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Orientation Dependence of Electron-Positron Pair Production in Single Crystals

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Abstract

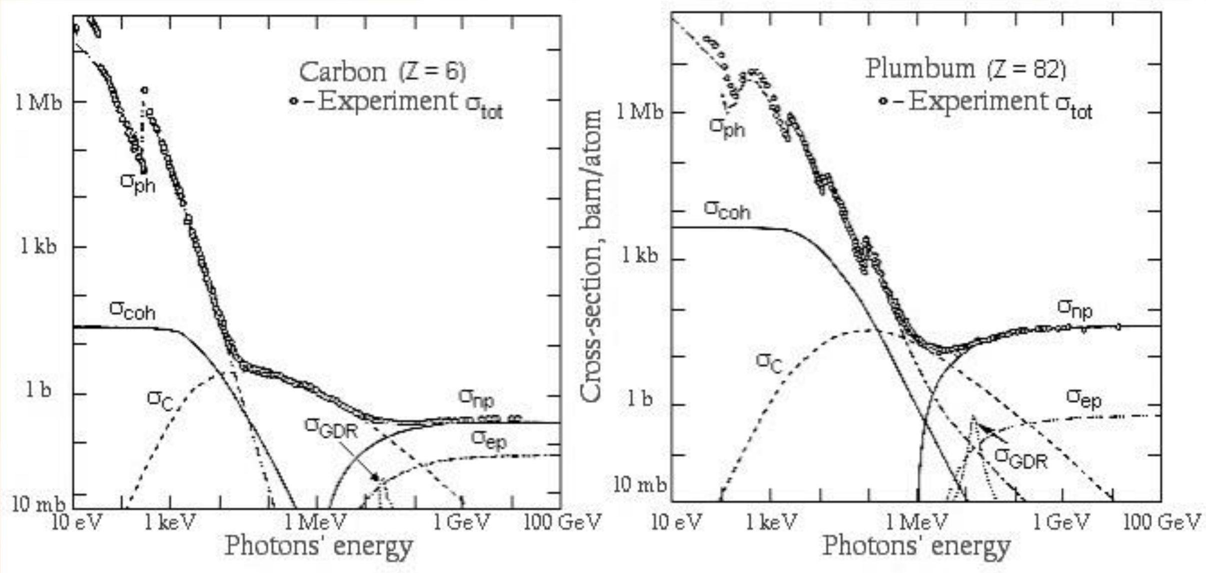
One of the important problems in present-day astrophysics and gamma-astronomy is the construction of detectors for high energy photons (more than 1GeV) with high angle resolution.

In this energy range the dominating effect in interaction of photons with matter is e^-e^+ pair production.

High angular resolution may be achieved using single crystals as an effective converter of photons into e^-e^+ pairs due to coherent pairs production in channeling regime.

e^-e^+ Pair Production in Photon Interaction with Matter

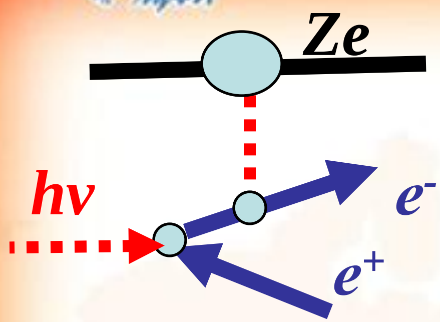
In the energy range $\hbar\omega$ > 1 GeV the main effect in photon interaction of with matter is the pairs production, colliding with atomic nucleus.



The cross-section of the photon interaction with matter (carbon (Z = 6) and plumbum (Z = 82)). σ_{ph} – photoeffect, σ_{coh} – Raleigh scattering, σ_C – Compton scattering, σ_{GDR} – nuclear photoabsorbtion, $_{ep}$ – pair production in collisions with electrons, σ_{np} - pair production in collisions with nucleus

Review of Particle Physics. Journal of Physics G. Nuclear and Particle Physics. v.37. Number 7A, July 2010. Article 075021.

e^-e^+ Pairs Production by High Energy Photons – Momentum and Energy Conservation Laws.



