



CONICAL EFFECT IN OPTICAL AND X-RAY DIFFRACTION RADIATION

Daria Sergeeva, Alexey Tishchenko, Mikhail Strikhanov

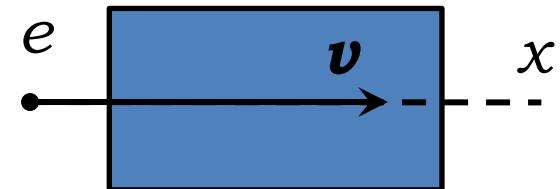
National Research Nuclear University "MEPhI"

Motivation

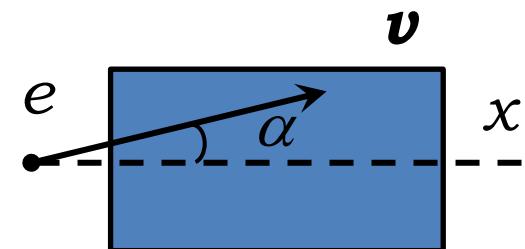
Non-divergent beam



Divergent beam



First step of investigation



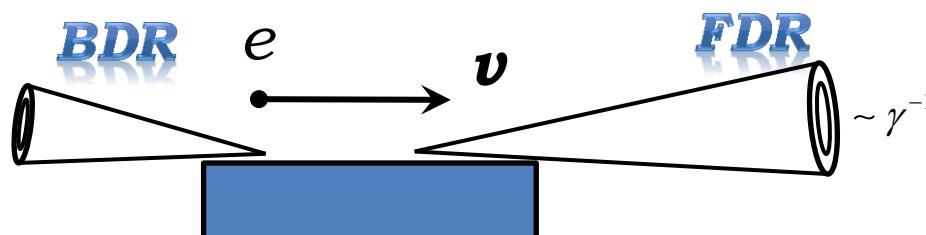
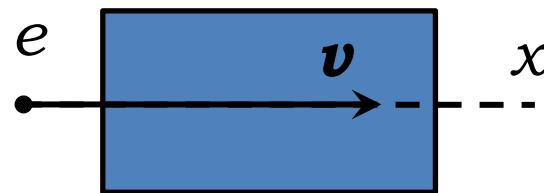
- Y. Shibata *et al.*, PRE **50** (1994) – TR, divergence
- N. Potylitsina-Kube and X. Artru, NIM B **201** (2003) – Oblique incidence analytical theory + divergent beam semi-numerically
- D.V. Karlovets and A.P. Potylitsyn, TPU Bulletin **308** (2005) - DR + vertical divergence
- V. Shpakov and S.B. Dabagov, J. of Phys.: Conf. Ser. **517** (2014) – Influence of divergence on form-factor

What you can expect ...

- What are the **general characteristics** of DR for arbitrary angle?
- How can it be proved **mathematically**?
- What is it in **optical** frequency region? 2 examples
- What is it in **X-ray** frequency region?
- How can it influence for **divergent** bunch?

Diffraction radiation

FOR $\alpha = 0$

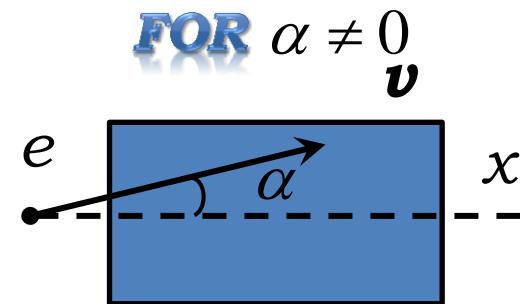
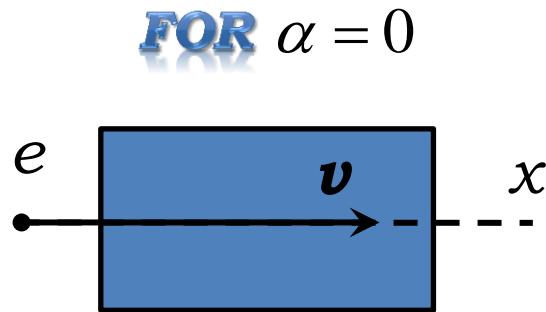


Transition radiation
↓
the same

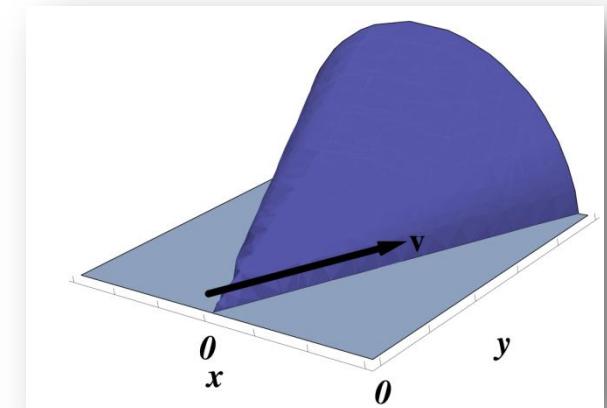
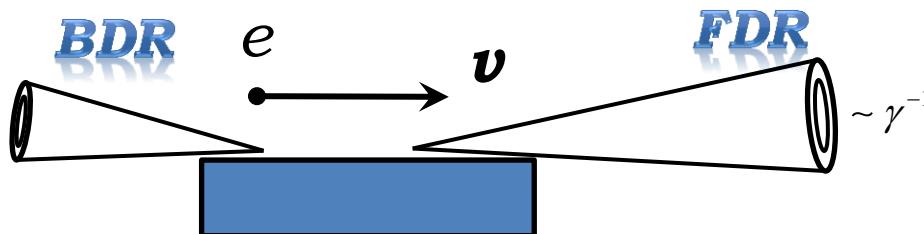
A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko,
Diffraction Radiation from Relativistic Particles, 2011

