Peculiarities of parametric gamma-rays in condition of anomalous transmission

A. Ahmadi*, I. Feranchuk, A. Lobko*****

*Physics Department, Malayer Branch, Islamic Azad University, Malayer, Iran
**Belarusian State University, Minsk, Belarus
***Institute for Nuclear Problems, Minsk, Belarus

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OUTLOOK

Possibility to generate resonant gamma-rays for the Mössbauer experiments - parametric gamma-radiation (**PGR**) - was proposed in our report before and specific features of this radiation were analyzed qualitatively.

PGR intensity is proportional to the absorption length which is rather small for the resonant gamma-radiation.

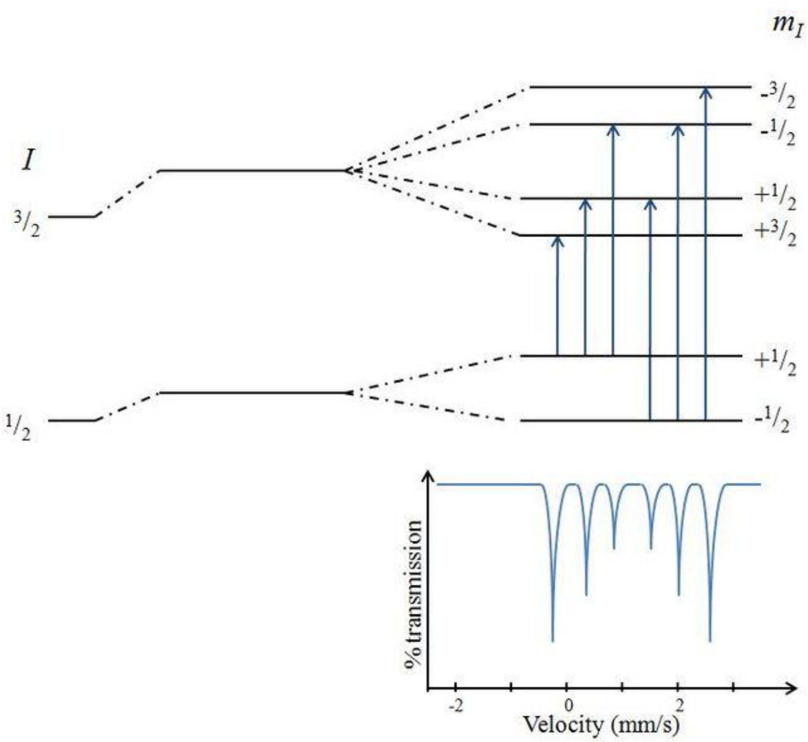
The absorption length increases essentially at the diffraction condition due to the Borrmann effect and this effect is very important for the resonant gamma-quanta moving in the Mössbauer crystals.

Characteristics of PGR in the case of the dynamical diffraction are considered in order to find the optimal conditions for increase of the PGR intensity.

Hyperfine splitting of the PGR peaks due to internal magnetic field in the Mössbauer crystals is considered

Mössbauer effect

In 1957, Rudolf Mössbauer discovered that, in some circumstances, if the nucleus is bound in a crystal lattice, the whole crystal recoils rather than the individual nucleus. Due to the much greater mass involved in recoil the energy of the emitted ray is very close to that of the difference in energy between the nuclear energy levels, and **resonant absorption is possible**.



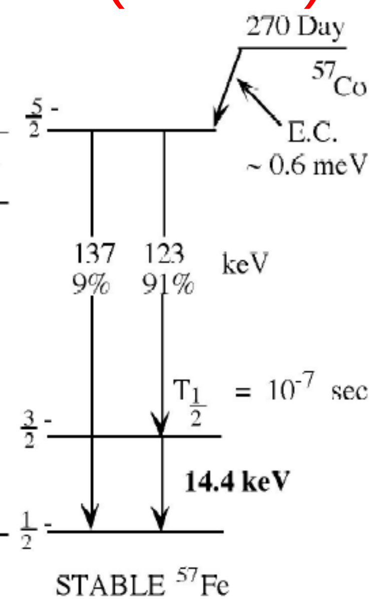
Magnetic splitting of the nuclear energy levels and the corresponding Mössbauer spectrum

Mössbauer spectroscopy is limited by the need for a suitable gamma-ray source. And samples must be solid (frozen)

