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SOFT X-RAY COHERENT BREMSSTRAHLUNG INDUCED BY HYPERSONIC WAVES

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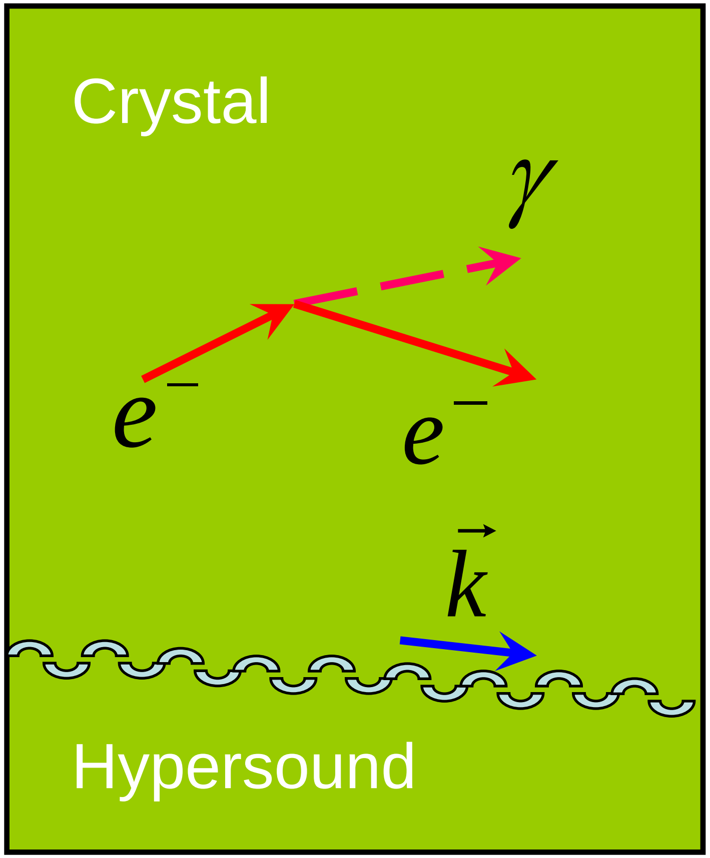
Outline

- Motivation
- Analysis of conditions for the influence of hypersound on the cross-section of bremsstrahlung
- Cross-section of the bremsstrahlung in crystals in presence of hypersonic vibrations
- Numerical results and discussion

Motivation

- In crystals the cross-sections of the high-energy electromagnetic processes can change essentially compared with the corresponding quantities for a single atom
- From the point of view of controlling the parameters of the high-energy electromagnetic processes in a medium it is of interest to investigate the influence of external fields (acoustic waves, temperature gradient) on the corresponding characteristics
- Investigation of bremsstrahlung by high-energy electrons is of interest from the viewpoint of the underlying physics and from the viewpoint of practical applications for generation of intense photon beams

Problem setting and notations



Photon energy ω

Photon momentum $\vec{k}$

Initial energy of electron E_1

Initial momentum of electron $\vec{p}_1$

Final energy of electron E_2

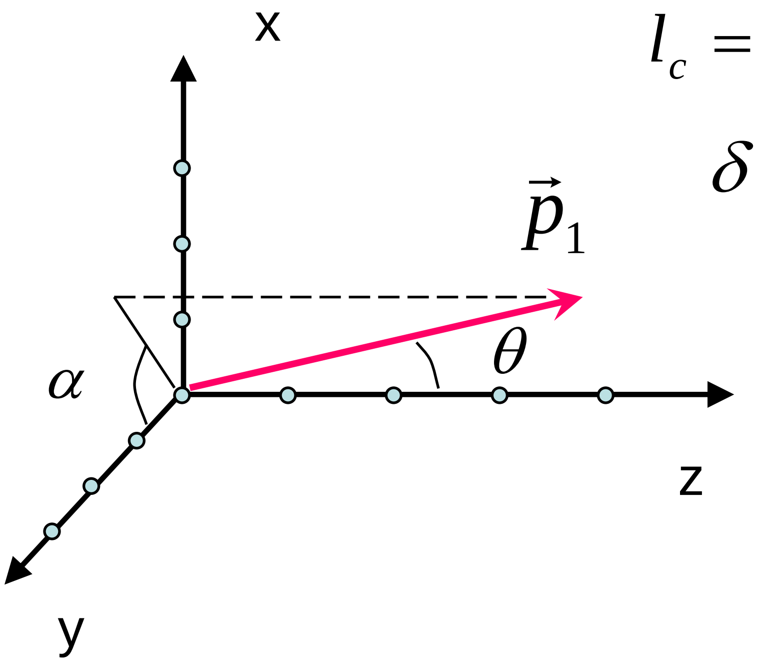
Final momentum of electron $\vec{p}_2$

Hypersound wave vector $\vec{k}_s$

Displacements of atoms due to the hypersound $\vec{u} = \vec{u}_0 f(\vec{k}_s \vec{r})$

Geometry of the problem

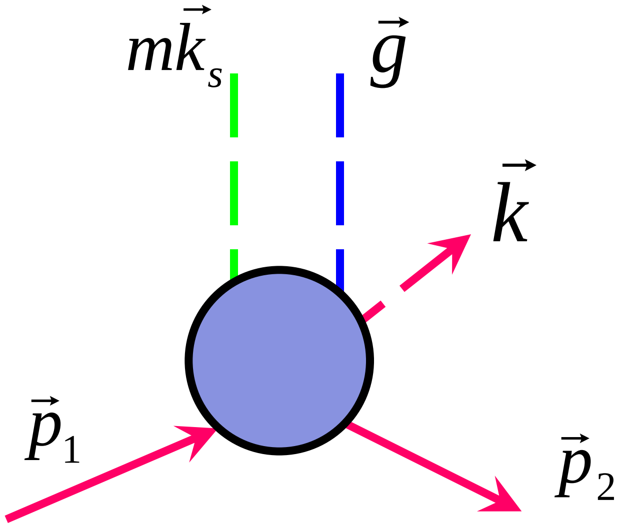
Coherence effects are essential if the electron enters into the crystal at small angle with respect to the crystallographic axis



$l_c = 2E_1E_2/\omega m_e^2$ ⇨ formation length

$\delta = 1/l_c$ ⇨ minimum longitudinal momentum transfer

Condition for the influence



- Momentum conservation

$$\vec{p}_1 = \vec{p}_2 + \vec{k} + \vec{g} - m\vec{k}_s$$

- Dominant contribution comes from

$$|m| \lesssim \lambda_s / a, \quad \lambda_s = 2\pi / k_s$$

(a — interatomic distance)

- Influence of the deformation field may be considerable if $|mk_{s\|}| \gtrsim \delta$

