IONIZATION EFFECT FROM ULTRA RELATIVISTIC ELECTRON-POSITRON PAIR IN THIN PLATE

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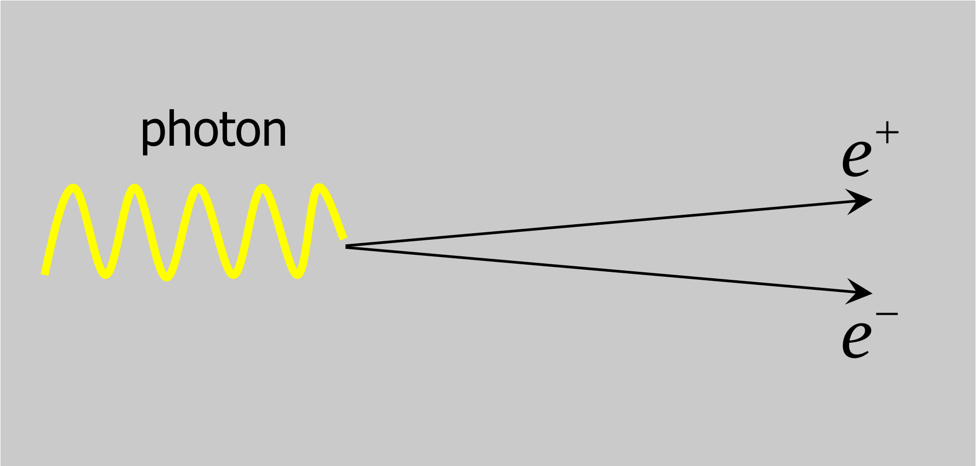
S.V. Trofymenko, N.F. Shul'ga // Phys. Lett. A. (2013)
N.F. Shul'ga, S.V. Trofymenko // Phys. Lett. A. (2014)

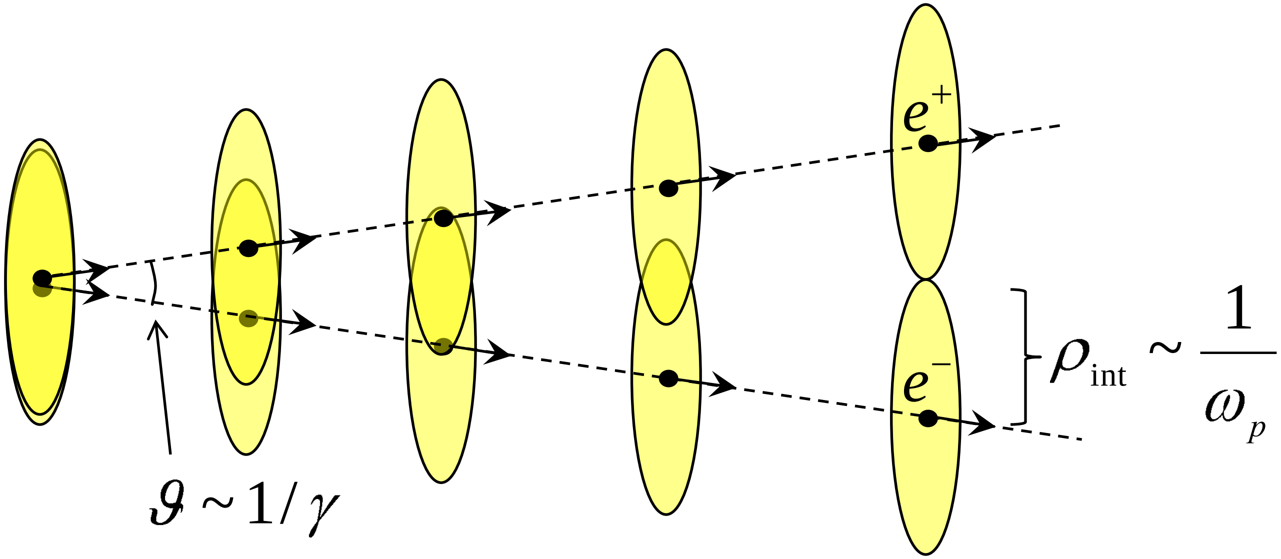
CHUDAKOV EFFECT
(ionization loss in boundless medium)



- *Chudakov A.E. // Izv. AS USSR, 1955*
- *Perkins D. // Phil.Mag., 1955*

Interference length:

$$L_{\text{int}} \approx \frac{\rho_{\text{int}}}{\vartheta} \sim \frac{\gamma}{\omega_p}$$