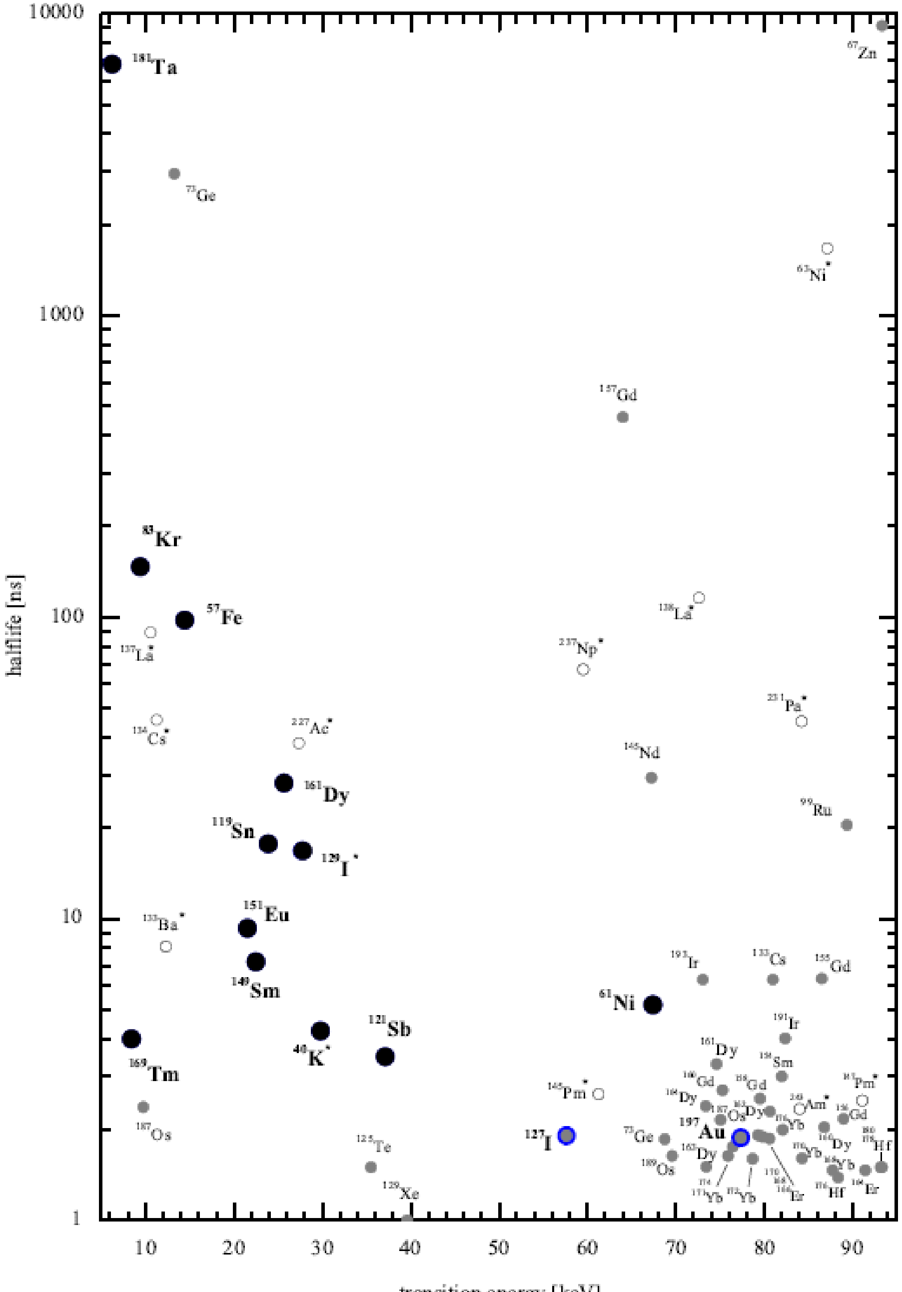
PROPERTIES OF ^{57}Fe α

	Ground state	First excited state
Energy (keV)	0	14.36
Spin and parity	$\frac{1}{2}$ -	$\frac{3}{2}$ -
Magnetic moment (nm)	0.0903	-0.153
Quadrupole moment (barns)	0	0.29
Mean life (sec)	Stable	1.4×10^{7}

α Internal conversion coefficient: 9.7 ± 0.2.



Family of Mossbauer isotopes



Small grey circles represent the isotopes examined with the standard Mossbauer effect.

From: T. Roth, PhD Thesis, 2005

PROSPECTS

Mossbauer spectroscopy can probe the hyperfine fields, measuring local properties of materials (structural and magnetic)

Hyperfine interactions: interactions between a nucleus and its environment (isomer shift, quadrupole splitting, magnetic splitting)

Conversion electron Mossbauer spectroscopy allows to study magnetic properties of nanostructure of smono-layer thickness

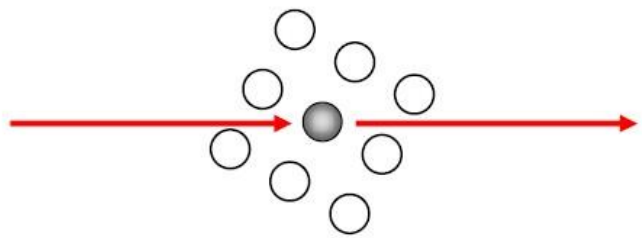
Accelerator-based Mossbauer spectroscopy can provide spectra in time domain to addition of energy domain (standard MS)

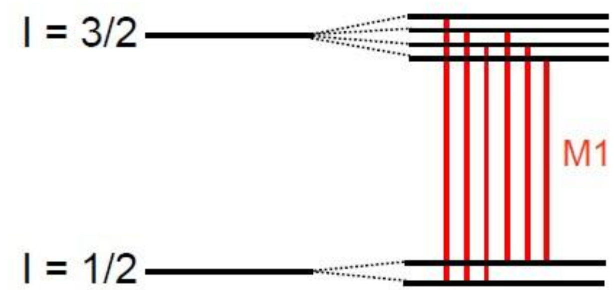
Accelerator-based Mossbauer spectroscopy can provide polarized photons necessary for magnetic measurements

Accelerator-based Mossbauer spectroscopy can deal with non-conventional Mossbauer isotopes