A. Potylitsyn, NIM B **145** (1998)

Diffraction radiation



MAXIMAL RADIATION



A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko,
Diffraction Radiation from Relativistic Particles, 2011

A. Potylitsyn, NIM B **145** (1998)

Cones of radiation

The current density:

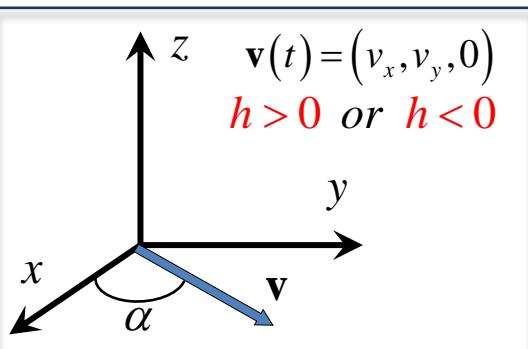
$$\mathbf{j}_0(\mathbf{r}, t) = e\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t - h\mathbf{e}_z)$$

Fourier Coulomb field:

$$\mathbf{E}_0(\mathbf{q}, \omega) = -\frac{ie}{2\pi^2} \frac{\mathbf{q} - \mathbf{v}\omega/c^2}{q^2 - k^2} \exp(-ihq_z) \delta(\omega - \mathbf{q}\mathbf{v})$$

Spectral-angular distribution:

$$\frac{dW}{d\hbar\omega d\Omega} = \exp(-2\rho h) W_0$$



Cones of radiation

The current density:

$$\mathbf{j}_0(\mathbf{r}, t) = e\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t - h\mathbf{e}_z)$$

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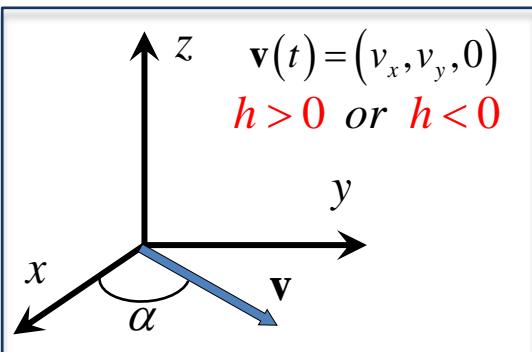
Spectral-angular distribution:

$$q_y = k_y$$

$$q_x = k_x \rightarrow q_z = i\rho \quad \rightarrow \quad \rho_{\min} = \frac{\omega}{c\beta\gamma}$$

$$\omega = \mathbf{q}\mathbf{v}$$

$$n_y = \beta_y / \beta^2$$



Cones of radiation

The current density:

$$\mathbf{j}_0(\mathbf{r}, t) = e\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t - h\mathbf{e}_z)$$

Fourier Coulomb field:

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Spectral-angular distribution:

$$q_y = k_y$$

$$q_x = k_x \rightarrow q_z = i\rho$$

$$q = k$$

$$\omega = \mathbf{q}\mathbf{v}$$

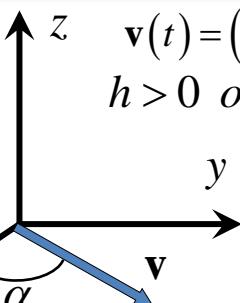
$$\rho_{\min} = \frac{\omega}{c\beta\gamma}$$

$$n_y = \beta_y / \beta^2$$

$$\frac{x^2}{1 - \frac{\beta_y^2}{\beta^4}} + \frac{z^2}{1 - \frac{\beta_y^2}{\beta^4}} - \frac{y^2}{\frac{\beta_y^2}{\beta^4}} = 0$$

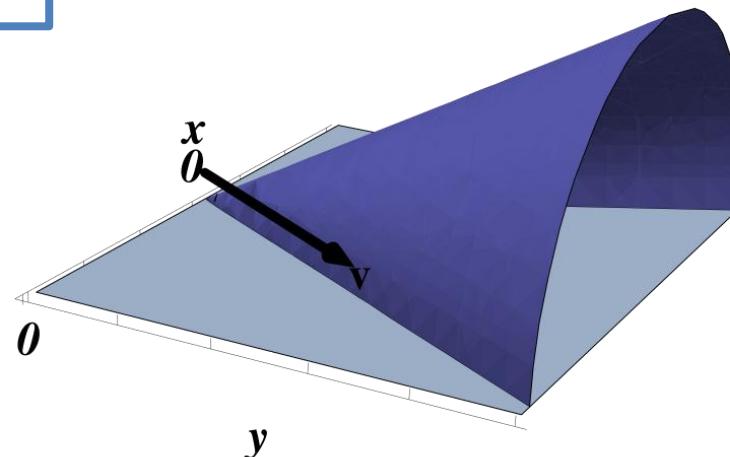
CONE!