$$\rho_{\text{int}} \sim \frac{1}{\omega_p}$$

$$\vartheta \sim 1/\gamma$$

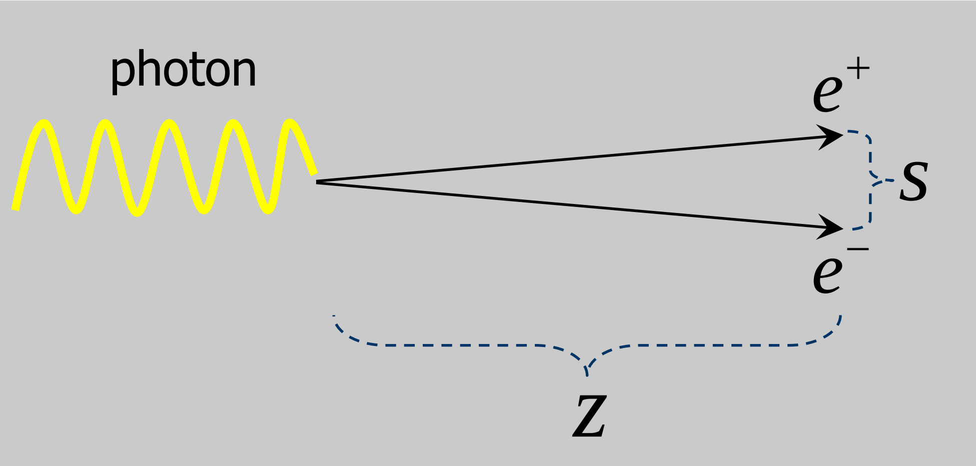
For $\varepsilon = 100\, GeV$

$$L_{\text{int}} \sim 1mm$$

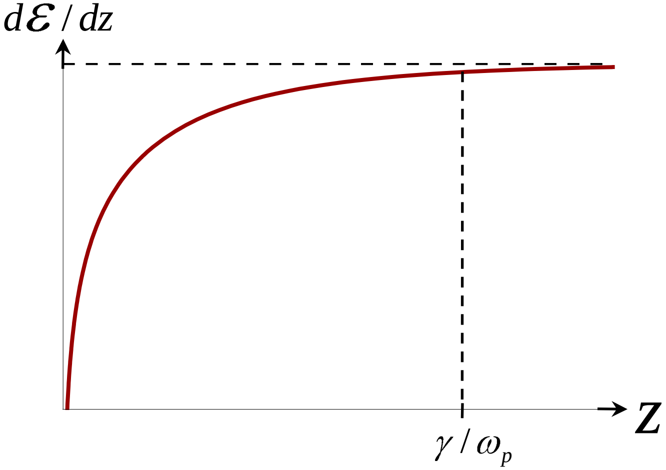
γ – Lorenz-factor of each particle

ω_p – plasma frequency of substance

CHUDAKOV EFFECT
(ionization loss in boundless medium)



Dependence of pair ionization loss on distance from its creation point



For $z<\gamma/\omega_p$ (which corresponds to $s<1/\omega_p$):

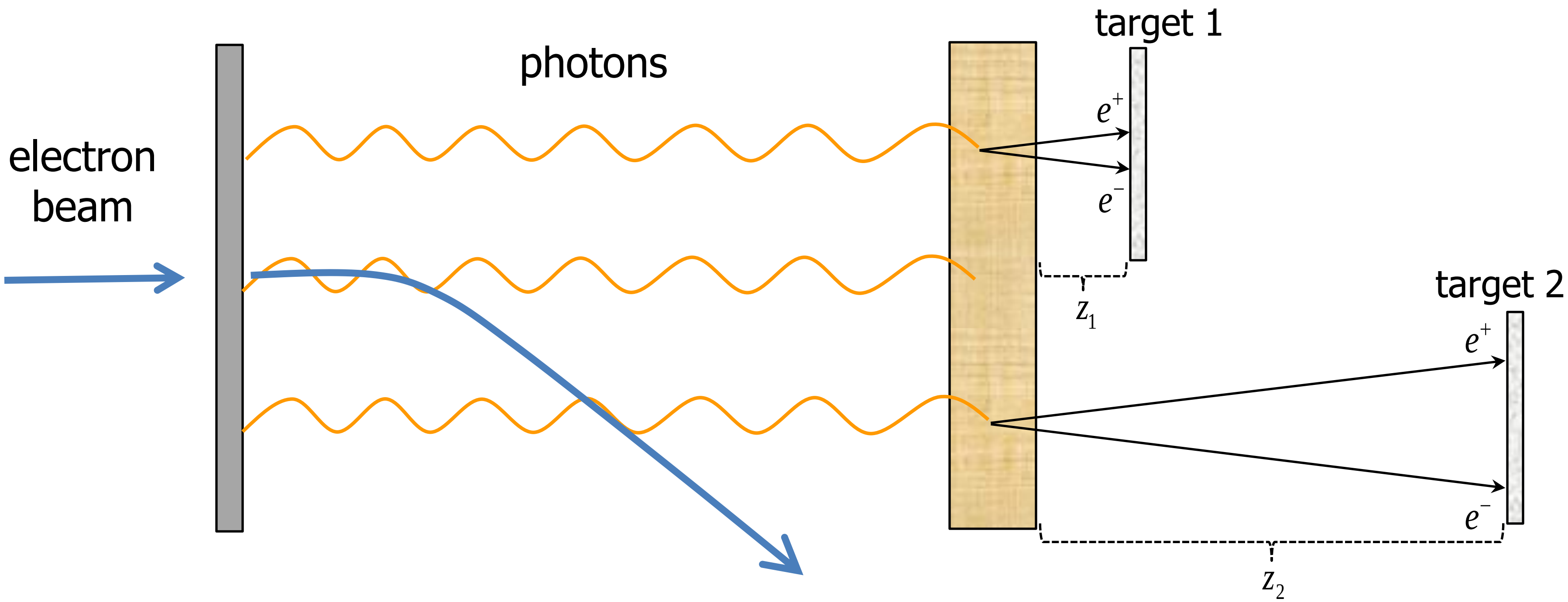
strong suppression of $d\mathcal{E}/dz$

- *Berestetskii V.B., Geshkenbain B.V. // JETP, 1956*
- *Yekutieli G. // Nuovo Cim., 1957*
- *Mito I., Ezawa H. // Progr. Theor. Phys., 1957*
- *Burkhardt G.H. // Nuovo Cim., 1958*

CERN (SPS) NA63 EXPERIMENT

T. Virkus, H.D. Thomsen, E. Uggerhøj et al. // Phys. Rev. Lett., 2008

H. D. Thomsen, U. I. Uggerhøj // Nucl. Instrum. Meth. B., 2011



The ratio of pair ionization losses in two plates $\sigma = \Delta\mathcal{E}_1 / \Delta\mathcal{E}_2$ as a function of the pair energy $\mathcal{E}$ was measured in the range $1\,GeV < \mathcal{E} < 100\,GeV$

For $L_{\text{int}} > z_1$ and $L_{\text{int}} > z_2$ $\longrightarrow$ $\sigma < 1$

