Channeling 2014
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QED aspects in Radiation by Relativistic Electrons in Matter or in External Fields

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Dedicated to V.N. Baier, V.M. Strakhovenko and M. Kumakhov

Main topics

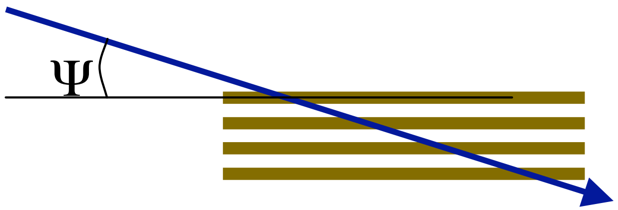
* I - Soft photon emission (ω << E) : coherence lenght, equivalent photons, crystal-assisted radiation

- II - QED in strong field : hard photon emission, Baier-Katkov formula, pair creation
- III - Particular aspects of radiation : tunnelling, impact parameter, electron side-slipping

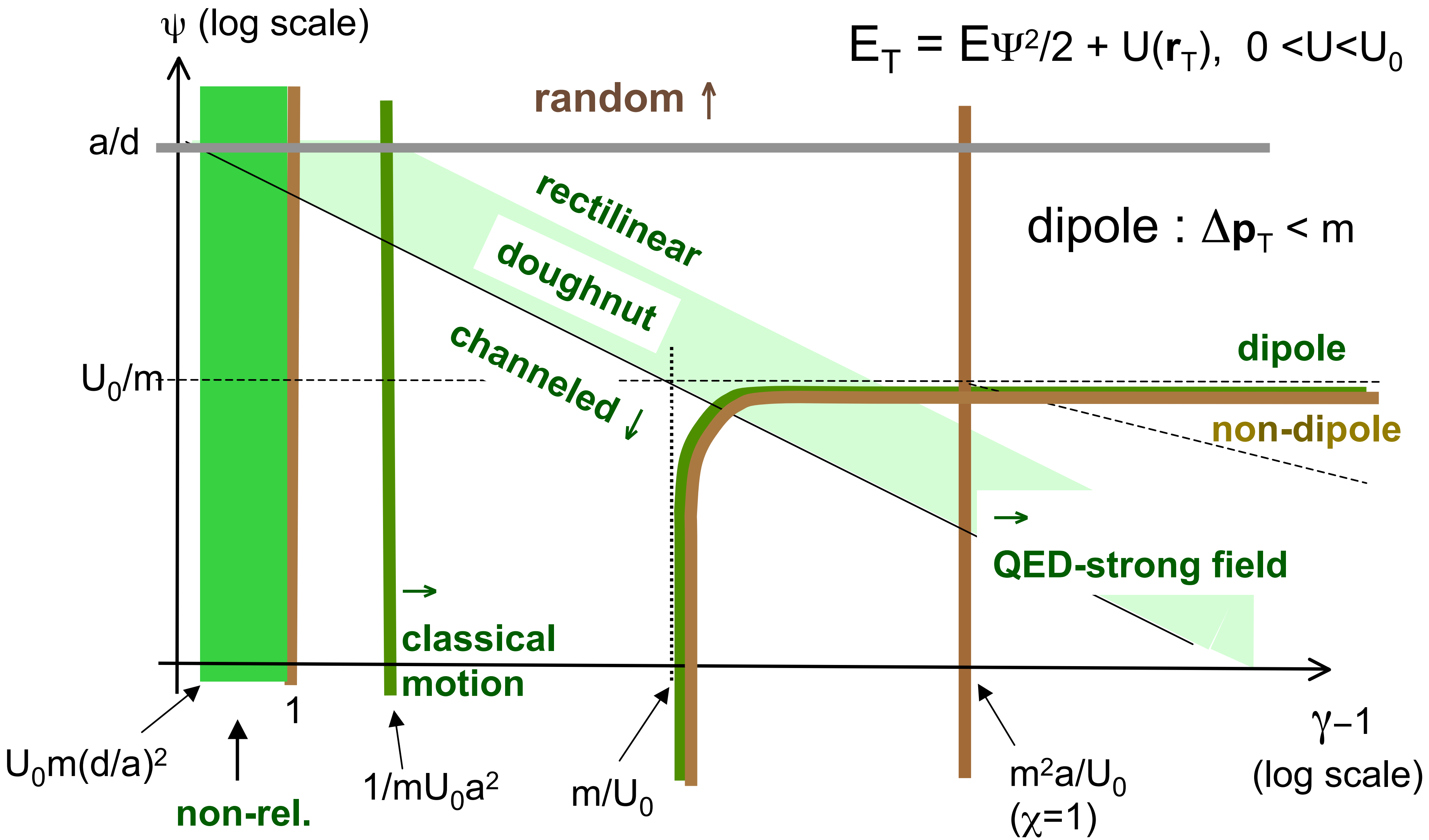
Natural units systems $\hbar = c = 1$; $\alpha = 1/137$

m = 511 KeV $E/m \equiv \gamma = (1-v^2)^{-1/2}$ $(v \equiv \beta)$

Channeling regimes in the (γ,ψ) lanscape

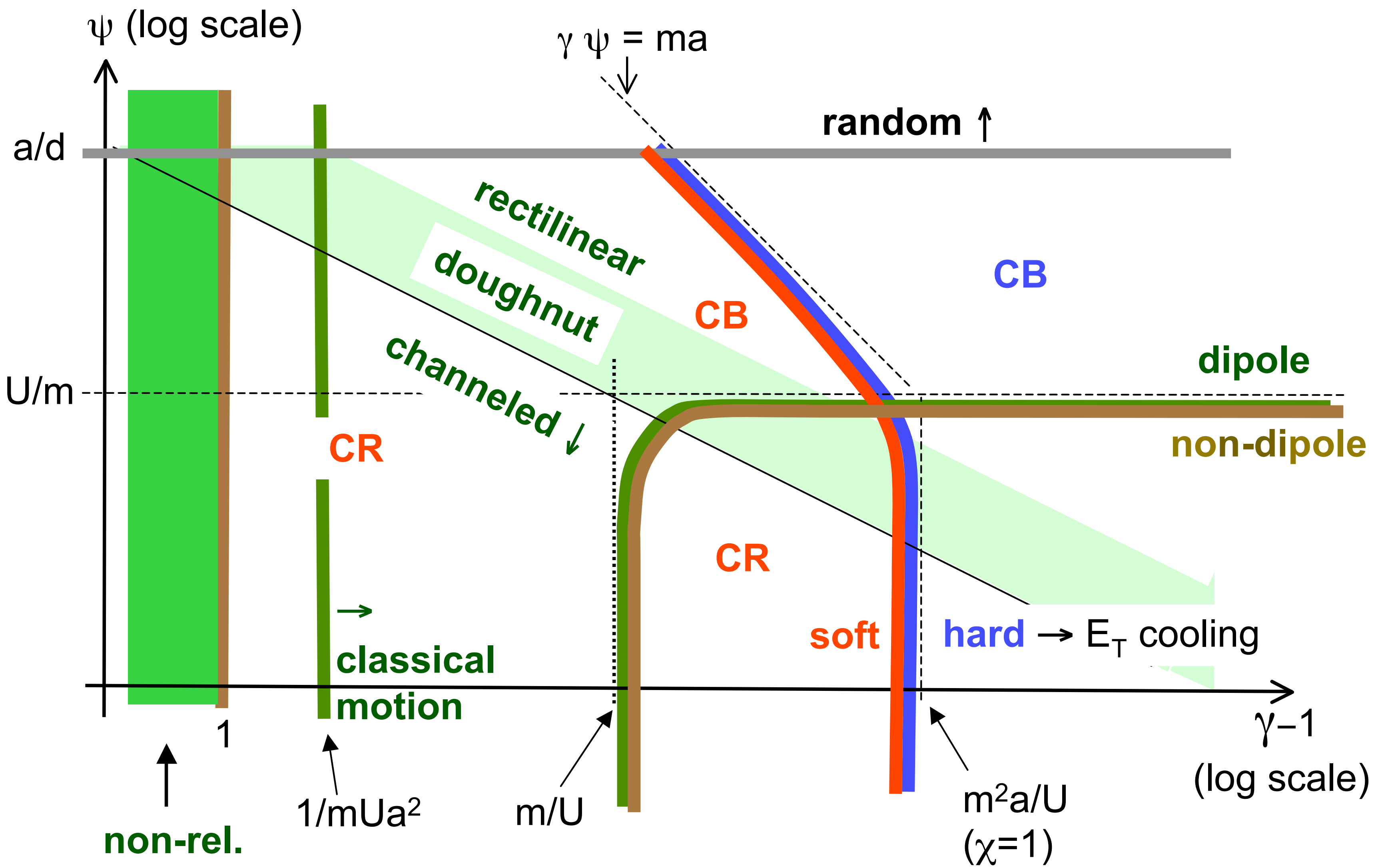


$E_T = E\Psi^2/2 + U(\mathbf{r}_T), \; 0 < U < U_0$



dipole : $\Delta\mathbf{p}_T < m$

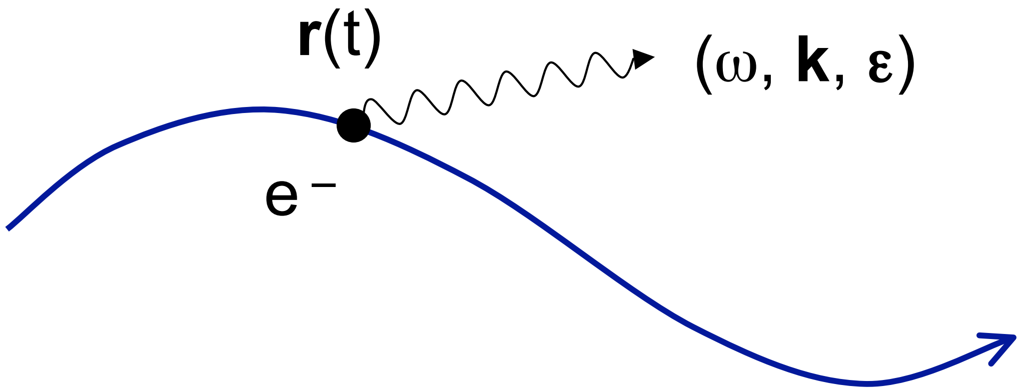
Radiation regimes



I - Soft photon emission
($\omega << E$)

- Classical radiation formula
- Coherence length
- Infrared contributions
- Equivalent photons ; applications
- Semi-bare electron
- Shadowing

Classical radiation formula in vacuum (1/3)



applies to :

- Synchrotron radiation in **weak** uniform or *non-uniform* field, e.g. undulator radiation
- **Soft** Compton effect (Thompson regime)
- **Soft** coherent Bremsstrahlung (ω << E)
- Channeling Radiation (classical regime)