- Condition for the influence of the hypersound to be essential

$$u_0/\lambda_s \gtrsim a/(4\pi^2 l_c)$$

Cross-section

- Cross-section $d\sigma = N_0(d\sigma_n + d\sigma_c)\,,\ N_0$ number of atoms
- Coherent part of the cross-section

$$\frac{d\sigma_c}{d\omega} = \frac{e^2 N}{N_0 E_1^2 \Delta} \sum_{m,\vec{g}} \frac{g_{m\perp}^2}{g_{m\parallel}^2} \left|F_m(\vec{g}_m \vec{u}_0)\right|^2 \left|S(\vec{g}_m, \vec{g})\right|^2 \times$$

$$\left[1 + \frac{\omega^2}{2E_1E_2} - 2\frac{\delta}{g_{m\parallel}}\left(1 - \frac{\delta}{g_{m\parallel}}\right)\right] \qquad \delta = 1/l_c$$

N number of cells

Δ unit cell volume

$\vec{g}_m = \vec{g} - m\vec{k}_s$

$m = 0, \pm 1, \pm 2, \ldots$

Reciprocal lattice vector

$g_{m\parallel}$ and $g_{m\perp}$ are the parallel and perpendicular components of $\vec{g}$ with respect to the photon momentum

- Summation goes under the constraint $g_{m\parallel} \geq \delta$

Cross-section

$$F_m(x) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{ixf(t)-imt}\,dt\,,\ S(\vec{g})$$ structure factor of the crystal

- For $u_0 = 0$ one has $F_m = \delta_m^0$ and from the general formula the bremsstrahlung cross-section in an undeformed crystal is obtained
- Sinusoidal deformation field $f(z) = \sin(z+\varphi_0)$

$$F_m(z) = e^{im\varphi_0} J_m(z)$$

Bessel function

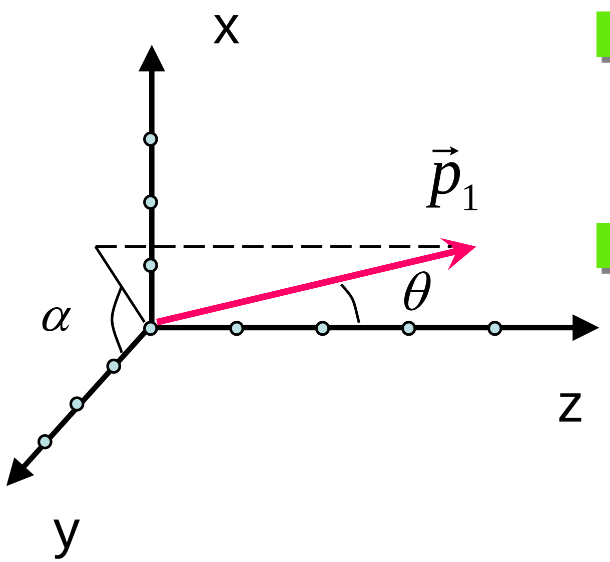
- In the case of the presence of the hypersound the formula for the cross-section differs from the corresponding formula for an undeformed crystal by the replacement $\vec{g} \to \vec{g}_m$, and additional summation over *m* with weights $\left|F_m(\vec{g}_m\vec{u}_0)\right|^2$

Qualitatively different cases

- Orthogonal crystal lattice with the reciprocal lattice vector components

$$g_i = 2\pi n_i / a_i,\ n_i = 0, \pm 1, \pm 2, \ldots,\ i = 1, 2, 3$$

- Coherent effects appear when the electron enters into the crystal at small angles θ



- Main contribution to the cross-section give the terms with $g_z = 0$

- Qualitatively different cases

  a) Angles α and $\pi/2-\alpha$ are not small

  b) Angle α is small and $\delta \sim 2\pi\theta / a_2$

  c) Angle α is small and $\delta \sim 2\pi\theta\alpha / a_1$

Qualitatively different cases

- Angles α and $\pi/2-\alpha$ are not small

$$\sum_{g_x, g_y} \rightarrow (a_1 a_2/4\pi^2) \int dg_x g_y$$

- Angle α is small and $\delta \sim 2\pi\theta/a_2$

$$g_{m\|} \approx -mk_{s\|} + \theta g_y \geq \delta\,,\ \sum_{g_x} \rightarrow (a_1/2\pi) \int dg_x,$$

Formula for the cross-section is further simplied in the case when the amplitude of the deformation field is perpendicular to the crystallographic *x*-axis

$$\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{2\pi E_1^2 a_2 a_3 N_0} \sum_{m, g_y} \left[1 + \frac{\omega^2}{2E_1 E_2} - 2\frac{\delta}{g_{m\|}}\left(1 - \frac{\delta}{g_{m\|}}\right)\right]$$

$$\times \frac{|F_m(\mathbf{g}_m \mathbf{u}_0)|^2}{g_{m\|}^2} \int dg_x g_\perp^2 |S(\mathbf{g}_m, \mathbf{g})|^2$$

Effective structure factor

Qualitatively different cases

- Angle α is small and $\delta \sim 2\pi\theta\alpha/a_1$

  Dominant contribution comes from the terms $g_y = 0$

$$g_{m\|} \approx -mk_{z\|} + \psi g_x, \qquad \psi \equiv \alpha\theta,$$

Numerical calculation

- Numerical calculations have been performed for $\mathrm{SiO_2}$ single crystal at low temperatures

- For the Fourier transforms of the atomic potentials the Moliere parametrization is used

$$u_q^{(j)} = \sum_{i=1}^{3} \frac{4\pi Z_j e^2 \alpha_i}{q^2 + (\chi_i/R_j)^2}, \; \alpha_i = \{0.1, 0.55, 0.35\}, \; \chi_i = \{6.0, 1.2, 0.3\},$$