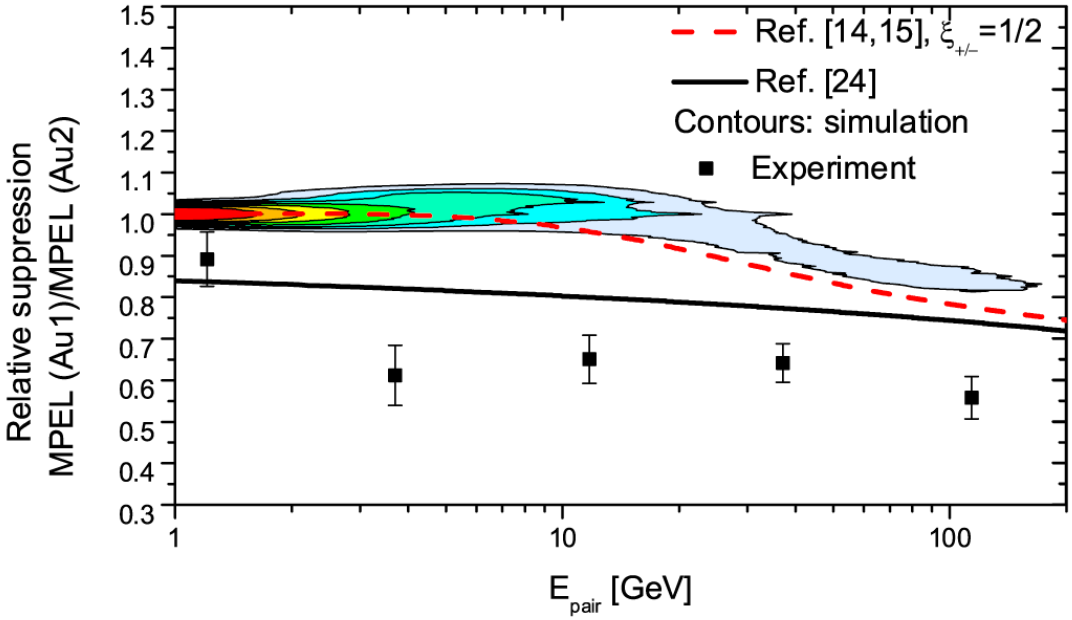
For $L_{\text{int}} << z_1$ and $L_{\text{int}} << z_2$ $\longrightarrow$ $\sigma = 1$

CERN (SPS) NA63 EXPERIMENT

T. Virkus, H.D. Thomsen, E. Uggerhøj et al. // Phys. Rev. Lett., 2008

H. D. Thomsen, U. I. Uggerhøj // Nucl. Instrum. Meth. B., 2011

$\Delta\mathcal{E}_1 / \Delta\mathcal{E}_2$ as a function of the pair energy $\mathcal{E}$



Ref.[14]: V.B. Berestetskii, B.V. Geshkenbain // JETP, 1956

Ref.[15]: P. Sigmund // Particle Penetration and Radiation Effects, 2006

Ref.[24]: G.H. Burkhardt // Nuovo Cim., 1958

PAIR FIELD EVOLUTION IN VACUUM

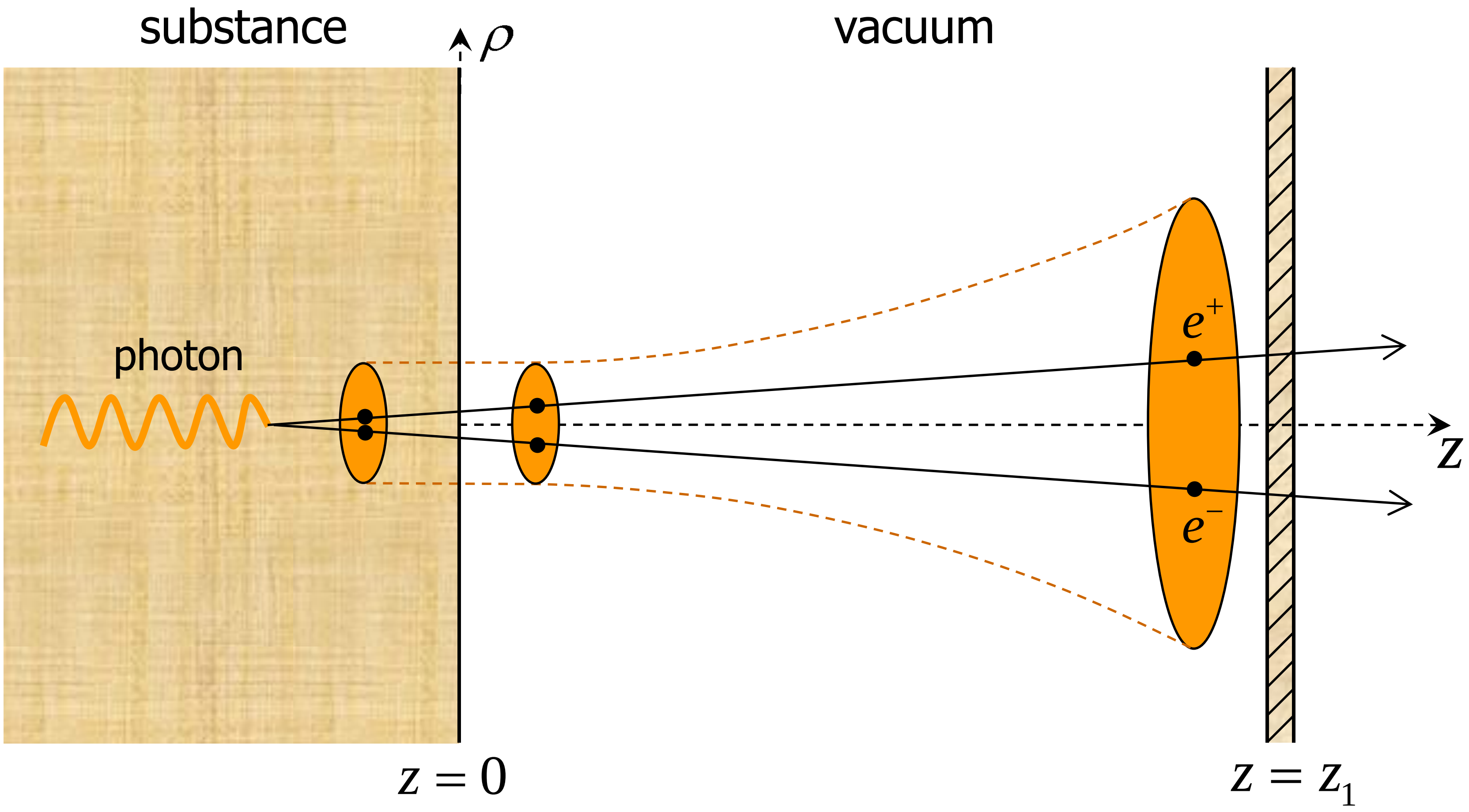
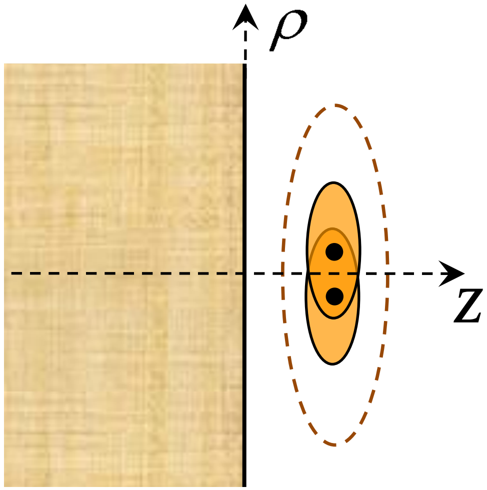


