Interference effects in the radiation of the relativistic electron in the structure "amorphous matter layers - single crystal"

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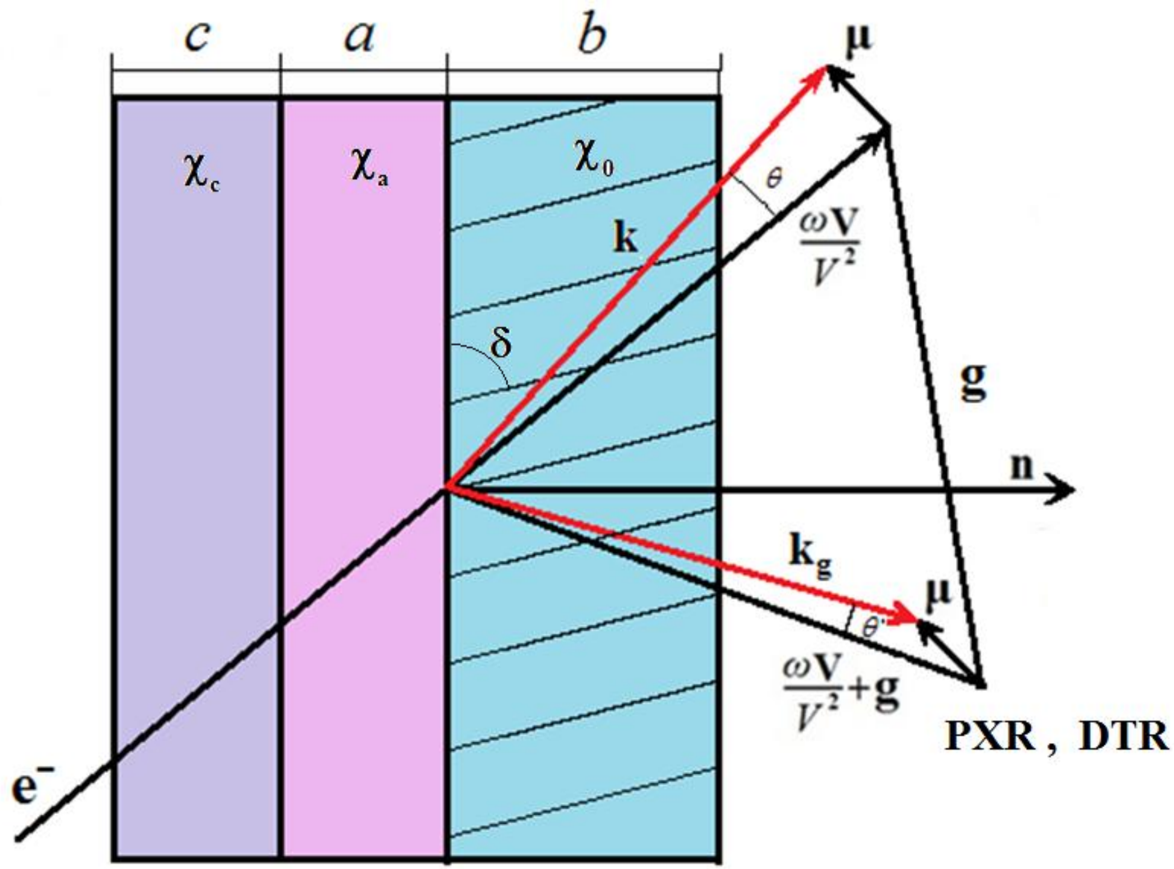
- In the present work a theory of coherent radiation of a relativistic electron moving at a constant speed in a combined target, consisting of several amorphous matter layers and a monocrystalline layer is built.

Geometry of radiation processes



- **Fig. 1.**

$$E_{\mathbf{g}}^{(s)Rad} = E_{DTR}^{(s)} + E_{PXR}^{(s)}$$

The fields of the diffracted radiation

$$E_{DTR}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} e^{i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{(c+a+b)}{\gamma_g}} \frac{\omega^2 \chi_{\mathbf{g}} C^{(s)}}{2\omega \frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)}\right)} \left(e^{i\frac{\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b} - e^{i\frac{\lambda_{\mathbf{g}}^{(2)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b} \right) \times$$

$$\times \left[\left(\frac{1}{\theta^2+\gamma^{-2}-\chi_c} - \frac{1}{\theta^2+\gamma^{-2}} \right) e^{-i\frac{\omega c}{2\gamma_0}\left(\gamma^{-2}+\theta^2-\chi_c\right)-i\frac{\omega a}{2\gamma_0}\left(\gamma^{-2}+\theta^2-\chi_a\right)} + \right.$$

$$+ \left(\frac{1}{\theta^2+\gamma^{-2}-\chi_a} - \frac{1}{\theta^2+\gamma^{-2}-\chi_c} \right) e^{-i\frac{\omega a}{2\gamma_0}\left(\gamma^{-2}+\theta^2-\chi_a\right)} +$$

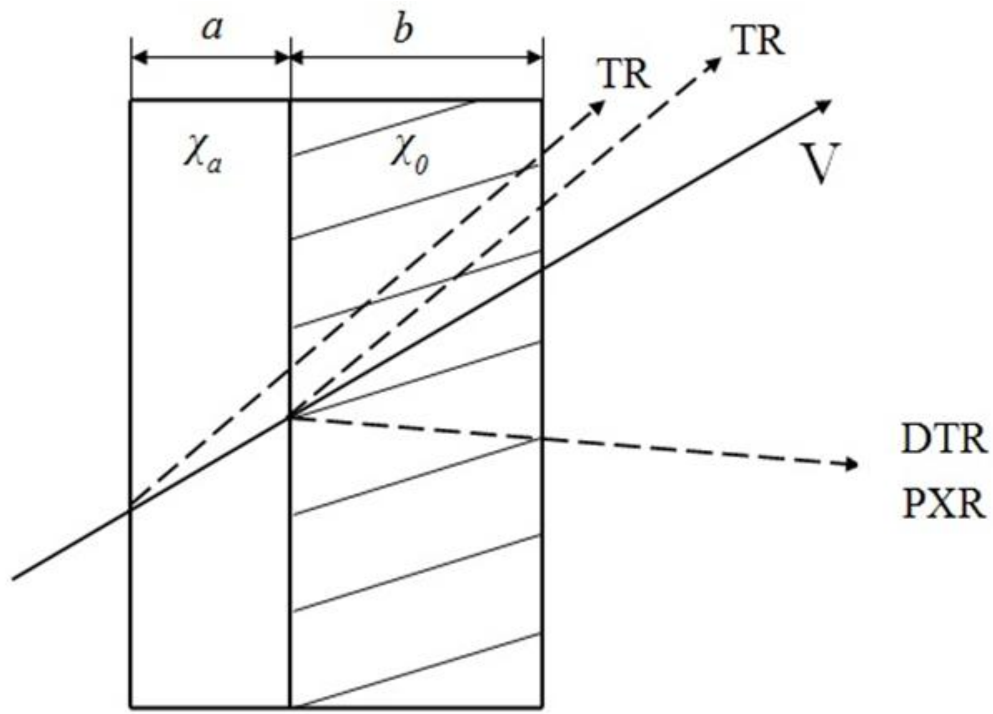
$$\left. + \frac{1}{\theta^2+\gamma^{-2}-\chi_0} - \frac{1}{\theta^2+\gamma^{-2}-\chi_a} \right]$$

The field of parametric X-ray radiation

$$E_{PXR}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}e^{i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{(c+a+b)}{\gamma_g}}\ \frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)}\right)}\left[\left(\frac{1}{\chi_0-\theta^2-\gamma^{-2}}+\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}\right)}\right)\times\right.$$

$$\left.\times\left(e^{i\frac{\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b}-1\right)-\left(\frac{1}{\chi_0-\theta^2-\gamma^{-2}}+\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}\right)}\right)\left(e^{i\frac{\lambda_{\mathbf{g}}^{(2)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b}-1\right)\right]$$

Spectral-angular density of the radiation from two-layer target



- **Fig.2.** Let us assume that $c = 0$

Spectral-angular density of PXR

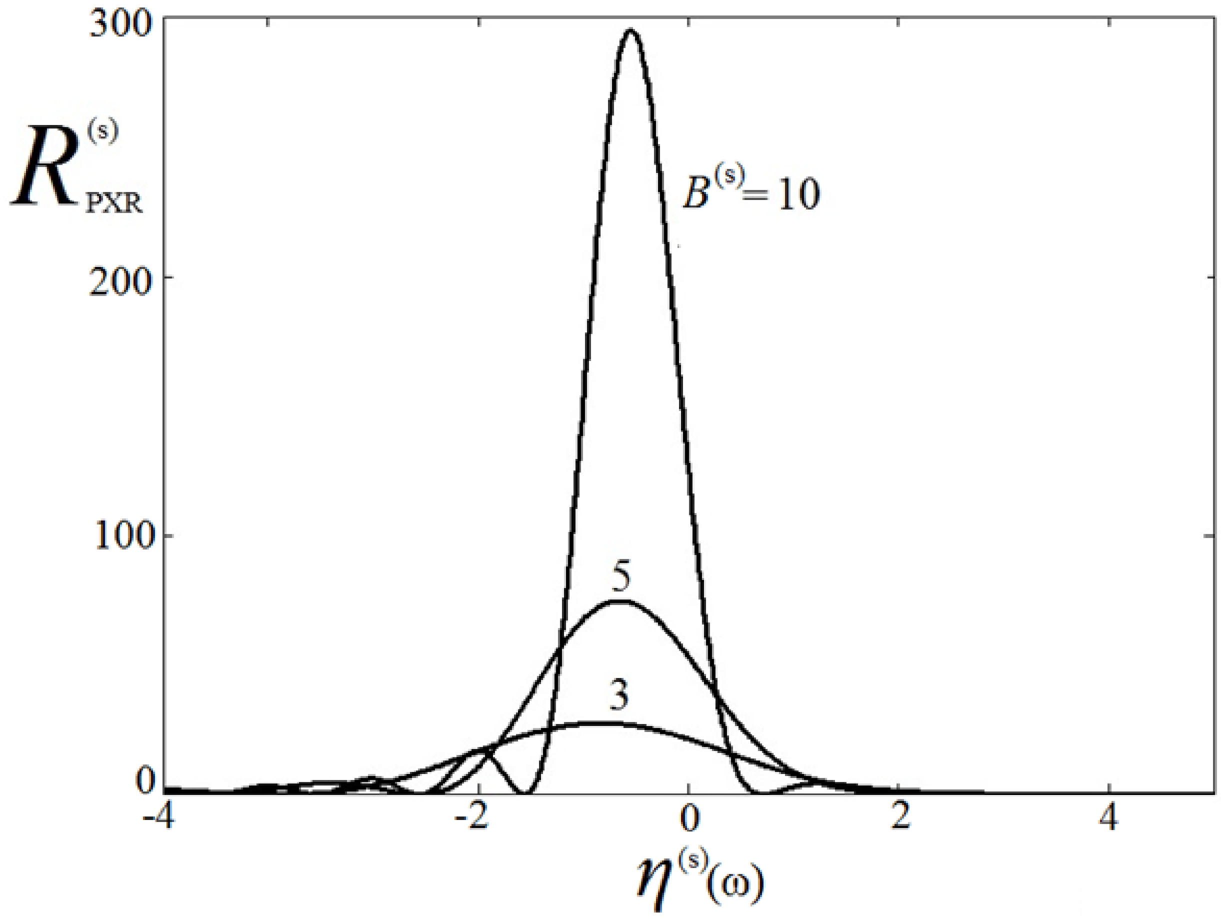
$$\omega \frac{d^2 N}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E_{\mathbf{g}}^{(s)Rad} \right|^2$$

