assume (see fig.2) that only in the area 2 the dielectric function is differ from unity, equal to ε_ω and take into account only the time dispersion. In the areas 1, 2, 3 we have different solutions to the electric displacement potential, which we write as a superposition of two samples of linear independent terms

$$\Phi_\omega = \sum_{k,m} e^{im\varphi}\left(R^+_{\omega km}(r)e^{ikz} + R^-_{\omega km}(r)e^{-ikz}\right). \quad (3)$$

Here the wave number k assumes to be non-negative. For us is important to get the all sample of independent solutions to the wave equation, in particular, for different signs of the longitudinal component of the wave vector k. In the expression (3) this circumstance is taken into account explicitly. As we know, the functions $R^{\pm}_{\omega km}(r)$ obey the Bessel equation

$$\frac{d^2 R^{\pm}_{\omega km}}{dr^2} + \frac{1}{r}\frac{dR^{\pm}_{\omega km}}{dr} - \left(k^2 + \frac{m^2}{r^2}\right)R^{\pm}_{\omega km} = -4\pi\rho^{(ext)\pm}_{\omega km}(r) \ . \quad (4)$$

On the boundaries $r = a, b$ the continuity of radial derivatives of the potential (3) should be fulfilled and at the same time the continuity of quantities $R_{\omega km} / \varepsilon_\omega$ should be ensured.

In following we use the approach which divide all the electromagnetic fields on two independent classes – longitudinal and vortex. This division is useful in the vacuum electrodynamics (as clearly demonstrated in the book [7]), and may be just more important in the condensed medium. It was used at calculations of the stopping power for the projectile moving in the dielectric cylinder [8] and based on the two series of Maxwell's equations, for the potential fields,

$$\begin{cases} \mathrm{div}\vec{D}^{(p)} = 4\pi\rho_{ext}; \quad \dfrac{\partial \vec{D}^{(p)}}{\partial t} = -4\pi\,\vec{j}^{(p)}_{ext}; \quad \dfrac{\partial \vec{E}^{(p)}}{\partial t} = -4\pi(\vec{j}^{(p)}_{pol} + \vec{j}^{(p)}_{ext}) \\ \vec{D}^{(p)} = \hat{\varepsilon}^{(e)}\vec{E}^{(p)} = \vec{E}^{(p)} + 4\pi\vec{P}^{(p)}; \quad -\mathrm{div}\vec{P}^{(p)} = \rho^{(p)}_{pol}; \quad \vec{E}^{(p)} = -\mathrm{grad}\,\varphi^{(p)} \end{cases}.$$

and for the vortex fields

$$\begin{cases} \mathrm{rot}\vec{E}^{(v)} = -\dfrac{1}{c}\dfrac{\partial \vec{B}}{\partial t}; \quad \mathrm{rot}\vec{H} = \dfrac{1}{c}\dfrac{\partial \vec{D}^{(v)}}{\partial t} + \dfrac{4\pi}{c}\vec{j}^{(v)}; \quad \mathrm{div}\vec{B} = 0; \\ \mathrm{div}\vec{E}^{(v)} = 0; \quad div\vec{H} = 0; \quad \vec{D}^{(v)} = \hat{\varepsilon}^{(v)}\vec{E}^{(h)}; \quad \vec{B} = \hat{\mu}\vec{H}. \end{cases}$$

In framework of this decomposition in this paper we would like find an exact solution to the potential fields.