Plate thickness $a \leq I / \eta_p^2$

η_p – plasma frequency of the plate

I – mean ionization potential

PAIR FIELD EVOLUTION IN VACUUM



Total field around the pair in vacuum:

$$\vec{E} = \vec{E}_C^{+} + \vec{E}_C^{-} + \vec{E}^{F}$$

Total electric field Fourier-component:

$$\vec{E}_{\omega\perp}(\vec{r}) = -\frac{ie}{\pi v}\int d^2q\,\vec{q}\left\{\ Q_c(q)e^{i\omega z/v} + Q_f(q)e^{i\omega z(1-q^2/2\omega^2)}\ \right\} e^{i\vec{q}\vec{\rho}}(1-e^{-i\vec{q}\vec{s}})$$

where: $Q_f(q) = \frac{1}{q^2+\omega_p^2+\omega^2/v^2\gamma^2} - \frac{1}{q^2+\omega^2/v^2\gamma^2}$ and $Q_c(q) = \frac{1}{q^2+\omega^2/v^2\gamma^2}$

PAIR IONIZATION LOSS IN PLATE

(the plate is situated on distance z_1 from the substance)

$$\frac{d\mathcal{E}}{dz} = 2\eta_p^2 e^2 \left\{ \underbrace{\ln\frac{q_0 v\gamma}{I} - \frac{1}{2}}_{\text{ionization by particles' own coulomb fields}} + \underbrace{\ln\frac{\omega_p v\gamma}{I} - 1}_{\text{ionization by transition radiation}} + \underbrace{F(z_1)}_{\text{influence on ionization by interference of the particles' fields with each other and with transition radiation field}} \right\}$$

where:

$$F(z_1) = \lambda_\gamma Si\lambda_\gamma + Ci\lambda_\gamma + \cos\lambda_\gamma - \cos\lambda_p Ci(\lambda_p + \lambda_\gamma) - \sin\lambda_p Si(\lambda_p + \lambda_\gamma) -$$

$$-2K_0\left(2\frac{Iz_1}{\gamma^2}\right) + 2\frac{Iz_1}{\gamma^2}K_1\left(2\frac{Iz_1}{\gamma^2}\right) + K_0\left(2\frac{\omega_p z_1}{\gamma}\right) + \frac{\omega_p z_1}{\gamma}K_1\left(2\frac{\omega_p z_1}{\gamma}\right) +$$

$$+2\omega_p^2\int_0^\infty dq q^3 \frac{J_0(2qz_1/\gamma)\cos(\lambda_\gamma + q^2\lambda_p/\omega_p^2)}{(q^2+\omega_p^2+\omega^2/\gamma^2)(q^2+\omega^2/\gamma^2)^2}$$