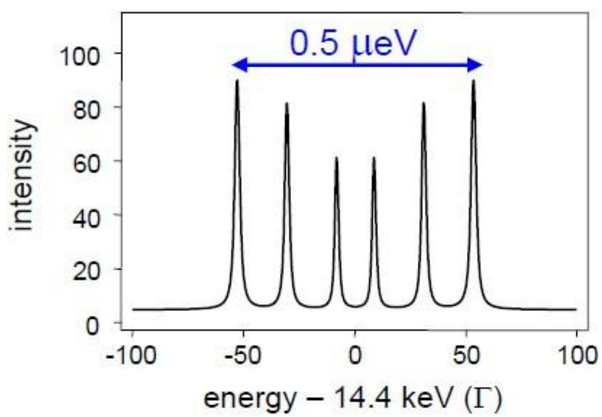
Mossbauer spectra in energy and time domains

Nucleus embedded in lattice:

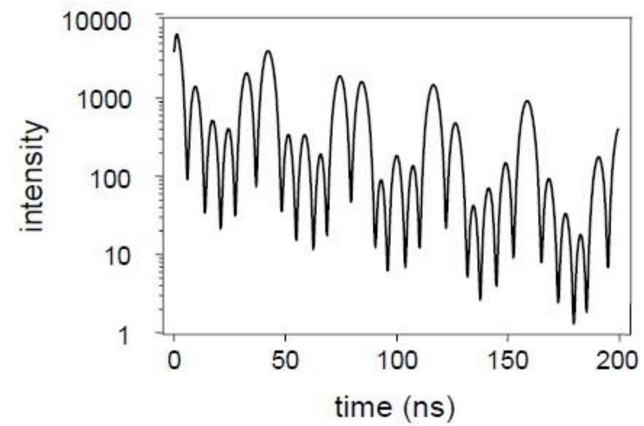




Energy domain :



Time domain :

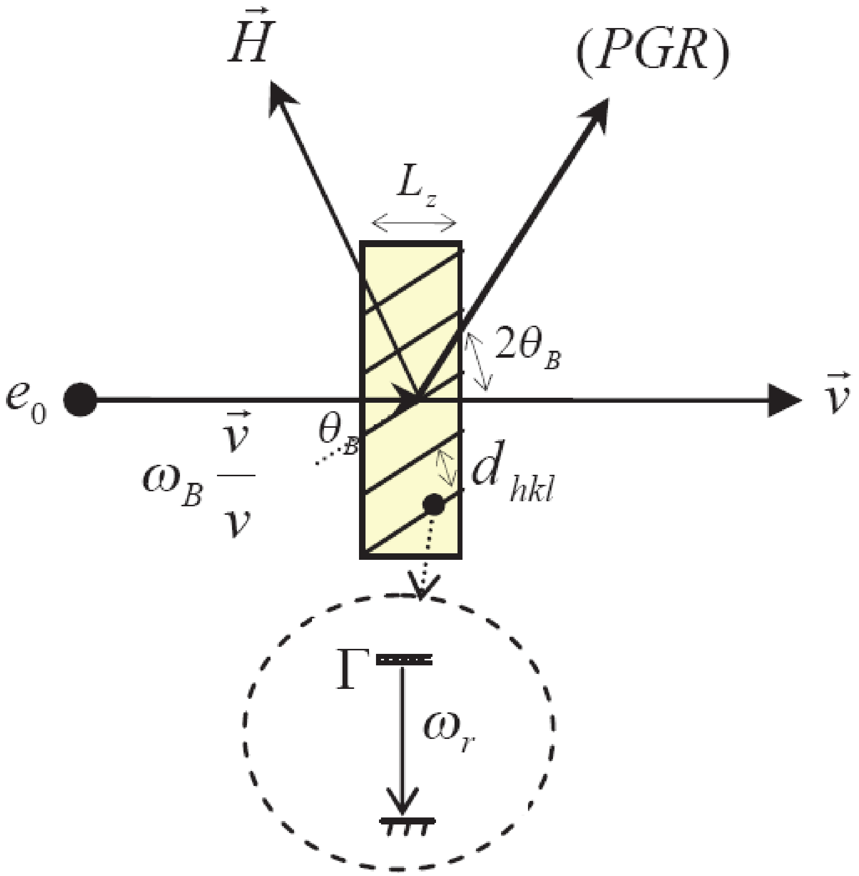


quantum beats

due to hyperfine splitting of nuclear states

FORMATION OF PGR

$$\vec{k}_B = \omega_{\mathbf{B}} \frac{\vec{v}}{v} + \vec{H}$$



Main parameters and vectors for formation of PGR

$$\omega_B = \frac{|\vec{H}|}{2\sin\theta_B} \approx \omega_r;$$

$$\sin\theta_r \approx \sin\theta_B = \frac{|\vec{H}|}{2\omega_r}$$

$$\frac{\Delta\omega}{\omega_r} \approx \frac{1}{\omega_r L_z}$$

PGR CHARACTERISTIC PARAMETERS

$$\frac{\partial N_r}{\partial \omega} \approx \frac{e_0^2 \cos 2\theta_r}{4 \sin 2\theta_r} \gamma L_z |\chi(\mathbf{H}, \omega)|^2;$$

$$\chi(\mathbf{H}, \omega) = \chi_e(\mathbf{H}, \omega_r) + \chi_n(\omega),$$

$$\gamma = \frac{E}{m};$$

Electronic part of the polarisability

$$\chi_e(\mathbf{H}, \omega_r) = \frac{4\pi S(\mathbf{H})}{\Omega_0 \omega_r^2} f_e(\mathbf{H}), \quad f_e(\mathbf{H}) = -\frac{e_0^2}{m} F_a(\mathbf{H})$$