along infinite side of the target (y)
and open angle is determined by \mathbf{v}



$$\mathbf{v}(t) = (v_x, v_y, 0)$$

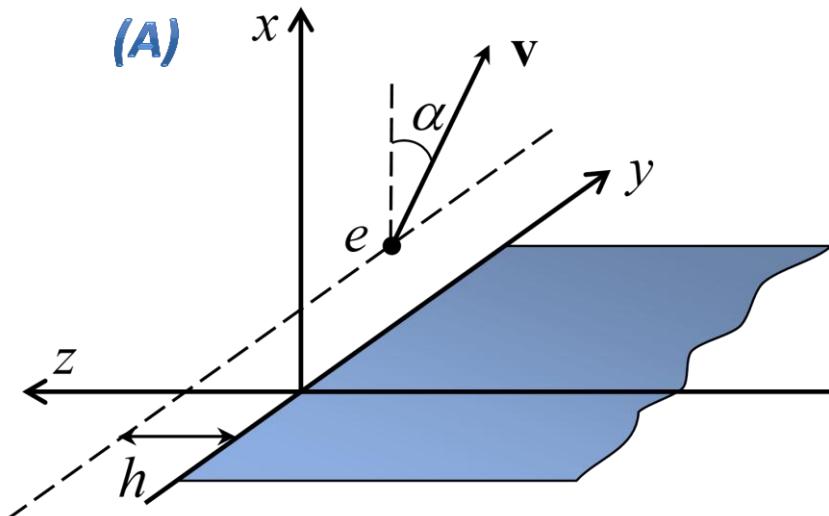
$$h > 0 \text{ or } h < 0$$



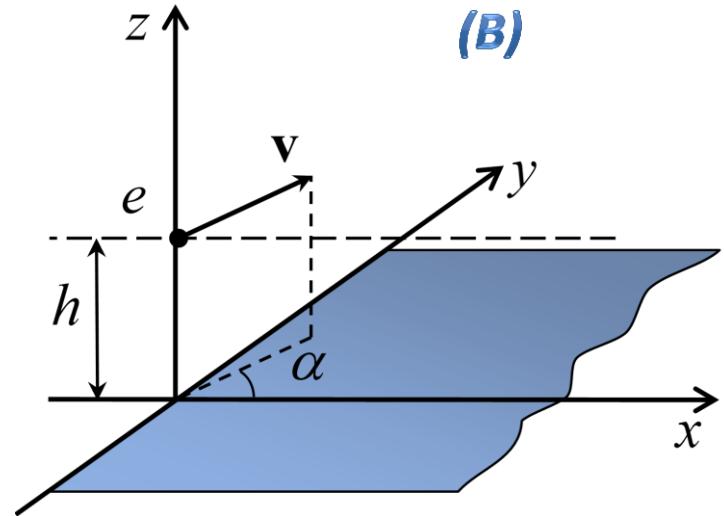
**surface
of maximal
radiation**

Geometry

$$\mathbf{v}(t) = (v_x, v_y, 0)$$



plane of the vector-velocity
perpendicular
to the plane of the target

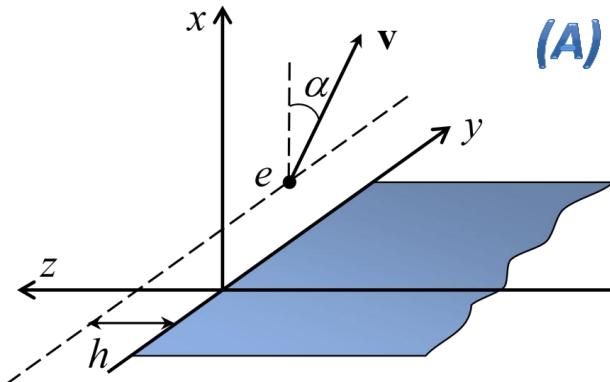


plane of the vector-velocity
parallel
to the plane of the target

X-Ray and Optics

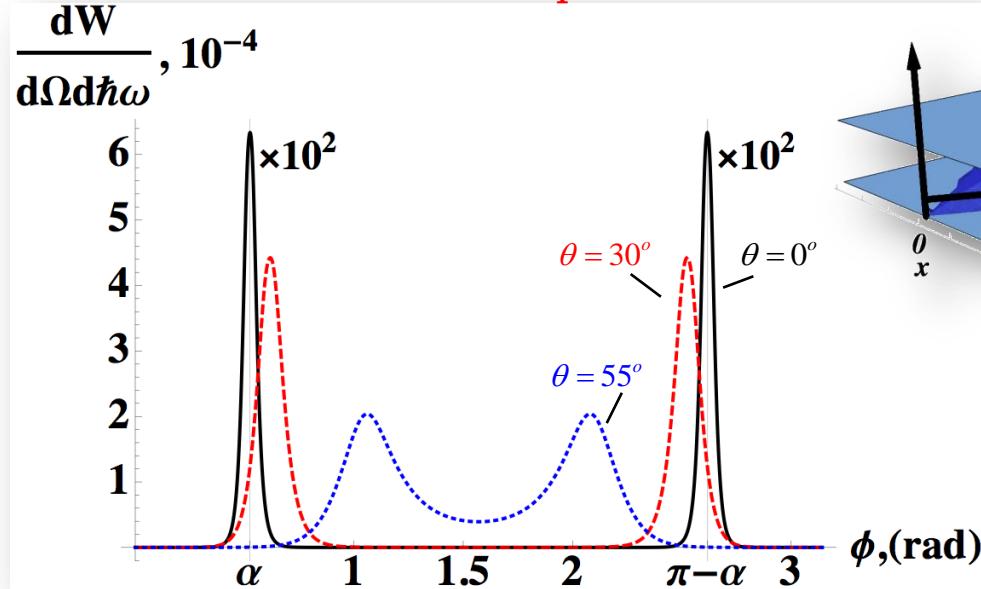
Optics

perpendicular



(A)

Forward and backward peaks are the same!



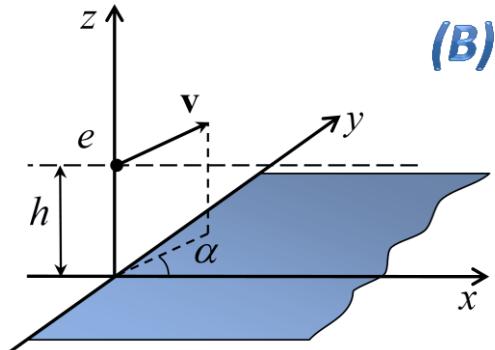