Screening radius of the *j*-th atom

- Sinusoidal transversal acoustic wave of the S-type

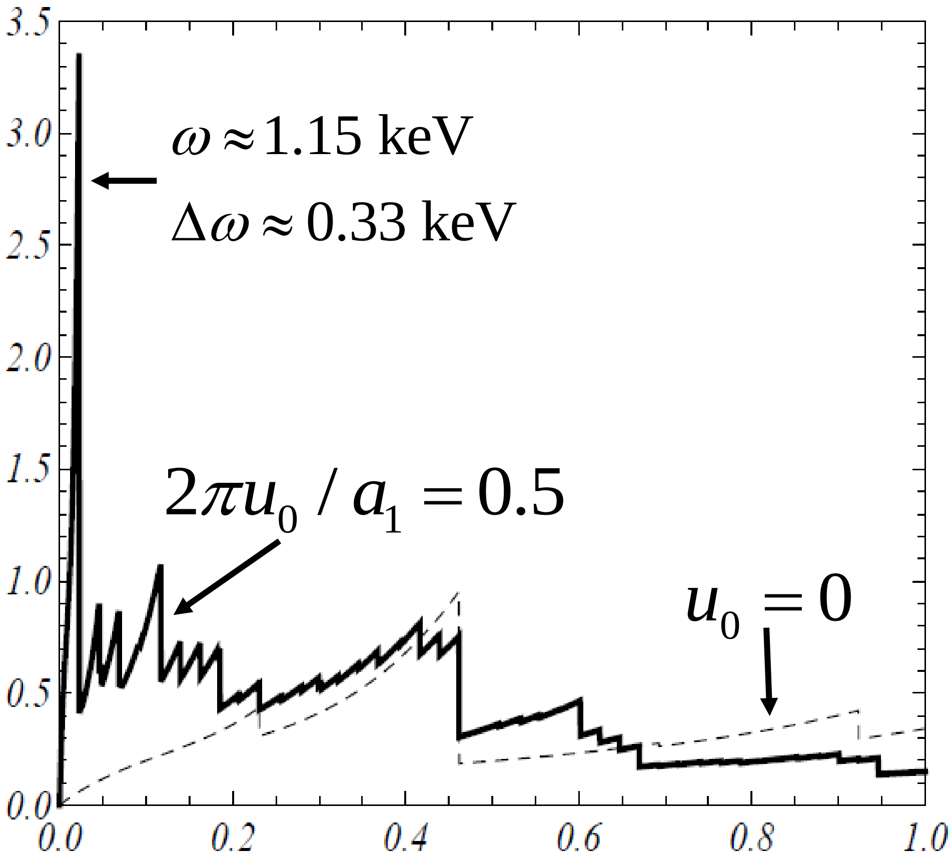
  The vector determining the direction of the hypersound propagation lies in the *yz*-plane and forms the angle 0.295 rad with the *z*-axis

- Numerical calculations show that, in dependence of the values for the parameters the external excitation can either enhance or reduce the bremsstrahlung cross-section

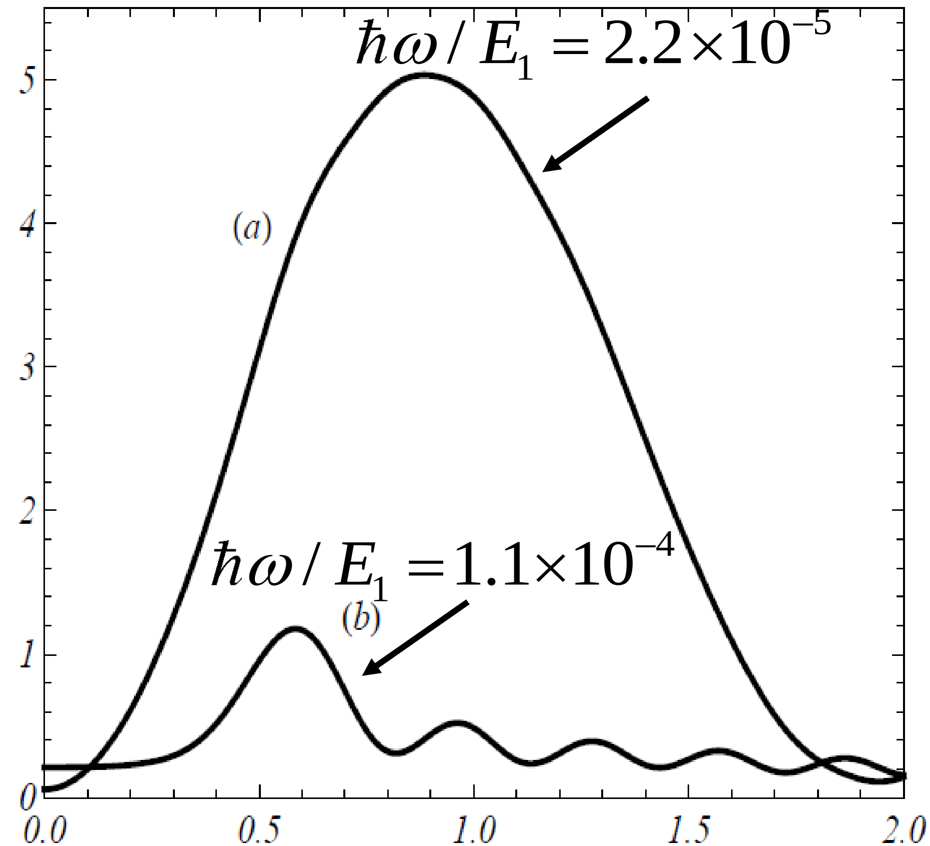
Coherent part of the cross-section: Numerical examples