$$\omega \frac{d^2 N_{\mathrm{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} \frac{P^{(s)^2}}{\left|\chi_0'\right|} T_{PXR}^{(s)}$$

$$T_{\mathrm{PXR}}^{(s)} = \frac{\Omega^2}{\left(\Omega_0^2 + 1\right)^2} R_{\mathrm{PXR}}^{(s)}$$

$$R_{\mathrm{PXR}}^{(s)} = 4\left(1 - \frac{\xi}{\sqrt{\xi^2 + \varepsilon}}\right)^2 \frac{\sin^2\left(\frac{B^{(s)}}{2}\left(\sigma^{(s)} + \frac{\xi - \sqrt{\xi^2 + \varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)} + \frac{\xi - \sqrt{\xi^2 + \varepsilon}}{\varepsilon}\right)^2}$$

where $\varepsilon = \frac{\sin(\delta + \theta_B)}{\sin(\delta - \theta_B)}$, $\Omega_0^2 = \Omega^2 + \Gamma^2$, $\Gamma = \frac{1}{\gamma\sqrt{\left|\chi_0'\right|}}$, $\Omega = \frac{\theta}{\sqrt{\left|\chi_0'\right|}}$, $B^{(s)} = \frac{1}{2\sin\left(\delta - \theta_B\right)} \frac{b}{L_{ext}^{(s)}}$



- **Fig.3.** PXR spectrum under different values of the crystalline layer thickness (parameter $B^{(s)}$): $\varepsilon = 3$, $\Omega = 0.3$, $\Gamma = 0.3$, $\nu^{(s)} = 0.8$

The contribution of PXR, DTR and their interference term into resulting spectrum

$$T^{(s)} = T^{(s)}_{\mathrm{DTR}} + T^{(s)}_{\mathrm{PXR}} + T^{\mathrm{int}(s)}_{\mathrm{PXR,DTR}}$$

$$\varepsilon = \frac{\sin(\delta + \theta_B)}{\sin(\delta - \theta_B)}$$

$$\Omega = \frac{\theta}{\sqrt{|\chi_0'|}}$$

$$\nu^{(s)} = \frac{\chi_{\mathbf{g}}' C^{(s)}}{\chi_0'}$$

$$B^{(s)} = \frac{1}{2\sin(\delta - \theta_B)} \frac{b}{L^{(s)}_{ext}}$$

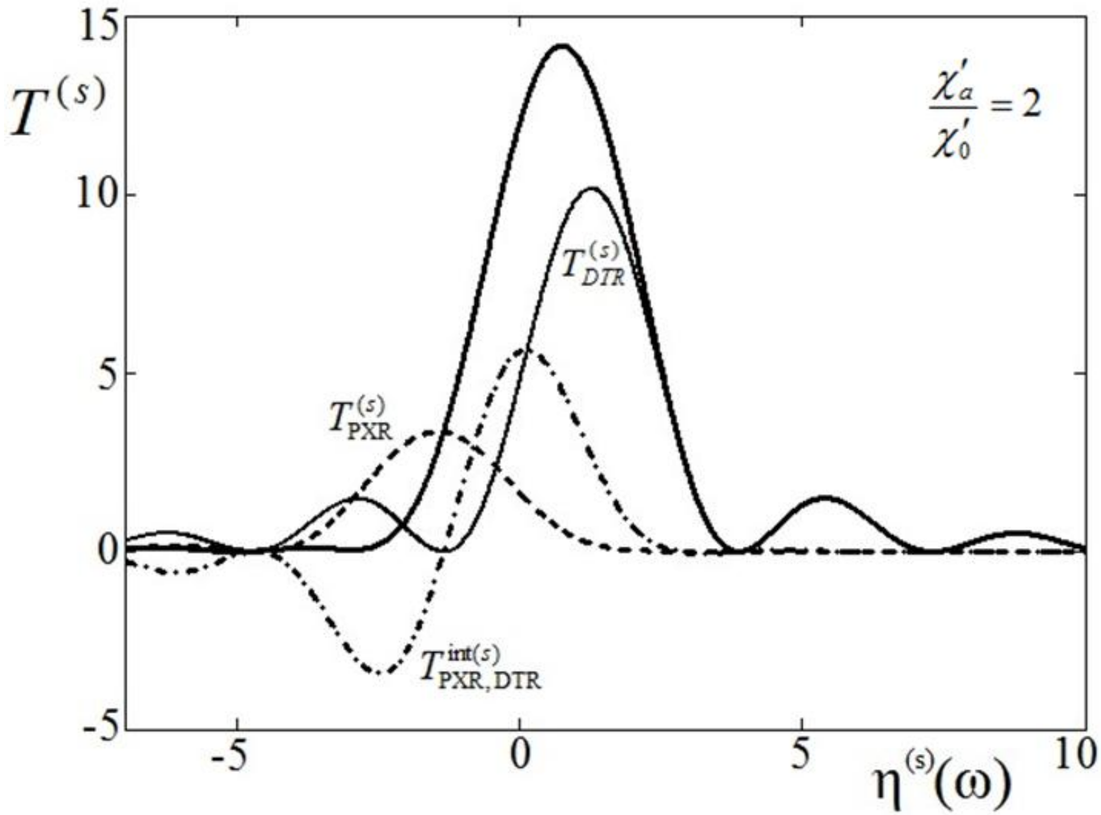


Fig.4 The contributions of PXR and DTR and interference term to the resulting spectrum: $\Omega = 0.5$, $\Gamma = 0.5$, $\nu^{(s)} = 0.8$, $B^{(s)} = 3$, $\frac{\chi_a'}{\chi_0'} = 2$, $\frac{a}{b} = 1$, $\varepsilon = 3$.