Nuclear part of the polarisability

$$\chi_n(\omega) = \frac{4\pi S(\mathbf{H})}{\Omega_0 \omega_r^2} \eta f_n(\omega),$$

$$f_n(\omega) = -\frac{1}{\omega_r(1+\alpha_c)} \frac{\Gamma/2}{\omega - \omega_r + i\Gamma/2}.$$

DYNAMICAL THEORY OF PGR (1)

PGR spectral density in thick crystal is defined by formula (Borrmann effect is most essential for sigma-polarization)

$$\frac{\partial N_\sigma^g}{\partial \omega} = \frac{e_0^2}{\hbar\omega\pi^2 c} 2 \int_0^\infty \theta d\theta \int_0^{\pi/2} d\varphi \{\Phi[\psi(\theta), \alpha_B(\theta,\varphi)] + \Phi[\psi(\theta), \alpha_B(\theta, \varphi + \pi/2)]\};$$

$$\Phi[\psi(\theta), \alpha_B(\theta,\varphi)] = \left(\theta^2 \sin^2\varphi + \frac{1}{2}\theta_s^2\right) \frac{|\gamma_0|\beta^2|\chi_g|^2}{4|\epsilon_{1\sigma} - \epsilon_{2\sigma}|^2} \left|\sum_{\mu=1}^{2} (-1)^\mu \frac{(1 - e^{-ikLq_{\mu\sigma}^g \gamma_0^{-1}})}{q_{\mu\sigma}^g}\right|^2.$$

DYNAMICAL THEORY OF PGR (2)

$$\psi = \theta^2 + \gamma^{-2} + \theta_s^2(L) \equiv \theta^2 + \theta_0^2;$$

$$\alpha_B = 4\sin^2\theta_B \frac{\omega - \omega_B}{\omega_B} - 2\theta\cos\varphi\sin 2\theta_B.$$

$$q_{\mu s}^{(g)} = \frac{1}{k_B L_{\mu s}^{(g)}} = \frac{m^2 c^4}{E^2} + \theta_{(g)}^2 + \theta_s^2(L) - 2\epsilon_{\mu s};$$

$$\theta_s^2(L) = \frac{E_s^2}{E^2}\frac{L}{L_R}.$$

$$\epsilon_{\mu s} = \frac{1}{4}\left\{[\chi_0 + \beta\chi_0 - \beta\alpha_B] \pm \sqrt{[\chi_0 + \beta\chi_0 - \beta\alpha_B]^2 + 4\beta[\chi_0\alpha_B - (\chi_0^2 - c_s^2\chi_{\vec{g}}\chi_{-\vec{g}})]}\right\}$$

MAXIMUM OF THE SPECTRAL DENSITY

Maximum of the spectral corresponds to the angular parameters, that are defined from the equations:

$$\frac{\partial D}{\partial \psi} = \frac{\partial D}{\partial \alpha_B} = 0; \; D(\psi, \alpha) = [\Re q^g_{\mu\sigma}]^2 + [\Im q^g_{\mu\sigma}]^2$$

Increase of the PGR intensity due to anomalous transmission (Borrmann effect) in comparison with kinematical theory is defined by the value:

$$\xi(x) = \frac{[\chi_0''(x)]^2}{[\Im q^g_{2\sigma}(x, \psi_0, \alpha_{01})]^2} \left[1 - \exp\left(-\frac{\omega_r L}{c\beta} |\Im q^g_{2\sigma}(x, \psi_0, \alpha_{01})|\right)\right]^2.$$

$$x = \frac{2\hbar(\omega - \omega_r)}{\Gamma};$$

NUMERICAL RESULTS FOR FE CRYSTAL WITHOUT SPLITTING OF RESONANT LEVELS (1)

Polarizabilities of the crystal near resonance:

$$\chi_0 = \chi(0, \omega_r) = \chi'_{0e}\left[1 + \frac{Ax}{x^2+1}\right] + i\left[\chi''_{0e} - \frac{A}{x^2+1}\chi'_{0e}\right]$$

$$\chi_{\vec{g}} = \left[\chi'_{\vec{g}e} + \frac{Ax}{x^2+1}\chi'_{0e}\right] + i\left[\chi''_{\vec{g}e} - \frac{A}{x^2+1}\chi'_{0e}\right];$$

$$x = \frac{2\hbar(\omega - \omega_r)}{\Gamma}; \quad A = \eta\frac{mc^3}{e_0^2\omega_r(1+\alpha_c)Z}.$$