$$\frac{dW}{d\hbar\omega d\Omega} = \frac{1}{137} \frac{\beta_x (1 - \beta_y n_y)}{4\pi^2 \rho^2} \left(\frac{\omega}{c\beta_x} \right)^2 e^{-2h\rho} \xi \frac{\left(\frac{c\beta_x}{\omega} \right)^2 \rho^2 (1 + n_z \xi) + (\beta_y - n_y)^2 (1 - n_z \xi)}{\left((1 - \beta_y n_y)^2 - \beta_x^2 n_x^2 \right)}$$

$$\rho = \sqrt{1 + \gamma^2 (\beta_y^2 - 2n_y \beta_y + n_y^2 \beta^2)} \omega / (c \beta_x \gamma), \quad \xi = 1 / \sqrt{n_x^2 + n_z^2}, \quad \phi = (1 - n_x \beta_x - n_y \beta_y) \omega / (c \beta_x)$$

N. Potylitsina-Kube and X. Artru, NIM B **201** (2003)
coincidence with A. Potylitsyn, NIM B **145** (1998)

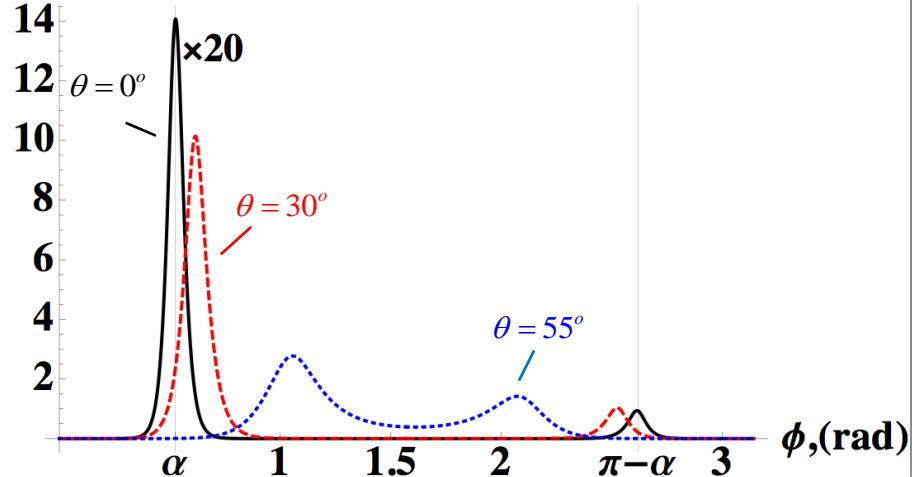
Optics

parallel



$\frac{dW}{d\Omega d\hbar\omega}, 10^{-4}$ Forward and backward peaks are different!