$$\varepsilon = \frac{\sin(\delta + \theta_B)}{\sin(\delta - \theta_B)}$$

$$\Omega = \frac{\theta}{\sqrt{|\chi'_0|}}$$

$$\Gamma = \frac{1}{\gamma\sqrt{|\chi'_0|}}$$

$$\nu^{(s)} = \frac{\chi'_{\mathbf{g}} C^{(s)}}{\chi'_0}$$

$$B^{(s)} = \frac{1}{2\sin(\delta - \theta_B)} \frac{b}{L_{ext}^{(s)}}$$

$$\frac{\chi'_a}{\chi'_0} = \frac{Z_a n_a}{Z_0 n_0}$$

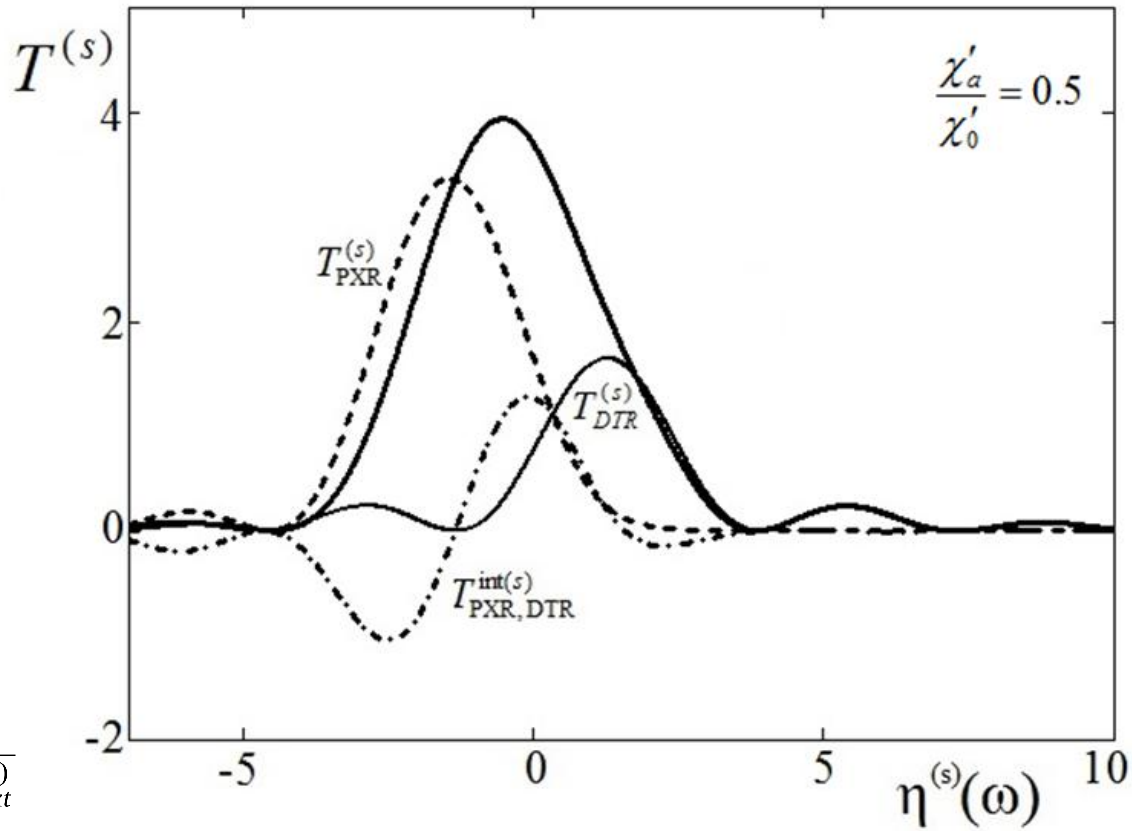
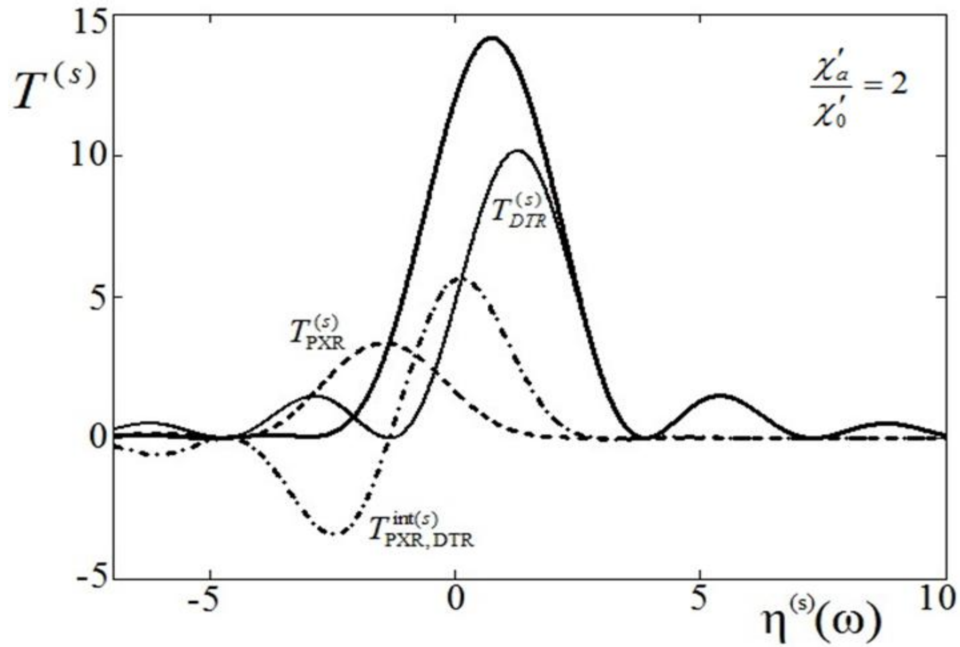
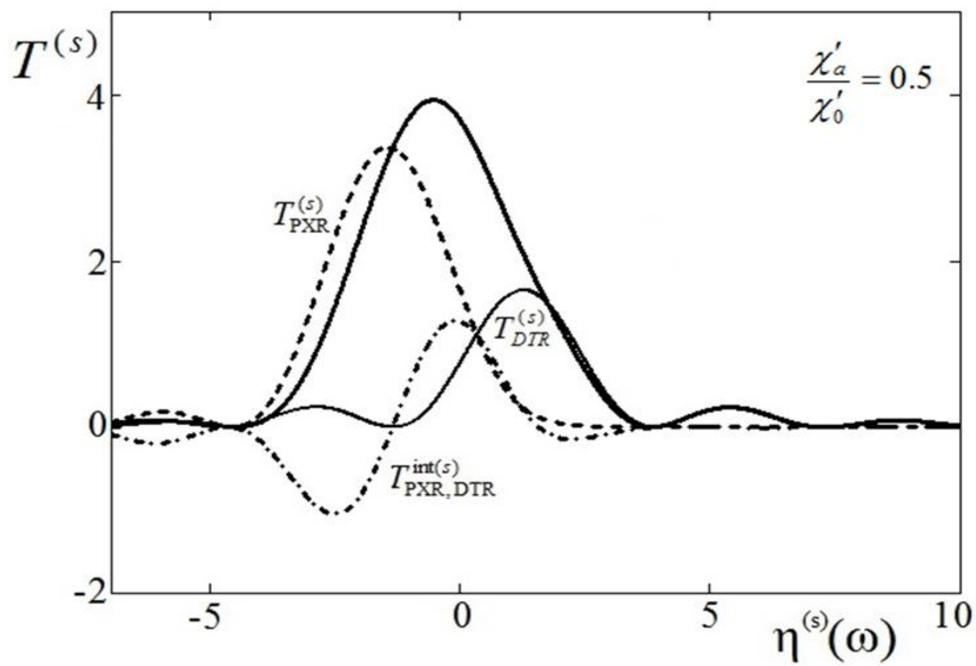


Fig. 5. The same as in Fig.4, but $\frac{\chi'_a}{\chi'_0} = 0.5$

($\Omega = 0.5$, $\Gamma = 0.5$, $\nu^{(s)} = 0.8$, $B^{(s)} = 3$, $\frac{\chi'_a}{\chi'_0} = 0.5$, $\frac{a}{b} = 1$, $\varepsilon = 3$)



$$\frac{\chi'_a}{\chi'_0} = \frac{Z_a n_a}{Z_0 n_0}$$



$$\omega \frac{d^2 N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} \frac{P^{(s)^2}}{|\chi_0'|} T_{DTR}^{(s)}$$

$$T_{DTR}^{(s)} = T_1^{(s)} + T_2^{(s)} + T_{\mathrm{int}}^{(s)}$$

$$T_1^{(s)} = \Omega^2 \left(\frac{1}{\Omega_0^2} - \frac{1}{\Omega_0^2 + \frac{\chi_a'}{\chi_0'}} \right)^2 R_{DTR}^{(s)}$$

$$T_2^{(s)} = \Omega^2 \left(\frac{1}{\Omega_0^2 + \frac{\chi_a'}{\chi_0'}} - \frac{1}{\Omega_0^2 + 1} \right)^2 R_{DTR}^{(s)}$$

$$T_{\mathrm{int}}^{(s)} = 2\Omega^2 \left(\frac{1}{\Omega_0^2} - \frac{1}{\Omega_0^2 + \frac{\chi_a'}{\chi_0'}} \right) \left(\frac{1}{\Omega_0^2 + \frac{\chi_a'}{\chi_0'}} - \frac{1}{\Omega_0^2 + 1} \right) \cos\left(B^{(s)} \cdot \frac{a}{b} \cdot \frac{1}{\nu^{(s)}} \left(\Omega_0^2 + \frac{\chi_a'}{\chi_0'} \right) \right) R_{DTR}^{(s)}$$

$$R_{\mathrm{DTR}}^{(s)} = \frac{4\varepsilon^2}{\xi^2 + \varepsilon} \sin^2 \left(\frac{B^{(s)} \sqrt{\xi^2 + \varepsilon}}{\varepsilon} \right)$$

