Levon Sh. Grigoryan

Hrant F. Khachatryan

Institute of Applied Problems in Physics
Yerevan

Transition radiation formed on a smoothly varying boundary between a medium and vacuum:

exact solution of the problem

1. Introduction

1.
The report focuses on the study of

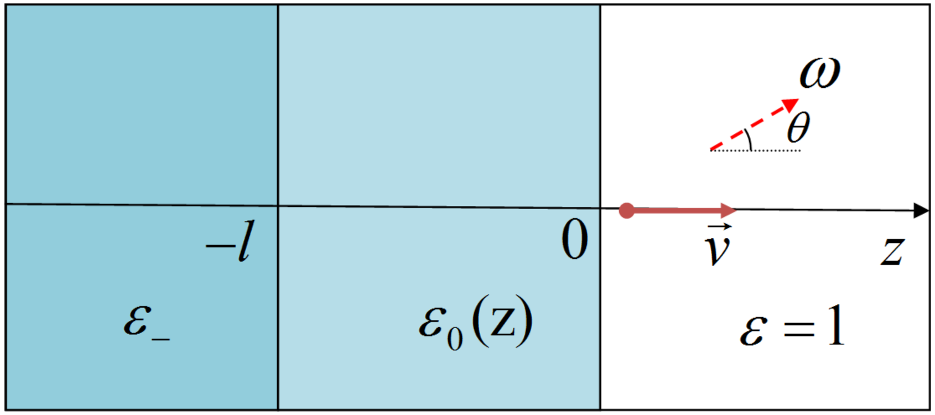
the radiation from a charged particle passing through a smoothly varying boundary between the medium and vacuum.

2.
The impact of a smoothly varying boundary between the medium and vacuum for Transition Radiation (TR) from a relativistic particle has been previously studied by approximate methods [1,2]: eikonal approximation, approximation of geometrical optics, etc., which are not applicable in the case of strong influence of the blurred intemadiate layer.

3.
The purpose of our report is to go beyond these approximations, which opens up the prospect of practical applications of this phenomenon.

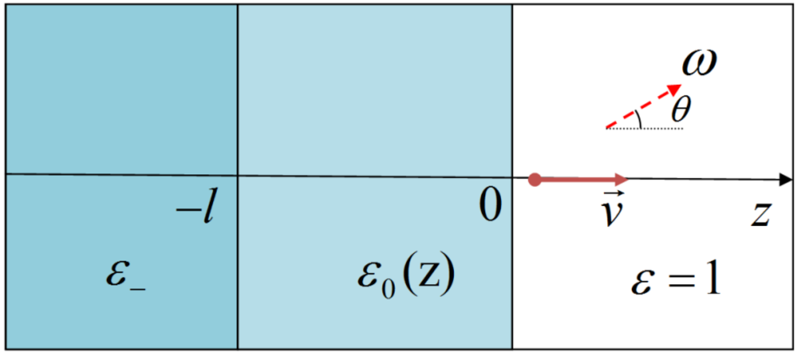
2. Formulation of the problem

Consider a charged particle uniformly moving along Z axis under assumption that the particle first travels inside a semi-infinite homogeneous medium (range z<-*l*), then passes an intermediate layer of inhomogeneous matter (-*l*<z<0) and eventually to vacuum (z>0).



$$\varepsilon = \begin{cases} \varepsilon_9 & \mathrm{z} \le -l \\ \varepsilon_0(\mathrm{z}) & -l < \mathrm{z} < 0 \\ 1 & \mathrm{z} \ge 0 \end{cases}$$

The length of interlayer is equal to l



We'll assume that within the interlayer dielectric permittivity and magnetic permibiality of the matter are arbitrary smooth functions of z.

Let us
a) consider the radiation from a particle in flight through of a smoothly varying boundary and

b) investigate the features of spectral-angular distribution of the energy

$$W = \int I(\omega,\theta)\,d\omega\,d\theta$$

radiation propagating in vacuum

during the whole time of particle motion.

3. The final formula

The spectral-angular distribution of the radiation energy is determined by the well known expression [1,2]

$$I(\omega,\theta) = \frac{2q^2}{\pi c}\frac{\cos^2\theta}{\sin\theta}\left|a(\omega,u)\right|^2$$

$a(\omega,u)$ is the amplitude describing the free field (the radiation field).

$$u = \frac{\omega}{c}\sin\theta$$

We derived an anlytical expression for the amplitude

$$a(\omega,u)$$

which is applicable for an arbitrary smooth change of the dielectric permittivity and the magnetic premibiality of the matter in the intermediate layer.

I shall omit the final formula for brevity.

If the contribution of the intermediate layer in the radiation of a particle is small, one can use a simple approximate expressions, which are well-known [1,2].

In this report, we are interested in the opposite case, when the contribution of the intermediate layer in a particle radiation is large.

3. Numerical results and comments

Below we shall confine ourselves to the cases when

1\.

the real part of the dielectric permittivity of the matter

$$\varepsilon(\omega) > 1$$

(the frequency region below the X-ray),

$$\mu(\omega) = 1$$

And

2.

the substance of semi-infinite medium is optically so dense that Chrenkov Radiation (ChR) generated from relativistic particle does not escape into the vacuum as experienced Total Internal Reflection (TIR) at a plane interface between matter and vacuum.

3.

In numerical calculations we'll assume that dielectric permittivity of semi-infinite medium

$$\varepsilon_{-} = 3 + 0.01i$$

and consider two values of the Lorentz factor of the charged particle (electron):

$$\gamma = 100$$

(a relativistic particle) and

$$\gamma = 2$$

(a moderately relativistic particle).

A smoothly varying boundary can be described by different dielectric functions

4. $\varepsilon_0(z)$

We use different model dielectric functions in numerical calculations.