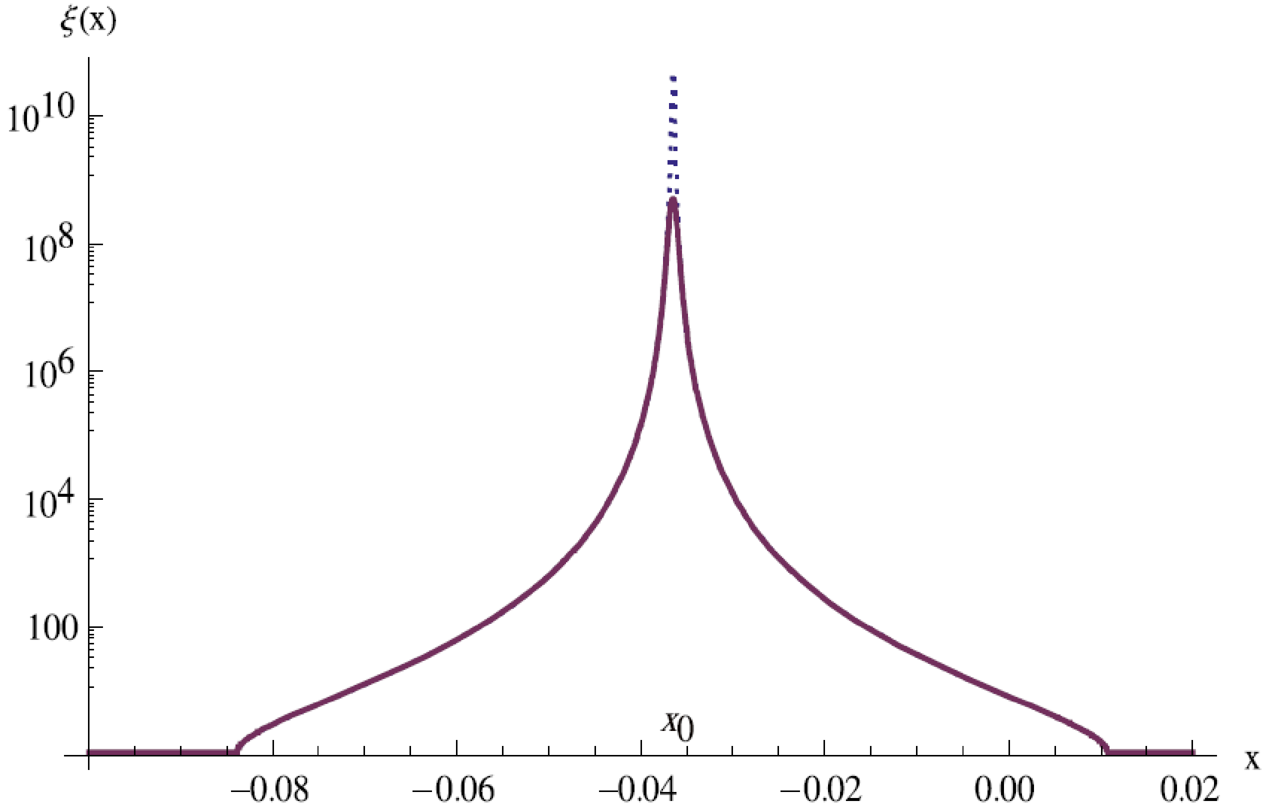
$$\hbar\omega_r = \hbar\omega_B = 14.41 \text{ keV};\ \Gamma = 4.66 \times 10^{-12} \text{ keV};$$

$$\theta_B = 12.26^o;\ \beta = \gamma_0 = 0.9099;\ A = 19.0$$

$$\chi_{0e} = -0.1487 \times 10^{-4} + i0.6921 \times 10^{-6};$$

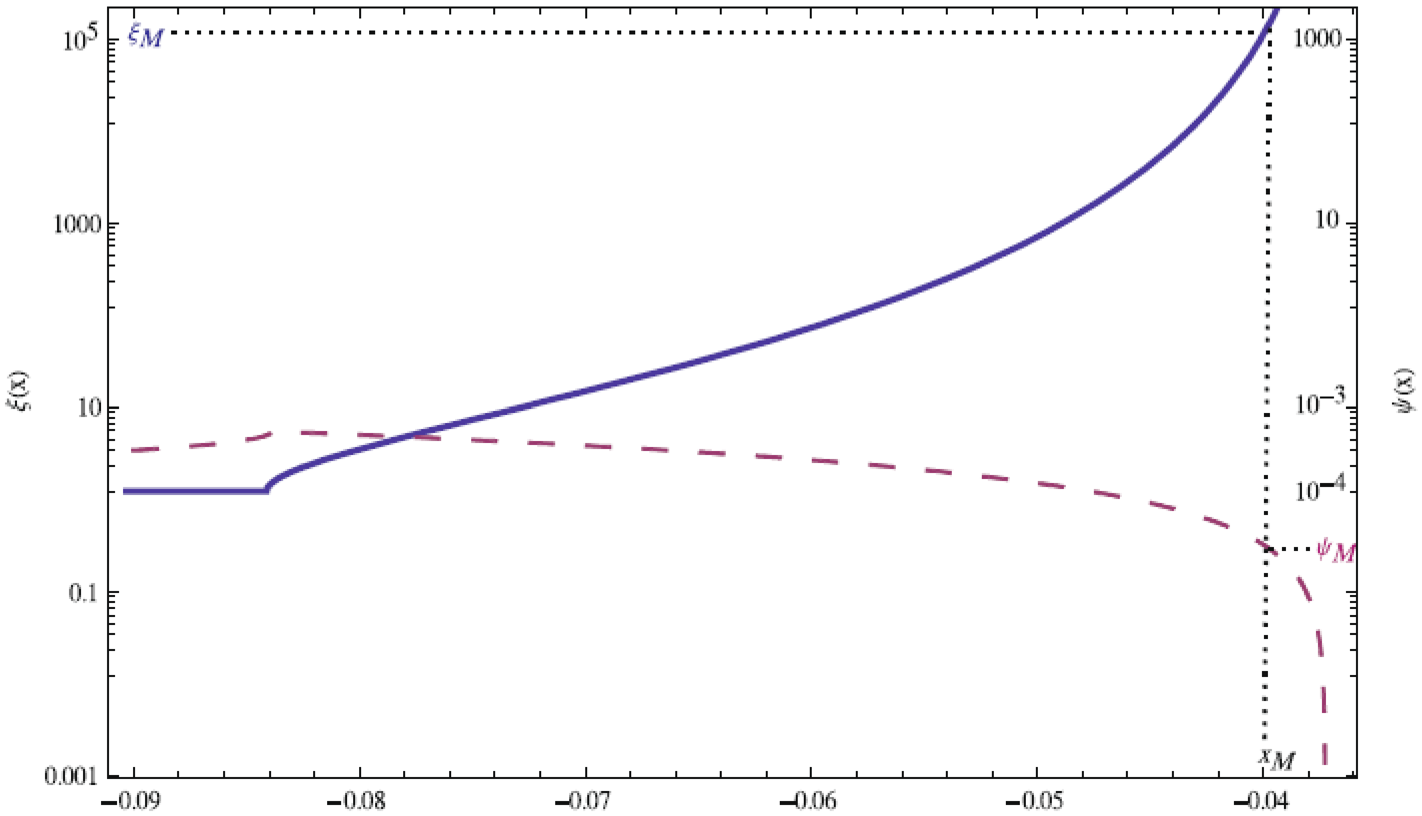
$$\chi_{\vec{g}e} = -0.1030 \times 10^{-4} + i0.6690 \times 10^{-6}.$$