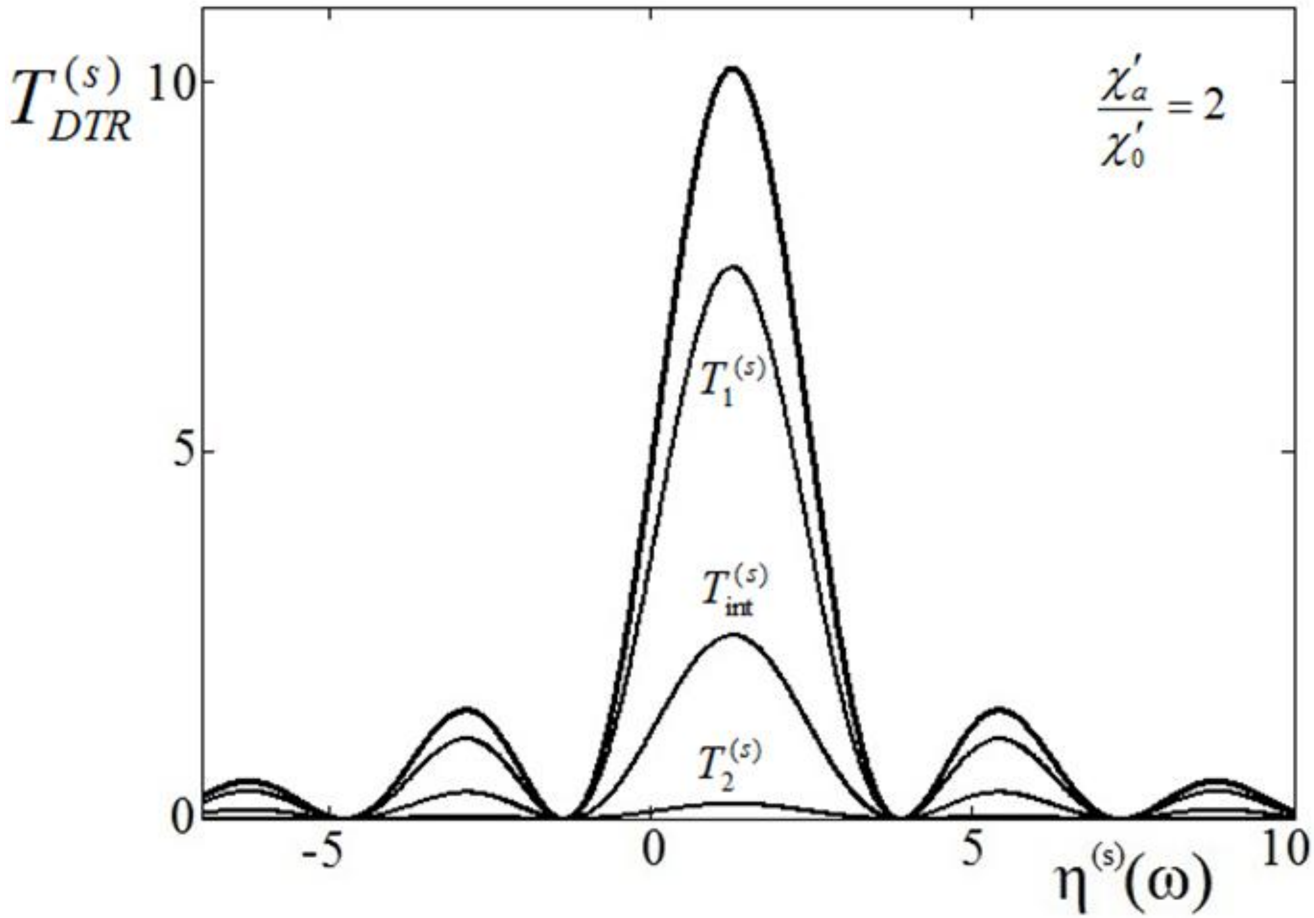
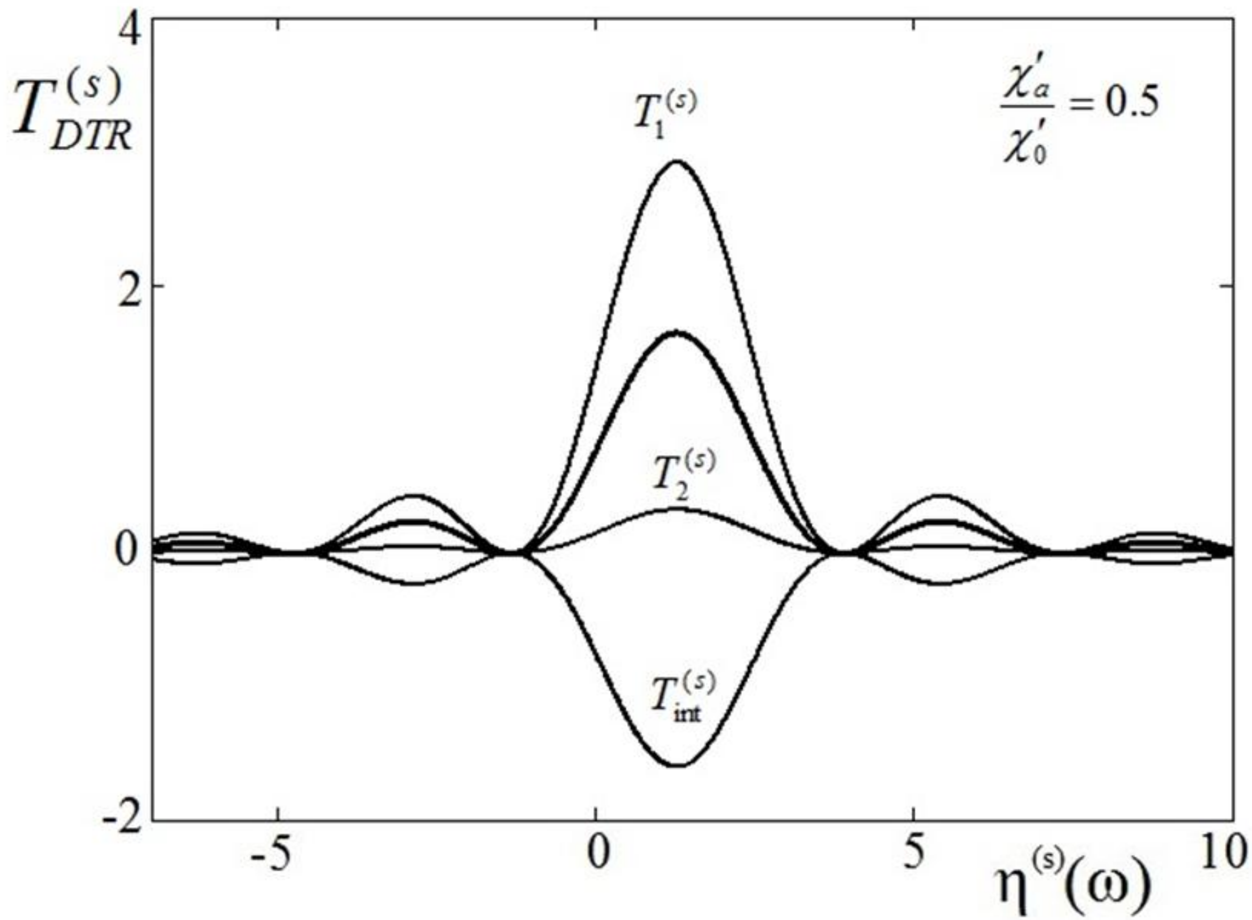


Fig. 6. Contributions of TR generated on the first and second boundaries of the target and interference term to DTR:

$$\Omega=0.5\ ,\ \Gamma=0.5\ ,\ \nu^{(s)}=0.8\ ,\ B^{(s)}=3\ ,\ \frac{\chi'_a}{\chi'_0}=2\ ,\ \frac{a}{b}=1\ ,\ \varepsilon=3\ .$$



- **Fig. 7.** Contributions of TR generated on the first and second boundaries of the target and interference term to DTR: $\Omega = 0.5$, $\Gamma = 0.5$, $\nu^{(s)} = 0.8$, $B^{(s)} = 3$, $\frac{\chi'_a}{\chi'_0} = 0.5$, $\frac{a}{b} = 1$, $\varepsilon = 3$.

Angular densities of PXR and DTR and effect of their interference

$$\omega \frac{d^2 N}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E_{\mathbf{g}}^{(s)Rad} \right|^2$$

$$\omega \frac{d^2 N_{\mathrm{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} \frac{P^{(s)^2}}{\left|\chi_0'\right|} T_{PXR}^{(s)}$$

$$T_{\mathrm{PXR}}^{(s)} = \frac{\Omega^2}{\left(\Omega_0^2 + 1\right)^2} R_{\mathrm{PXR}}^{(s)}$$

$$F_{1,2,\mathrm{int}}^{(s)}(\theta) = \nu^{(s)} \int_{-\infty}^{+\infty} T_{1,2,\mathrm{int}}^{(s)} d\eta^{(s)}(\omega),$$

$$\frac{dN_{INT}^{(s)}}{d\Omega} = \frac{e^2 P^{(s)^2}}{8\pi^2 \sin^2 \theta_B} F_{INT}^{(s)}(\theta),$$

$$F_{INT}^{(s)}(\theta) = \nu^{(s)} \int_{-\infty}^{+\infty} T_{\mathrm{PXR,DTR}}^{\mathrm{int}(s)} d\eta^{(s)}(\omega)$$

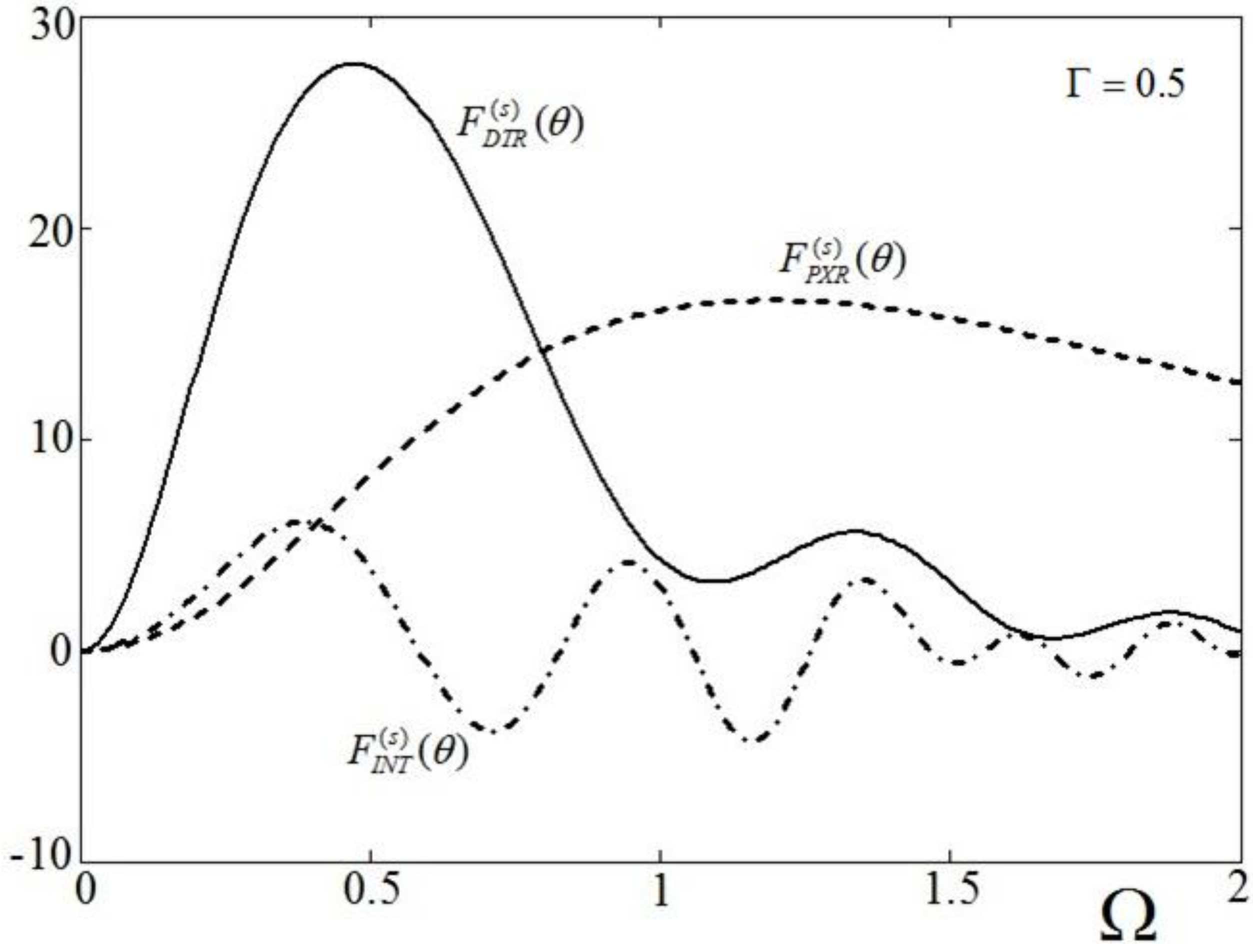


Fig. 8. The angular density of PXR and DTR and the interference term; $\Omega = 0.5$, $\Gamma = 0.5$, $\nu^{(s)} = 0.8$, $B^{(s)} = 3$, $\dfrac{\chi_a'}{\chi_0'} = 2$, $\dfrac{a}{b} = 1$.