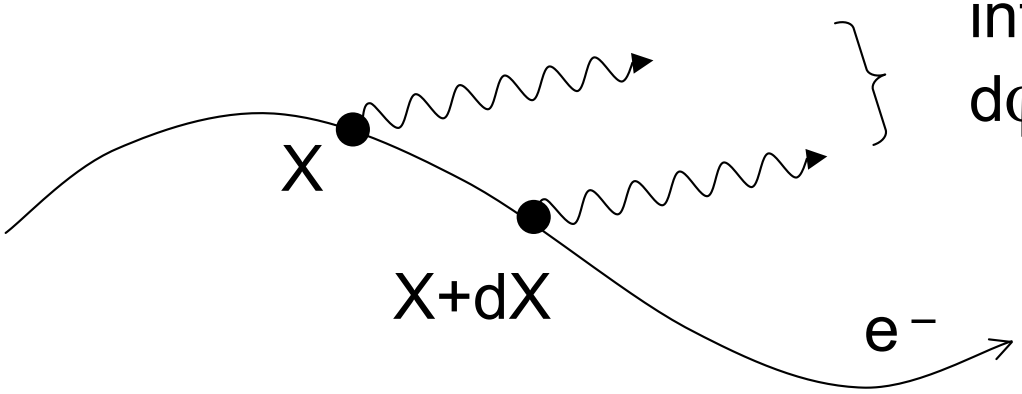
- **Spin-blind** and without **recoil effect**

Classical radiation formula (2/3) : **covariant form**

$$dN_{phot} = (\alpha/4\pi^2)\ \ d^3\mathbf{k}/\omega\ \ |A\cdot\boldsymbol{\varepsilon}^*|^2$$

$A = \int dX \exp(i\phi)$; $\phi = -\, k\cdot X$ = emission phase

4-vectors : $X = (t,\mathbf{r})$, $k = (\omega,\mathbf{k})$, $A = (A^0,\mathbf{A})$, $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}^0,\boldsymbol{\varepsilon})$



interfere, with the phase

$d\phi = -\, k\cdot dX\ = -\, k\cdot p\ dt/E$

Metric : $-+++$

Classical radiation formula (3/3) : **non-covariant** form

$dN = (\alpha\omega/4\pi^2)\, d\omega d\Omega\, |\mathbf{A}\cdot\boldsymbol{\varepsilon}^*|^2$ gauge $\boldsymbol{\varepsilon}^0=0$

$\mathbf{A} = \int \exp(i\phi)\, d\mathbf{r}_\perp$; $\phi = \omega\tau$

$\mathbf{r}_\perp$ ($\neq \mathbf{r}_T$) : perp. to $\mathbf{k}$; $\tau = t - \mathbf{n}\cdot\mathbf{r}$ = detector time ; $\mathbf{n} = \mathbf{k}/\omega$

Ultrarelativistic approximation $d\tau = (\gamma^{-2} + \mathbf{v}_\perp{}^2)\, dt\, /2$

$\mathbf{A} = \int d\tau \exp(i\omega\tau)\, d\mathbf{r}_\perp/d\tau$

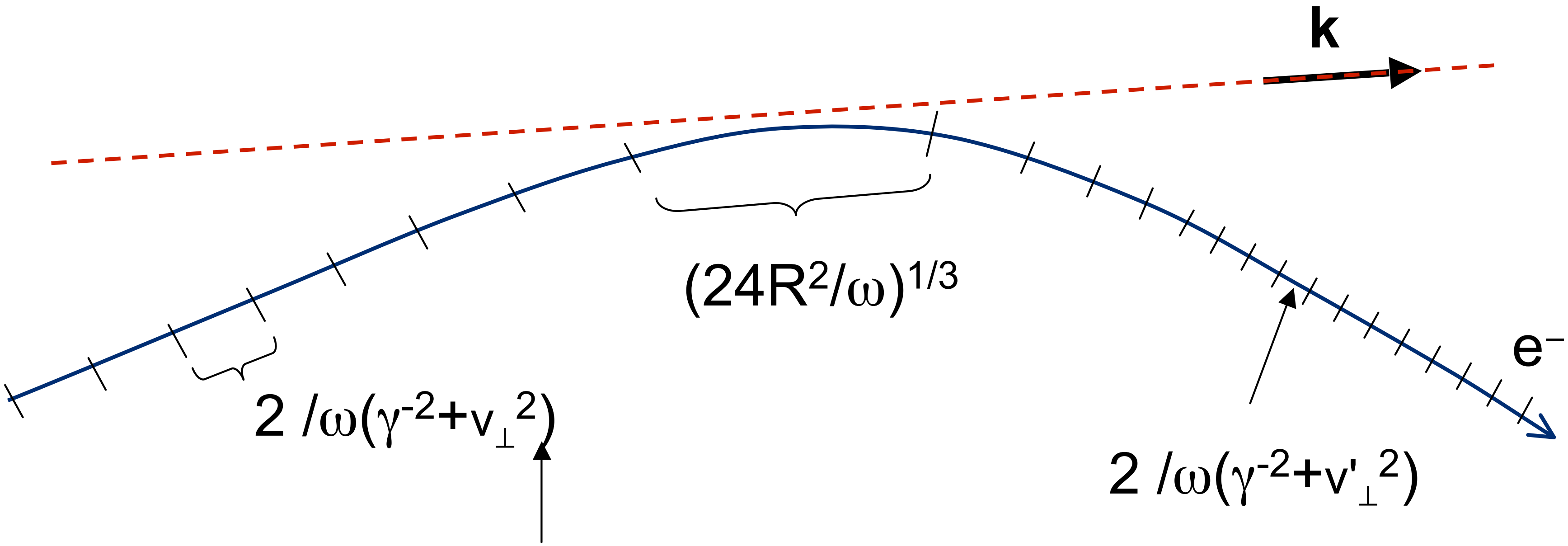
apparent perpendicular velocity

$= (i/\omega) \int d\tau \exp(i\omega\tau)\, d^2\mathbf{r}_\perp/d\tau^2$

apparent perp. acceleration

Coherence lengths

L_{coh} = distance over which ϕ changes by 1 radian



$(24R^2/\omega)^{1/3}$

$2\ /\omega(\gamma^{-2}+v_{\perp}^{2})$

$+\omega_P^2/\omega^2$

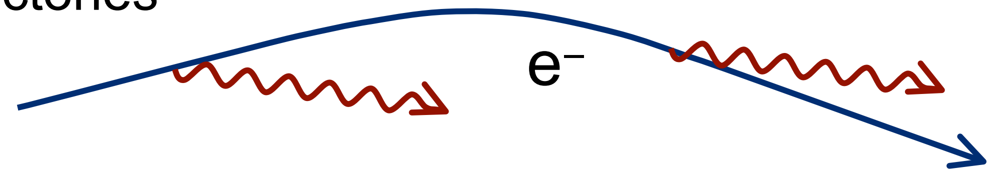
for density effect

$2\ /\omega(\gamma^{-2}+v'^{2}_{\perp})$

$v_{\perp} \sim \theta_{\text{electron-photon}}$

Infrared-divergent contributions

For **straight** asymptotic trajectories



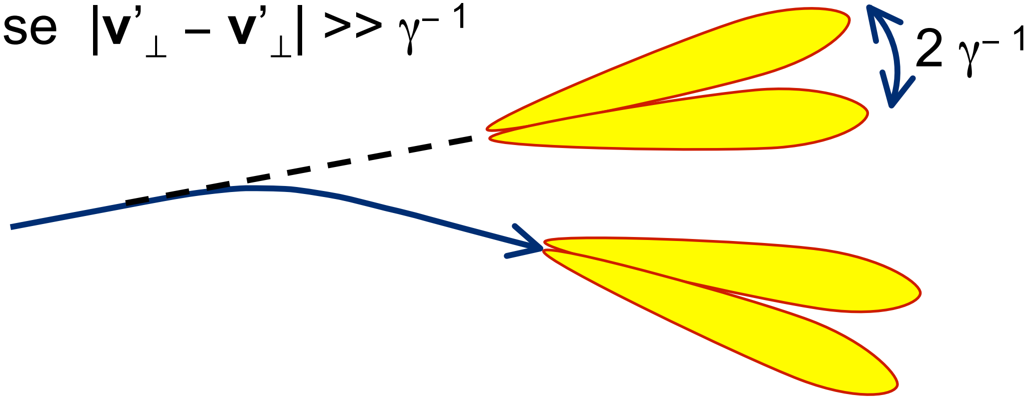
Recall : $\mathbf{A} = (i/\omega)\ \int d\tau\ \exp(i\omega\tau)\ d^2\mathbf{r}_\perp/d\tau^2$, $\quad d\tau = (\gamma^{-2} + \mathbf{v}_\perp{}^2)\ dt/2$

For very small ω → $\mathbf{A} = (2i/\omega)\ (\text{ initial } d\mathbf{r}_\perp/d\tau - \text{final } d\mathbf{r}_\perp/d\tau\ )$

$dN/d\omega d\Omega = (\alpha/\pi^2)\ \omega^{-1}\ \ |\ \mathbf{v}'_\perp/(\gamma^{-2}+v'_\perp{}^2\ ) - \mathbf{v}_\perp/(\gamma^{-2}+v_\perp{}^2\ )\ |^2$

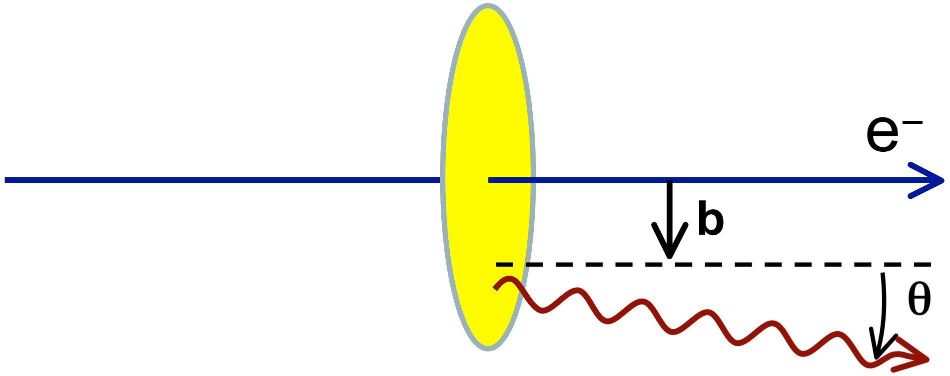
$\rightarrow\ N_{\text{photons}} = \infty$

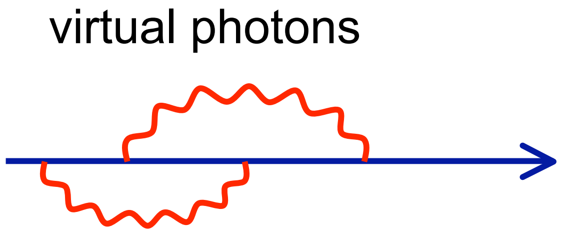
Case $|\mathbf{v}'_\perp - \mathbf{v}'_\perp| >> \gamma^{-1}$



Radial polarization
~ TR

Equivalent Photons





Moving Coulomb field ≈ cloud of **quasi-real** photons

(Fermi, Weiszäker, Williams)