$\lambda = 0.5 \mu m$
 $h = 1.2 \mu m$
 $\gamma = 30$
 $\alpha = 30^\circ$



$$\frac{dW}{d\hbar\omega d\Omega} = \frac{1}{137} \frac{\beta_x (1 - \beta_y n_y)}{4\pi^2 \rho^2 \varphi^2} \left(\frac{\omega}{c\beta_x} \right)^4 \exp(-2\rho h) \times \\ \times \left[\left(\frac{c\beta_x}{\omega} \right)^2 \rho^2 \left(\xi + \frac{\beta_x}{1 - \beta_y n_y} \right) (1 - n_x \xi) + (\beta_y - n_y)^2 \left(\xi - \frac{\beta_x}{1 - \beta_y n_y} \right) (1 + n_x \xi) \right]$$

$$\rho = \sqrt{1 + \gamma^2 (\beta_y^2 - 2n_y \beta_y + n_y^2 \beta^2)} \omega / (c \beta_x \gamma), \quad \xi = 1 / \sqrt{n_x^2 + n_z^2}, \quad \varphi = (1 - n_x \beta_x - n_y \beta_y) \omega / (c \beta_x)$$

N. Potylitsina-Kube and X. Artru, NIM B **201** (2003)

X-Rays, Polarization current method

$$\omega \gg \omega_p \quad \varepsilon(\omega) = 1 + \chi'(\omega) + i\chi''(\omega), \quad \chi'' \ll |\chi'(\omega)|, \quad \chi'(\omega) = -\omega_p^2/\omega^2.$$

Polarization current density

$$\mathbf{j}(\mathbf{r}, \omega) = \frac{\omega}{4\pi i} [\varepsilon(\omega) - 1] \mathbf{E}_0(\mathbf{r}, \omega)$$

Field of radiation

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{i\omega}{c^2} \frac{e^{ikr}}{r} \left(\mathbf{n} \times \mathbf{n} \times \int_V d^3 r \mathbf{j}(\mathbf{r}, \omega) e^{-ik \cdot \mathbf{r}} \right)$$

Spectral-angular distribution

$$\frac{dW(\mathbf{n}, \omega)}{d\Omega d\omega} = cr^2 |\mathbf{E}(\mathbf{r}, \omega)|^2$$

A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko, *Diffraction Radiation from Relativistic Particles*, 2011

X-Rays, Spectral-angular distribution

$$\frac{d^2E(\mathbf{n},\omega)}{d\Omega d\hbar\omega} = \frac{1}{137} \left(\frac{\varepsilon(\omega) - 1}{4\pi\beta_x\varphi} \right)^2 e^{-2\rho h} \left| 1 - \exp \left(-b\rho + i\sqrt{\varepsilon(\omega)}b \frac{\omega}{c} n_z' \right) \right|^2 \frac{\omega^4}{c^4} \frac{\left[\mathbf{n}' \times \mathbf{n}' \times \left(\frac{\mathbf{A}}{\rho} - i\mathbf{e}_z \right) \right]^2}{\left| \rho - i\sqrt{\varepsilon(\omega)} \frac{\omega}{c} n_z' \right|^2} 4 \sin^2 \left(\frac{a\varphi}{2} \right)$$

$$\varphi = \frac{\omega}{c} \frac{1}{\beta_x} \left(1 - \sqrt{\varepsilon(\omega)} n'_x \beta_x - \sqrt{\varepsilon(\omega)} n'_y \beta_y \right)$$

$$\rho = \frac{\omega}{c \beta_x \gamma} \sqrt{1 + \gamma^2 (\beta_y^2 - 2n_y \beta_y + n_y^2 \beta^2)}$$

$$\mathbf{A} = \frac{\omega}{\beta_x c} (1 - n_y \beta_y - \beta_x^2, n_y \beta_x - \beta_x \beta_y, 0)$$

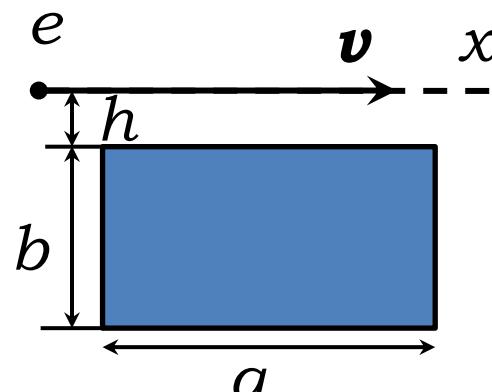
\mathbf{n}' – *in the matter*

$$a \gg b$$

$$n'_z \gg n'_x \frac{b^*}{a}, \quad b^* = \min \left\{ \frac{\gamma \beta \lambda}{2\pi} - h, b \right\}$$

$$\sqrt{\varepsilon(\omega)} \mathbf{n}' = (n_x, n_y, \sqrt{\varepsilon(\omega) - 1 + n_z^2})$$

$$n_z^{\min} = \sqrt{1 - \varepsilon(\omega)}$$



$$a \ll b$$

$$n'_z \ll n'_x \frac{b^*}{a}, \quad b^* = \min \left\{ \frac{\gamma \beta \lambda}{2\pi} - h, b \right\}$$

$$\sqrt{\varepsilon(\omega)} \mathbf{n}' = (\sqrt{\varepsilon(\omega) - 1 + n_x^2}, n_y, n_z)$$