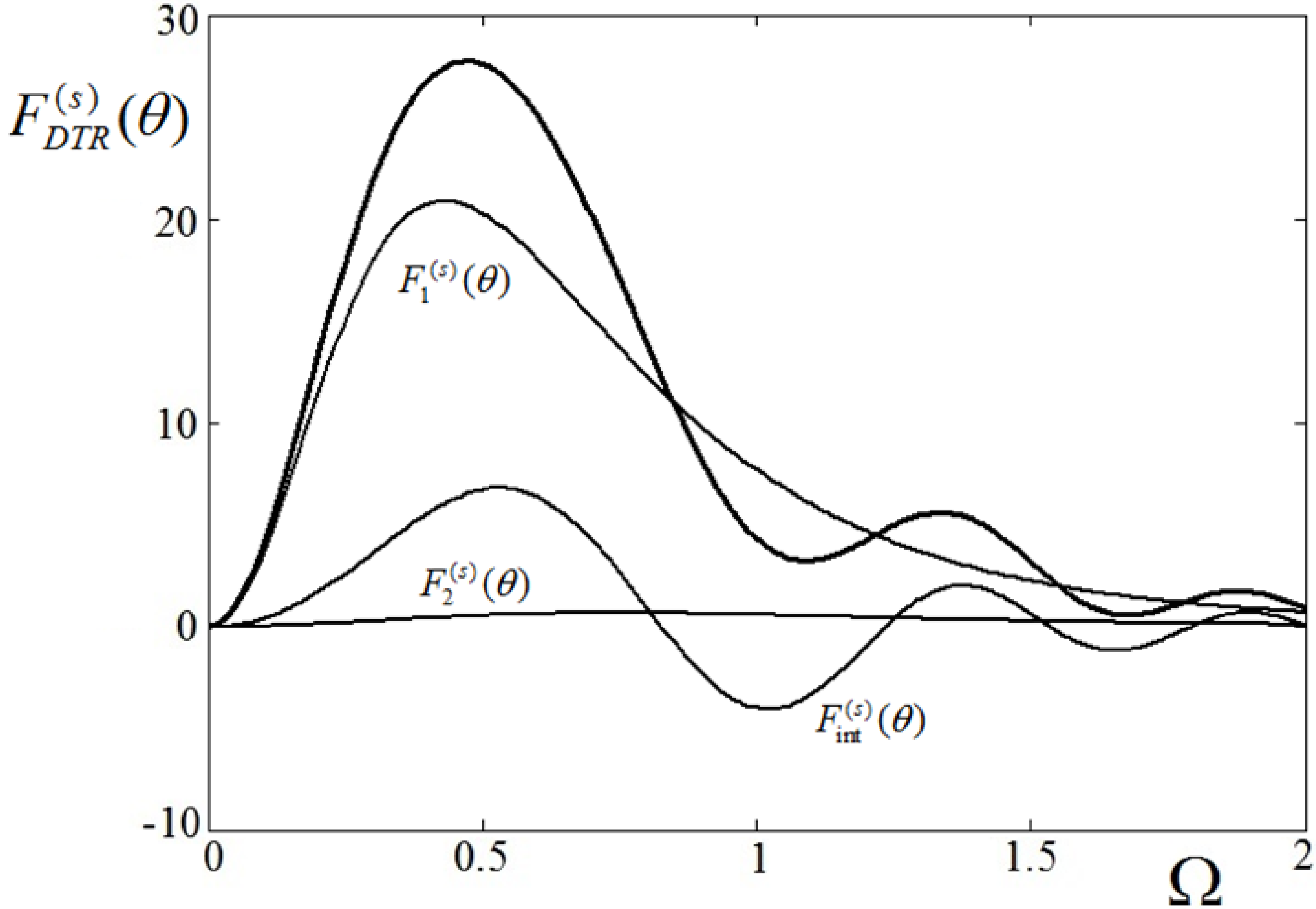
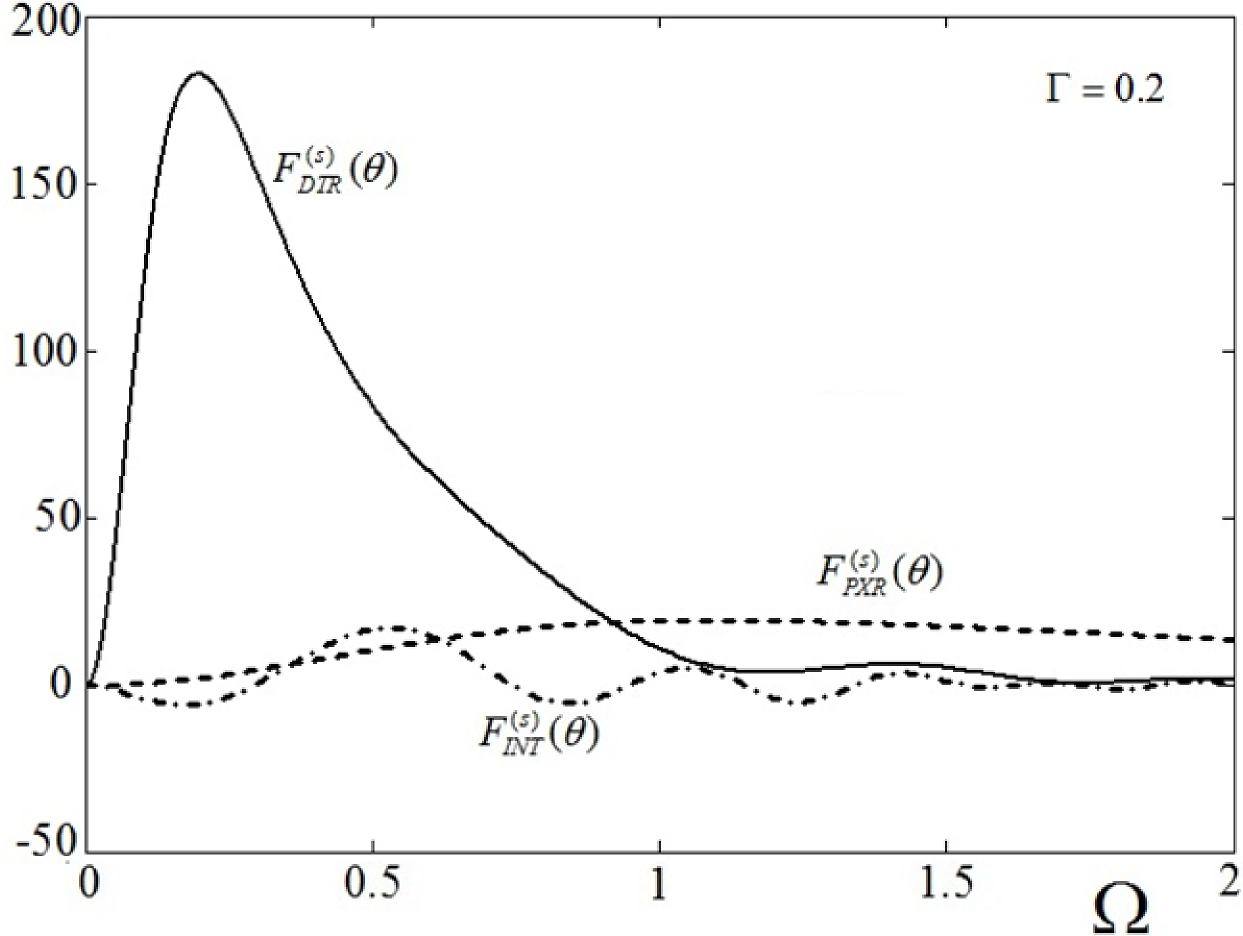


Fig. 9. Contribution of TR from the first and second boundaries $F_1^{(s)}$, $F_2^{(s)}$ and interference term $F_{\mathrm{int}}^{(s)}$ to DTR $\Omega = 0.5$, $\Gamma = 0.5$, $\nu^{(s)} = 0.8$, $B^{(s)} = 3$, $\frac{\chi'_a}{\chi'_0} = 2$, $\frac{a}{b} = 1$, $\varepsilon = 3$.



$$\Gamma = \frac{1}{\gamma\sqrt{|\chi_0'|}}$$

Fig. 10. Angular density of PXR, DTR and interference term in the conditions of constructed interference , $\Omega = 0.5$, $\Gamma = 0.2$, $\nu^{(s)} = 0.8$, $B^{(s)} = 3$, $\dfrac{\chi_a'}{\chi_0'} = 2$, $\dfrac{a}{b} = 1$, $\varepsilon = 3$.

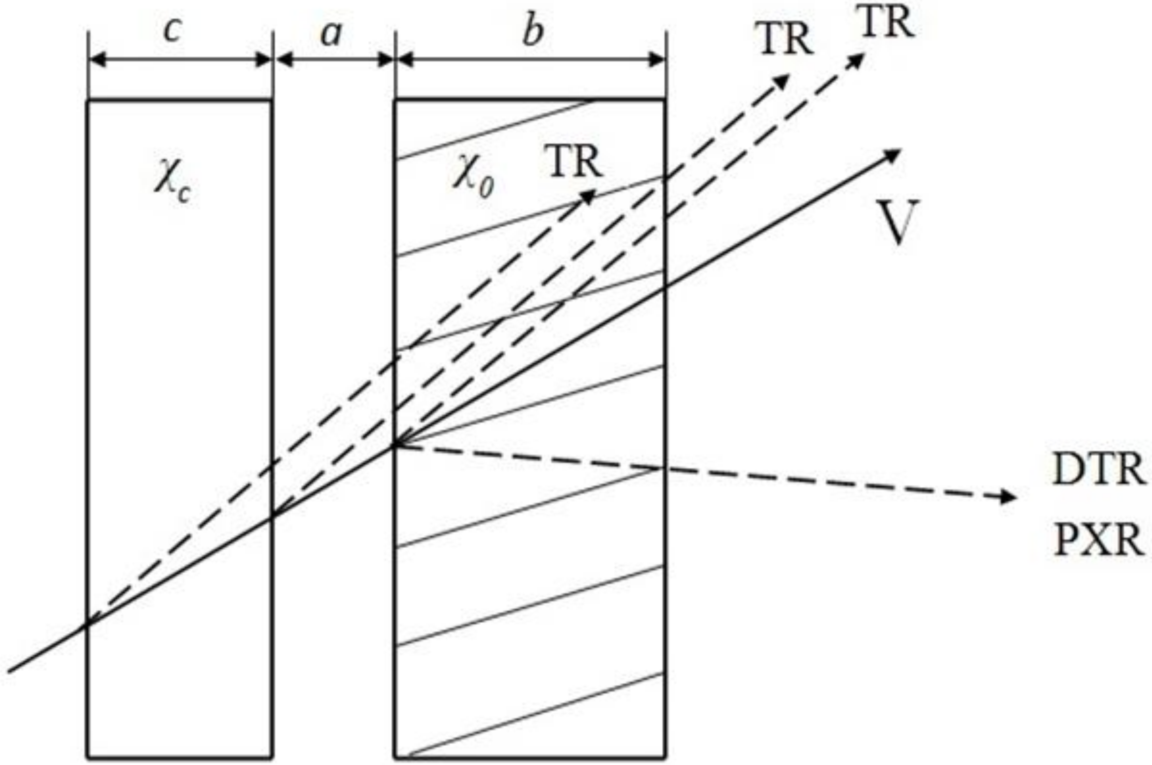


Fig. 11. Scheme of radiation of a relativistic electron in the structure "amorphous – vacuum – crystal"