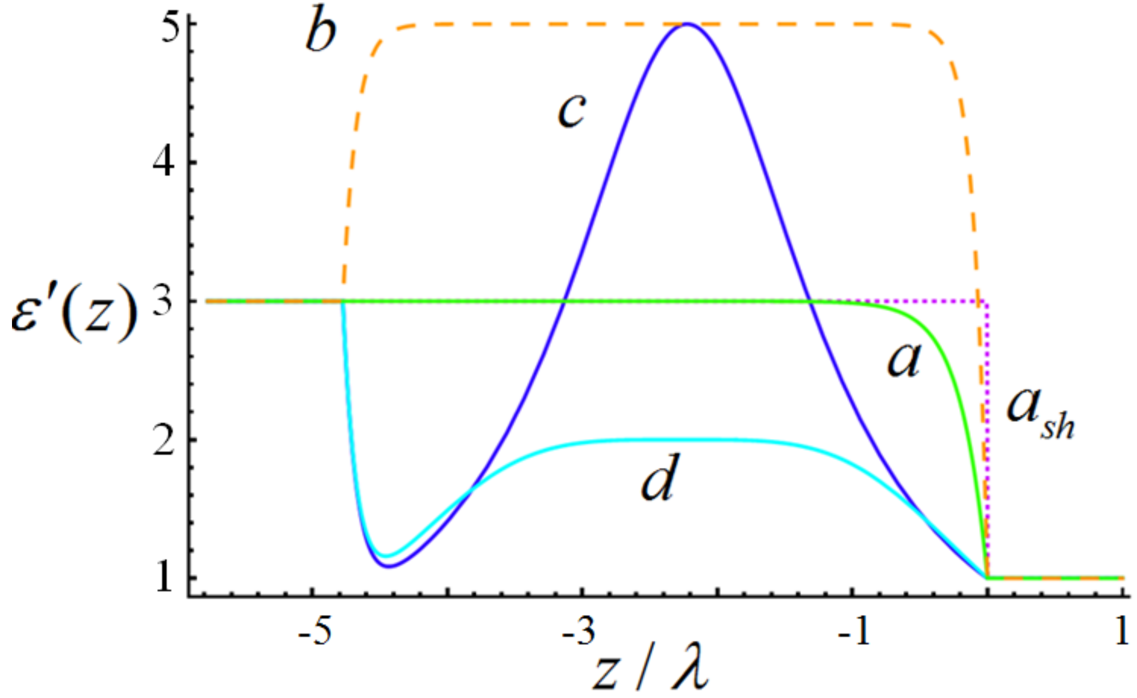


Fig.1b.

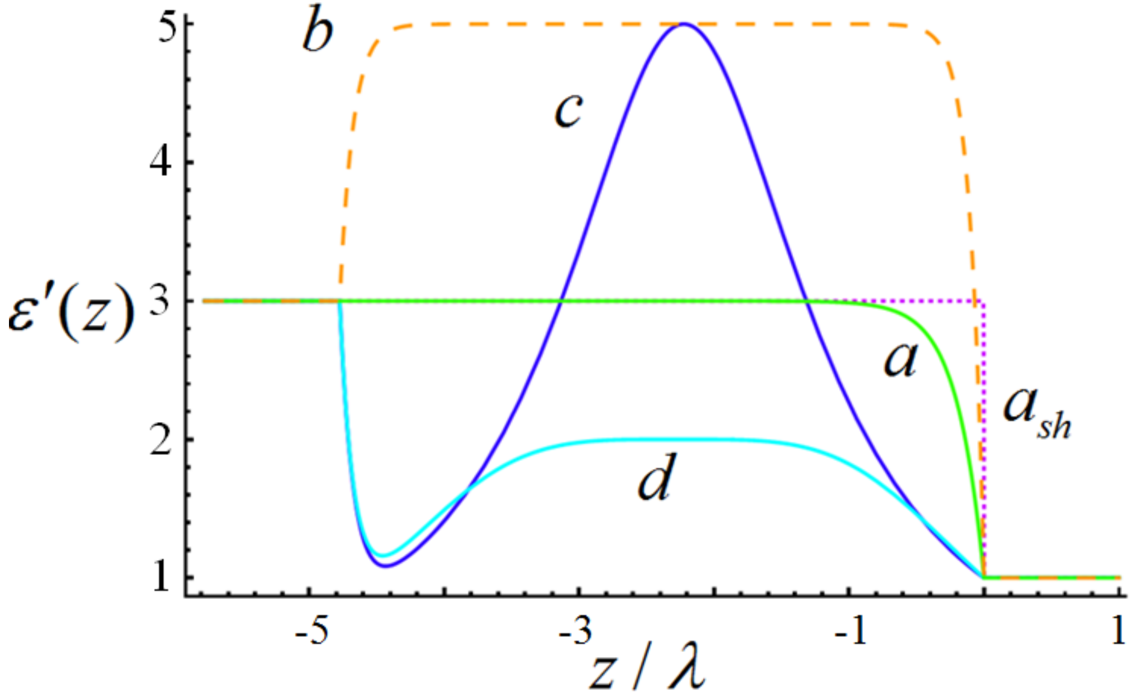


Fig1b. Four (a,b,c,d) model dieelctric functions describing the real part of the dielectric permittivity of the medium.

λ is the wavelength of radiation in vacuum. The frequency of radiation ω is fixed.

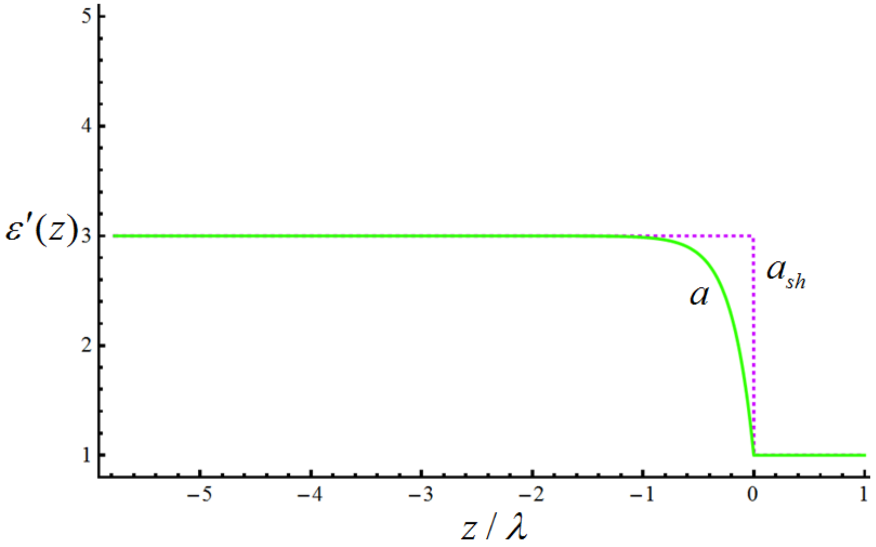
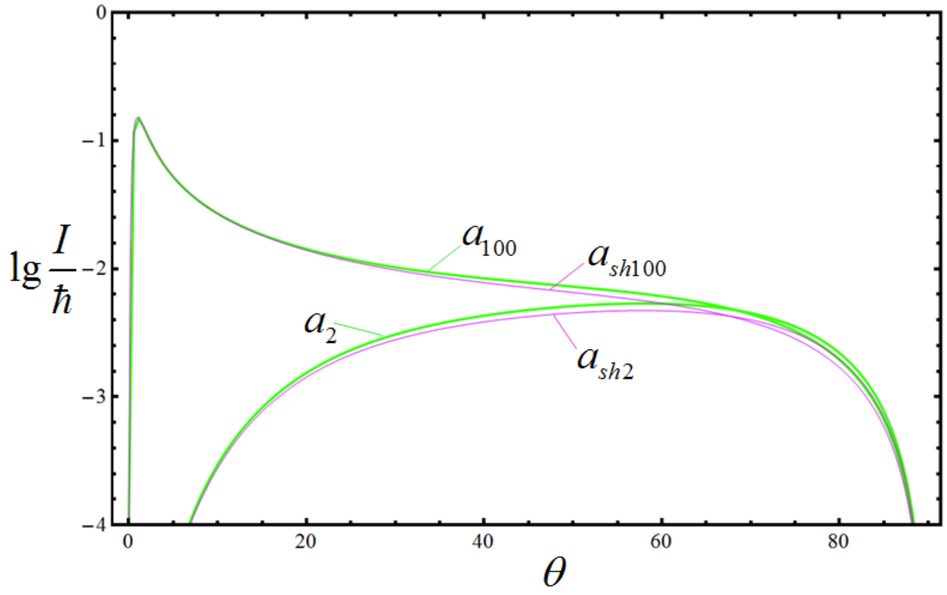