Assume the point charge is found inside the tube on a distance r_0 $(0 < r_0 < a)$ from the axis,

$$\Delta\Phi = -4\pi\rho^{(ext)} = -4\pi Z\delta^{(3)}(\vec{x} - \vec{x}_0(t)), \quad \vec{x}_0(t) = \left(\vec{r}_0(t), z_0(t)\right),$$

where $\vec{r}_0(t) = (r_0(t), \varphi_0(t), z_0(t))$, $(0 < r_0(t) < a)$ - are the current coordinates of the moving point charge Z. After the Fourier transformation we have

$$\Delta\Phi_\omega = -4\pi\rho^{(ext)}_\omega(\vec{x}) = -4\pi Z \int_{-\infty}^{\infty} \delta^{(3)}(\vec{x} - \vec{x}_0(t))e^{i\omega t}dt \ . \tag{6}$$

Equation (6) presumes the definition of the particle's trajectory to the all times and only for the motion with a constant velocity $\vec{v}$ (in our case directed along the tube's axis, in a case of the appropriate choice of the coordinate system origin, when $\delta^{(3)}(\vec{x} - \vec{x}_0(t)) = \delta^{(2)}(\vec{r} - \vec{r}_0)\delta(z - vt)$, $\delta^{(2)}(\vec{r} - \vec{r}_0) = \dfrac{1}{r_0}\delta(r - r_0)\delta(\varphi - \varphi_0)$) obtains a comparatively simple form (4), where $\rho^{(ext)\pm}_{\omega km}(r) = \dfrac{Z_1}{r_0}\delta(r - r_0)e^{-im\varphi_0}\delta(\omega \pm kv)$. Then

$$R^{\pm}_{\omega km}(r) = R^{(ext)\pm}_{\omega km}(r) + \begin{cases} D^{\pm}K_m(|k|r) + G^{\pm}I_m(|k|r), & 0 \le r \le a \\ B^{\pm}K_m(|k|r) + C^{\pm}I_m(|k|r), & a < r \le b \end{cases}. \tag{7}$$

Here

$$R_{\omega km}^{(ext)\pm}(r) = -4\pi\int_0^r \left[I_m(|k|r)K_m(|k|r') - I_m(|k|r')K_m(|k|r)\right]\rho_{\omega km}^{(ext)\pm}(r')r'dr'$$

is the partial solution to the inhomogeneous equation (7). The condition of the regularity in the origin and $R_{\omega km}^{(ext)\pm}(0) = 0$ enforced us to set $D^{\pm} = 0$. Taking into account also the conditions on the boundaries $r = a, b$ we get the polarization field, which could be calculated by subtracting the field in the vacuum, at $\varepsilon = 1$. This quantity been calculated within the region between the charge and the inner boundary of the tube takes the form:

$$\Phi_m^{(pol)} = \frac{Z}{\pi}\int_0^{\infty} dk I_m(kr)\left(\frac{g_m^+(k)}{\Delta_m^+(k)}e^{ik(z-vt)} + \frac{g_m^-(k)}{\Delta_m^-(k)}e^{-ik(z-vt)}\right)e^{im(\varphi-\varphi_0)}, \quad r_0 < r < a \quad . \tag{8}$$

Here

$$g_m^{\pm} = (\varepsilon^{\pm} - 1)kaI_m(kr_0)\Big\{K_m(|k|a)K_m'(|k|a) - \frac{b}{a}K_m(kb)K_m'(kb) +$$
$$(\varepsilon^{\pm} - 1)kbK_m(kb)K_m(|k|a)\left[K_m'(|k|a)I_m'(kb) - K_m'(kb)I_m'(ka)\right]\Big\}$$

and

$$\Delta_m^{\pm} = 1 - ka\Big\{(\varepsilon_{\omega}^{\pm}(k) - 1)\left[I_m(ka)K_m'(|k|a) - \frac{b}{a}K_m(kb)I_m'(kb)\right] +$$
$$\left(\varepsilon_{\omega}^{\pm}(k) - 1\right)^2 kbK_m(kb)I_m(ka)\left[-K_m'(kb)I_m'(ka) + K_m'(ka)I_m'(kb)\right]\Big\}.$$

In this expression we have used that due to the translation symmetry of the tube the dependence of the dielectric function on the longitudinal component of the wave vector could be easily taken into calculations. Because the dielectric function involves as in the denominator of the integrand in (9) as well in the nominator, in the same power, the singularities of the reciprocal dielectric function does not give the singularities of the integrand. Therefore, within the considered case the volume plasmons are not arise in course of the charge motion.