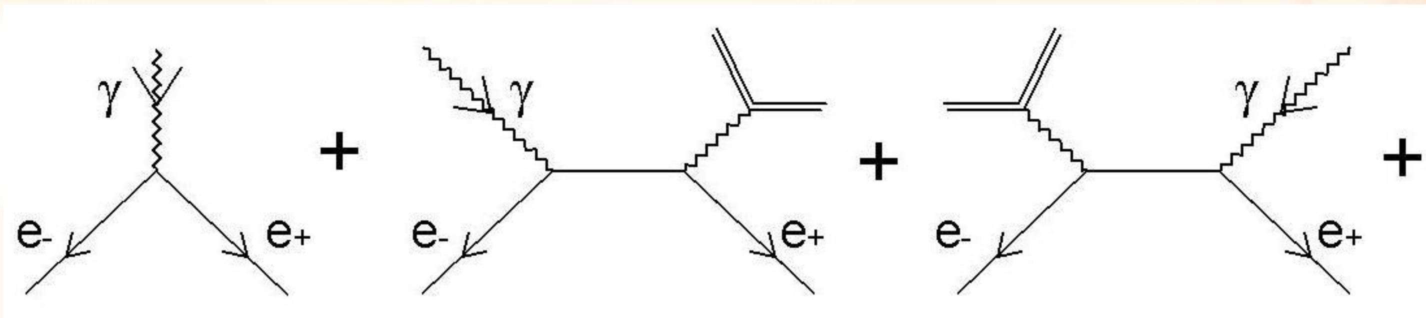
$$\frac{h\upsilon}{mc} = \frac{c}{\sqrt{1-\frac{v_-^2}{c^2}}} + \frac{c}{\sqrt{1-\frac{v_+^2}{c^2}}}$$

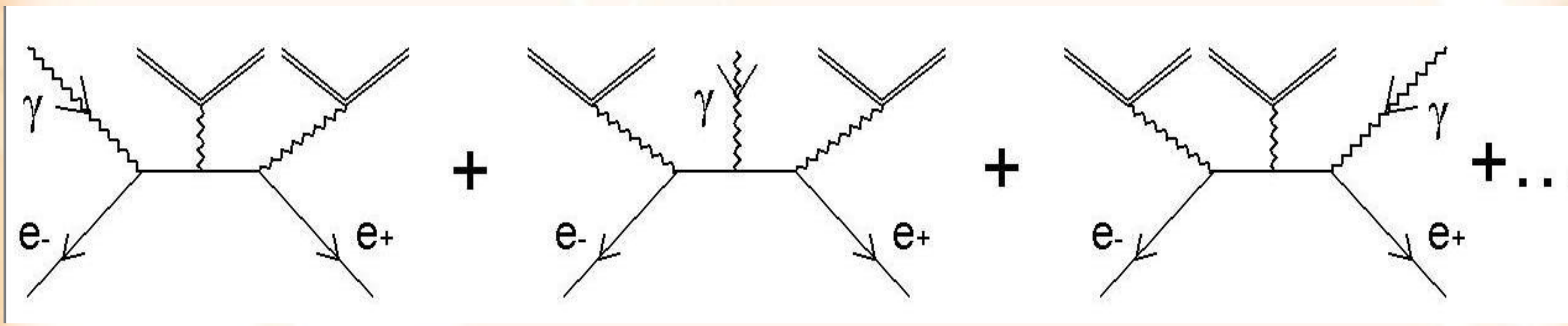
$$\frac{h\upsilon}{mc} = \frac{v_{-x}}{\sqrt{1-\frac{v_-^2}{c^2}}} + \frac{v_{+x}}{\sqrt{1-\frac{v_+^2}{c^2}}} + \Delta P_x$$

$$\frac{h\upsilon}{mc} \neq \frac{v_{-x}}{\sqrt{1-\frac{v_-^2}{c^2}}} + \frac{v_{+x}}{\sqrt{1-\frac{v_+^2}{c^2}}}$$

$$\Delta P_x \sim \frac{mc}{\sqrt{1-\frac{v_\pm^2}{c^2}}} \sim mc\left(\frac{mc^2}{h\upsilon}\right) << mc$$