Fig.2a

Distribution of the real part of dielectric function of the medium along the direction of electron motion (axis Z).

Dotted line (curve "a_sh") is the case of a sharp boundary between the medium and vacuum [1,2]. Curve "*a*" is the case of $l = \lambda$

Fig.2b

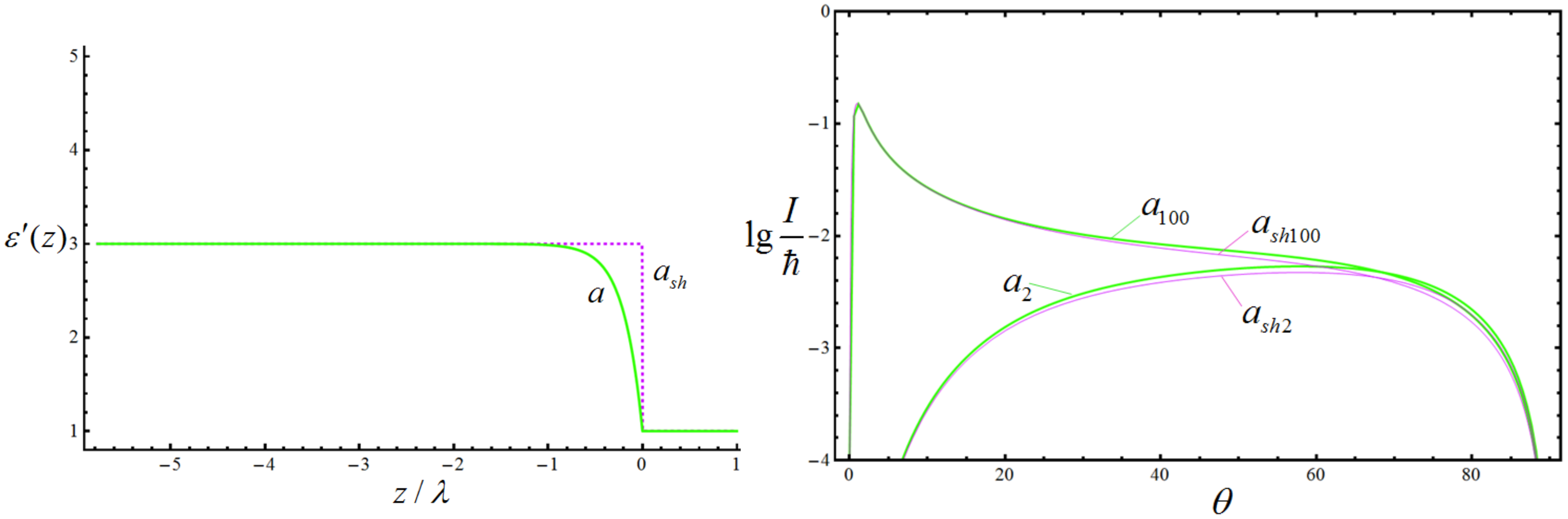


Angular distribution of TR generated by a a relativistic and moderately relativistic electrons at a given frequency.

The Lorentz factor

$\gamma = 100$ (curves a_sh100, a100)

$\gamma = 2$ (curves a_sh2, a2)



Comparing the data shown in Figs.2a,b we arrive at the following

Conclusion 1

The angular distribution of the energy of TR generated by a fast particle practically coincides with the results of similar calculations for the case of a sharp interface between media and vacuum: *l* =0 [1,2], if a blurred boundary (between the semi-infinite medium and vacuum) is thin enough: $l \leq \lambda$