NUMERICAL RESULTS FOR FE CRYSTAL WITHOUT SPLITTING OF RESONANT LEVELS (2)



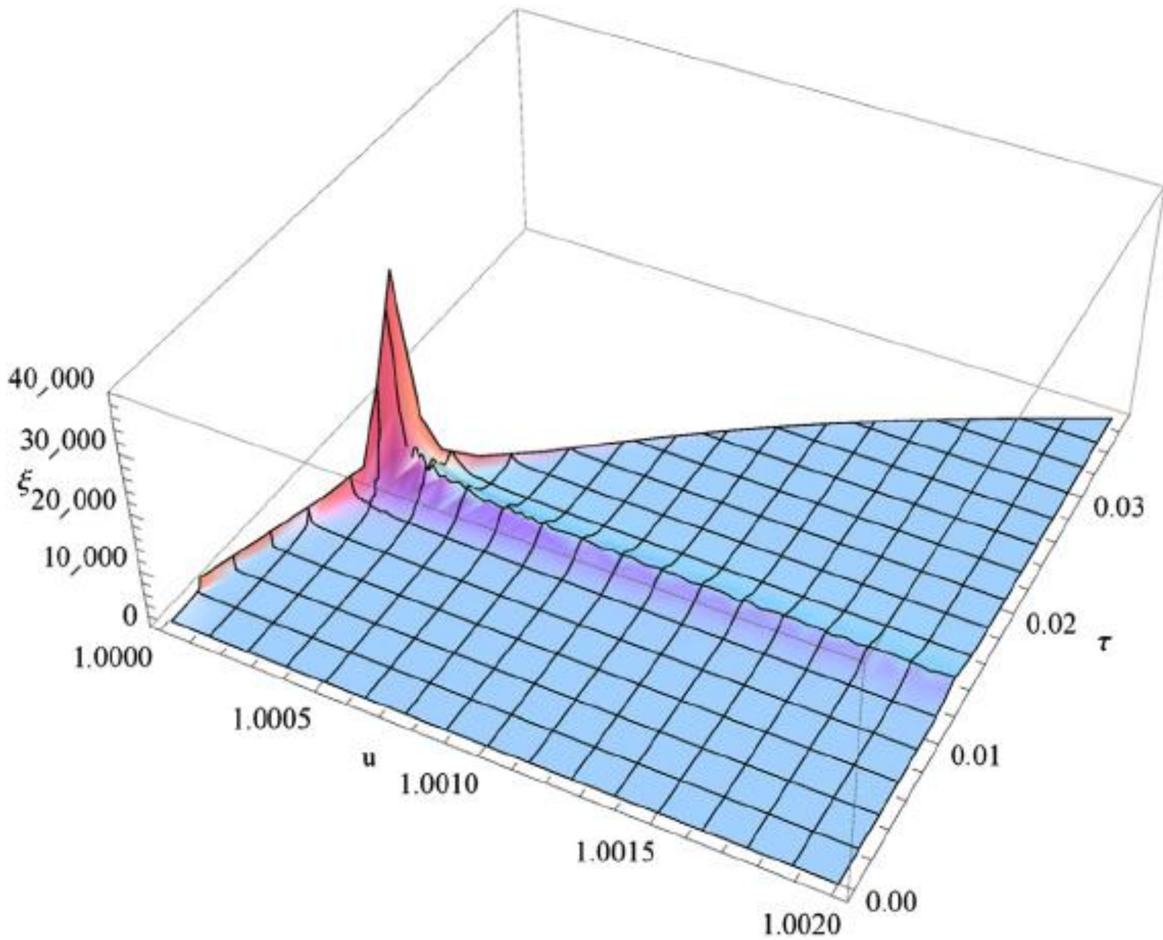
Formal solution of the equations; crystal thickness L = 0.1 cm (solid line); L=1.0 cm (dashed line)

NUMERICAL RESULTS FOR FE CRYSTAL WITHOUT SPLITTING OF RESONANT LEVELS (3)



Relative increase of the PGR intensity in the physically permitted range; dashed line corresponds to the electron energy E =855 MeV; crystal thickness L = 0.1 cm

NUMERICAL RESULTS FOR FE CRYSTAL WITHOUT SPLITTING OF RESONANT LEVELS (4)



$$\psi = u\theta_0^2;\ t = \alpha_B = \tau\theta_0.$$

$$\theta_0^2 = \gamma^{-2} + \theta_s^2(L)$$

Angular distribution of the PGR intensity at the resonance

x = - 0.04

X-RAY CRYSTAL POLARIZABILITY WITH SPLITTING (1)