$$10^{-4}\frac{m_e^2\omega}{e^6}\frac{d\sigma_c}{d\omega}$$

$$E_1 = 50\ \text{MeV},\ \nu_s = 5\ \text{GHz}$$



$$10^3 \hbar\omega / E_1$$



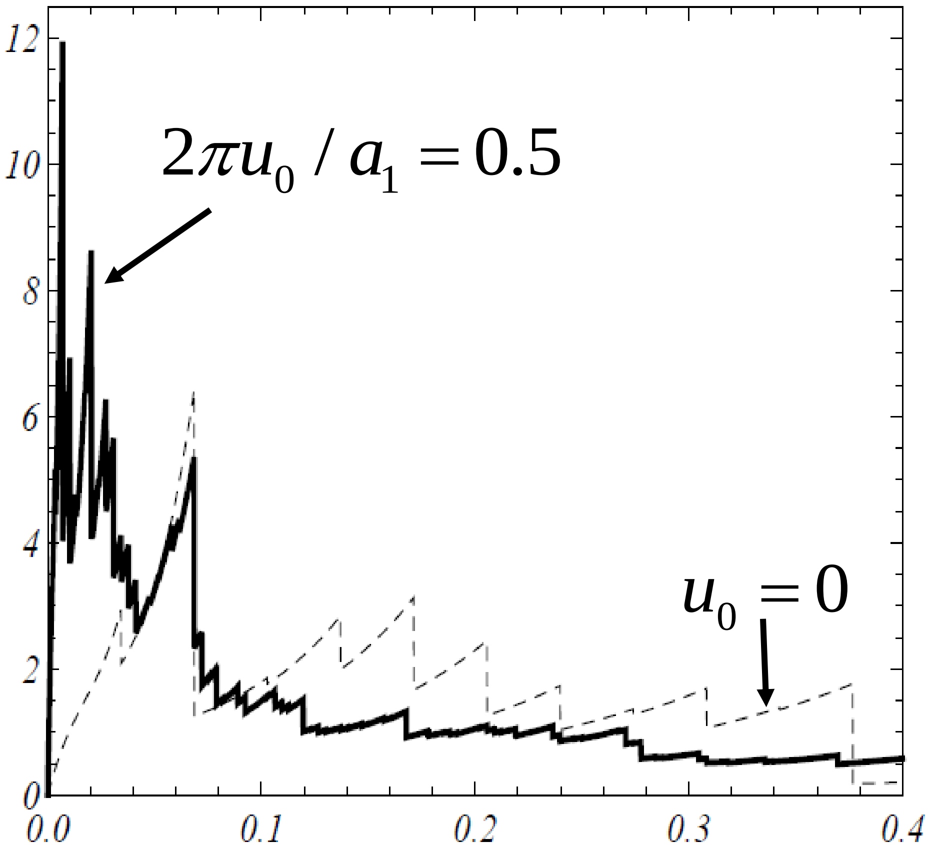
$$2\pi u_0 / a_1$$

$$\psi = \alpha\theta = 0.000263$$

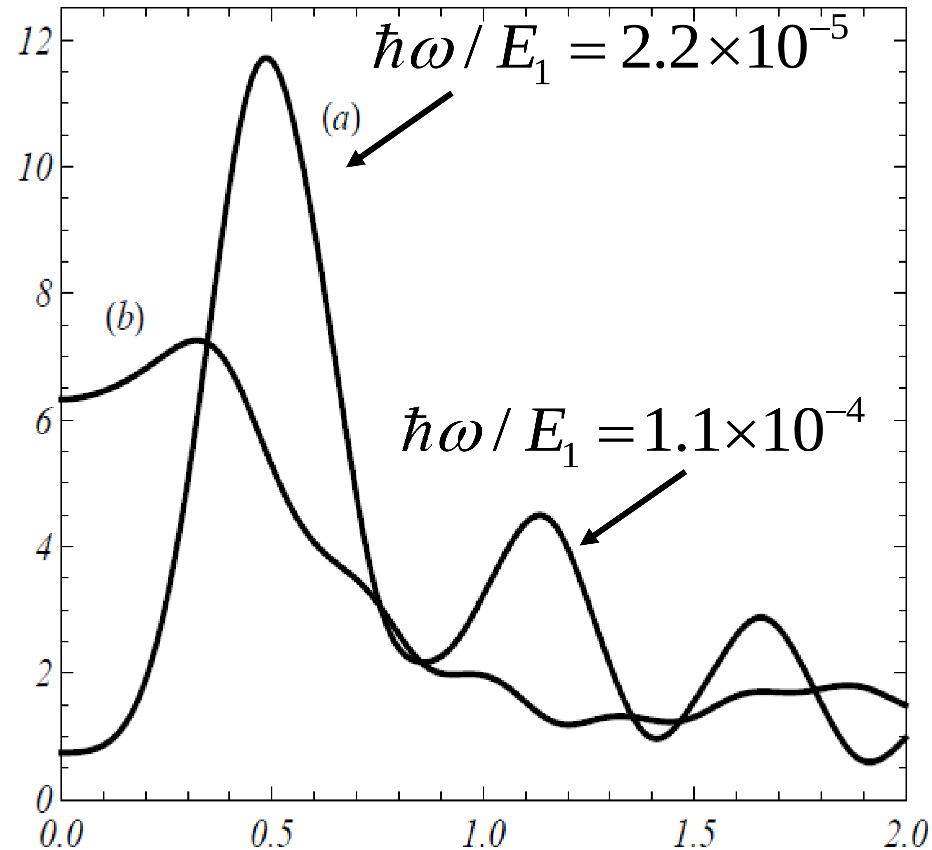
Numerical examples

$$10^{-4}\,\frac{m_e^2\omega}{e^6}\frac{d\sigma_c}{d\omega}$$

$$E_1 = 50\ \mathrm{MeV},\ \nu_s = 1.024\ \mathrm{GHz}$$



$$10^3\hbar\omega / E_1$$



$$2\pi u_0 / a_1$$

$$\psi = 0.000039$$

Experiments planned

For the experimental investigation of the effects discussed above, experiments are planned on the LINAC of the Yerevan Physics Institute

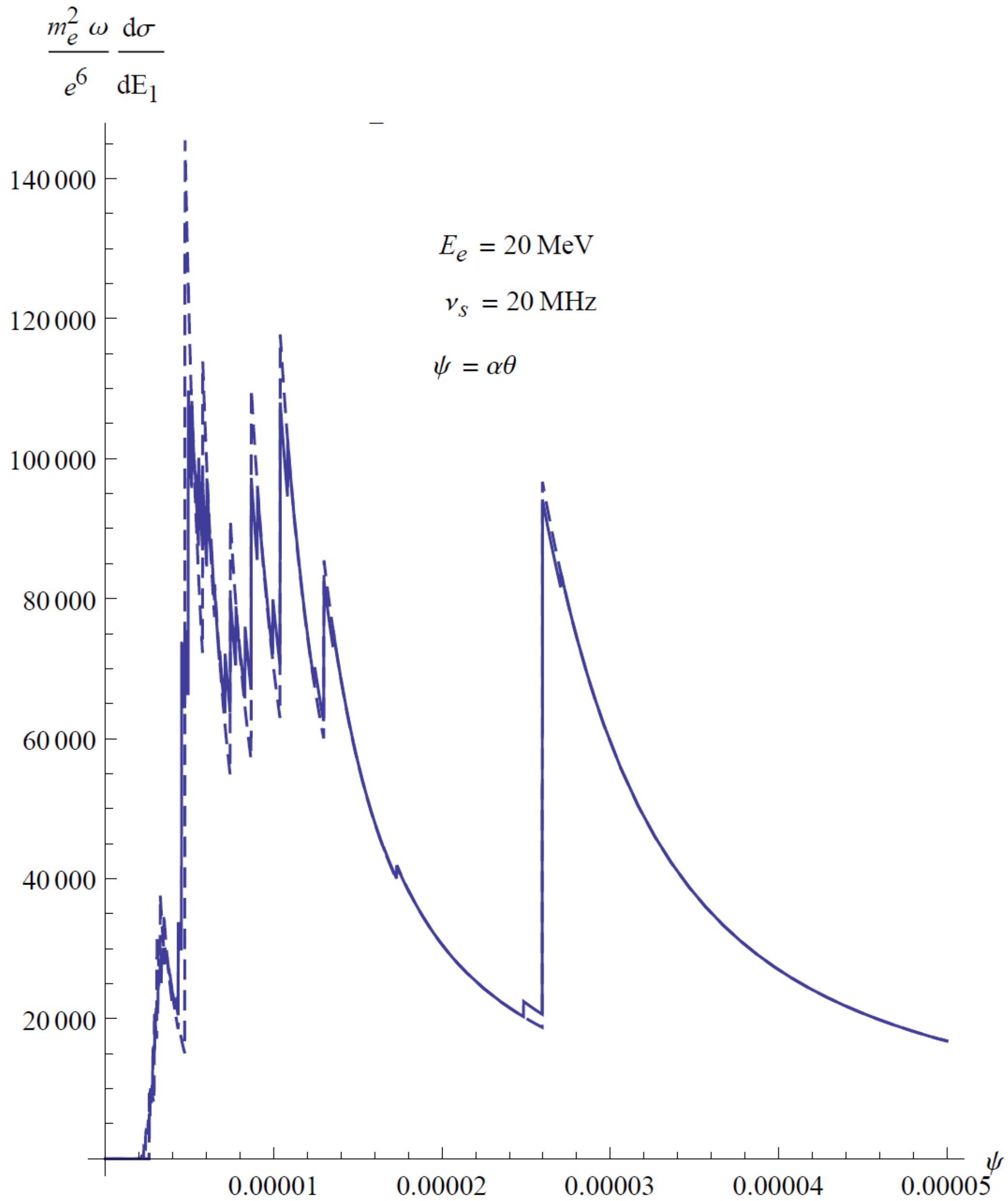
Parameters:

Beam energy	E=50 MeV, ΔE=E(FWHM)=0.02
Beam intensity	1μA
Repetition rate	50 Hz
Pulse length	0.5-1.4 μsec
RF frequency	2.7973 GHz

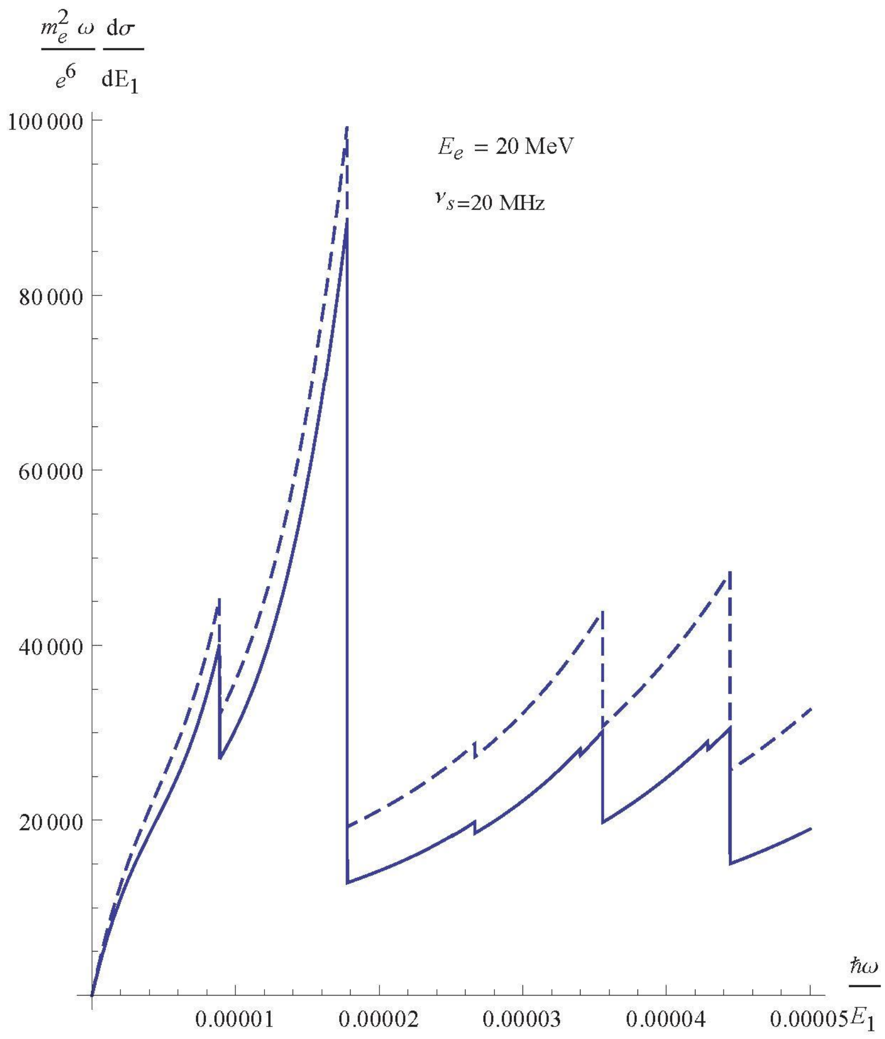
Conclusions

- Formula is derived for the coherent part of the differential cross-section for the bremsstrahlung in crystals in the presence of hyperacoustic vibrations
- Conditions are specified under which the influence of the hypersound is essential
- In dependence of the parameters the hypersonic waves can either enhance or reduce the cross-section
- Presence of hypersonic wave may lead to the appearance of new strong peaks in the cross-section at low frequencies (soft X-rays)