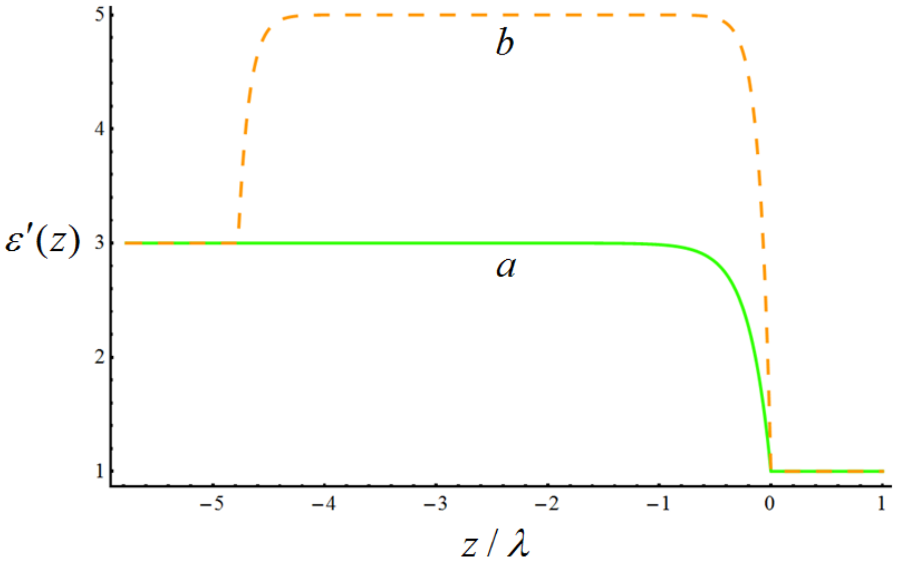


Fig. 3a

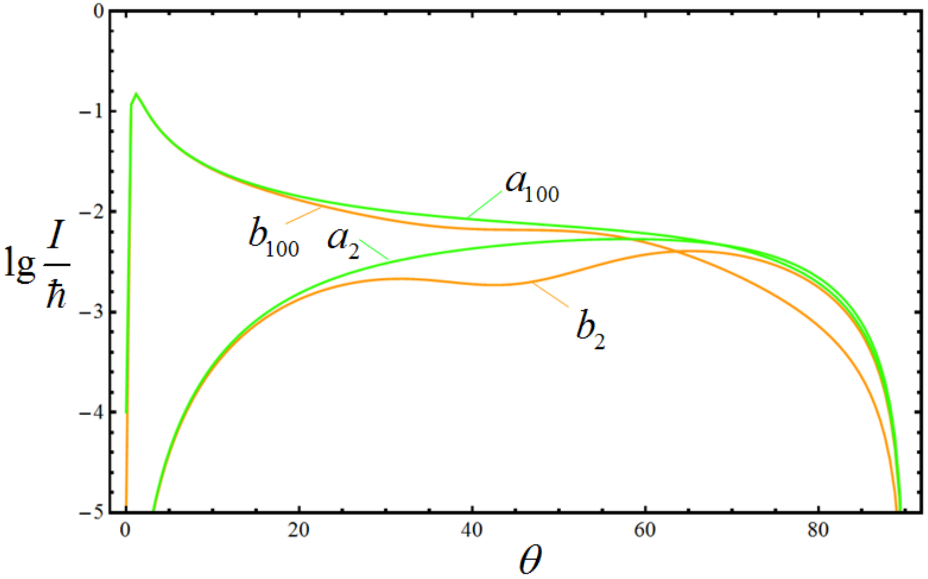
Distribution of the real part of dielectric function of the medium along the direction of electron motion (axis Z).

Dotted curve “b” is the case of non-thin $l = 5\lambda$

and curve “a” is the case of a thin $l = \lambda$

blurred boundary between media and vacuum.

Fig. 3b

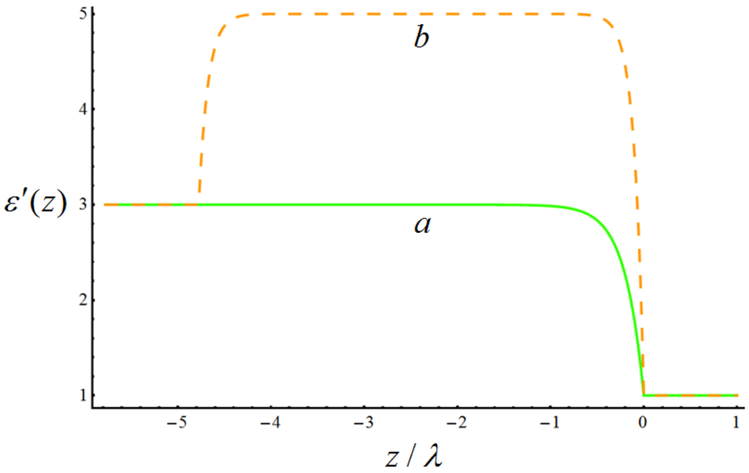


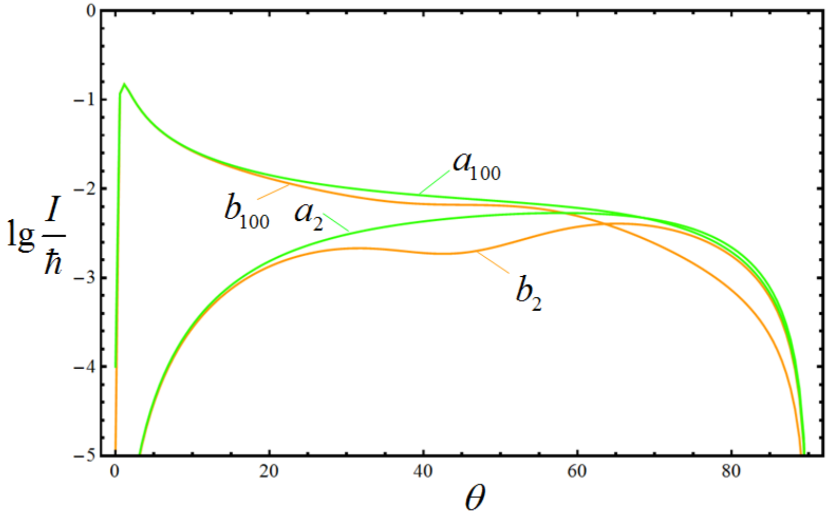
Angular distribution of TR generated by a relativistic and moderately relativistic electrons at a given frequency.

The Lorentz factor

$$\gamma = 100$$ (curves b100, a100)

$$\gamma = 2$$ (curves b2, a2)





Comparing the data shown in Figs.3a,b we arrive at the following

Conclusion 2

The impact of an interlayer with $l = 5\lambda$ practically doesn’t change the angular distribution of the energy of TR, even if the fast particle generates ChR inside the diffuse layer, but it does not go out of a substance in vacuum, due to the TIR at a plane interface between media and vacuum.

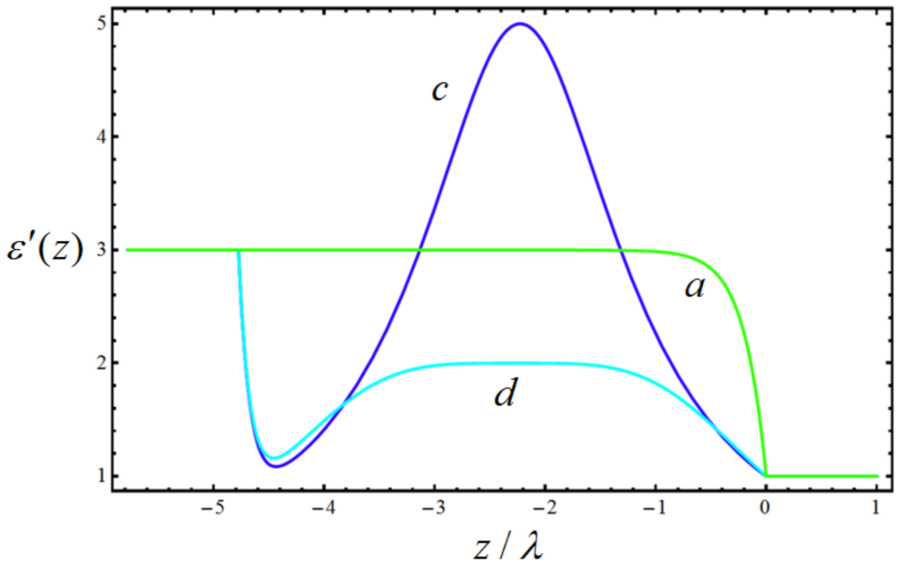


Fig. 4a

Distribution of the real part of dielectric function of the medium along the direction of electron motion (axis Z) for three cases: a,c,d.