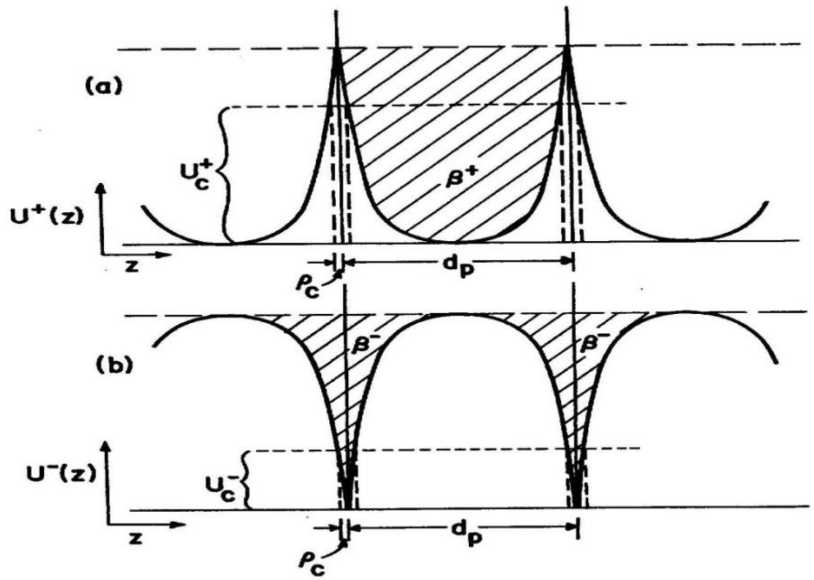
e^-e^+ Pairs Production by High Energy Photons – Matrix Elements





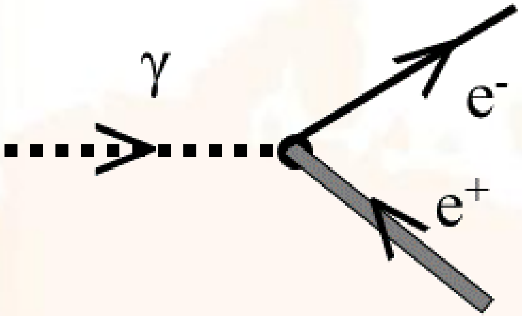
The Feynman diagrams series for pair production by gamma-photons. The first diagram in this series (without participating of the 3-rd body) is forbidden by conservation laws in case when the wave functions of leptons are plane waves.

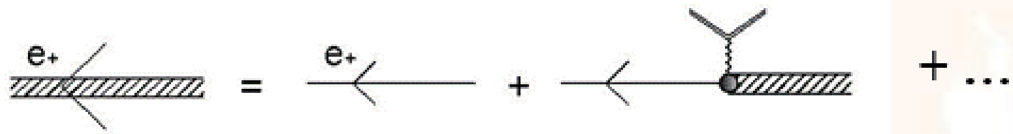
Effective Planar Channeling Potential for Electrons and Positrons



Schematic illustration of the planar continuum potentials for (a) positrons and (b) electrons. The shaded areas correspond to the potential wells in which channeled particles can move. $U_C{}^+$and $U_C{}^-$ are potentials evaluated at the critical distance ρ_C from a plane. (Note that $U_C{}^+$ represents a upper limit for the transverse energy at which stable positron channeling is possible, while $U_C{}^-$ is corresponding lower limit for electrons.)