Mössbauer crystal Fe includes six resonant lines with splitting:

$$\Delta E = \mu_N B_{int}(3g_e + g_0)$$

It leads to 6 peaks in the x-ray polarizability

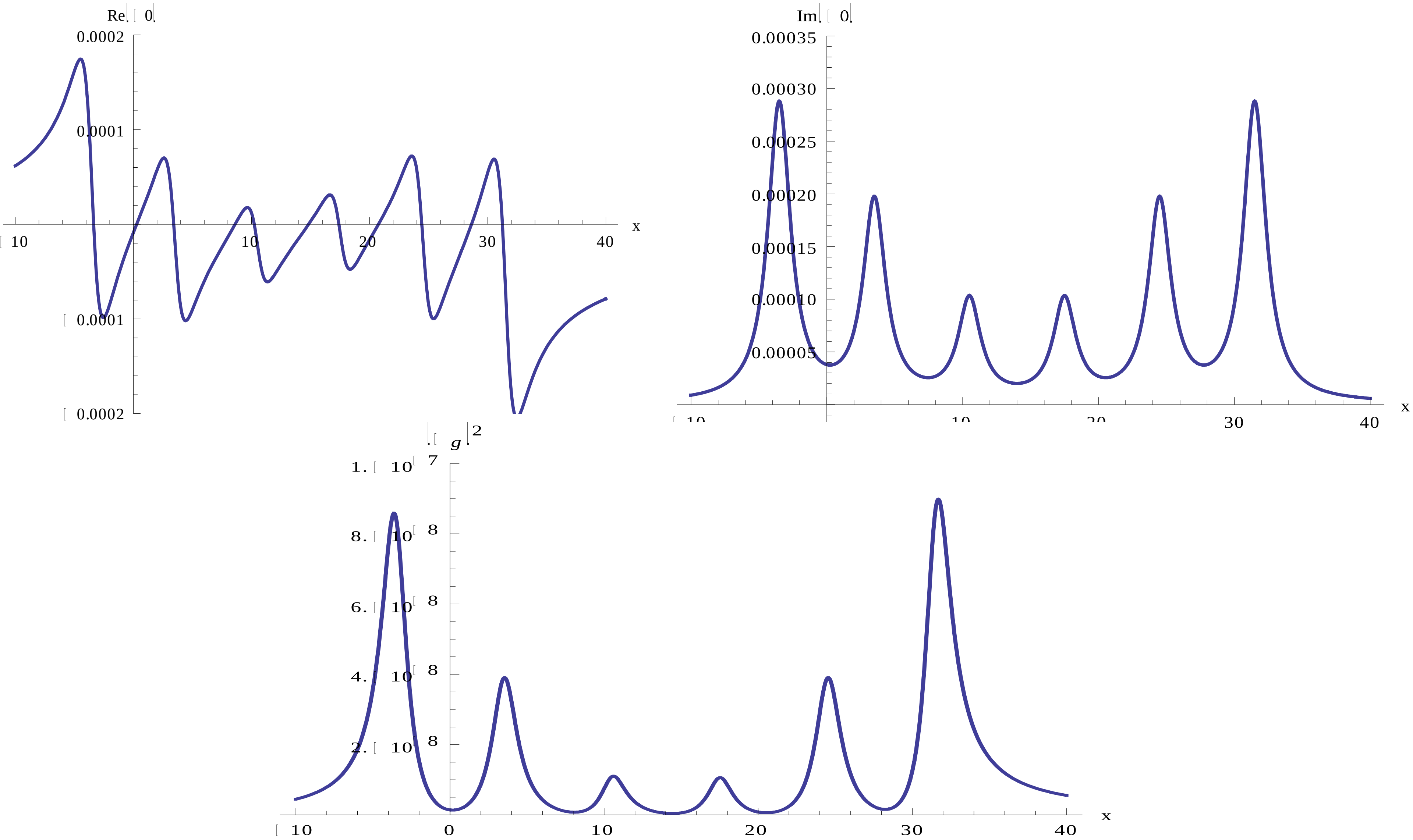
$$\chi_0 = \chi'_{0e}[1 + A\sum_{m=1}^{6}\frac{\lambda_m(x - x_m)}{(x - x_m)^2 + 1}] + i[\chi''_{0e} - A\sum_{m=1}^{6}\frac{\lambda_m}{(x - x_m)^2 + 1}\chi'_{0e}];$$

$$\chi_{\vec{g}} = [\chi'_{\vec{g}e} + A\sum_{m=1}^{6}\frac{\lambda_m(x - x_m)}{(x - x_m)^2 + 1}\chi'_{0e}] + i[\chi''_{\vec{g}e} - A\sum_{m=1}^{6}\frac{\lambda_m}{(x - x_m)^2 + 1}\chi'_{0e}];$$