Equations $\Delta_m^{\pm} = 0$ gives us a possibility to calculate the frequencies of a free electric oscillations of a tube. In this calculation we consider the frequency ω as the independent variable and set $\omega = kv$ only after calculating the eigenfrequencies. These oscillations could exist in the tube at the absence of the external electric charges. But the knowing the frequencies of the elementary oscillations don't gives us the knowing the intensities of the free oscillations generating by the external charges. Within the pure quantum-mechanical approach one need firstly to proceed the procedure of quantization the free fields and then to get the quantum-mechanical representation of potentials of free fields. For this procedure we need the correct estimation of the energy of electric field in dispersed medium (this one is famous only for homogeneous media). With the help of this result one will be able to write the Hamiltonian of interaction between free fields and the external charges. In this article we are following to another approach based on the purely

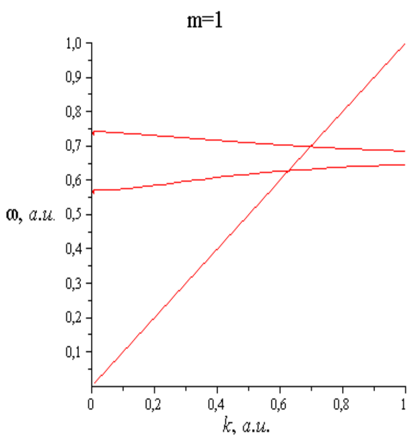


Fig.3

classical solution of the problem and only then go to the quantum-mechanical treatment of the result. In simplest cases the both considerations gave the same result but it is not to be ever. In particular, for the here considering case this approach isn't applied.

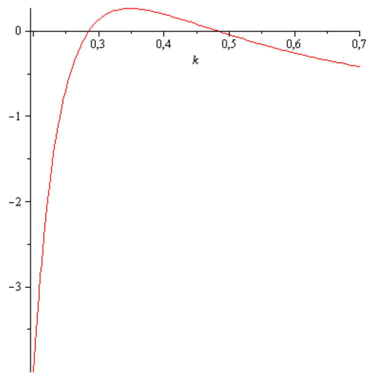


Fig.4

For the simple one-mode model for the dielectric function (1) we get two branches of the surface plasmons in the opposite to the homogeneous cylinder case. The dielectric function is set to be the simple one-mode model with the frequency of free plasma oscillations $\omega_0 = \sqrt{3/r_s^3} \approx 0.5794$. Each branch apparently relates to the corresponded surface of the tube (inner or outer). In the Fig.3 the crossing of two branches of surface fields with the straight line $\omega = kv$ at $v = 1$ or Al tube (a=4, b=6) at m=1 is depicted. In the Fig. 4. the function $\varDelta_1^+(k)$ at the same conditios is presented. The derivatives $\varDelta'_m(k_a) = \frac{\partial}{\partial k}\left[\varDelta_m(k)\right]_{k=k_a}$ and the roots for different m should be calculated in a special procedure. As we see, the left derivatives are positive but the right - negative. This difference in the signs arises apparently in consequence of opposite charging of two surfaces due to the polarization. At increasing m the distance between the left and right roots diminishes but allways to be nonzero.

The same two surface-mode result was obtained in a series of papers (see, e.g., [1-6]). Our result that the volume eigenfrequencies are not arisen in the case of cylindrical tube, when the external charge moves in vacuum near the dielectric, apparently, concerns with the analogous conclusion in the work [5].