It is a special case when the second layer of the target is vacuum ($\chi'_a = 0$)

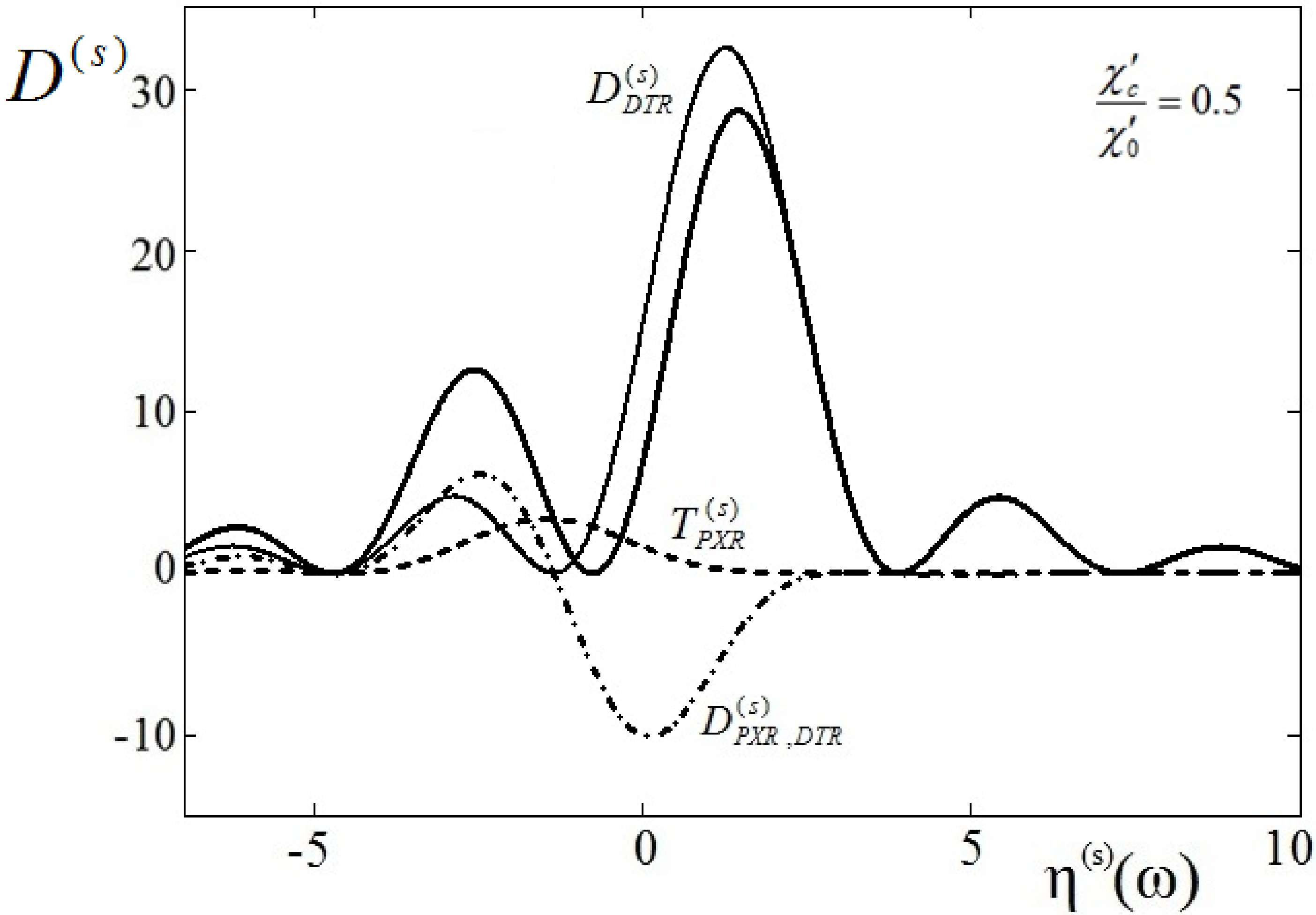


Fig.12. The contribution PXR , DTR and interference term to resulting spectrum in the condition of constructive interference. $\Omega=\Gamma=0.5$, $\chi_c'/\chi_0'=0.5$, $\varepsilon=3$, $\nu^{(s)}=0.8$, $B^{(s)}=3$, $a/b=1.676\cdot(2m+1)$, $c/b=0.838\cdot(2n+1)$.

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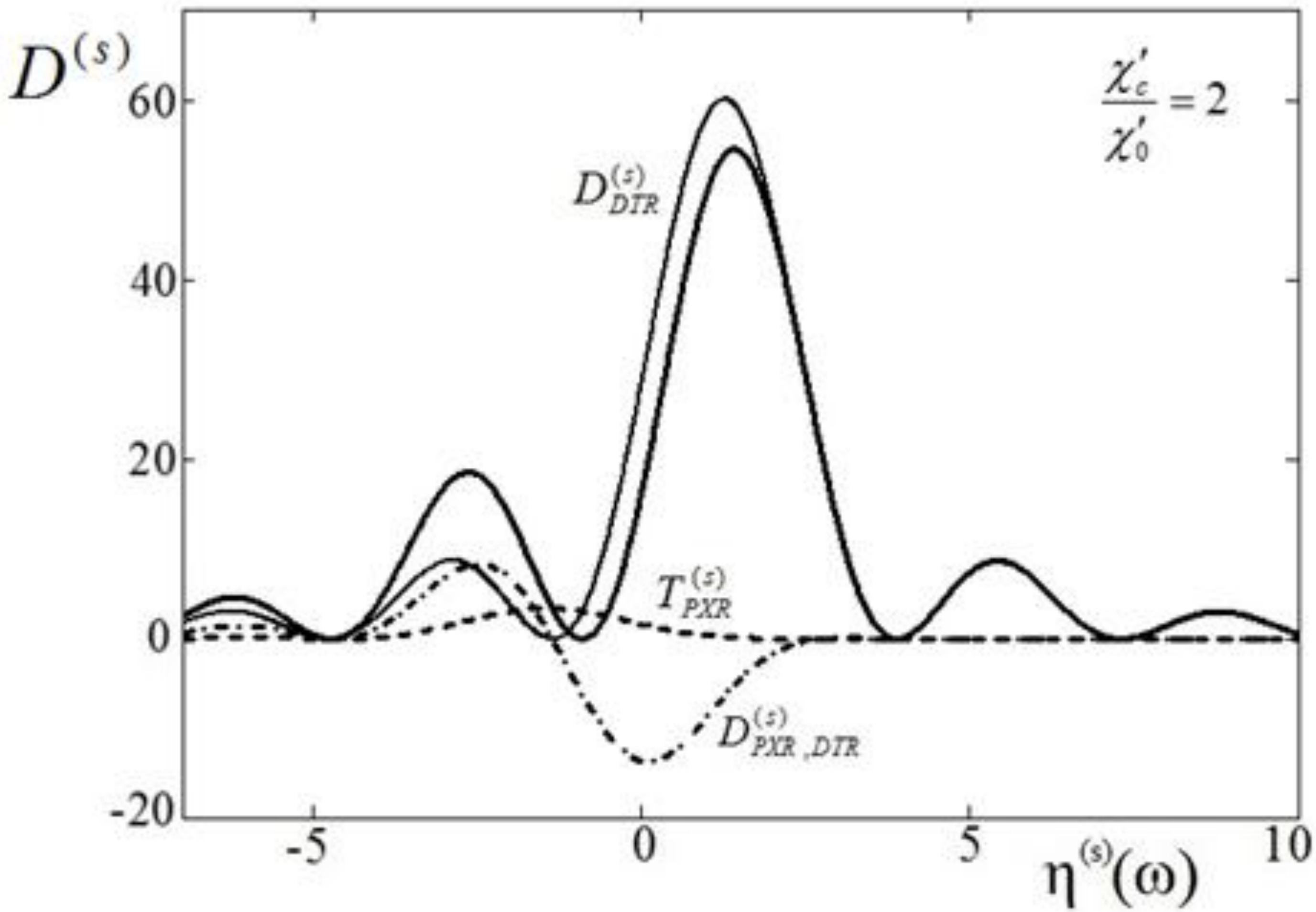

Fig.13. The same as in Fig. 12. , but $\chi'_c / \chi'_0 = 2$ and $c/b = 0.335 \cdot (2n+1)$