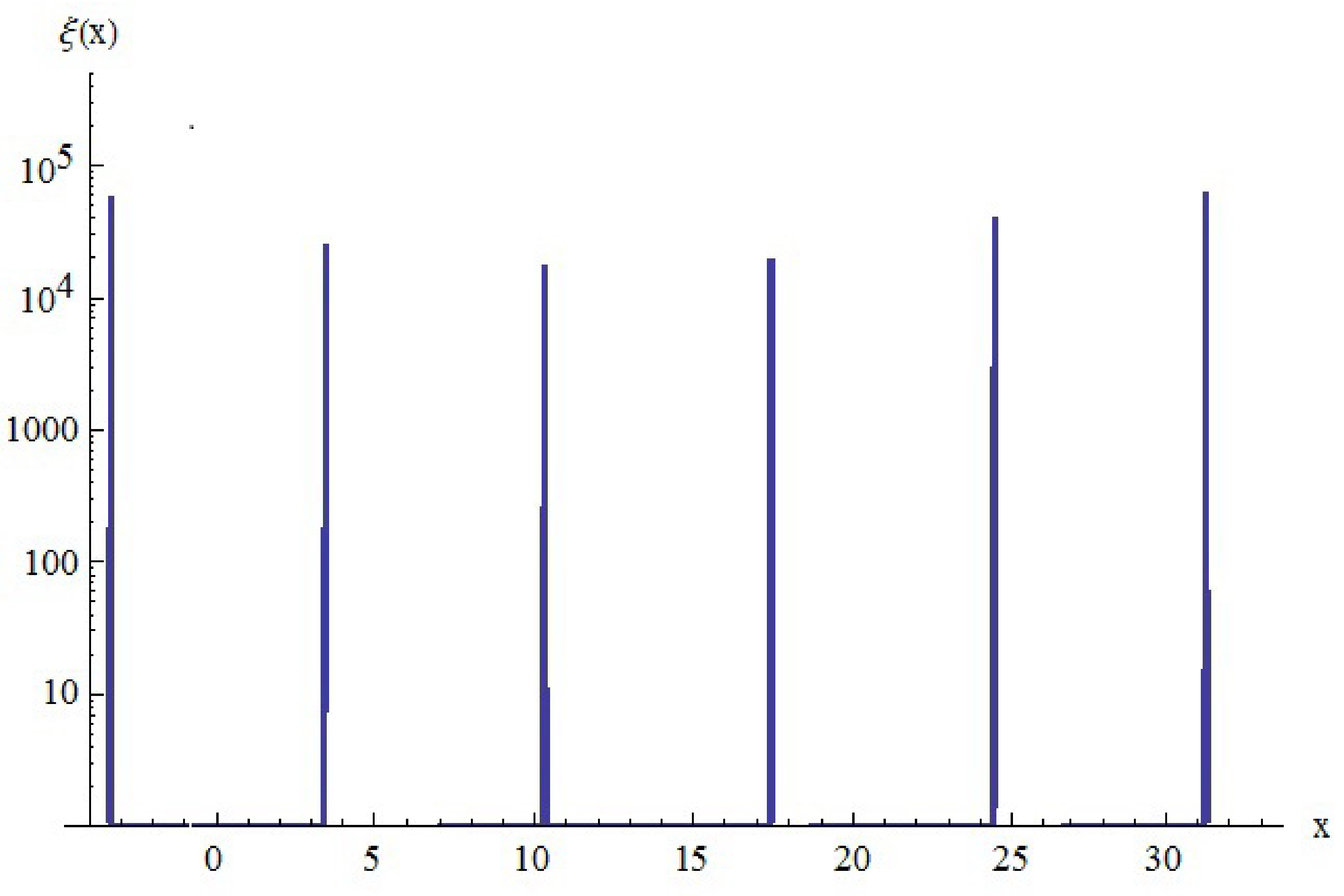
$$x = \frac{2(\omega - \omega_r)}{\Gamma}; \quad A = \eta\frac{mc^3}{e_0^2\omega_r(1 + \alpha_c)Z}; \; \lambda_1 = \lambda_6 = 1; \; \lambda_2 = \lambda_5 = \frac{2}{3};$$

$$\lambda_3 = \lambda_4 = \frac{1}{3}.$$

X-RAY CRYSTAL POLARIZABILITY WITH SPLITTING (2)



PGR PEAKS IN THE SPECTRAL INTENSITY



These peaks demonstrate that PGR can be applied for Mössbauer spectroscopy in energy domain

COMPARISON OF SR AND PGR SPECTRAL BRIGHTNESS

$$\Upsilon_{SR}\ [\text{photons}\cdot \text{s}^{-1}\cdot \text{mrad}^{-2}\cdot (0.1\%\ \text{BW})^{-1}] \approx 1.3\times 10^{13}\ E^2\ [\text{GeV}]\ J\ [\text{A}],$$

$$\Upsilon_{PGR}\ [\text{photons}\cdot \text{s}^{-1}\cdot \text{mrad}^{-2}\cdot (0.1\%\text{BW})^{-1}] \approx 3.1\times 10^{16}\ E^2\ [\text{GeV}]\ J\ [\text{A}].$$

One can see that the brightness of the PGR Mössbauer source could exceed analogous value for the SR Mössbauer source if the same electron current is considered. Unfortunately the characteristic electron current in the linear accelerators used for PGR is several order less than the average electron current in the storage rings used for SR. However, advantage of the PGR source can be conditioned by the fact that the same parameters of the radiation are obtained with essentially less electron energy than for SR.

CONCLUSION

Role of the dynamical diffraction effects in PGR was investigated in detail. The influence of the anomalous decrease of the absorption of the emitted resonant quanta on the PGR formation was considered.

It was shown that this effect leads to increase of the intensity and to decrease of the angular width of a PGR peak.

The spectral brightness of the PGR source at "low" energy accelerator is comparable with the synchrotron Mossbauer source at 5 GeV beam energy.

PGR source can provide similar information as SMS with much lower dose deposit at the target, that is extremely important for biomedical samples

Thank you for attention