3. Calculation of potential and forces

The expression (8) turns to zero into vacuum. With the help of taking the derivatives of the potential we could calculate all components of polarization forces applying to the projectile. The singularities in the integrand give important contributions to the measured quantities. The main singularities are roots of the denominator $\varDelta_m^\pm$. This roots have to be calculated separately in a special calculation procedure. The quantities $\varDelta_m^\pm$ at a specific m could be considered as dielectric functions of the tube.

Represent the function $I_m(kr)g_b^\pm(k)e^{ik(z-vt)}/\varDelta_m^\pm(k)$ which is analytic along the real half-axis $0 \le k < \infty$, tends to zero at $k \to \infty$, in a form of the Laurent series within the half-axis. The crawl rule poles defined by the causality principal (the rule Kramers-Kronig relations). We get:

$$\frac{I_m(kr)g_m^\pm(k)}{\varDelta_m^\pm(k)}e^{ik(z-vt)} = \wp\left[\frac{I_m(kr)g_m^\pm(k)}{\varDelta_m^\pm(k)}e^{ik(z-vt)}\right] \mp i\pi\sum_a \frac{I_m(k_a r)g_m^\pm(k_a)}{\partial\left(\varDelta_m^\pm(k)/\partial k\right)_{k=k_a}}e^{ik_a(z-vt)},$$

where $\varDelta_m^\pm(k_a) = 0$ and the symbol $\wp$ denotes the principal part of the integral. Applying it to the potential, we get

$$\Phi_m^{(pol)} = \frac{Z}{\pi} \wp \int_0^\infty dk I_m(kr) \left(\frac{g_m^+(k)}{\Delta_m^+(k)} e^{ik(z-vt)} + \frac{g_m^-(-k)}{\Delta_m^-(k)} e^{-ik(z-vt)} \right) e^{im(\varphi-\varphi_0)} -$$

$$iZ_1 e^{im(\varphi-\varphi_0)} \sum_a \left[\frac{I_m(k_a r) g_m^+(k_a)}{\partial \left(\Delta_m^+(k)/\partial k\right)_{k=k_a}} e^{ik(z-vt)} - \frac{I_m(k_a r) g_m^-(k_a)}{\partial \left(\Delta_m^-(k)/\partial k\right)_{k=k_a}} e^{-ik(z-vt)} \right], \ r_0 < r < a$$

Or, if $\dfrac{g_m^+(k)}{\Delta_m^+(k)} = \dfrac{g_m^-(k)}{\Delta_m^-(k)} = \dfrac{g_m(k)}{\Delta_m(k)}$,

$$\begin{aligned} \Phi_m^{(pol)} &= \frac{2Z}{\pi} \wp \int_0^\infty dk I_m(kr) \frac{g_m(k)}{\Delta_m(k)} \cos\left(k(z-vt)\right) e^{im(\varphi-\varphi_0)} - \\ &2Z_1 e^{im(\varphi-\varphi_0)} \sum_a \frac{I_m(k_a r) g_m(k_a)}{\partial \left(\Delta_m(k)/\partial k\right)_{k=k_a}} \sin\left(k_a(z-vt)\right), \ r_0 < r < a \end{aligned} .$$

After summing over m with different signs the potential takes the form

$$\begin{aligned} \Phi_m^{(pol)} &= \frac{4Z}{\pi} \cos(m(\varphi-\varphi_0)) \wp \int_0^\infty dk I_m(kr) \frac{g_m(k)}{\Delta_m(k)} \cos\left(k(z-vt)\right) - \\ &4Z_1 \cos(m(\varphi-\varphi_0)) \sum_a \frac{I_m(k_a r) g_m(k_a)}{\partial \left(\Delta_m(k)/\partial k\right)_{k=k_a}} \sin\left(k_a(z-vt)\right), \ r_0 < r < a \end{aligned} .$$

The total main part:

$$\wp\, \Phi_m^{(pol)} = \frac{4Z}{\pi} \cos(m(\varphi-\varphi_0)) \wp \int_0^\infty \frac{dk}{\Delta_m(k)} \left[I_m(kr) g_m(k) \cos\left(k(z-vt)\right) \right]. \tag{9}$$