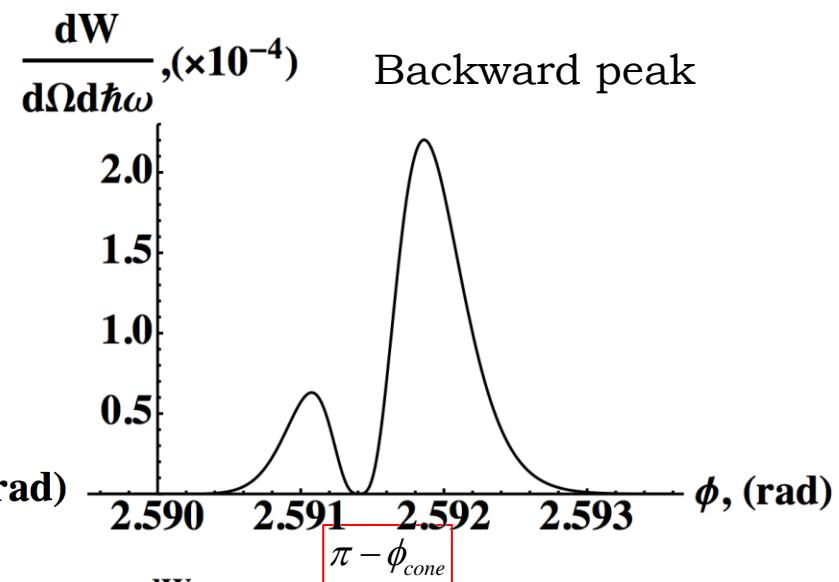
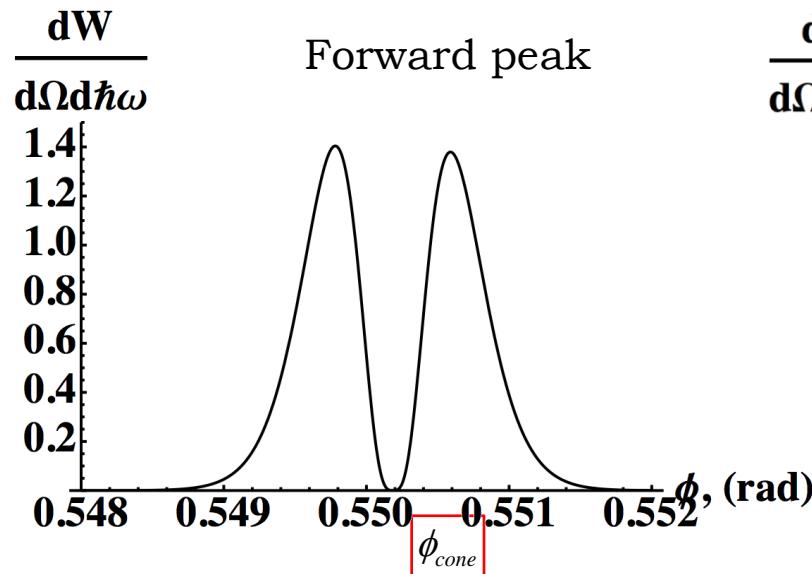
$$n_x^{\min} = \sqrt{1 - \varepsilon(\omega)}$$

D.Y. Sergeeva, A.A. Tishchenko,
Proc. of FEL 2014, Basel, Switzerland, TUPO14 - *the laws of refraction for both edges*

X-Rays, Two geometries

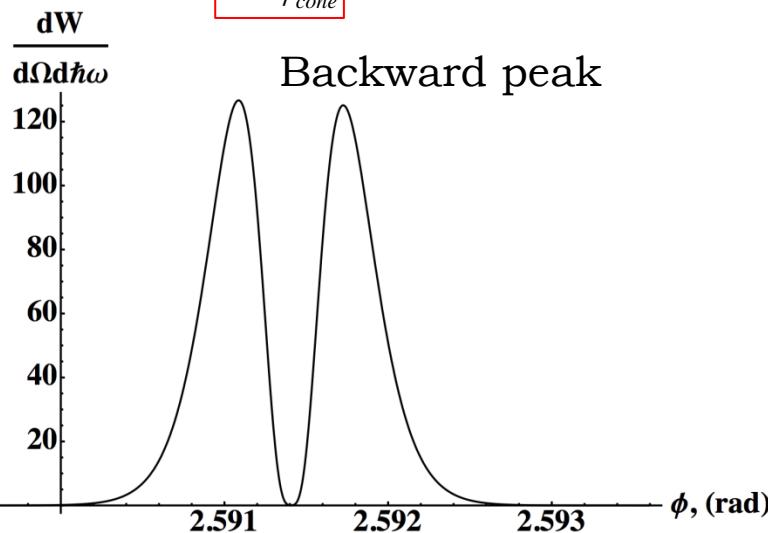
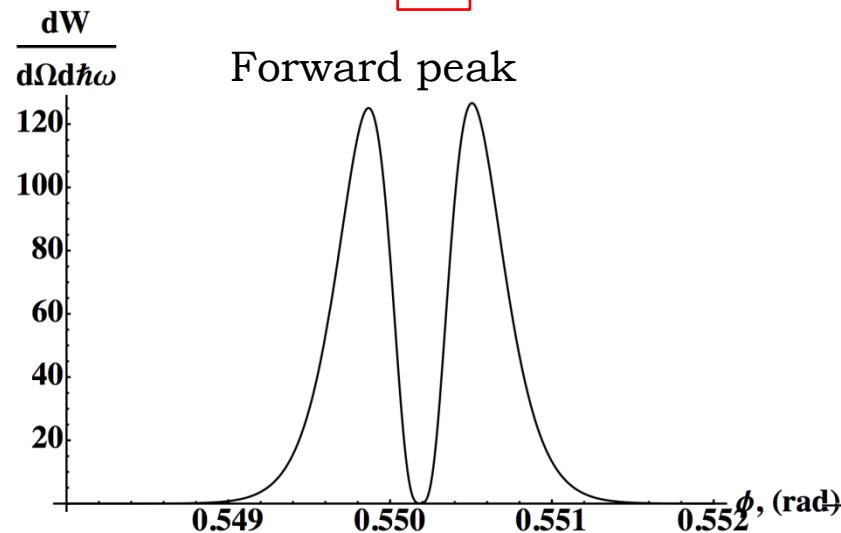
$a \gg b$