Momentum space: $dN/d\omega d\Omega = (\alpha\omega/\pi^2)\,|\mathbf{k}_T/(\omega^2\gamma^{-2} + \mathbf{k}_T^2)|^2$

Impact param. space : $dN/d\omega d^2\mathbf{b} \approx (\alpha/\omega\pi^2)\,|\omega/\gamma\; K_1(b\omega/\gamma) - b^{-1}\, J_0(b\omega)|^2$

$\approx (\alpha/\omega\pi^2)\, b^{-2}$ between $b_{min} = 1/\omega$ and $b_{max} = \gamma/\omega$

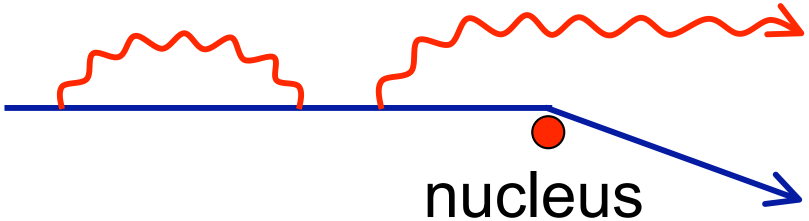
Screening in medium : replace $\mathbf{k}_T^2$ by $\mathbf{k}_T^2 + \omega_P^2$

photon mass

$b_{max} = \mathrm{Inf}\{\, \gamma/\omega,\ 1/\omega_P\}$

Application of Equivalent Photons

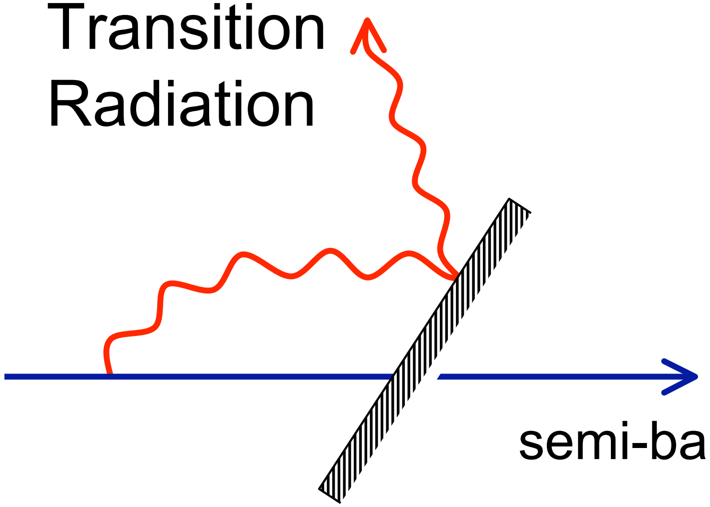
Initial State
Bremsstrahlung



nucleus

semi-bare e^-

Backward
Transition
Radiation

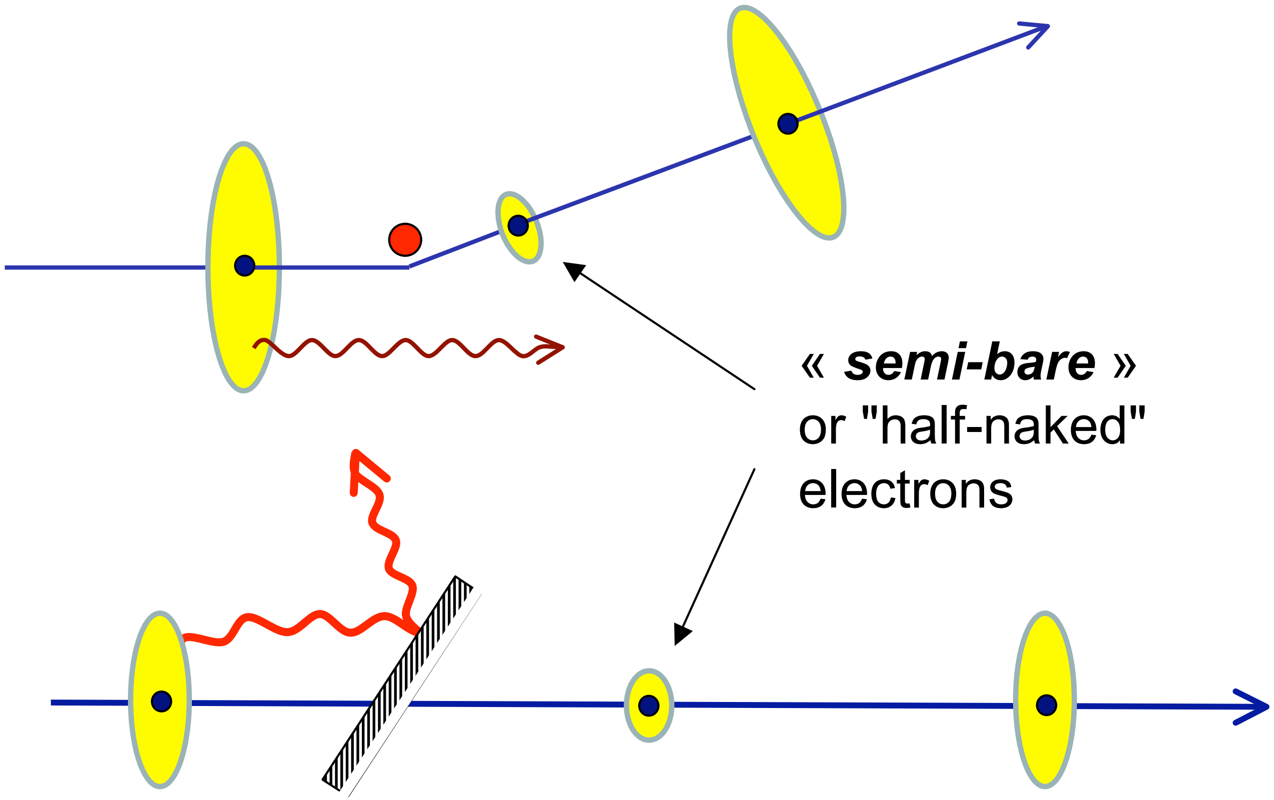


semi-bare e^-

(see analysis by Nikola Shul'ga)

Semi-bare electron

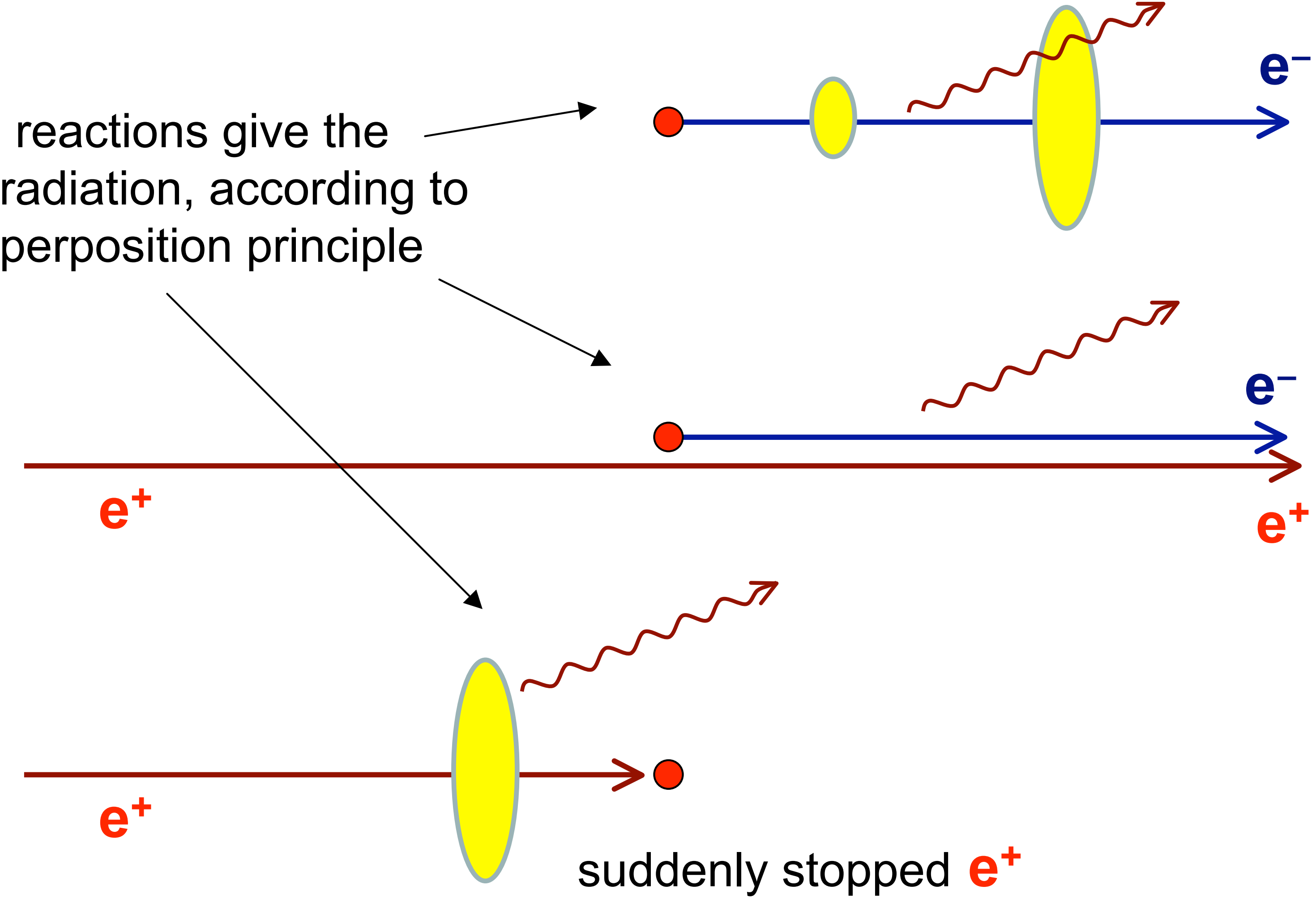
The electron is fully dressed by the equivalent photon cloud only after a **uniform** motion in **homogenous** medium of length > L_{coh} .