This integral converges as at the upper limit as well as in zero. At r=0 only m=0 contribution does not vanish. Emphasize the longitudinal polarization force (stopping) is well defined only by the contribution from the poles:

$$F_z = -Z \left[\frac{\partial \Phi^{(pol)}}{\partial z} \right]_{z=vt} = -4Z^2 \sum_m \sum_a k_a \frac{I_m(k_a r_0) g_m(k_a)}{\partial \left(\Delta_m(k)/\partial k\right)_{k=k_a}} . \tag{10}$$

At the same time, the transversal force contains the contributions only from the main part of the integral,

$$F_r = -Z\left[\frac{\partial \Phi^{(pol)}}{\partial r}\right]_{\substack{z=vt,\\ r=r_0}} = -\frac{4Z^2}{\pi}\sum_m \wp\int_0^\infty \frac{kdk}{\Delta_m(k)} I'_m(kr_0) g_m(k) \quad . \tag{11}$$

4. Effect of the spatial dispersion on the forces in a tube

At the presence of a spatial dispersion, when the Fourier components of the dielectric function $\varepsilon_\omega(\vec{k}_\perp, k)$ depends on the all components of the wave vector, the expressions should be changed. In this case the action of the reciprocal dielectric operator on the field functions gives

$$\Omega_m(kr) = \frac{1}{\hat{\varepsilon}} K_m(kr) = \int_0^\infty \widetilde{K}_m(k, k_\perp) \frac{\cos(k_\perp r)}{\varepsilon_\omega(k_\perp, k)} \frac{dk_\perp}{\pi} ;$$

$$\Xi_m(kr) = \frac{1}{\hat{\varepsilon}} I_m(kr) = \int_0^\infty \widetilde{I}_m(k, k_\perp) \frac{\cos(k_\perp r)}{\varepsilon_\omega(k_\perp, k)} \frac{dk_\perp}{\pi} .$$

Here with a tilde are denoted the Fourier transformations of the functions. The conditions on the boundaries $r = a, b$ give the system of four linear equations for four constants A,B,C,F:

$$\begin{cases} B\Omega_m(ka) + C\Xi_m(ka) - FI_m(ka) = -4\pi Z_1 e^{-im\varphi_0}\delta(\omega - kv)\big(I_m(ka)K_m(kr_0) - K_m(ka)I_m(kr_0)\big) \\ BK'_m(ka) + CI'_m(ka) - FI'_m(ka) = -4\pi Z_1 e^{-im\varphi_0}\delta(\omega - kv)\big(I'_m(ka)K_m(kr_0) - K'_m(ka)I_m(kr_0)\big) \\ AK_m(kb) - B\Omega_m(kb) + C\Xi_m(kb) = 0 \\ AK'_m(kb) - BK'_m(kb) + CI'_m(kb) = 0 \end{cases} \tag{12}$$

In course of the solution to this equations we should set $\omega = kv$ in all expressions . The zeros of the determinant of this system give the eigenfrequencies of electric oscillations of a tube. The corresponding equation is the integral equation and its solutions have the form $\omega = \omega_s(k_\perp, k)$. Here the index s represents a number of the corresponding branch of the solutions.

Within a simplest dispersion model (1) we are be able to take into account the cut-off in the wave vector space in cylindrical coordinates. In consequence of the angular momentum is expressed as $\vec{l} = \vec{r} \times \vec{p}$ then the cut-off in the momentum space leads to the cut-off in the angular momentum space at $l_c = r \cdot p_c$ and therefore to the critical wave vector k_c correspond the critical angular momentum $m_c = [r \cdot k_c]$ (here the square brackets denote the integer part of the number). This momentum depends on the radius in analogous way as the new introduced "dielectric functions" depend on the radius. The momenta which exceed the critical value, should be excluded from calculations of the polarization potential in the inner part of the tube. In consequence of this result at the estimation of the image of the point charge in the inner part of the tube the terms with $m > m_c$ should be eliminated from consideration. For example, if the inner radius of a tube less than $a < k_c^{-1}$ then in the image potential should be taken into account only one symmetrical term which corresponds to $m = 0$.