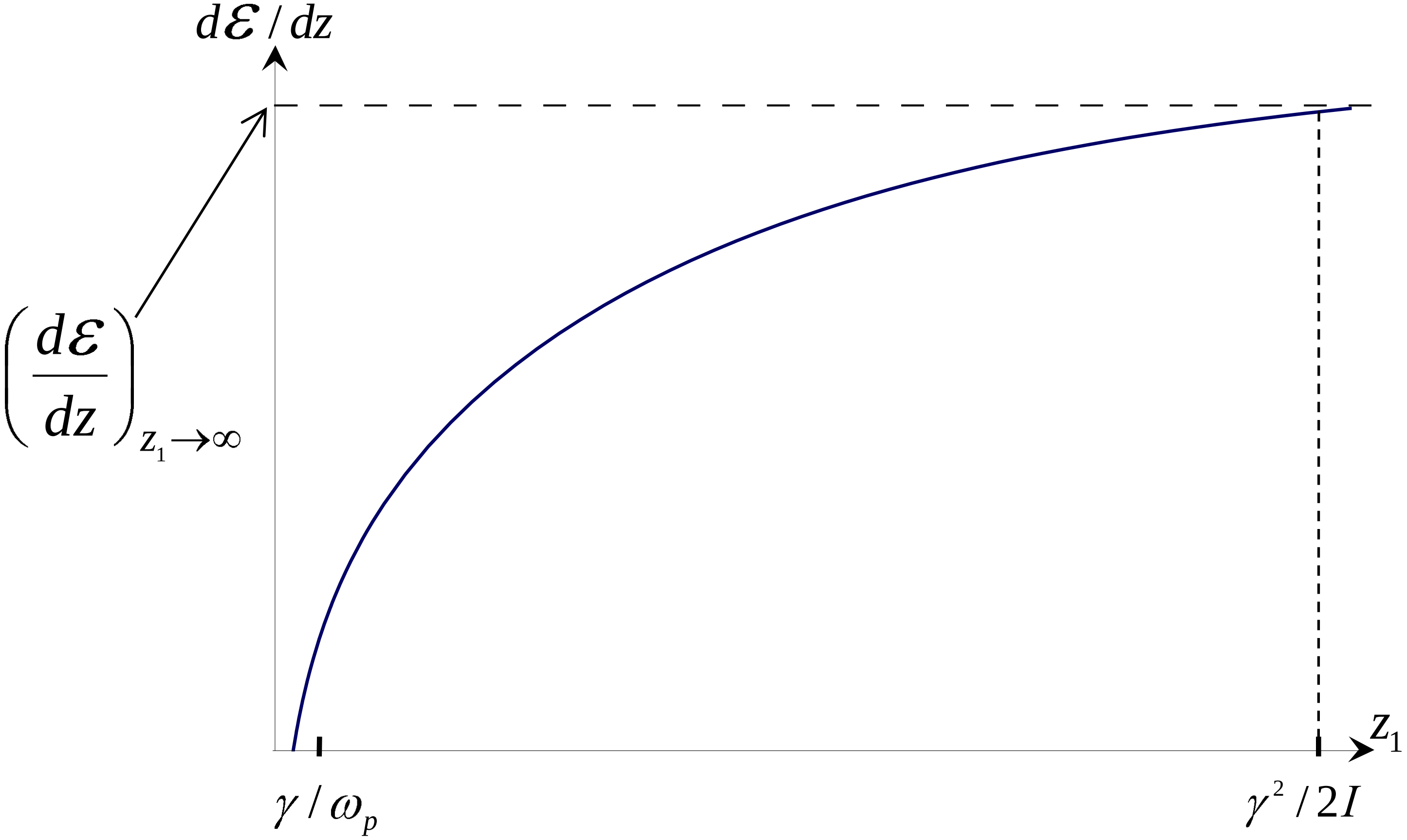
$$\lambda_\gamma = \frac{Iz_1}{2\gamma^2}$$

$$\lambda_p = \frac{\omega_p^2 z_1}{2I}$$

$K_i(x)$ – Macdonald function

PAIR IONIZATION LOSS IN PLATE

(the plate is situated on distance z_1 from the substance)



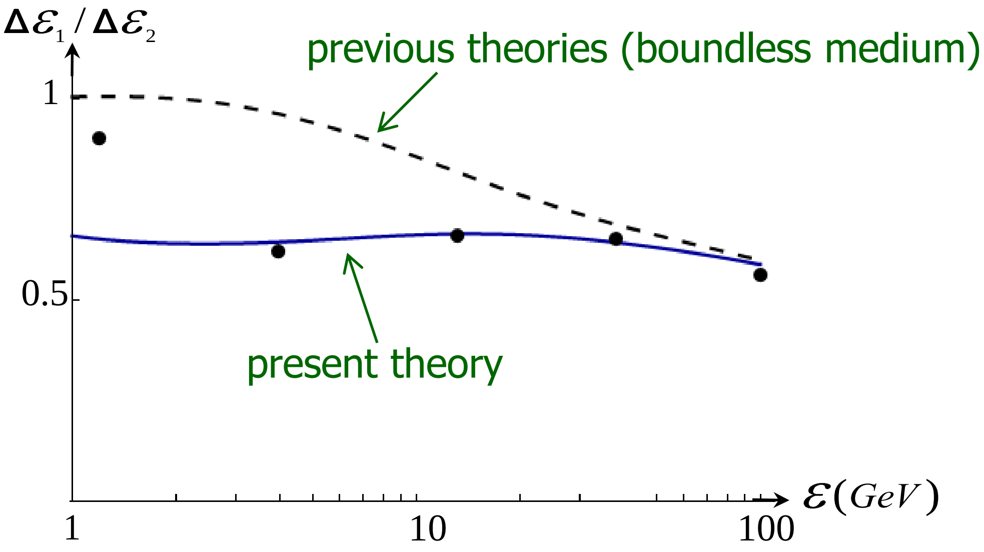
For:

$\mathcal{E} = 100\,GeV$

$\frac{\gamma^2}{I} \sim 10\,m$**!**

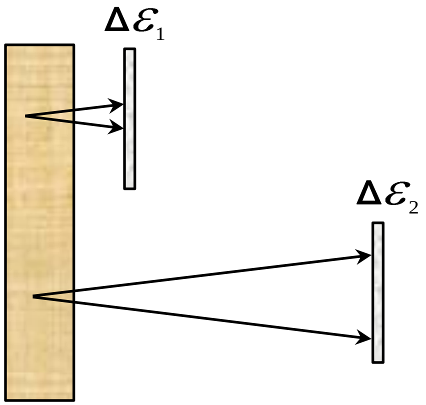
Interference effects exist on distances $z_1 \sim \gamma^2 / I$, which are much larger than the corresponding distances $z_1 \sim \gamma / \omega_p$ in the case of the pair motion in boundless medium

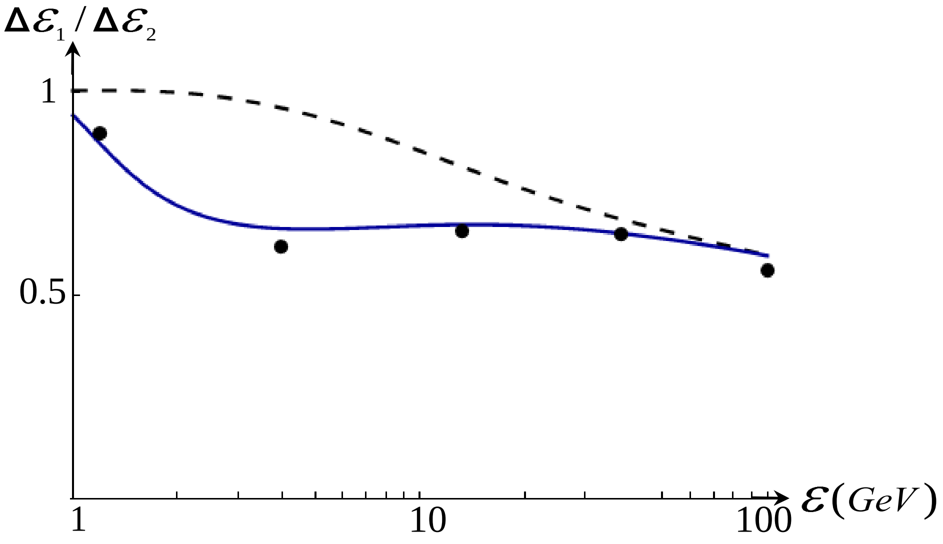
RATIO OF PAIR IONIZATION LOSSES IN TWO PLATES (as a function of pair energy)