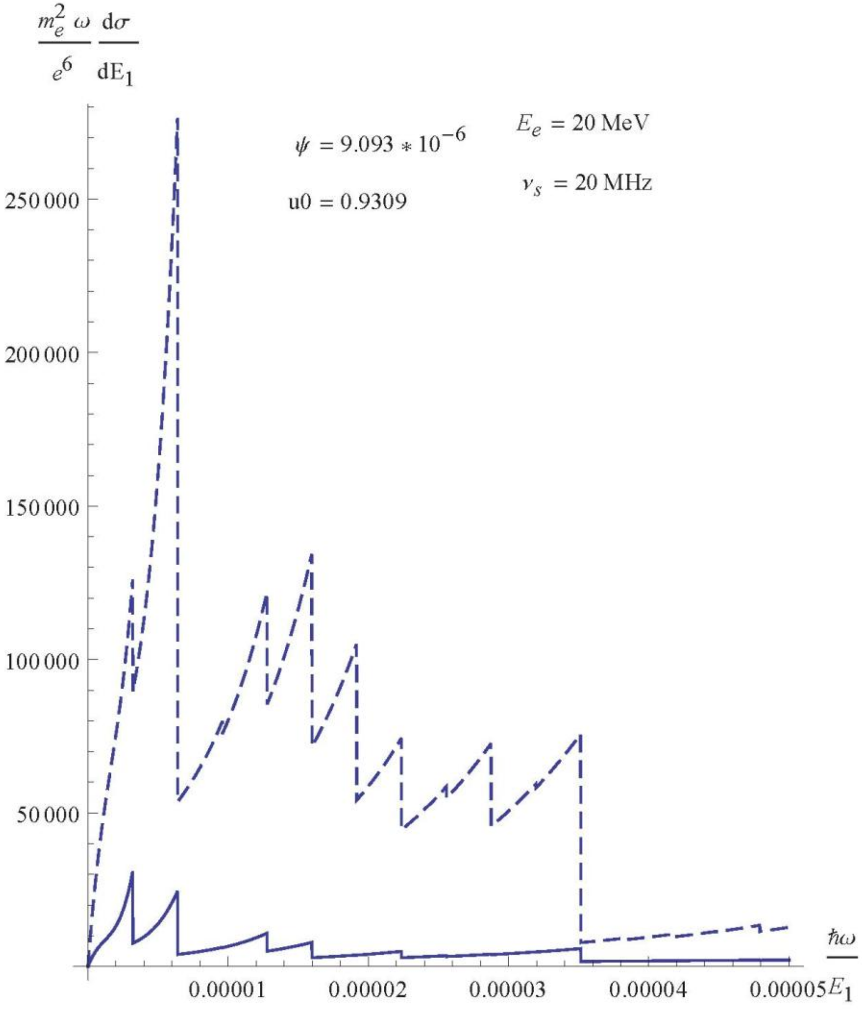
Numerical examples



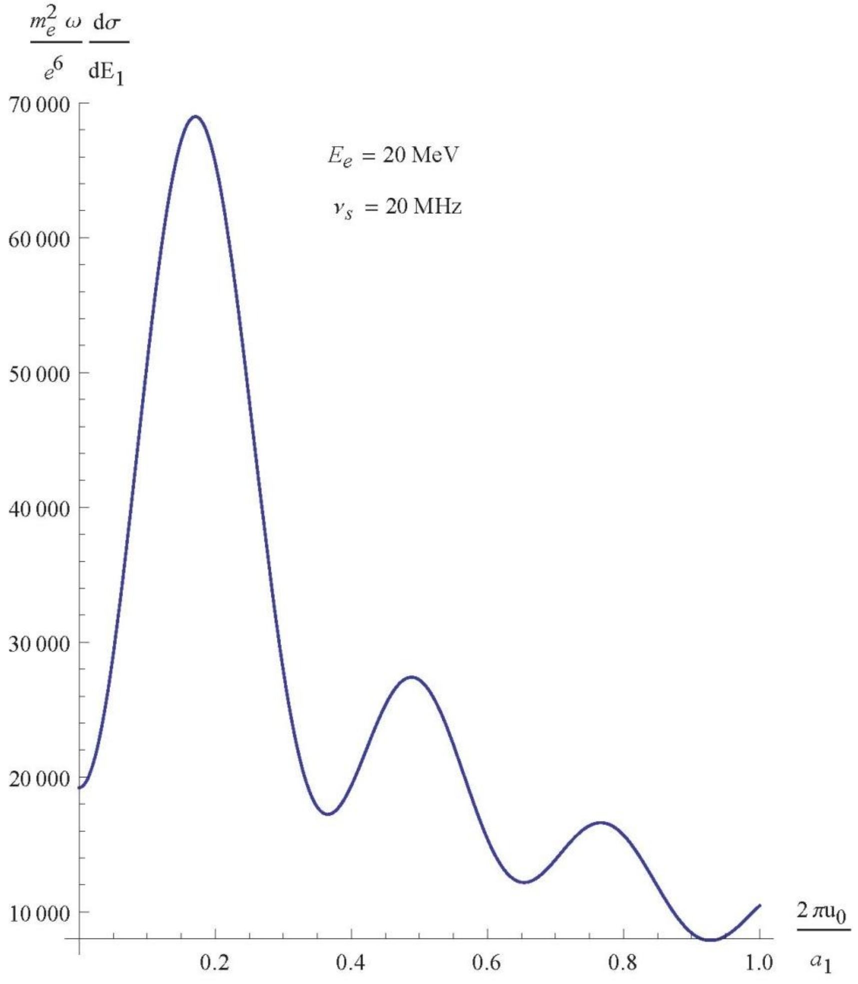
Numerical examples



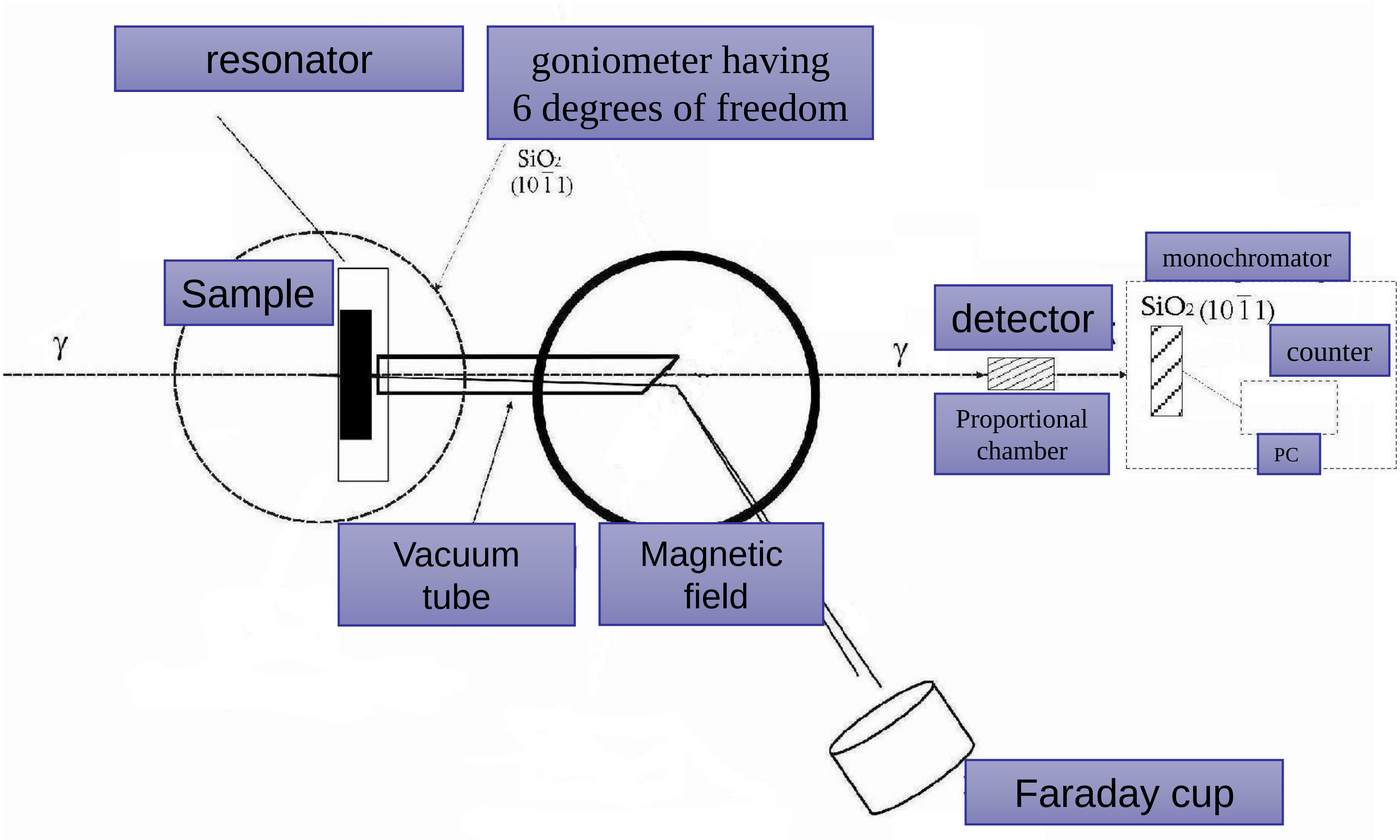
Numerical examples



Numerical examples

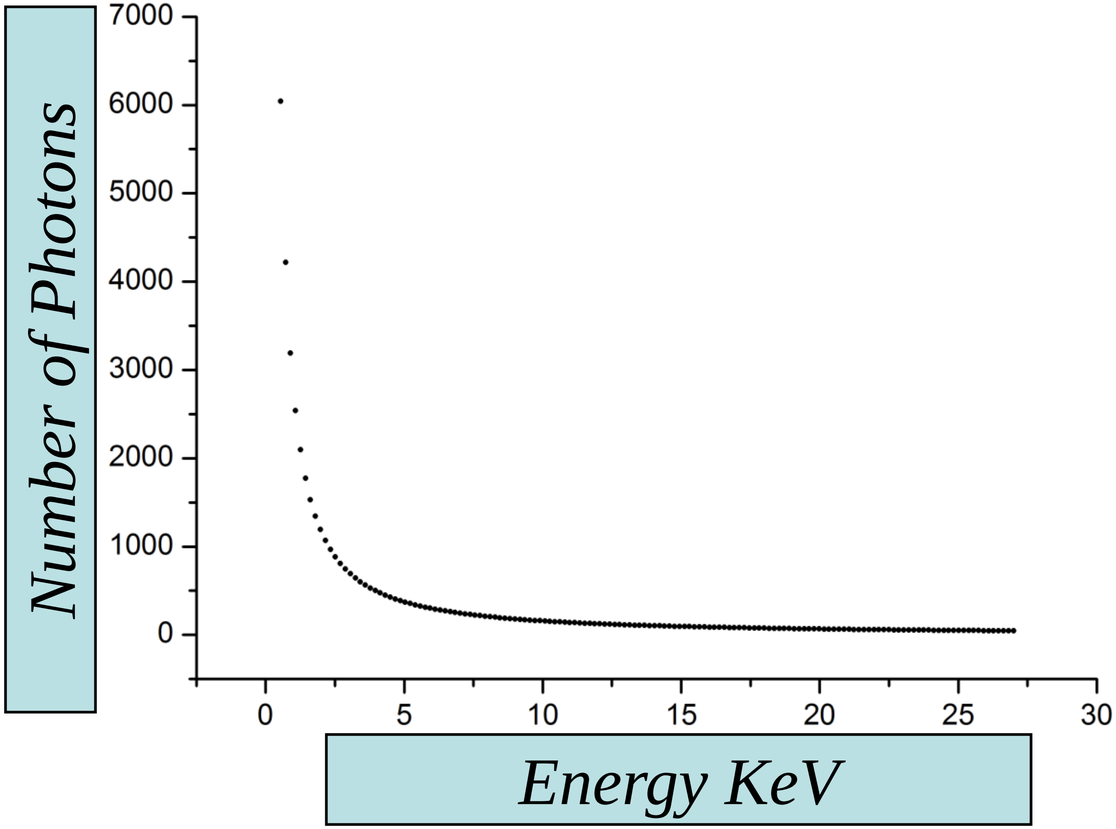