$\lambda = 12 \text{ nm}$
 $h = 10 \mu\text{m}$
 $\gamma = 4 \cdot 10^4$
 $\alpha = 30^\circ$
 $\theta = 17^\circ$
 $a = 0.9 \mu\text{m}$
 $b = 0.1 \mu\text{m}$



$a \ll b$

$\lambda = 12 \text{ nm}$
 $h = 10 \mu\text{m}$
 $\gamma = 4 \cdot 10^4$
 $\alpha = 30^\circ$
 $\theta = 17^\circ$
 $a = 0.65 \mu\text{m}$
 $b = 6.5 \mu\text{m}$



Divergent beam

Polarization current method, but

$$\mathbf{E}_0(\mathbf{q}, \omega) = -\sum_{m=1}^M \frac{ie}{2\pi^2} \frac{\mathbf{q} - \mathbf{v}_m \frac{\omega}{c^2}}{q^2 - \frac{\omega^2}{c^2}} e^{-i\mathbf{q}\mathbf{r}_m} \delta(\omega - \mathbf{q}\mathbf{v}_m)$$

Field of radiation

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{\omega^2}{4\pi c^2} \frac{e^{ik'r}}{r} (\epsilon(\omega) - 1) \left[\mathbf{n}' \left[\mathbf{n}' \int d^3 r' e^{-ik'r'} \mathbf{E}_0(\mathbf{r}', \omega) \right] \right]$$

Spectral-angular distribution

$$\frac{dW_m(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} = \left\langle cr^2 |\mathbf{E}(\mathbf{r}, \omega)|^2 \right\rangle_{\alpha, \mathbf{r}}$$

The law of refraction for the
upper edge of the target

Function of distribution

$$f(\mathbf{r}, \mathbf{v}) = \frac{1}{2\pi^{3/2} \sigma_x \sigma_z \sigma_\alpha \sqrt{\sigma_y^2 + x^2 \tau^2}} \exp \left[-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2 + x^2 \tau^2} - \frac{\alpha^2}{\sigma_\alpha^2} \right]$$

Averaging \longrightarrow Integrating

$$\frac{dW_m(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} = \int d\mathbf{r} \int d\alpha [N W_{inc} + N(N-1) W_{coh}]$$

Laplace's method,
two critical points

Coherent and incoherent parts

$$W_{coh(inc)} \sim \int d\alpha \exp[Gauss] \exp[cones]$$

Divergent beam

$$\frac{dW_m}{d\Omega d\hbar\omega} = \frac{1}{137} \frac{(\varepsilon-1)^2}{(2\pi)^2} \frac{\omega^4}{c^4} (NW_{inc} + N(N-1)W_{coh})$$

$$W_{inc} = |B(\rho_1, \mathbf{A}_1, \varphi_1)|^2 \frac{sh(2\rho\sigma_z)}{2\rho\sigma_z} \sqrt{\frac{c\beta}{2h\omega\gamma}} \frac{1}{1-n_y^2\beta^2} \exp\left[-\frac{\arcsin^2(\beta n_y)}{\sigma_\alpha^2}\right] \frac{1}{\sigma_\alpha} \Delta(n_y) + |B(\rho_1, \mathbf{A}_1, \varphi_1)|^2 \frac{sh(2\rho\sigma_z)}{2\rho\sigma_z}$$

$$W_{coh} = \frac{1}{\xi^2} \exp\left[-\frac{\sigma_y^2 n_y^2}{2} \frac{\omega^2}{c^2} - \frac{\sigma_x^2}{4\xi^2 \beta^2} \frac{\omega^2}{c^2}\right] \left| B(\rho_1, \mathbf{A}_1, \varphi_1) \frac{sh(\rho\sigma_z)}{\rho\sigma_z} \frac{1}{\sqrt{1-n_y^2\beta^2}} \frac{1}{\sqrt{h\frac{\omega\gamma}{c\beta} + \frac{1}{2\beta^2} \frac{\omega^2}{c^2} \frac{\sigma_x^2}{\xi^2}}} e^{\frac{\sigma_y^2 \omega^2}{4\xi^2 c^2 n_y^2}} \frac{1}{\sigma_\alpha} e^{-\frac{\arcsin^2(n_y\beta)}{\sigma_\alpha^2}} \Delta(n_y) + B(\rho_2, \mathbf{A}_2, \varphi_2) \frac{sh(\rho\sigma_z)}{\rho\sigma_z} \right|^2$$