total ionization loss

$I \sim 100\, eV$ (mean ionization potential)



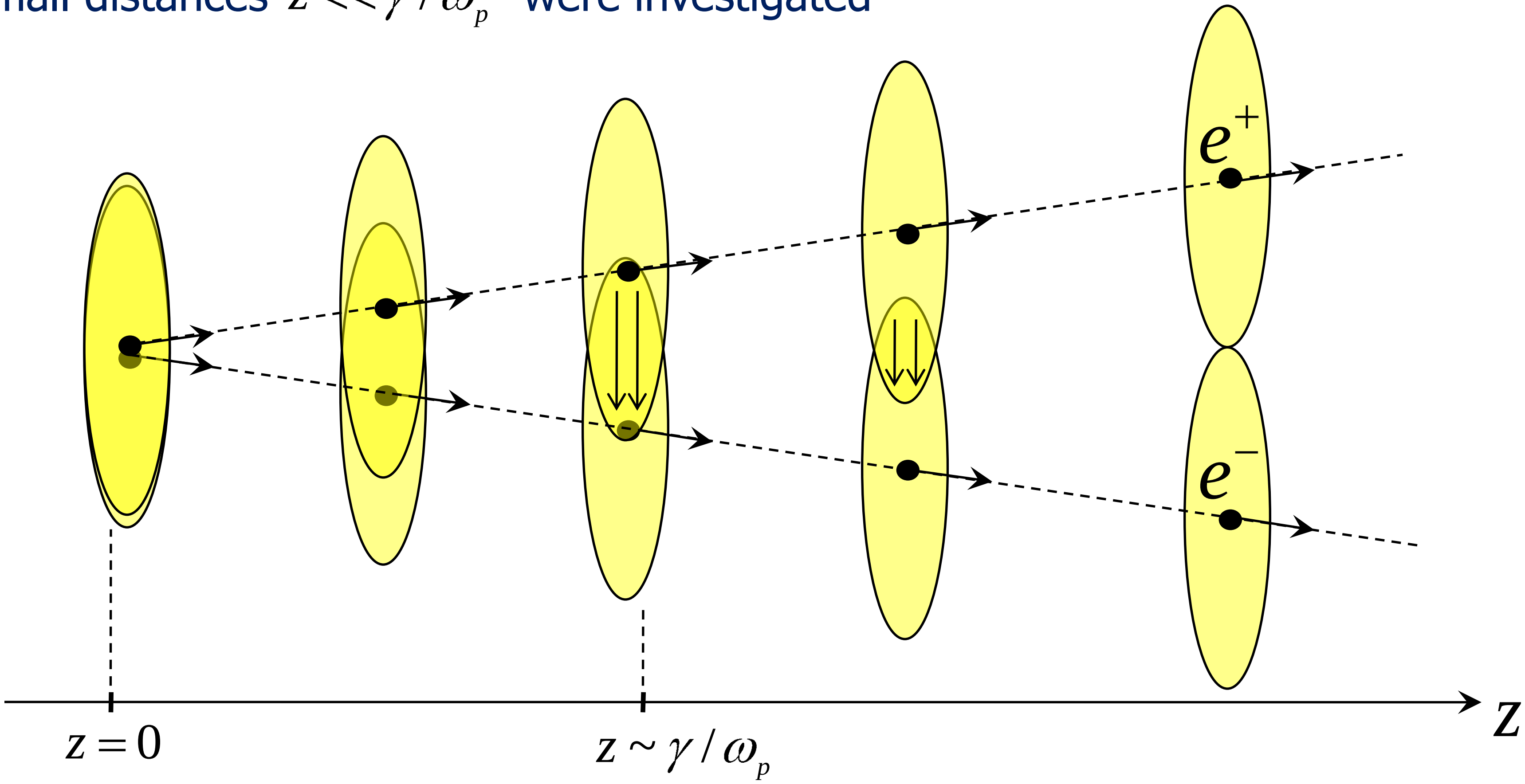


loss due to inner-shell (K-shell) excitation

$I_{in} \sim 2000\, eV$ (inner-shell ionization potential)

ANTI-CHUDAKOV EFFECT

Previously either total pair energy loss (ionization+cherenkov) on arbitrary distances from the pair creation point $0 < z < \infty$, or just ionization loss on small distances $z << \gamma / \omega_p$ were investigated



It is natural to expect increase of ionization loss at $z \sim \gamma / \omega_p$