Curves “c”,“d” represent non-homogeneous and non-thin $l = 5\lambda$

blurred boundaries between the medium and vacuum.

The curve “a” is the case of a thin boundary: $l = \lambda$

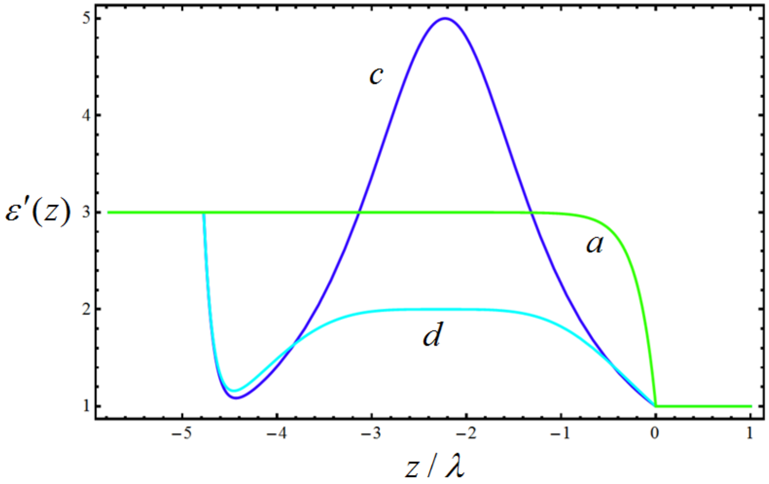


Fig.4a

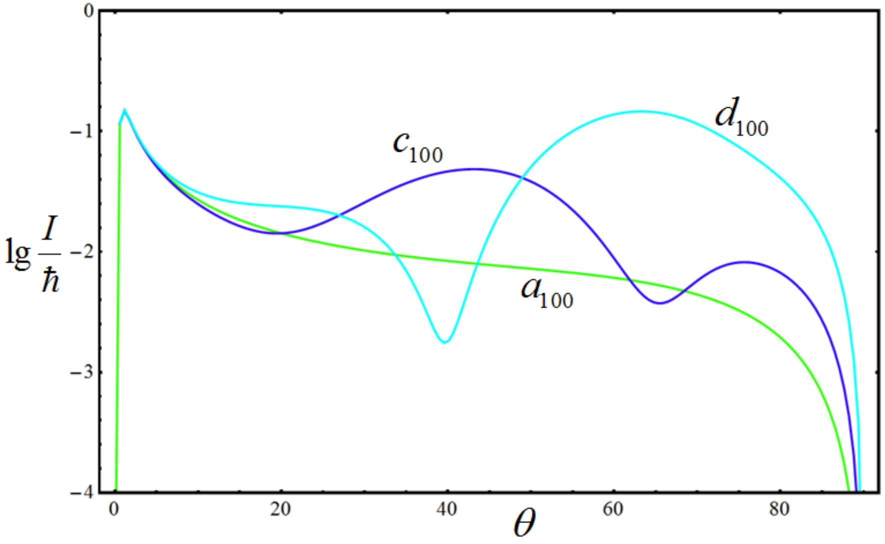


Fig.4b

Fig.4b. Angular distribution of radiation generated by a relativistic electron at a given frequency for the curves a,c,d.

$$\gamma = 100$$

The Lorentz factor

(curves a100, c100, d100)