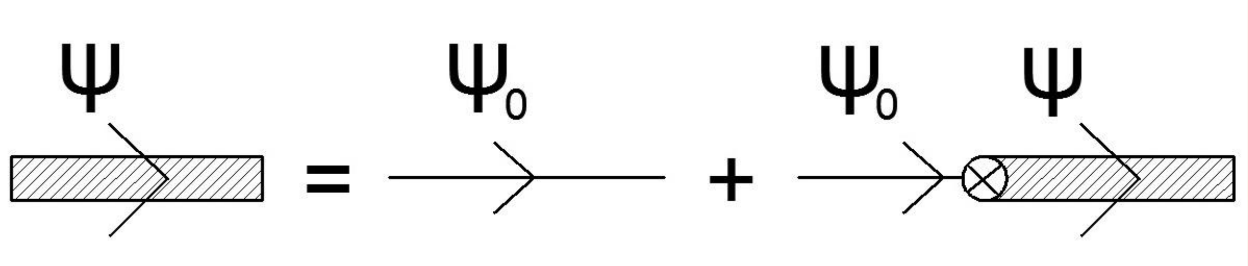
e^-e^+ PairsProduction by High Energy Photons – Matrix Element in Channeling Regime





The Feynman diagram for pair production, when positron is produced in channeling regime.

The single-vertex diagram is no longer forbidden by conservation laws, as the wave functions of the producing positron in the channeling regime are not plane waves.


Ψ
=
Ψ_0
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e^-e^+ Pair Production by High Energy Photons – Cross Section in Channeling Regime

$$M_{e\_e_+} = -2\pi i \frac{e}{\sqrt{2\omega}} \int \psi^{(+)*}(\vec{r})\, \exp(i\vec{k}\vec{r})\, (\vec{e}\vec{\alpha})\, \psi^{(-)}(\vec{r})\, \delta(\omega - \varepsilon_+ - \varepsilon_-) d^3 r$$

For photon and electron we may use the plane wave functions. The transverse component of the positron wave function $\varphi_k(x)$ must be obtained from the one dimensional Schrödinger equation with the relativistic mass:

$$\varphi''_k(x) + 2E(E_k - U(x))\varphi_k(x) = 0.$$

For estimations at $d \gg r_{at}$, we may approximate the potential $U(x)$ for positrons as a well with high walls and width d. For boundary conditions we will take $\varphi_k(x=d/2) = \varphi_k(x=-d/2)) = 0$. In this approximation the normalized wave functions and allowed transverse energies look like:

$$\varphi_k(x) = (2/d)^{1/2} \sin(\pi k x/d + \pi k/2),$$

$$E_k = (\pi k/d)^2/2E, \qquad k = 1,2,3,\ldots$$

e^-e^+ Pair Production by High Energy Photons – Cross Section in Channeling Regime

The matrix element with these functions looks like

$$M_{e\_e_+} = \sum_{n,\nu} \int \exp(iz(k_z - p_z^- - p_z^+)(\vec{e}\vec{\alpha})dz\, Q_{n,\nu}(\theta_0),$$