PAIR IONIZATION LOSS IN BOUNDLESS MEDIUM

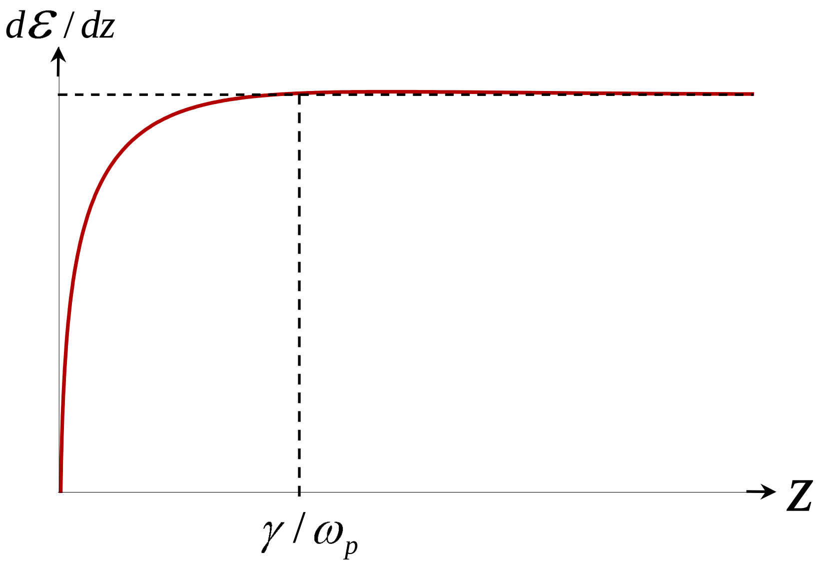
Total ionization per unit path:

$$\frac{d\mathcal{E}}{dz} = 2\omega_p^2 e^2 \left\{ \ln\frac{q_0}{\omega_p} - \frac{1}{2} - K_0(\lambda) + \frac{\lambda}{2} K_1(\lambda) \right\}$$

where $\lambda = 2\omega_p z / \gamma$

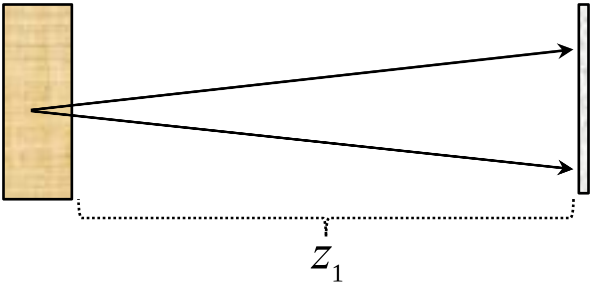
Pair divergence angle $\vartheta = 2/\gamma$

$K_i(x)$ – Macdonald function



PAIR IONIZATION LOSS IN PLATE
(from slide 9)

$$\frac{d\mathcal{E}}{dz} = 2\eta_p^2 e^2 \left\{ \ln\frac{q_0 v\gamma}{I} - \frac{1}{2} + \ln\frac{\omega_p v\gamma}{I} - 1 + F(z_1) \right\}$$



$$F(z_1) = (d\mathcal{E}/dz)_{z=z_1} - (d\mathcal{E}/dz)_{z\to\infty}$$

where:

$$F(z_1) = \lambda_\gamma Si\lambda_\gamma + Ci\lambda_\gamma + \cos\lambda_\gamma - \cos\lambda_p Ci(\lambda_p + \lambda_\gamma) - \sin\lambda_p Si(\lambda_p + \lambda_\gamma) -$$

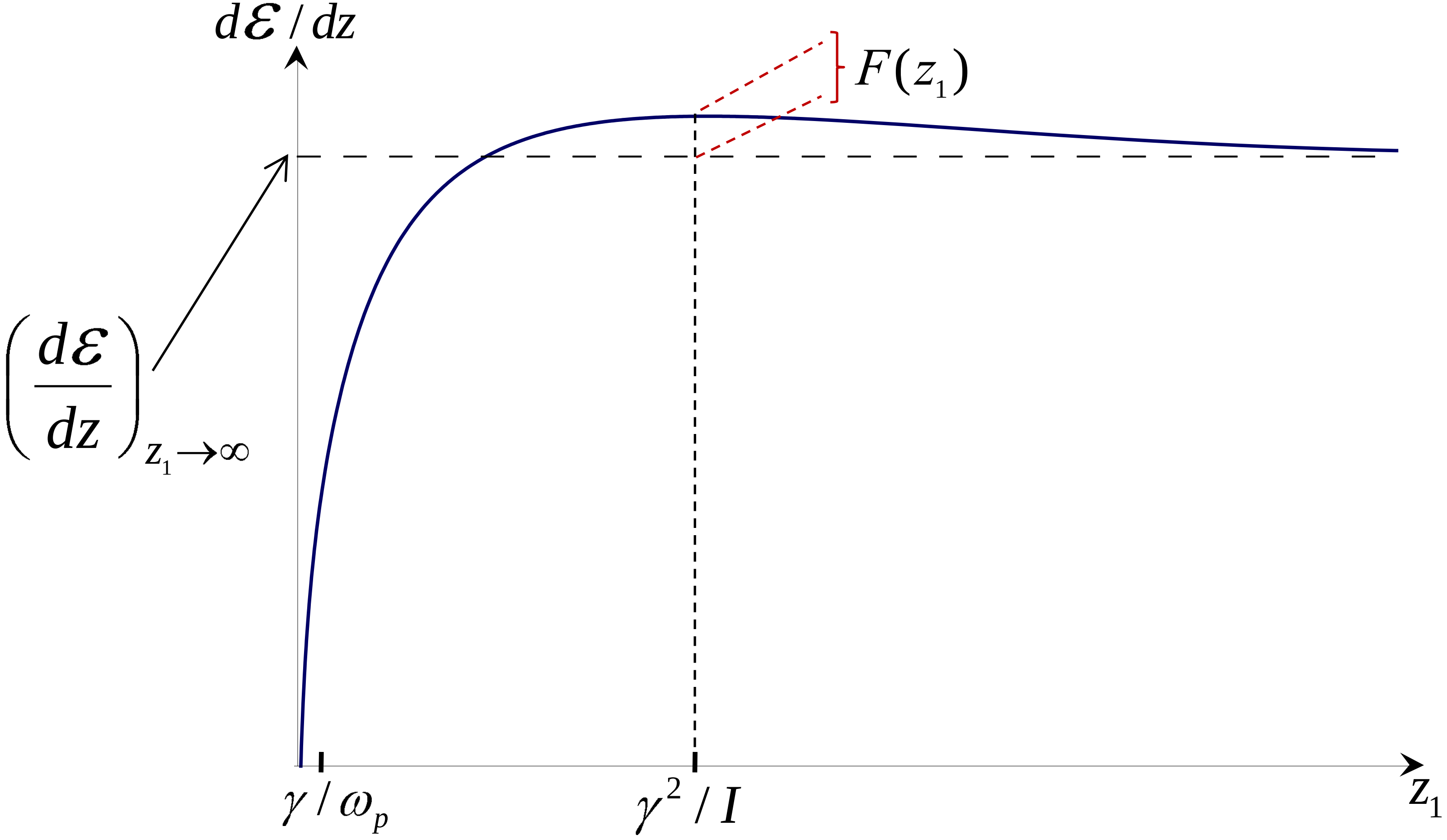
$$-2K_0\left(2\frac{Iz_1}{\gamma^2}\right) + 2\frac{Iz_1}{\gamma^2} K_1\left(2\frac{Iz_1}{\gamma^2}\right) + K_0\left(2\frac{\omega_p z_1}{\gamma}\right) + \frac{\omega_p z_1}{\gamma} K_1\left(2\frac{\omega_p z_1}{\gamma}\right) +$$