Numerical examples



Experimental setup for the study of coherent bremsstrahlung

Numerical examples

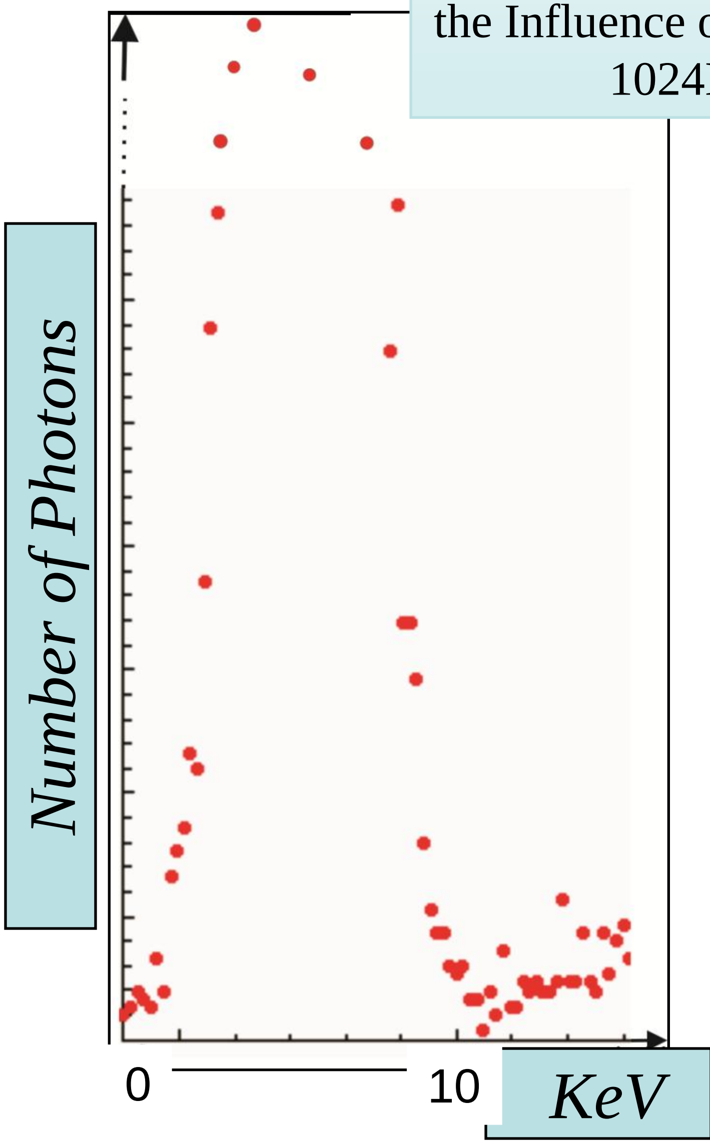


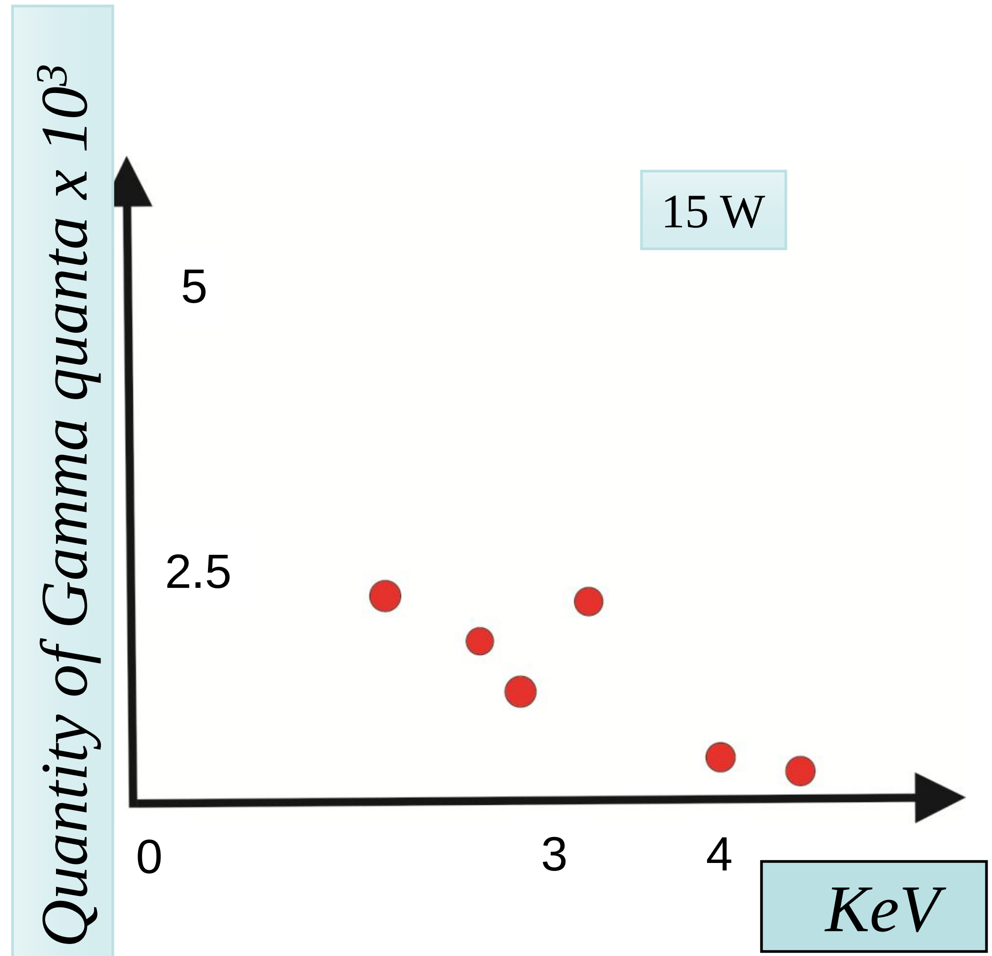
The number of photons incoherent bremsstrahlung, calculated using