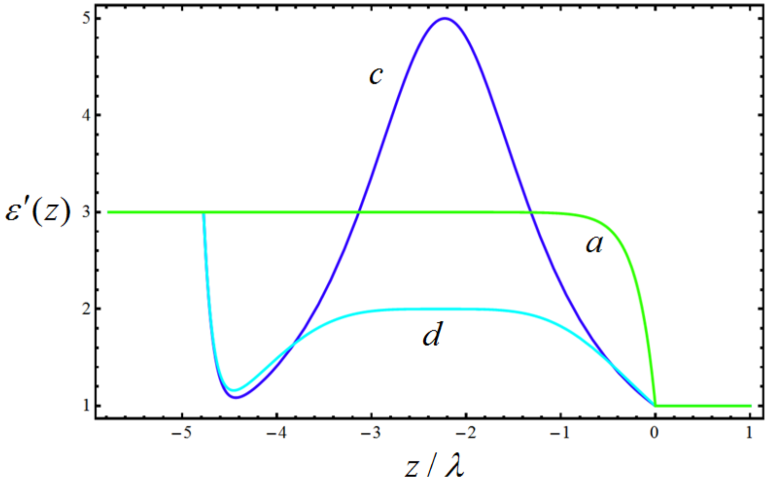


Fig.4a

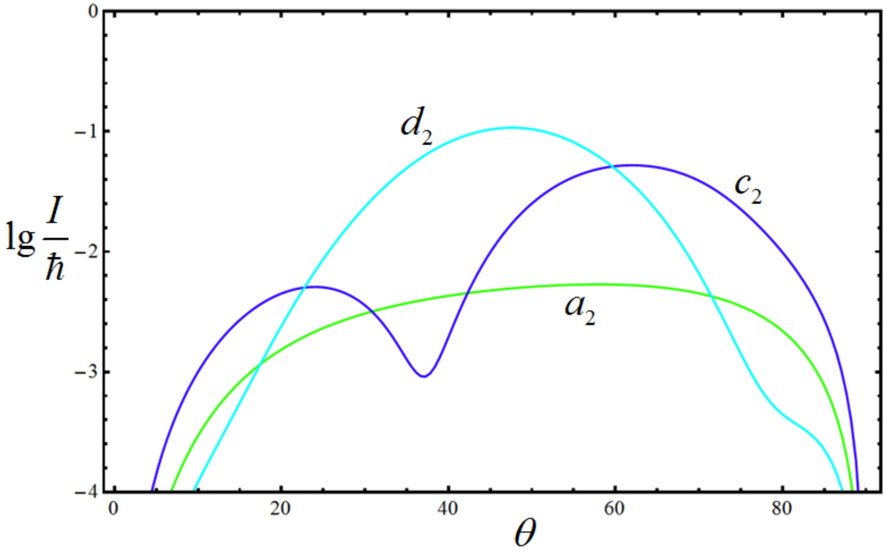


Fig.4c

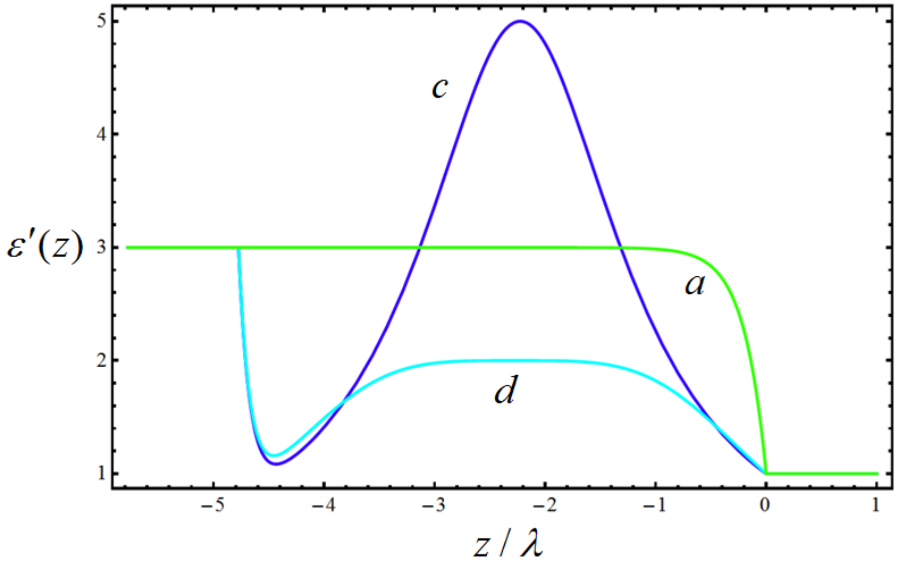
Fig.4c. Angular distribution of radiation generated by a moderately relativistic electron at a given frequency for the same curves a,c,d.

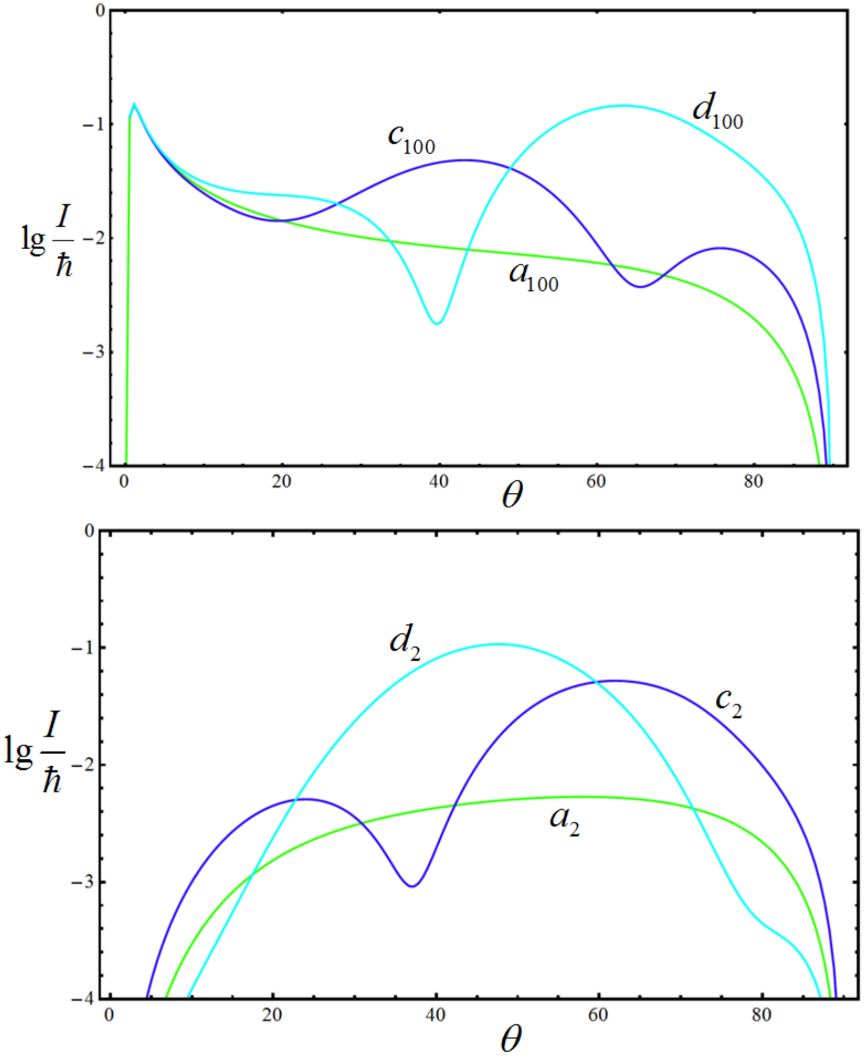
The Lorentz factor

$$\gamma = 2$$

(curves a2, b2, c2)