$$B(\rho, \mathbf{A}, \varphi) = e^{-\rho h} \frac{\sin\left(\frac{\varphi a}{2}\right)}{\varphi} \frac{1}{\beta} \frac{1-e^{-b\left(\rho-i\frac{\omega}{c}\sqrt{\varepsilon(\omega)-1+n_z^2}\right)}}{\rho-i\frac{\omega}{c}\sqrt{\varepsilon(\omega)-1+n_z^2}} \left(\mathbf{n}' \left(\frac{\mathbf{n}' \mathbf{A}}{\rho} - i n'_z \right) - \frac{\mathbf{A}}{\rho} + i \mathbf{e}_z \right) \quad \Delta(x) = \begin{cases} 0, & x=0 \\ 1, & x \neq 0 \end{cases} \quad \xi = \sqrt{1 + \left(\frac{\omega}{2c} \sigma_x n_y \tau \right)^2}$$

$$\rho_2 = \frac{\omega}{c\beta\gamma} \sqrt{1+\gamma^2 n_y^2 \beta^2} \quad \rho_1 = \frac{\omega}{c\beta\gamma}$$

$$\mathbf{A}_2 = \frac{\omega}{c\beta} \frac{1}{\gamma^2} \{1; \gamma^2 \beta n_y; 0\} \quad \mathbf{A}_1 = \frac{\omega}{c\beta} \frac{1}{\gamma^2} \{\sqrt{1-\beta^2 n_y^2}; \beta n_y; 0\}$$

$$\varphi_2 = \frac{\omega}{c\beta} (1 - n_x \beta) \quad \varphi_1 = \frac{\omega}{c\beta} (\sqrt{1-\beta^2 n_y^2} - \beta n_x)$$



Monday,
PS1-26
D.Yu. Sergeeva *et al.*

Conclusions

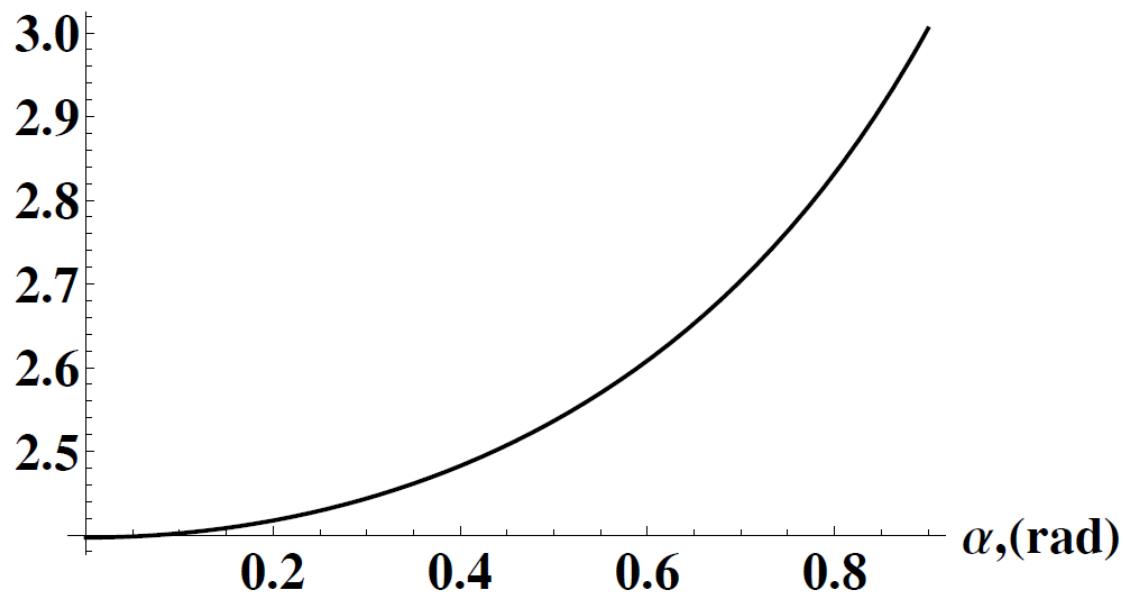
- Diffraction radiation in case of oblique incidence of a particles is redistributed over a **cone surface** – this is **strong spatial effect**. The open angle of the cone is determined by direction of particle velocity \mathbf{v} .
- **Analytical theory** is created both for optics and X-Rays.
- The intensities of **forward and backward DR** on the cone depend on the target orientation both for optics and X-Rays.
- The conical influences on intensity of radiation from divergent beam changing the **form-factor** of the bunch. Dependence of form-factor on divergence, predicted in work of V. Shpakov and S. Dabagov (2014), should be even stronger because of conical effect in spatial distribution of radiation.

A scenic coastal town built into a hillside overlooking the sea. The town is densely packed with white buildings with red roofs, nestled among green trees and bushes. A winding road leads down from the town towards a harbor where several boats are docked. The sky is clear and blue.

THANK YOU
FOR YOUR ATTENTION!

Optics, α - dependence

$\frac{dW}{d\Omega d\hbar\omega}, 10^{-4}$ perpendicular



$$\alpha \leq \arcsin(\beta \cos \theta)$$

N. Potylitsina-Kube and X. Artru, NIM B **201** (2003)