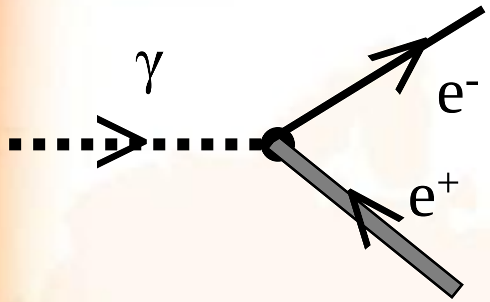
where

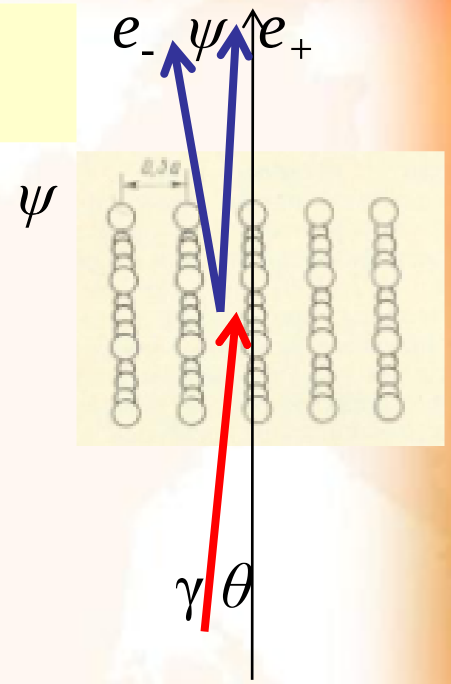
$$Q_{n,\nu}(\theta_0) = \int dx\, \varphi_n{}^*(\text{x-}\nu a) \exp(ix(k\theta_0 - p^-\theta^- - p^+\theta^+)$$

i.e. $Q_{n,\nu}(\theta_0)=$

$$= \sqrt{\frac{8}{d-2r_{at}}} \begin{cases} i\, \dfrac{p_0\, \theta_n \sin(2p_0\theta_0 r_{at})}{p_0^2\theta_n^2 - p^2\theta_0^2} & \text{if } (\mathrm{n} + \pi \mathrm{d} p_0 \theta_0) \text{ is an even number} \\ i\, \dfrac{p_0\, \theta_n \cos(2p_0\theta_0 r_{at})}{p_0^2\theta_n^2 - p^2\theta_0^2} & \text{if } (\mathrm{n} + \pi \mathrm{d} p_0 \theta_0) \text{ is an odd number} \end{cases}$$

Pair Production in Channeling Regime – Conditions and Threshold





Entrance angle: $\theta < \theta_L \sim (U/E)^{1/2} \sim 10^{-4} – 10^{-5}$

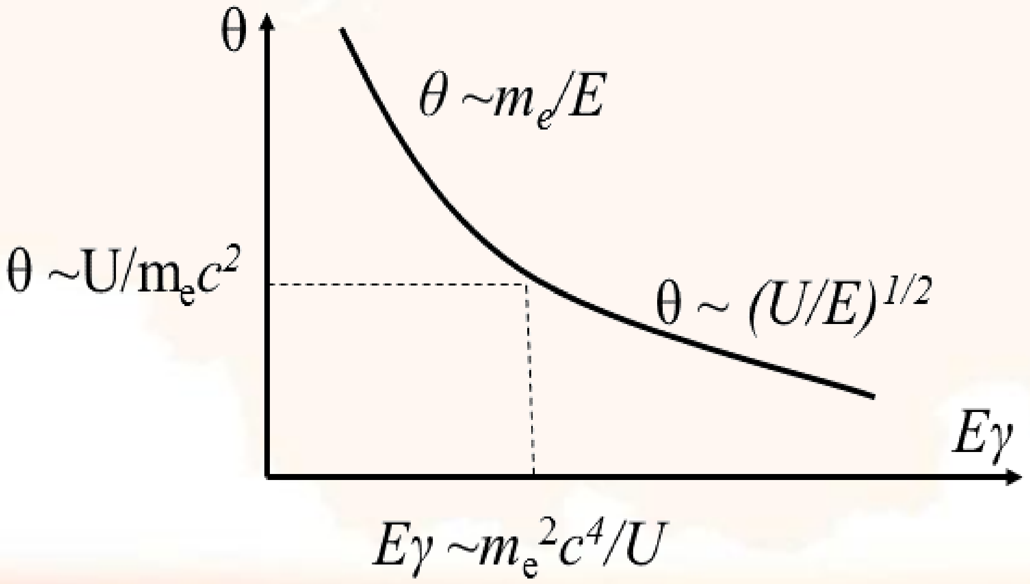
Angle $\psi \sim (mc^2/\hbar\omega) < \theta_L \Rightarrow \hbar\omega > (mc^2)^2/U \sim$ 5-10 GeV

Angular Resolution of a Single Crystal as a Gamma–Particles Detector.