« ***semi-bare*** »
or "half-naked"
electrons

Final State Bremsstrahlung and *Forward* TR

FSB and FTR are produced during electron **re-clothing**

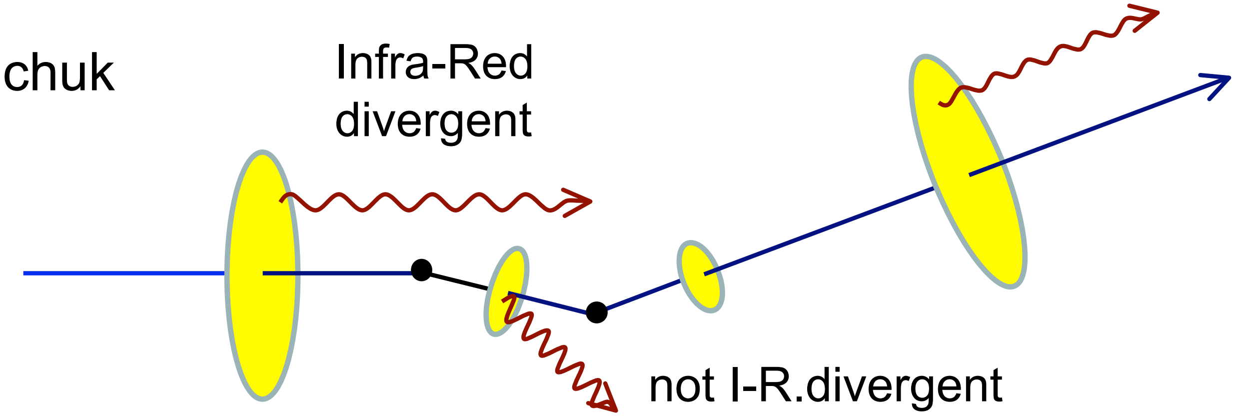


These reactions give the same radiation, according to the superposition principle

Manifestations of semi-bare electrons

- Landau - Pomeranchuk - Migdal effect

(see also Ternovskii, Shulga and Fomin)

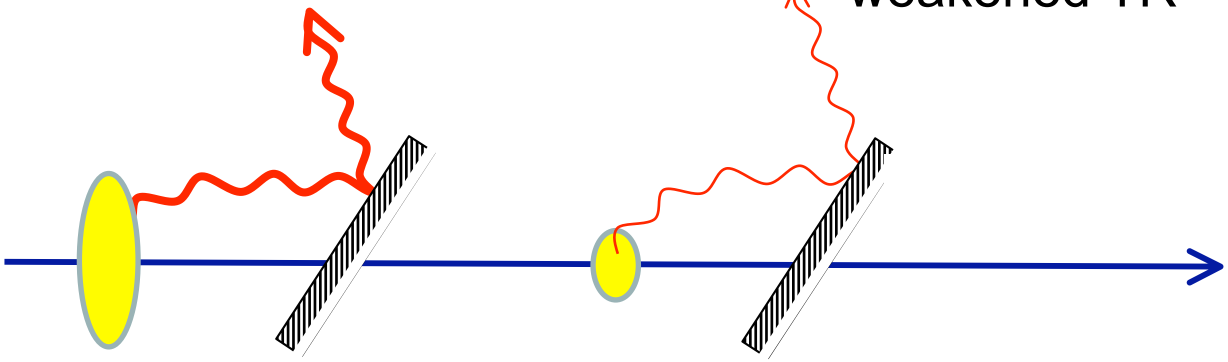


I-R.div.

Infra-Red divergent

not I-R.divergent suppression at small ω

- Formation zone effect in TR



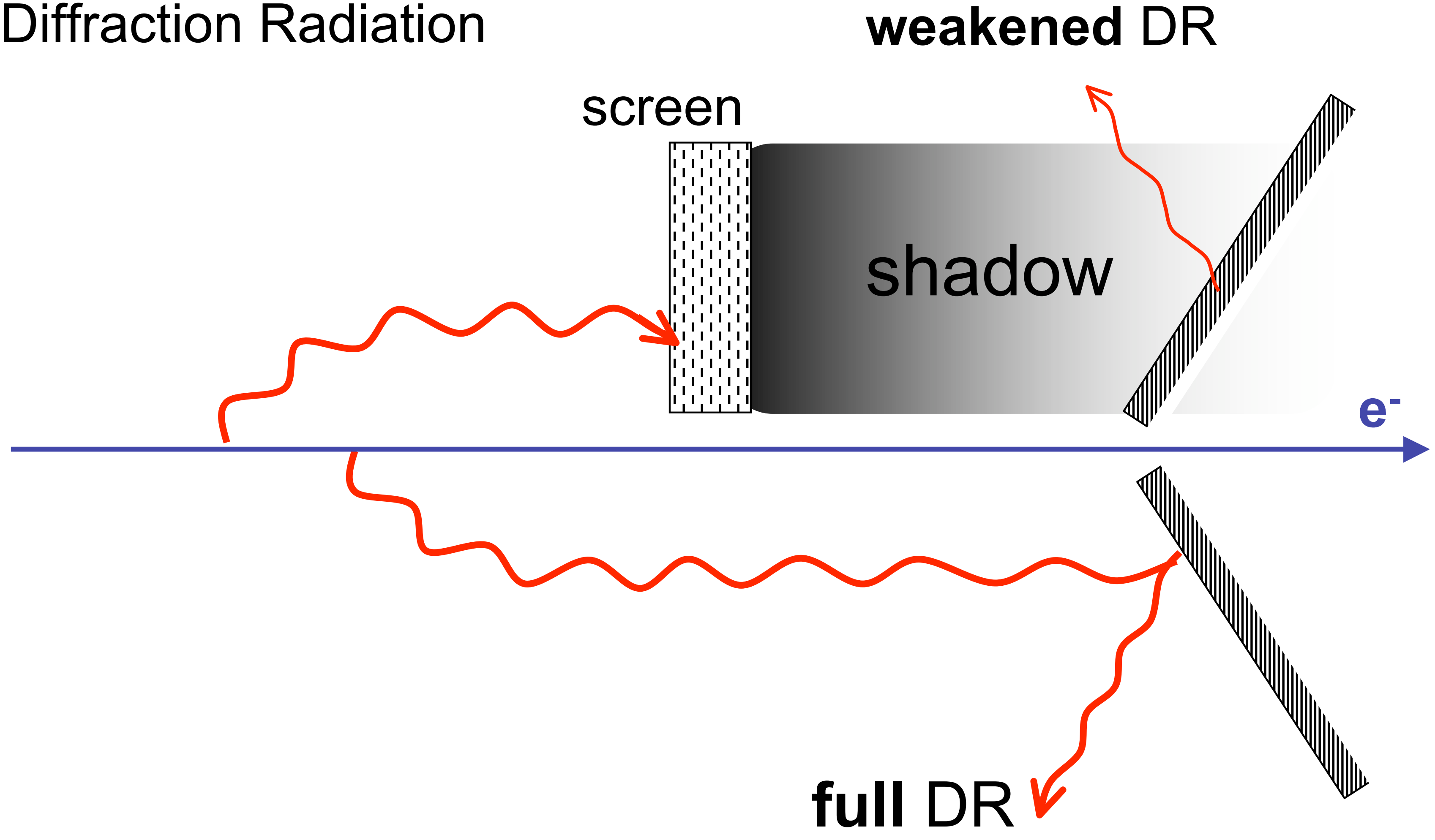
full TR

weakened TR

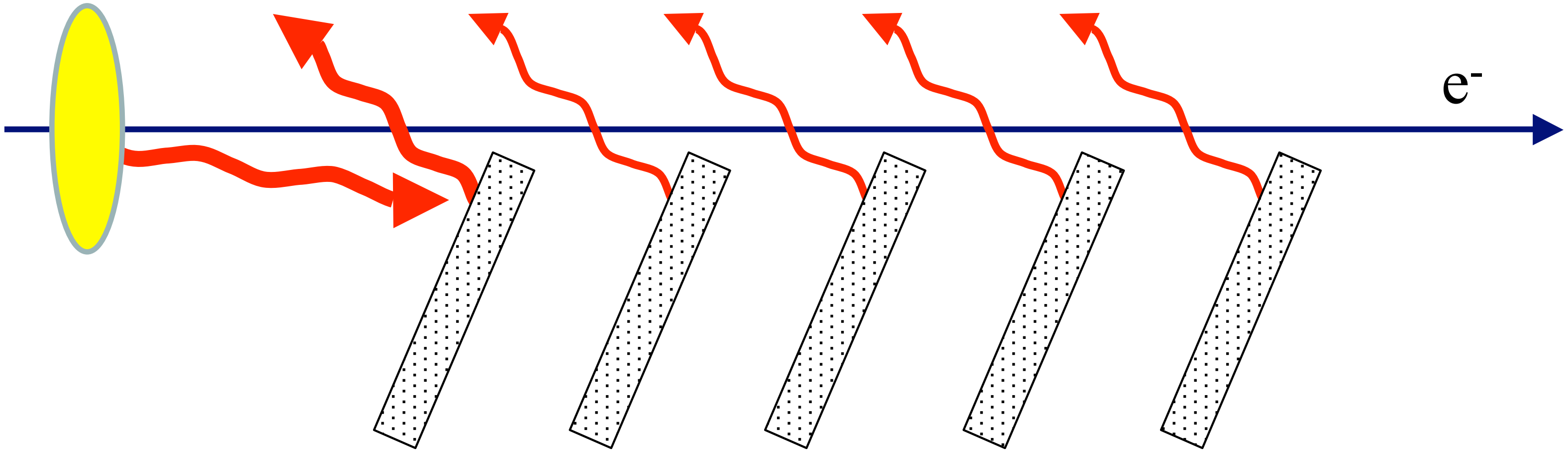
- reduced ionization loss (works by Shul'ga)

Shadowing

Tomsk experiment with
Diffraction Radiation



Shadowing effect is crucial in Smith-Purcell radiation



Shadowing suggests an **upper bound** for the Smith-Purcell radiation :