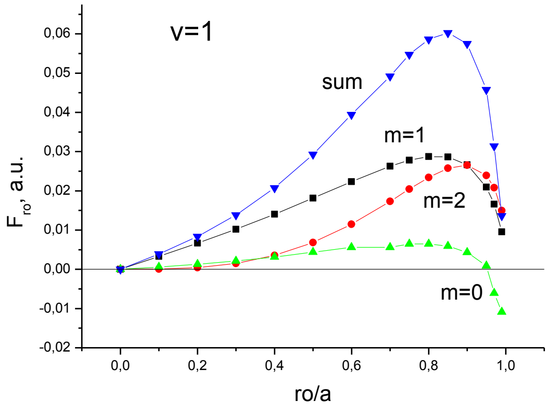


Fig.5

As an example, we perform the calculation of the transversal force for the point charge moving parallel to axis within the Aluminum tube (a=4, b=6). In this case the inner radius of a tube is equal to 4 but the cut-off momentum of plasmons calculated for Al in the Lindhard approach is equal to 0.68 (in atomic units). Then the maximal m for the plasmon excitations in this case is equal $m_c = \lfloor a \cdot k_c \rfloor = [2.72] = 2$. In the Fig.5 the contributions of three partial waves at $m = 0,1,2$ for the radial force acting on a point charge moving parallel to the tube's axis with the velocity $v = 1$ are depicted.

Appendix:

The expression

$$\left[\frac{1}{k - k_a + i0} \right] \tag{A1}$$

could be considered as proportional to the propagator of a quasiparticle, which associates with the elementary electric excitation of the tube

$$G_\omega(k) = \frac{1}{\omega - \omega_a + i0}$$

which expressed in the explicit form as a Fourier representation of the propagator

$$G(t - t') = -i \left\langle vac \middle| \hat{T}\left(\hat{b}_{\beta k}(t)\hat{b}^{+}_{\beta k}(t')\right) \middle| vac \right\rangle$$

where the index β denotes a type of the quasi-particle, k - is the longitudinal momentum. Within the theory considered this propagator appear in the point $\omega = kv, \omega_a = k_a v$ and after multiplying on the velocity returns to (A1). In this approach we get the new form of quantization of the electric excitations of the tube.

References

1. Ng. King-Yien, PR D. **42** (1990) **No.5,** 1819.
2. U. Schroeter, A. Dereux, Phys. Rev. **6 4** (2001) 125420.
3. O. Ste´phan, D. Taverna, M. Kociak, K. Suenaga, L. Henrard, and C. Colliex, Phys. Rev. B 66 (2002)155422.
4. W. J. Que, Phys. Condens. Matter 14 (2002) 5239.
5. Tu Y.H., C.M., Kwei, Y.C., C.J. Li Tung, Phys. Rev. B 74 (2006) 045403.
6. S. Segui, J.L. Gervasoni, N.R. Arista, Radiation Physics and Chemistry 76 (2007) 586.
7. A. S. Sabirov, Journal of Surface Investigation. X-ray, Synchrotron and Neutron Techniques, © Pleiades Publishing, Ltd., 8(2014) No. 2, 297.
8. C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, Photons and Atoms: Introduction to Quantum Electrodynamics, ISBN-10: 0471184330 (1997).
9. G.M. Filippov, Bulletin of the Russian Academy of Sciences. Physics, © Allerton Press, Inc. 76 (2012) No. 6, 678.
10. J. Jackson, Classical Electrodynamics, J. Wiley & Sons, New York, London, Sydney (1962).