$$+2\omega_p^2 \int_0^\infty dq q^3 \frac{J_0(2qz_1/\gamma)\cos(\lambda_\gamma + q^2\lambda_p/\omega_p^2)}{(q^2 + \omega_p^2 + \omega^2/\gamma^2)(q^2 + \omega^2/\gamma^2)^2}$$

$$\lambda_\gamma = \frac{Iz_1}{2\gamma^2}$$

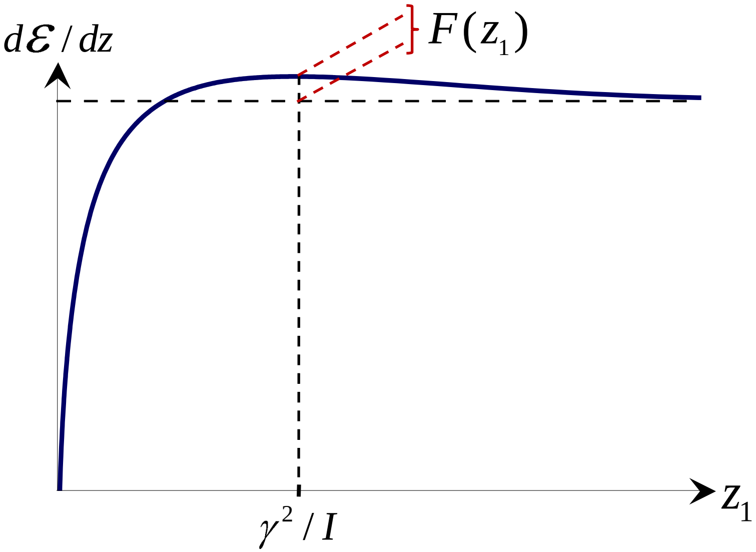
$$\lambda_p = \frac{\omega_p^2 z_1}{2I}$$

PAIR IONIZATION LOSS IN PLATE



The effect is most significant for $z_1 \approx \gamma^2/I$

ANTI-CHUDAKOV EFFECT



For $z_1 \sim \gamma^2/I$ **:**

$$F(z_1) = xSi(x) + Ci(x) + \cos x -$$

$$-2[K_0(4x) - 2xK_1(4x)] + \frac{4}{25}\frac{\sin 3x}{x}$$

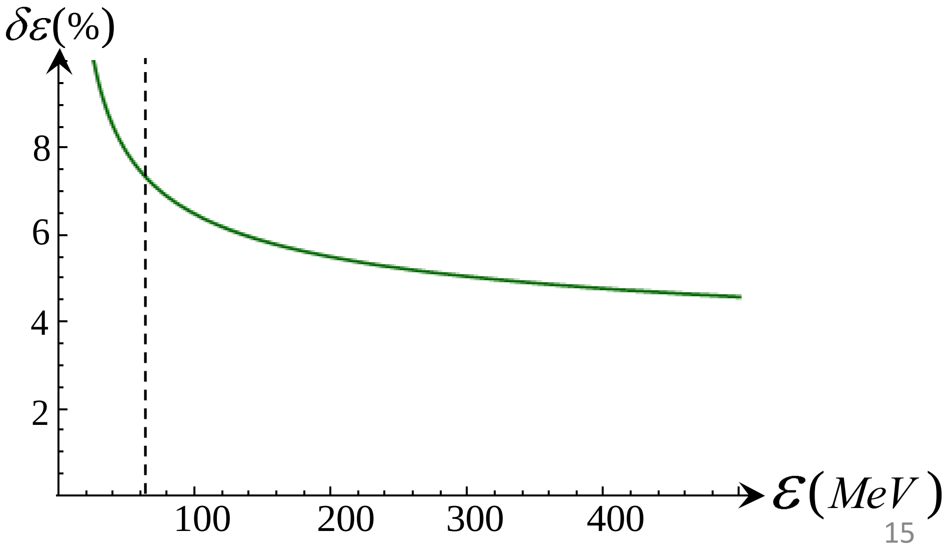
where $x = Iz_1/2\gamma^2$

Relative excess of ionization loss ($\gamma >> I/\omega_p$):

$$\delta\varepsilon = \frac{F(\gamma^2/I)}{(d\mathcal{E}/dz)_{z\to\infty}} = \frac{\alpha}{2\ln\gamma + \beta}$$

where: $\alpha \approx 1/2$

$$\beta \approx \ln(q_0\omega_p/I^2) - 3/2$$



CONCLUSIONS

- In thin targets interference effects in electron-positron pair ionization loss should be manifested **on much larger distances** from the pair creation point than in homogeneous infinite substance

- Together with Chudakov effect of electron-positron pair ionization loss reduction there **exists the opposite effect** (anti-Chudakov effect) of exceeding by the pair loss of the sum of single electron and positron losses.

- **In thin plates** anti-Chudakov effect is much more significant than in homogeneous infinite substance