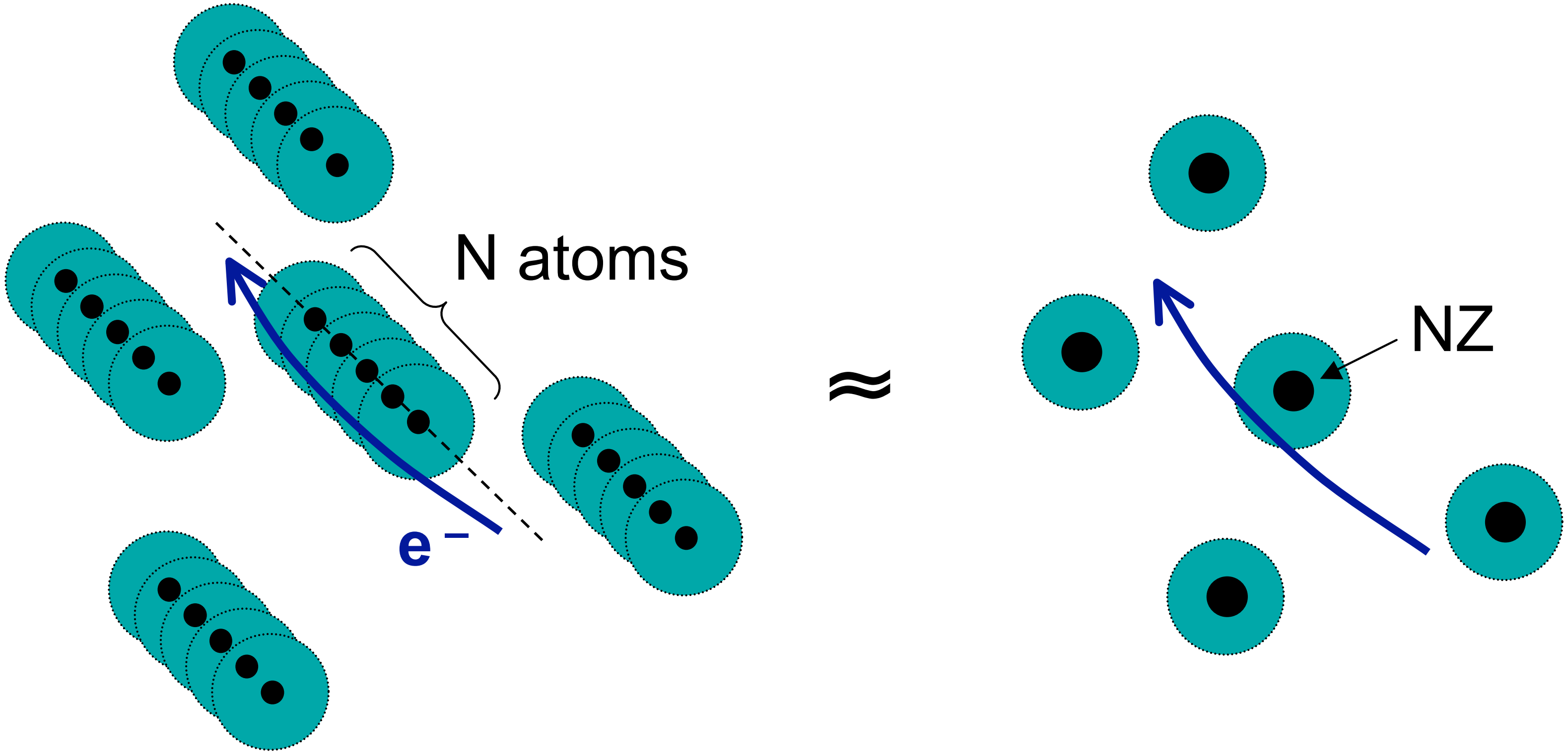
$dW/dL < \alpha / (2\pi b^2)$ (b = impact parameter)

Crystal-assisted radiation

(Coherent Bremsstrahlung, Channeling Radiation)

Why electrons radiate more energy
in aligned crystal
than in amorphous matter ?

a simplified answer : super-atoms

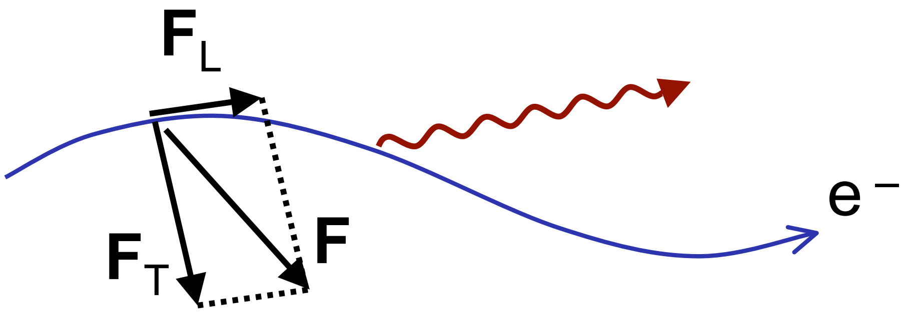


Ignoring channeling effects:

$$\theta^2_{m.s.}\text{ (aligned)} \sim N \times \theta^2_{m.s.}\text{ (random)}$$
$$W_{Brems}\text{ (aligned)} \sim N \times W_{Brems}\text{ (random)}$$

Limitation to N : $N\,d \sim \mathrm{Inf}\,\{a_{T\text{-}F}/\Psi\ ,\ L_{coh}\}$

Contradiction with the Larmor formula



Recall :

$$I(\omega,\mathbf{n}) = (\alpha/4\pi^2)\ |\int d\tau\ \exp(i\omega\tau)\ d^2\mathbf{r}_\perp/d\tau^2\ |^2$$

$\tau = t - \mathbf{n}\cdot\mathbf{r}$; $\mathbf{n} = \mathbf{k}/\omega$; $d^2\mathbf{r}_\perp/d\tau^2$ = apparent perp. acceleration

Parseval-Plancherel identity :

$$\int d\omega\ I(\omega,\mathbf{n}) = (\alpha/4\pi)\ \int d\tau\ (d^2\mathbf{r}_\perp/d\tau^2)^2$$

Integrating over **n** gives

$$W = (2/3)\ \alpha \int dt\ \ [\ (\gamma\mathbf{F}_T/m)^2 + (\mathbf{F}_L/m)^2\ ] \qquad \text{[Larmor]}$$

For a straight trajectory : $dW/dL = (16\pi\alpha^2/9m^2)\ \gamma^2\ \langle \mathbf{E}^2\rangle_{\text{microscopic}}$

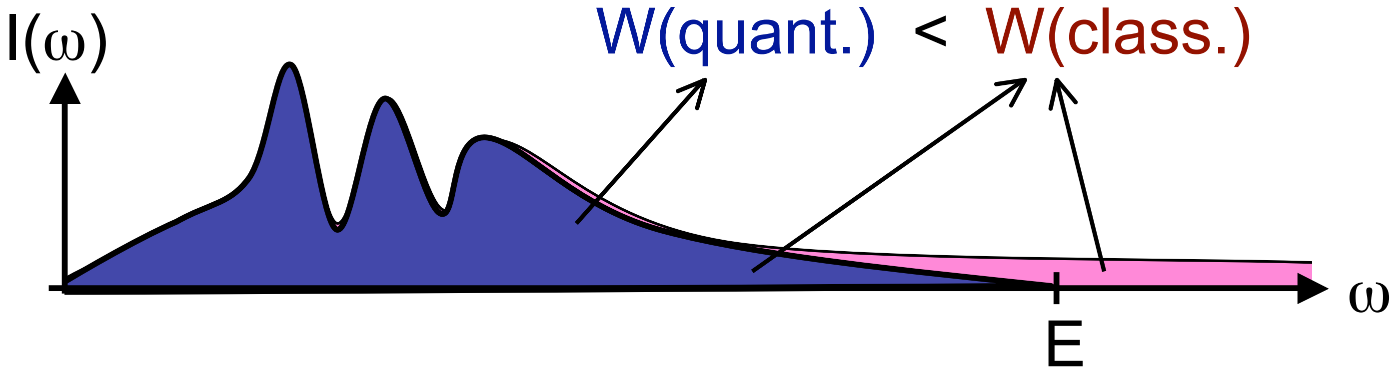
It should not depend on the crystal orientation…

= contrary to observation !

Failure of the Larmor formula

$$F(t) = \underset{\downarrow}{F_{slow}}(t) + \underset{\downarrow}{F_{fast}}(t)$$

soft photons $\omega << E$ hard photons $\omega \sim E$ (suppressed) or $> E$ (forbidden)

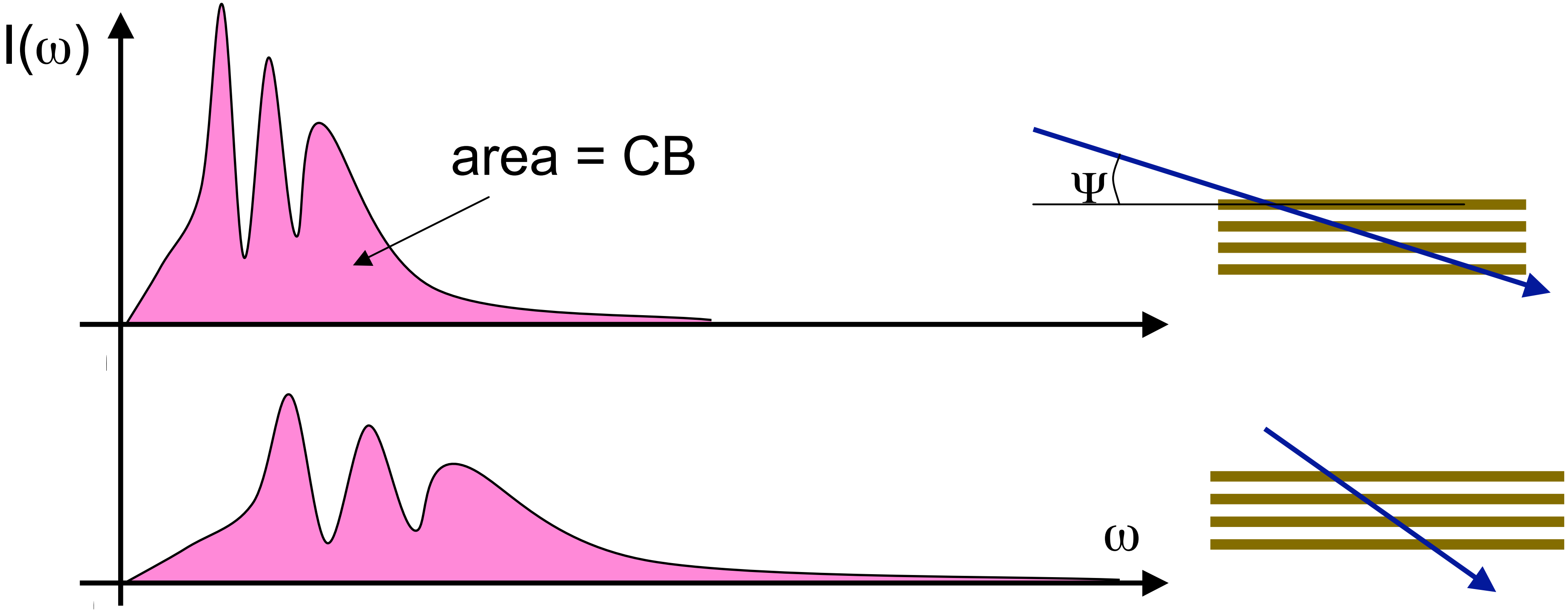


$\langle F^2_{slow}\rangle + \langle F^2_{fast}\rangle$ ~ independent of the crystal orientation,

but $\langle F^2_{slow}\rangle$(aligned) > $\langle F^2_{slow}\rangle$(random)

- F_{slow} (aligned)= $F_{Lindhard}$

… but Larmor formula works in CB



CB is due to F_{Lindhard} only ⇒ W_{CB} is independent on Ψ (checked by Gouanère et al),

- except for $\gamma\Psi > m\, a_{T\text{-}F}$: F_{Lindhard} becomes "fast"