Fig. 4a,b,c





$$l = 5\lambda$$

Comparing the data shown in Figs.4a,b,c one comes at the following Conclusion

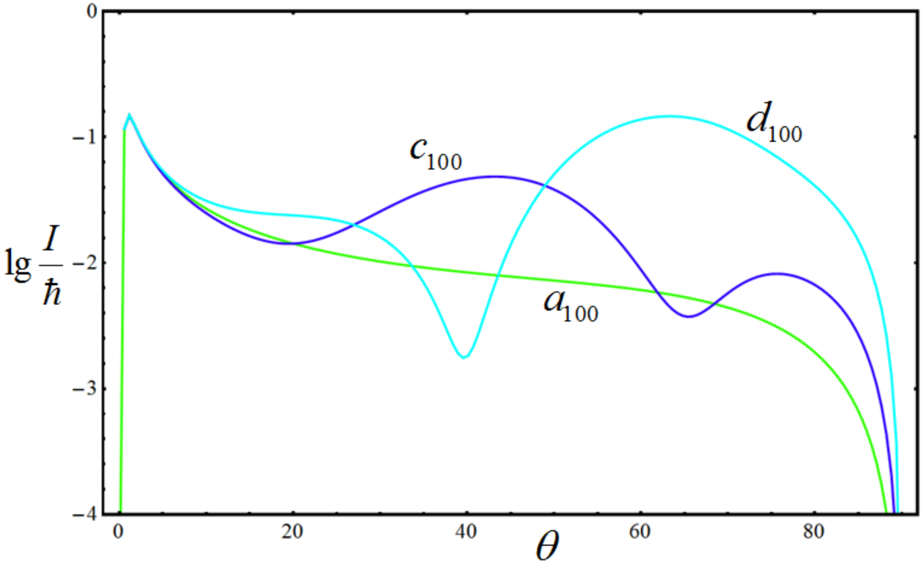


Fig. 4b,c

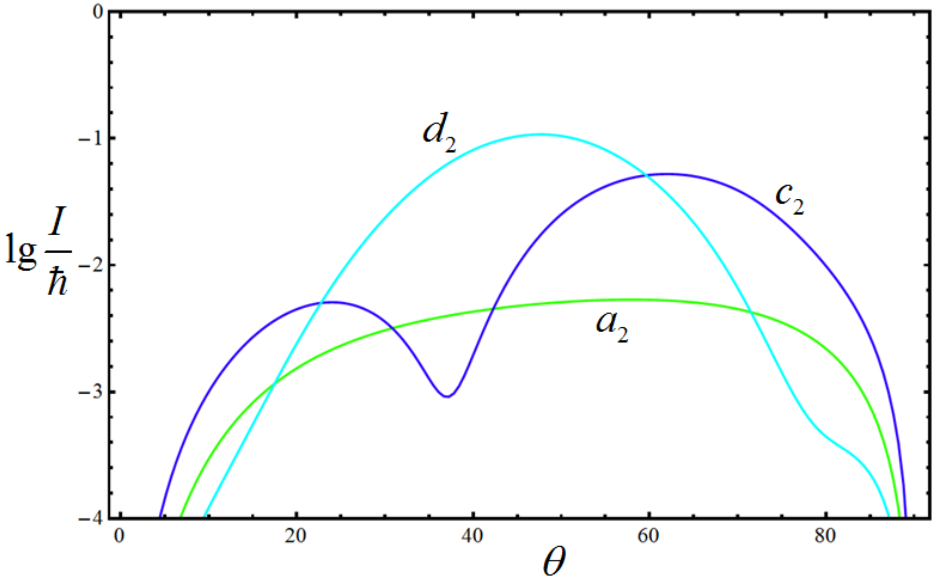
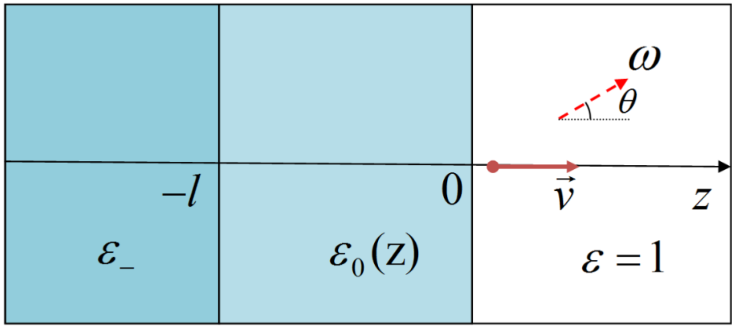


Fig. 4b,c

Conclusion 3

The presence of interlayer with $l = 5\lambda$ can increase many times the angular distribution of radiation generated by a fast particle at a given frequency.

4. Conclusions

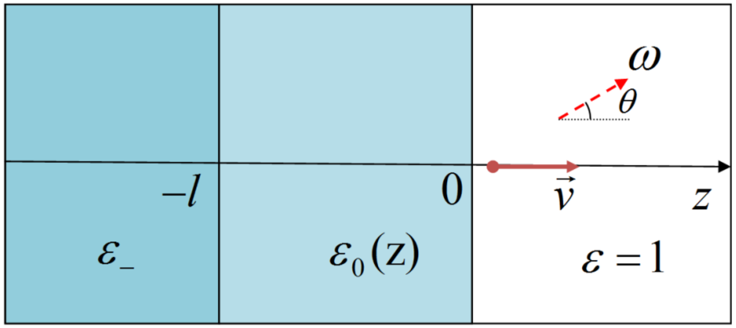


$$\varepsilon = \begin{cases} \varepsilon_9 & z \le -l \\ \varepsilon_0(z) & -l < z < 0 \\ 1 & z \ge 0 \end{cases}$$

In this report

the radiation from a charged particle passing through a smoothly varying intermediate layer between a semi-infinite medium and vacuum

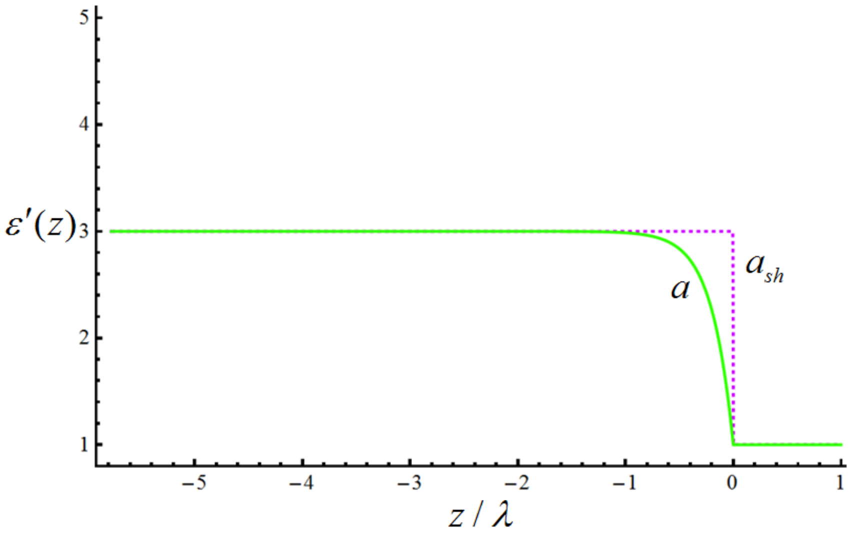
is investigated .

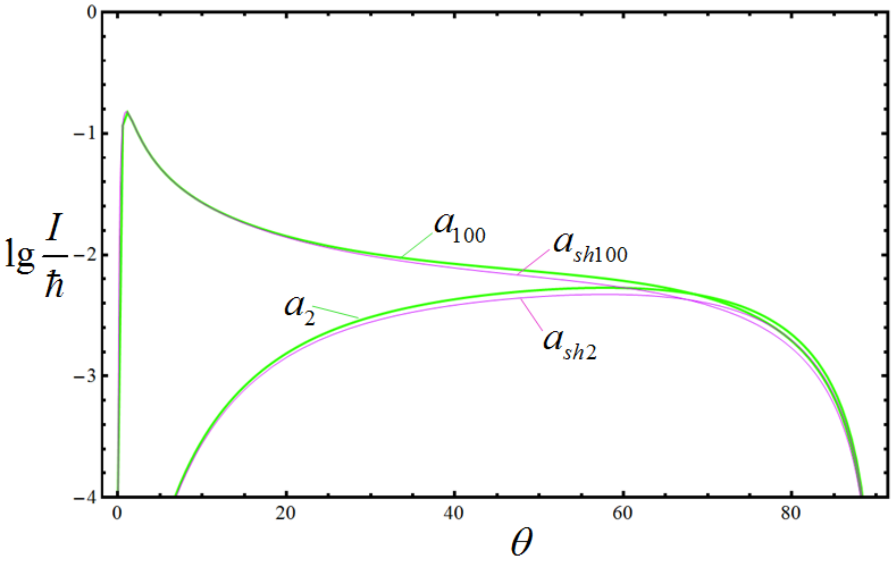


1.