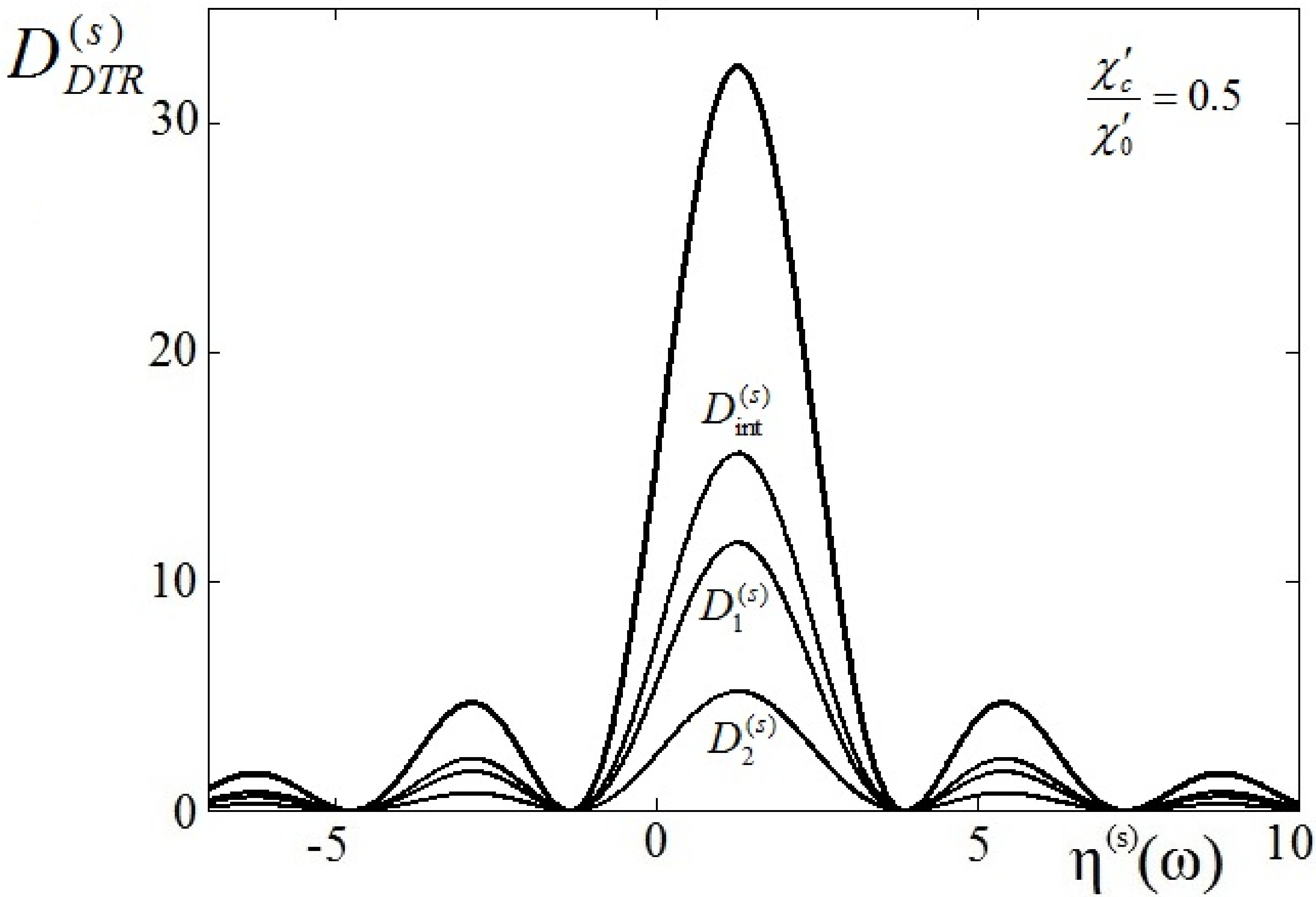

Fig. 14. The contribution of waves TR from amorphous layer ($D_1^{(s)}$) , from the entrance surface of crystalline layer ($D_2^{(s)}$) and interference term ($D_{\text{int}}^{(s)}$) to resulting spectrum of DTR ($D_{DTR}^{(s)}$) in the condition of constructive interference. All the parameters are the same as in **Fig. 12**

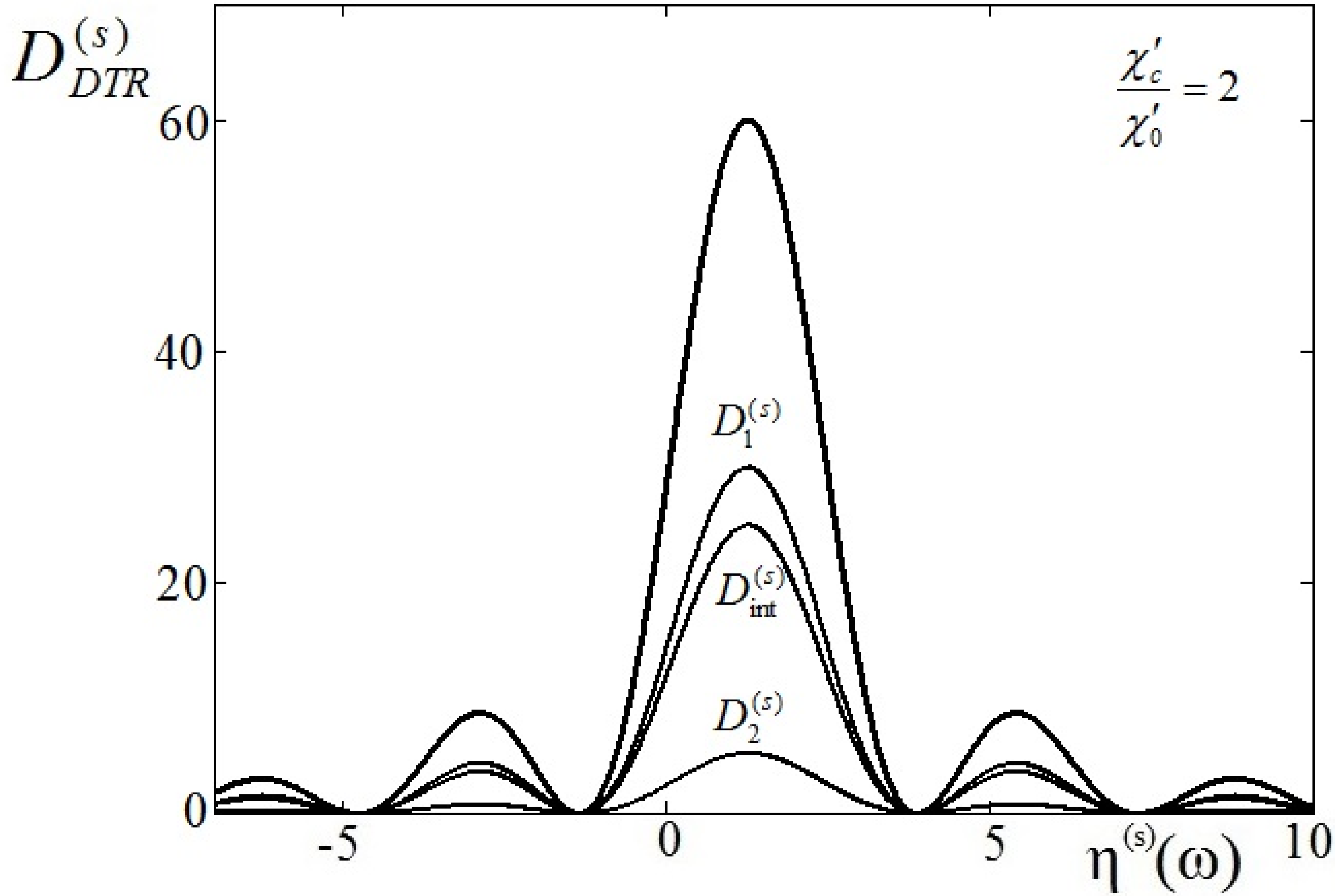

Рис. 15. Contribution of TR waves ($D_1^{(s)}$, $D_2^{(s)}$) and the interference term ($D_{int}^{(s)}$) to resulting spectrum of DTR ($D_{DTR}^{(s)}$). All the parameters are the same as in Fig. 13.