II - QED in Strong Field

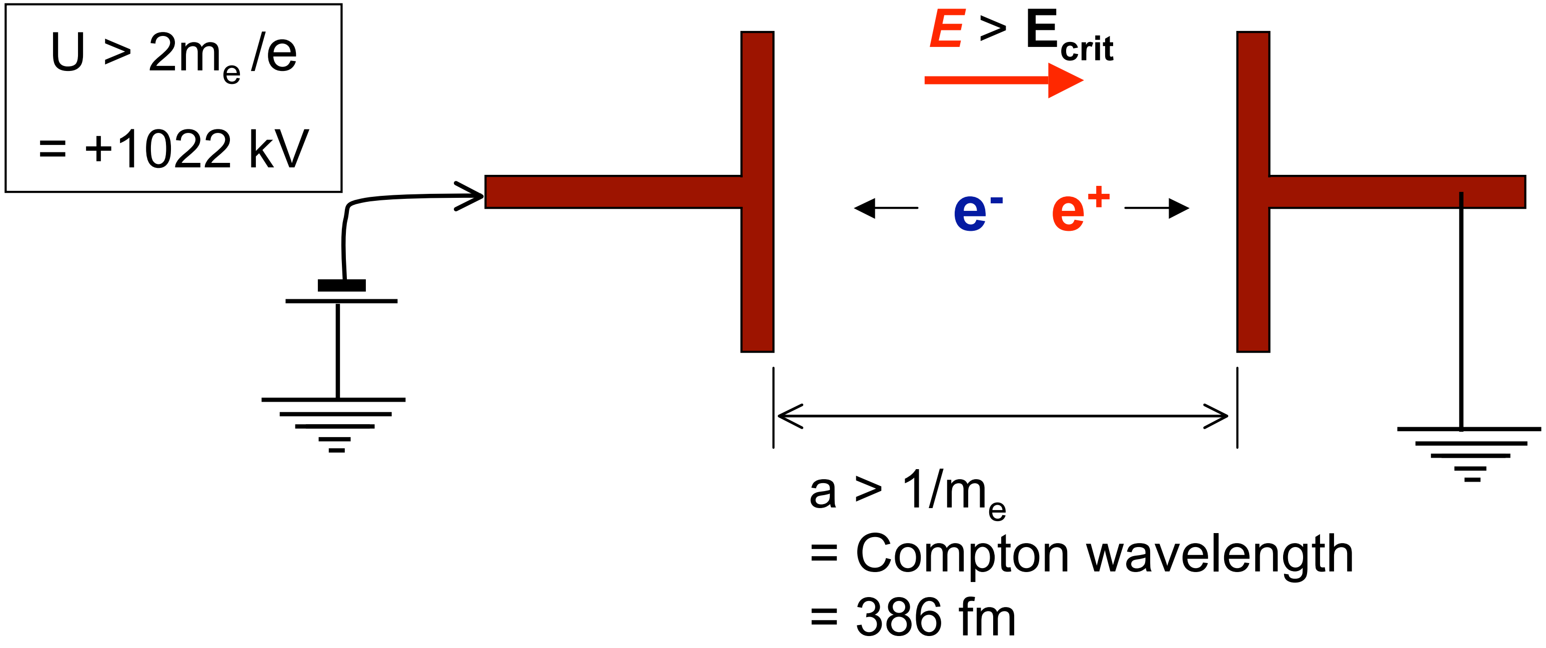
Critical QED field : $\mathbf{E_{crit}} = m^2/e = 1{,}32\ 10^{18}$ volt/m

$$e\ \mathbf{E_{crit}}\ \lambda_C = m$$

with λ_C = *Compton wavelength* = 1/m = 386 fermi

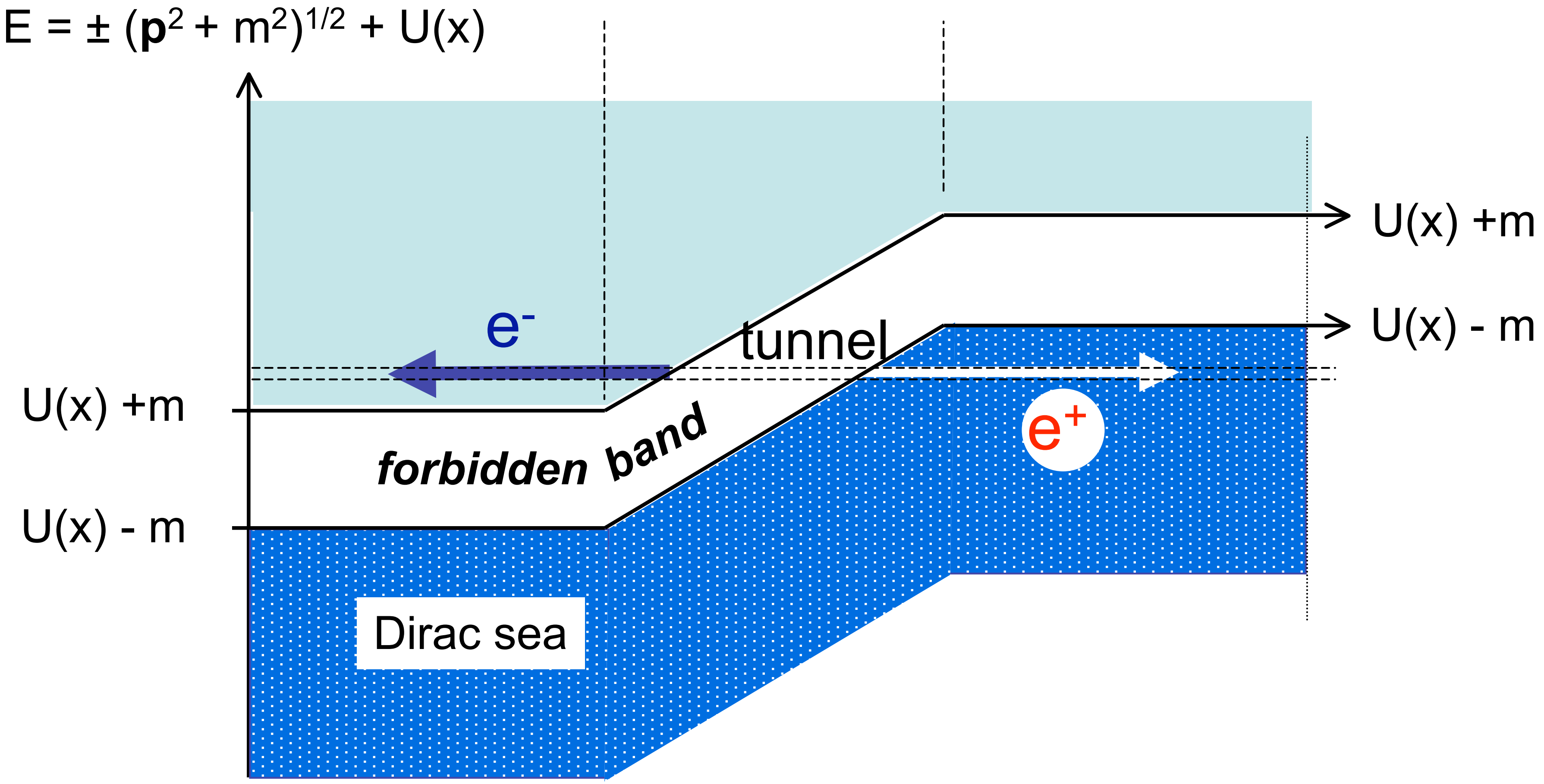
Schwinger process in QED critical field

In an electric field **E** ~ $\mathbf{E_{crit}}$, the vacuum is unstable against **spontaneous pair creation**



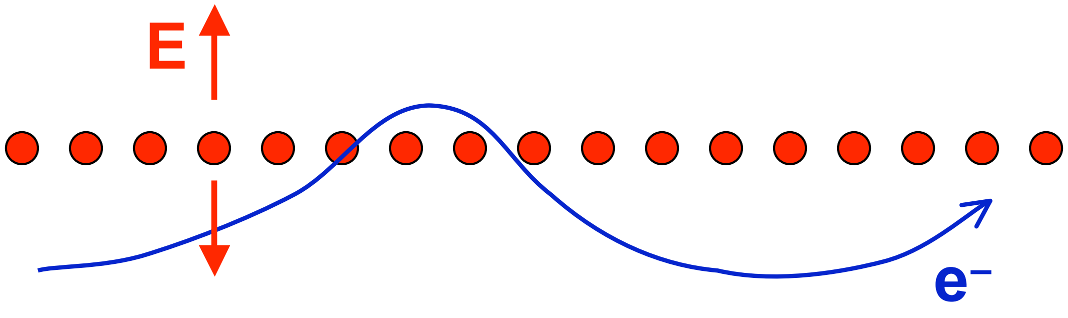
- Needs critical field **and enough volume** . It does not take place in the Coulomb field of a nucleus.

Schwinger process is a tunnelling effect

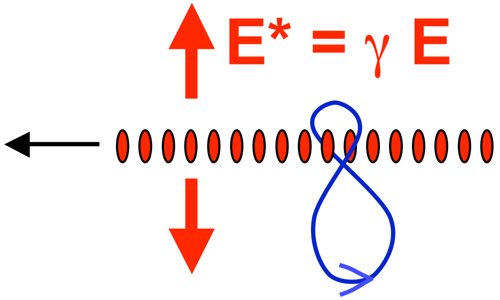


Tunnelling probability ~ $\exp(-\pi E_{crit}/E)$

Critical fields in channeling



lab. frame



e⁻ - comoving frame

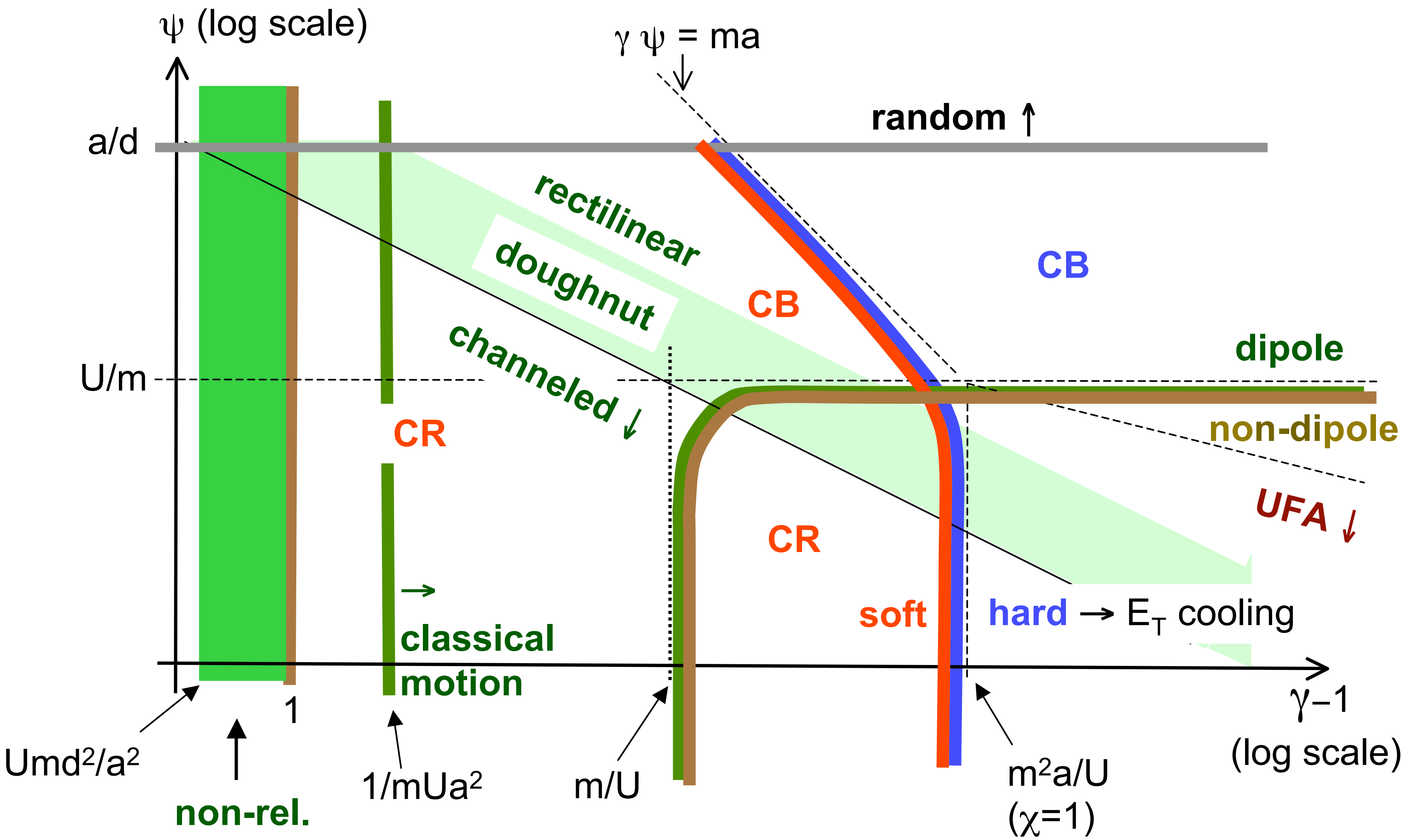
At high γ, **E*** can be over-critical. Critical parameter : χ = **E*/E**$_{crit}$