In case of coherent pair production it is necessary that photon should propagate along the row under the angles $\theta \leq m_e c^2/\hbar\omega\gamma$.

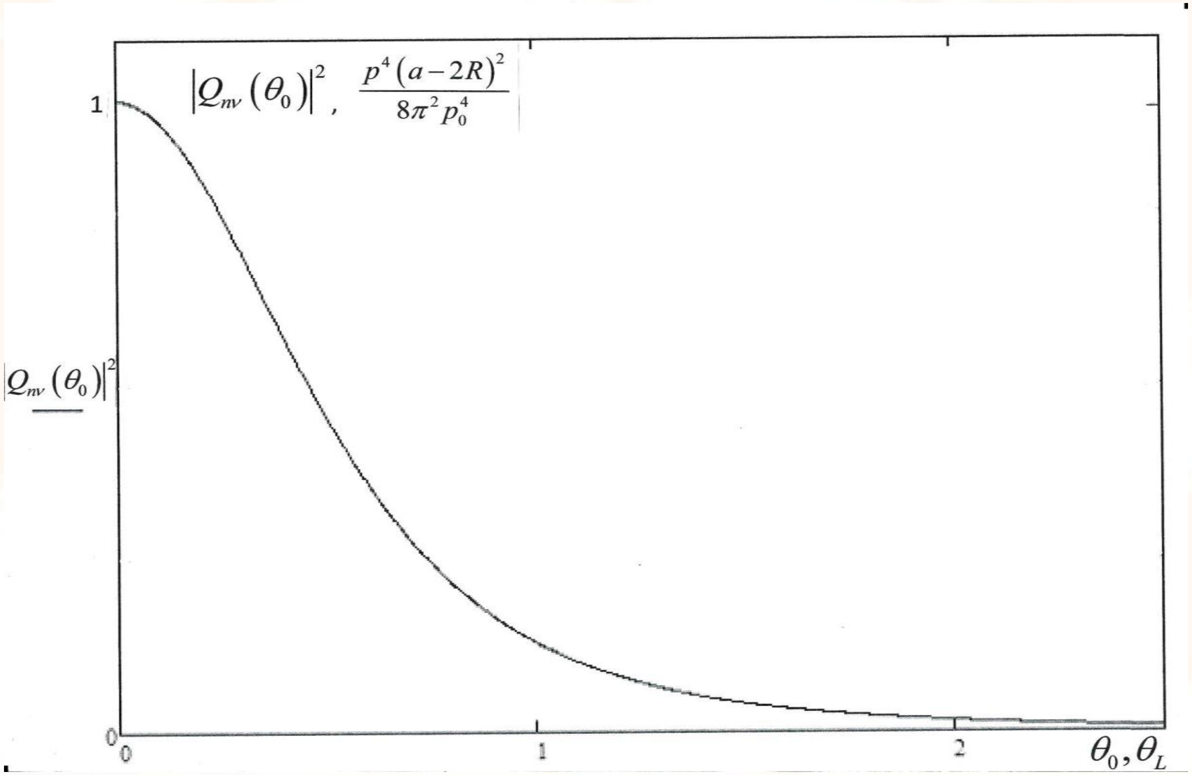
In case when $\theta < \theta_L \sim (2U/E_e)^{1/2}$ – the dominating effect shall be producing pairs in channeling regime. In this case the collimation angle coincides with the crytical Lindhard angle.

Quantitatively the expected dependence of collimation angle on the photon energy is shown below



Pair Production in Channeling Regime – Orientation Dependence of e^-e^+ - Pair Production

The function $|Q_{n,\nu}(\theta_0)|^2$ characterizes the orientation dependence of e^-e^+ - pair production in single crystals. The angular maximum half-width coincides by an order of magnitude with the crytical Lindhard angle.



Acknowledgements

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Thank You for Attention!