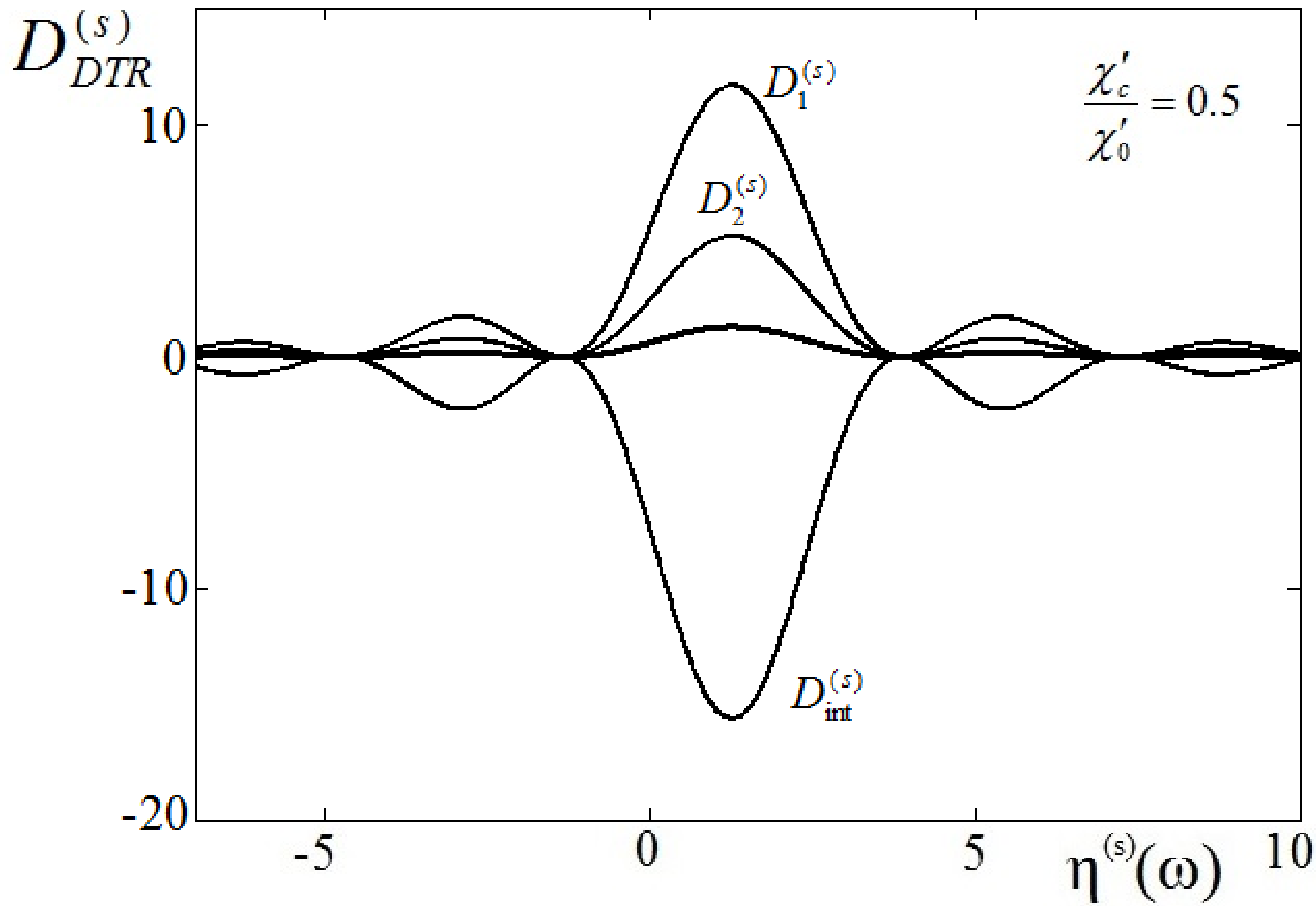

Fig. 16. Contribution of TR waves ($D_1^{(s)}$, $D_2^{(s)}$) and interference term ($D_{int}^{(s)}$) to resulting spectrum of DTR ($D_{DTR}^{(s)}$). All the parameters are the same as in Fig. 13., but $a/b = 3.352 \cdot (2m+1)$, - the curves are built in the condition of destructive interference.

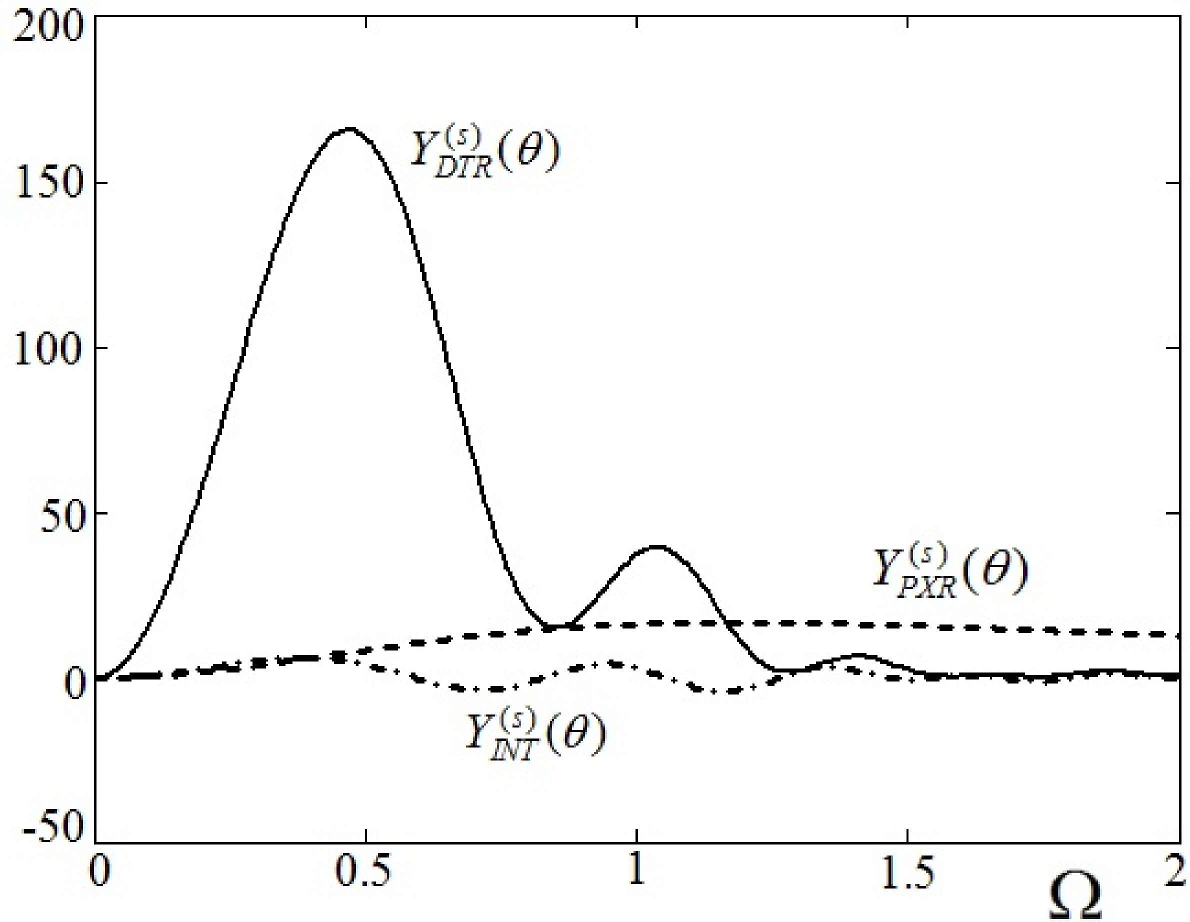

Fig. 17. Angular density of PXR, DTR and their interference term . The curves are built for such values of parameters: $\Gamma = 0.5$, $\chi'_c / \chi'_0 = 2$, $B^{(s)} = 3$, $\varepsilon = 3$, $\nu^{(s)} = 0.8$, $a/b = 1.676 \cdot (2m+1)$, $c/b = 0.335 \cdot (2n+1)$, which correspond to the conditions of constructed interference

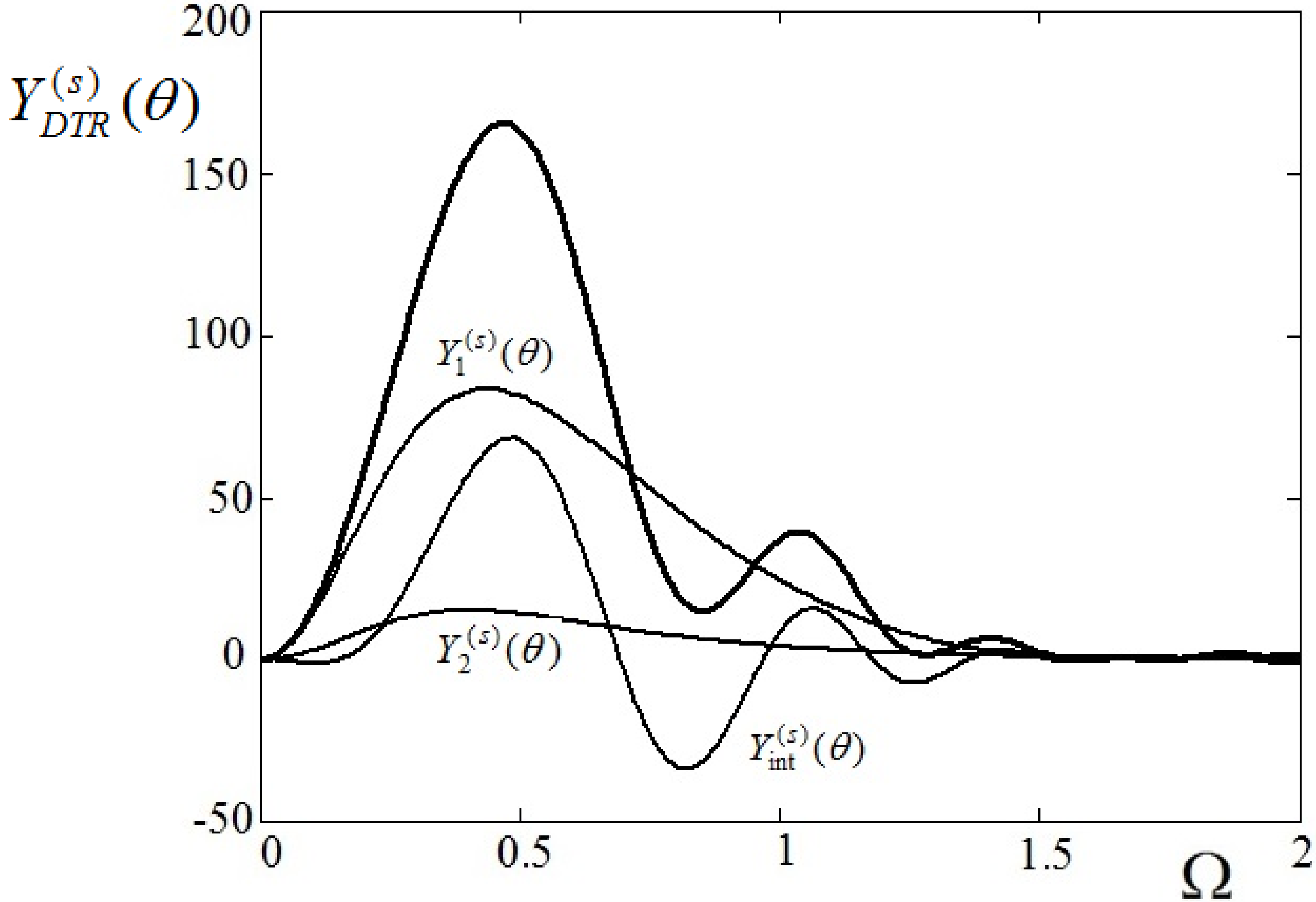

Fig. 18. Contribution of TR from amorphous layer $Y_1^{(s)}$ and the entrance surface of the crystal $Y_2^{(s)}$ and interference term $Y_{\text{int}}^{(s)}$ to resulting angular density of DTR $Y_{DTR}^{(s)}$. The parameters are the same as in Fig.17.

Conclusions

The expressions obtained in the present work allow optimizing parameters of three-layer radiator consisting of two amorphous and one monocrystalline layers. The use of such a radiator can give the increase of the radiation yield almost one order in comparison with the use of a monocrystalline plate. It opens the perspective of creation intensive source of X-ray radiation on the base of diffracted transition radiation of relativistic electron.

Thank you for your attention