For χ >1 : - "photon decay" $\gamma \rightarrow e^+ e^-$
- photon splitting $\gamma \rightarrow \gamma\,\gamma$
- **hard** channeling radiation : $\omega_{max} \sim E_{electron}$

A magnetic field **B*** = v **E*** prevents spontaneous vacuum decay

In Germanium, **E** ~ 1 kV / angstrom → χ = 1 for 50 GeV electrons (γ =10^5).
- A strong enhancement of pair creation is observed.
- Hard CR leads to rapid **transverse energy cooling** → Belkacem peak

(recall) Channeling regimes in (γ,ψ) plane



The "magic" Baier-Katkov formula (1/5)

Cures the defects of the classical radiation formula
and takes **recoil** and **spin** effects into account. Applies to :

- Synchrotron radiation in **strong** field
- Hard Compton effect (Klein-Nishina regime)
- Non-linear Compton effect
- Hard Coherent Bremsstrahlung
- Hard Channeling Radiation
- Incoherent Bremsstrahlung

Using crossing symmetry: pair creation in strong field (?)

The "magic" Baier-Katkov formula (2/5)

1) Recoil effect : replace $\phi = - k\cdot X$

by $\phi' = - (E/E')\, k\cdot X$

2) Helicity-dependent amplitude : $\mathbf{A}\cdot\boldsymbol{\varepsilon}^* \rightarrow \langle\lambda'|A_\Lambda|\lambda\rangle$

$\lambda, \lambda' = \pm 1/2$

Λ : photon helicity. "R" : $\Lambda = +1$, "L" : $\Lambda = -1$

Summing over spins :

$$dN = (\alpha\omega/8\pi^2)\, d\omega d\Omega \quad \mathbf{\{} \ (1+ \gamma^2/\gamma'^2)\ |\int \exp(i\phi')\ d\mathbf{r}_\perp|^2 + (1/\gamma' - 1/\gamma)^2\, |\int \exp(i\phi')\ dt\ |^2 \ \mathbf{\}}$$

The "magic" Baier-Katkov formula (3/5)

Helicity non-flip amplitudes

$$\left.\begin{aligned}\langle +|A_L|+\rangle &= \int \exp(i\phi') \,(dx+idy)\, /\sqrt{2} \\ \langle -|A_R|-\rangle &= \int \exp(i\phi') \,(dx-idy)\, /\sqrt{2}\end{aligned}\right\} = \text{as classical, but with } \phi'$$

$$\langle -|A_L|-\rangle = (E/E')\,\langle +|A_L|+\rangle$$

$$\langle +|A_R|+\rangle = (E/E')\,\langle -|A_R|-\rangle$$

Helicity-flip amplitudes

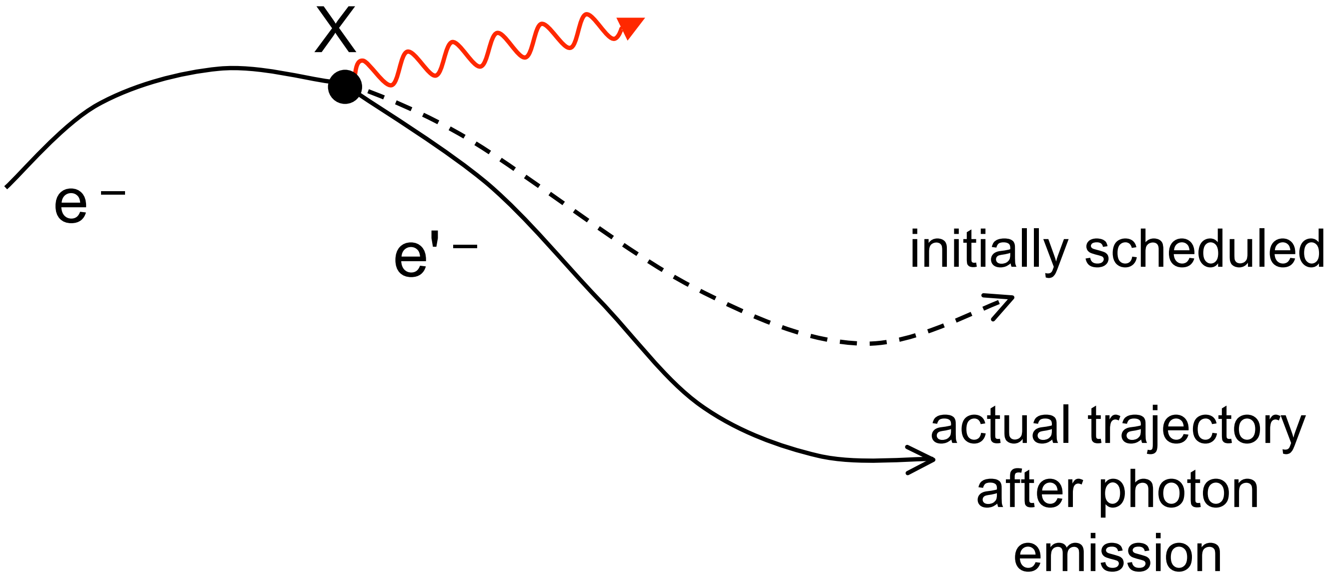
$$\langle -|A_R|+\rangle = -\langle +|A_L|-\rangle = 2^{-1/2}\,(1/\gamma'-1/\gamma)\,\int \exp(i\phi')\,dt$$

$$\langle -|A_L|+\rangle = -\langle +|A_R|-\rangle = 0$$

The "magic" B-K formula (4/5)

Why "magic" ?

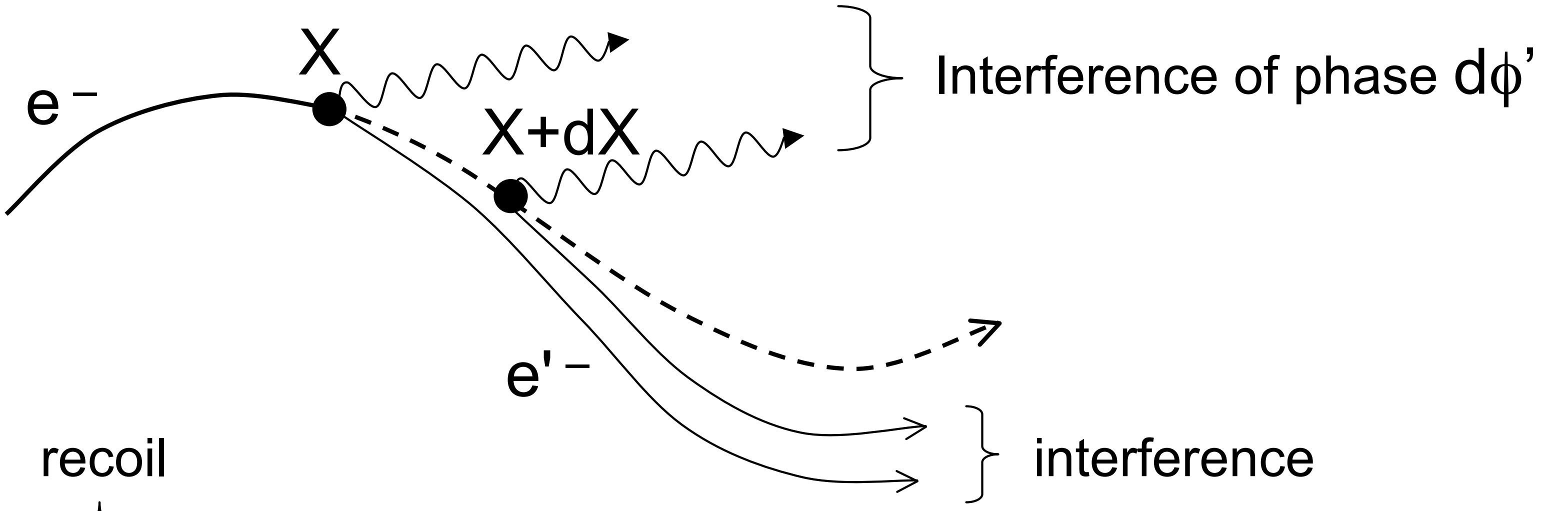
- it does not depend on the final electron trajectory !



The field must not vary too fast in the transverse directions.

The "magic" B-K formula (5/5)

Explanation of the recoil correction $\phi' = (E/E')\,\phi$



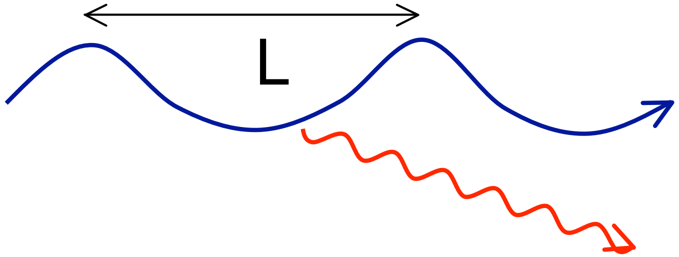
$$d\phi' = (\overbrace{p - p'}^{\text{recoil}} - k)\cdot dX \qquad \text{instead of } d\phi = -\,k\cdot dX$$

$$dX = p\,dt/E \qquad d\phi'/d\phi = (k+p'-p)\cdot p \,/\, k\cdot p \approx E/E'$$

(using local conservation of energy and transverse momentum)

Fine test of the B-K formula in channeling radiation

- Assume periodic trajectories and compare the spectral lines predicted by B-K with "exact" spectral lines.