We have determined the spectral angular distribution of radiation from a fast particle **with no limitations on the variation of the permittivity inside the intermediate layer.**

Our study is based on **exact solutions of Maxwell's equations.**





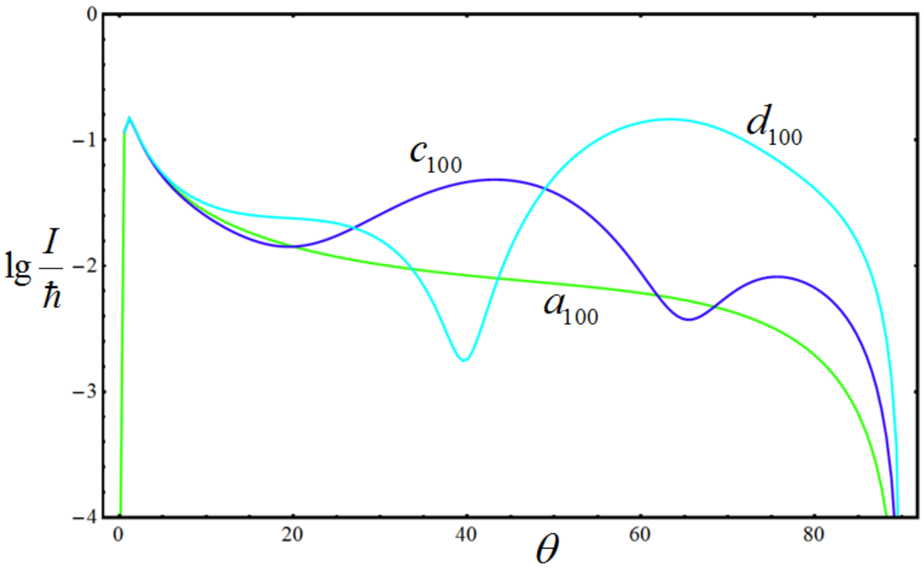
2.

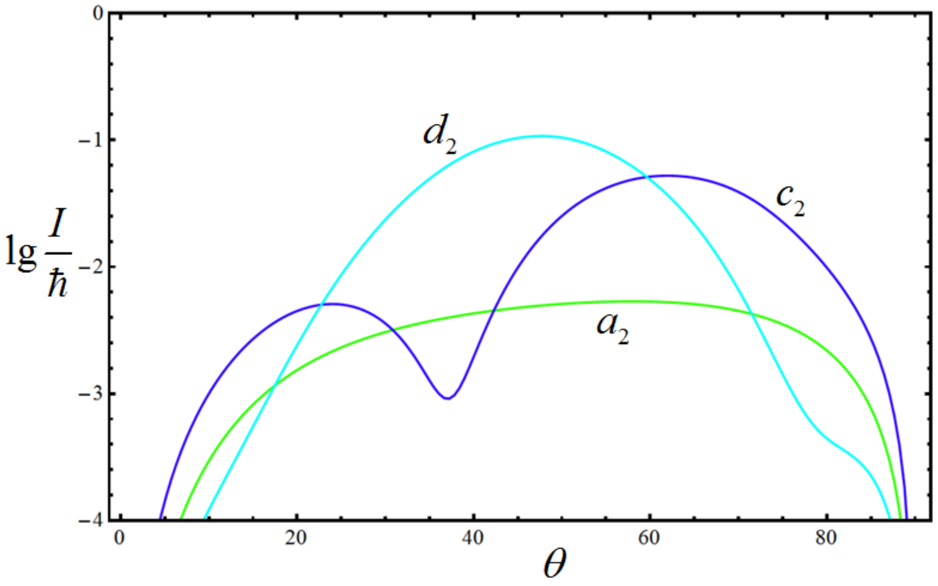
The angular distribution of the radiation energy at a given frequency **practically does not differ from** the analogous distribution for the case of **a sharp interface** between the medium and vacuum [1,2], **if the blurred boundary** (between the semi-infinite medium and vacuum**) is sufficiently thin**:

$$l \leq \lambda$$

If the substance of a semi-infinite medium is optically dense enough, from a semi-infinite medium in vacuum goes out only TR, whereas **ChR** generated by a relativistic particle **does not escape into vacuum**, undergoing total internal reflection at a plane interface matter-vacuum [1,2].

In this case, the numerical results lead to **the following conclusions**:





3. The presence of **interlayer** with $l = 5\lambda$ **can increase many times** the angular distribution of radiation generated by a fast particle at a given frequency **if** the particle **generates** ChR inside the blurred boundary **and** that ChR **goes into vacuum** (not testing TIR at a plane interface matter-vacuum).

The influence of the intermediate layer (blurred border) on the angular distribution of radiation **is mainly determined by** the following **three circumstances:**

A.
The way in which the dielectric permittivity and magnetic permeability of the matter **vary in the blurry (intermediate) layer.**

B.

Charged particle in a blurred layer generates or does not generate ChR?

More precisely:

Generates or does not generate “embryo “ of ChR ?

(Since the thickness of the blurred layer is small)

C

What is the thickness of the part of the diffuse layer, in which

(a) ChR is generated by the particle

and

(b) it goes out in vacuum?

This thickness is greater than a certain minimum value **or not?**

This phenomenon can be used
for **practical applications.**

For instance, one can use it
to probe the structure
of thin surface layers
of different media
with electron beams.

ChR generated by a relativistic particle,
in an optically dense matter,
goes into the vacuum
(without experiencing TIR at a plane interface
between matter and vacuum)
and leads to similar conclusions.

Consideration of this issue
is beyond the scope of our report.

REFERENCES

[1] G.M. Garibian, C. Yang, 1983, *X-Ray Transition Radiation* (Yerevan: AN Arm. SSR Press) (in Russian).

[2] V.L. Ginzburg, V.N. Tsytovich, 1990, *Transition Radiation and Transition Scattering* (Bristol: Adam Hilger).

Thank you indeed for your attention

Levon Grigoryan

levonshg@mail.ru