Conserv. of energy and P_L :

$$(\omega/\gamma\gamma' + \omega\theta^2)/2 = E_T - E'_T \qquad (1)$$

B-K formula for the ν^{th} harmonic

$$(\omega/\gamma\gamma' + \omega'\theta^2)/2 = 2\nu\pi/L - \omega'\langle \mathbf{v}_T^2 \rangle /2 \qquad (2)$$

$\nu = n - n'$ = decrease of transverse quantum number

n is given by the Bohr quantization rule $L \cdot E \cdot \langle \mathbf{v}_T^2 \rangle = 2n\pi$

$\Rightarrow$ The R-H-S of (1) and (2) are equal for $\omega << E$.

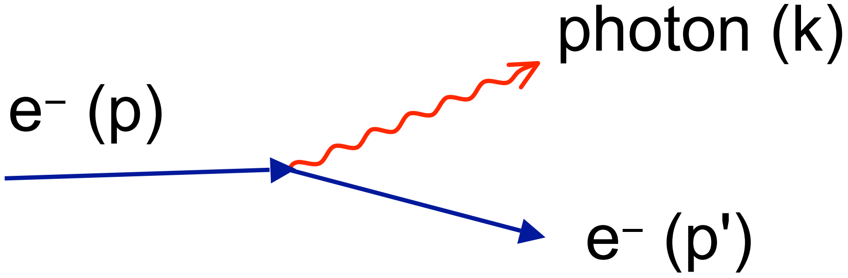
$\Rightarrow$ B-K formula does not give the exact detailed spectrum in the hard region of CR.

III - Particular aspects of radiation

- Radiation as a tunneling effect
- Impact parameter of synchrotron radiation
- Side-slipping of the electron

Tunneling as a remedy to classical forbidness

The reaction :



photon (k)

e^- (p)

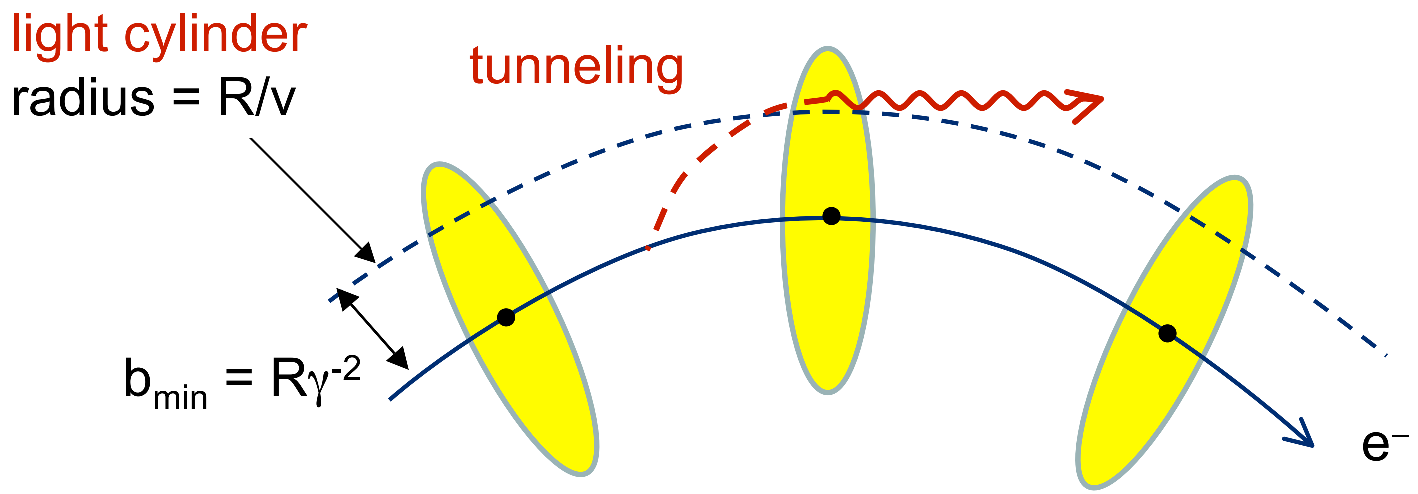
e^- (p')

is kinematically forbidden *as a* ***local*** process between ***classical particles***.
One cannot satisfy both p = p'+k
and the mass-shell conditions.

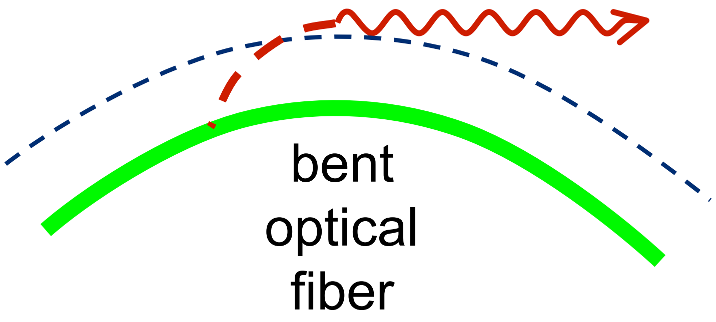
It can occur via a quantum **tunnelling effect**.

Impact parameter of Synchrotron radiation

Synchrotron radiation ~ **slow leakage** of the Coulomb field



Analogy :



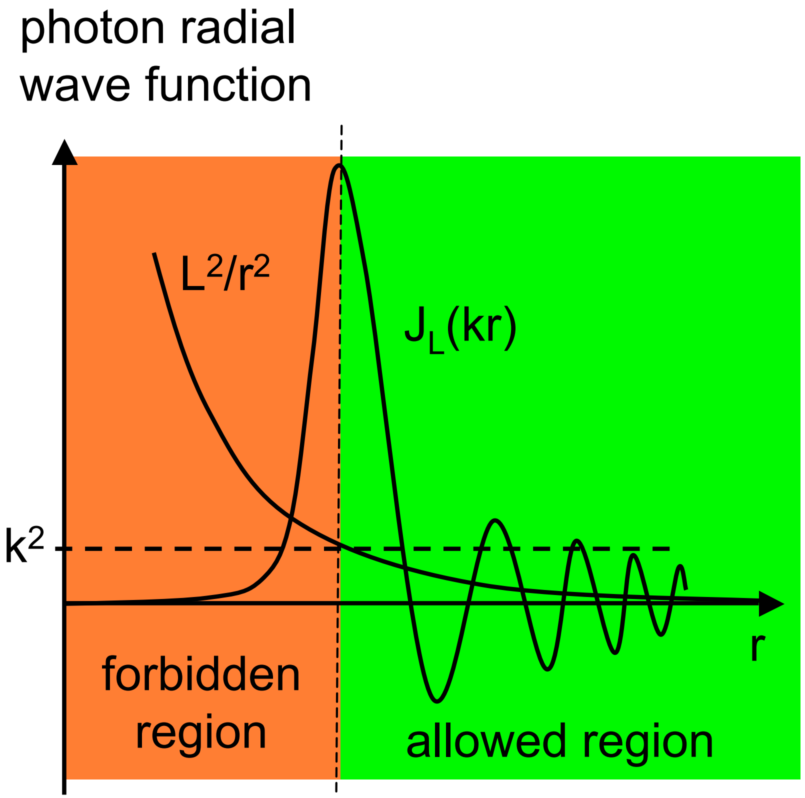
Tunnelling factor in Synchrotron Radiation

The tunnelling effect is more dramatic when the photon energy is above the cutoff $\omega_c = \gamma^3/R$.

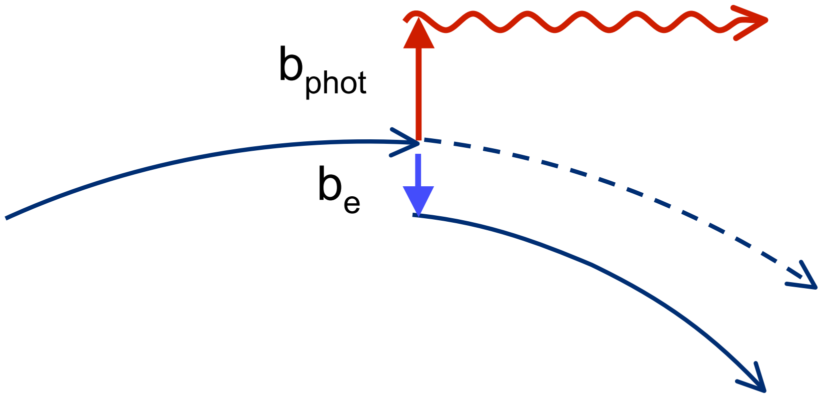
It leads to the exponential decrease in

$\exp\{-(\omega/\omega_c)(1+\gamma^2\theta^2_{out})^{3/2}/3\}$

of Synchrotron radiation.



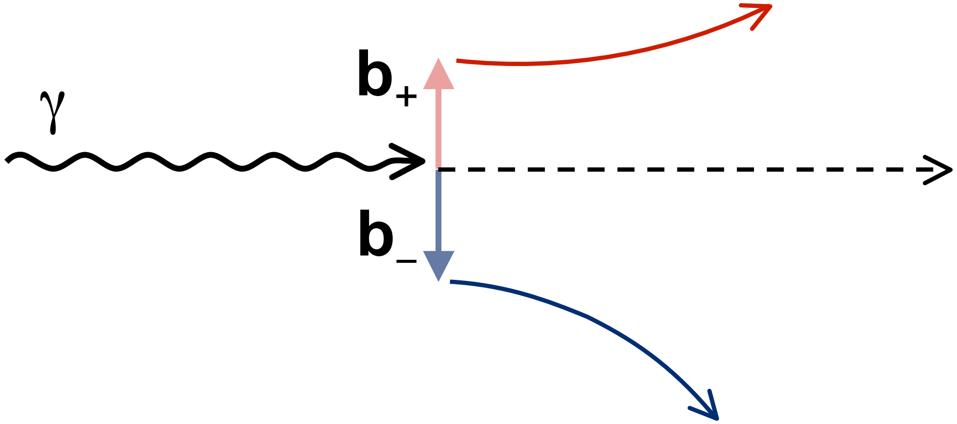
Side slipping of the electron



photon + electron
centre of mass
$E\ b_e = \omega\ b_{phot}$

- b_e ~ Compton wavelength = 400 fm
- Side-slipping is responsible for the d^3X/dt^3 term of the Abraham-Lorentz equation

Crossed process : pair creation in strong field



$e^+\ e^-$
centre of mass
$E_+\ b_+ = E_-\ b_-$

Summary

We have looked at some of the QED phenomena involved in radiation by relativistic electrons, in matter or in external fields :

- Infrared divergence
- escape of the equivalent photons
- semi-bare electrons, LPM effect, shadowing
- quantum recoil
- radiation in critical field
- impact parameter of the photon, tunnelling, side-slipping.

Our considerations were very qualitative, phenomenological, a little bit superficial, and covered a very small part of the subject. We hope they have some universal character.

Thank you for your patient attention !