the Bethe-Heitler low with amplitude (power) 0, 5, 10 и 15 W

Numerical examples

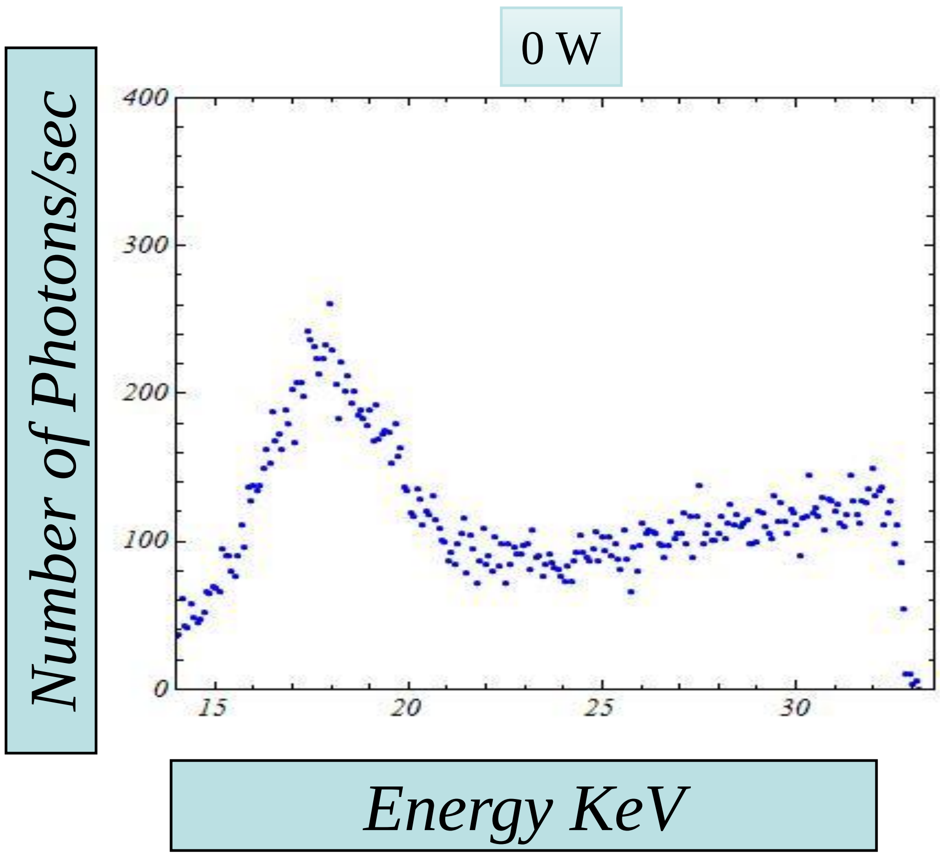
N = 75153

20 MeV X=Cut SiO_2, Under
the Influence of Hypersonic Waves
1024KhZ, 15W



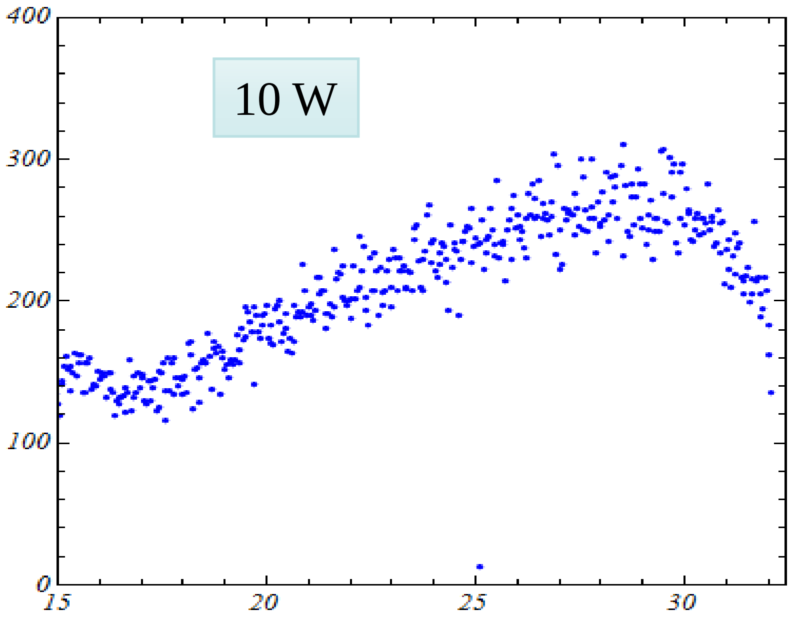


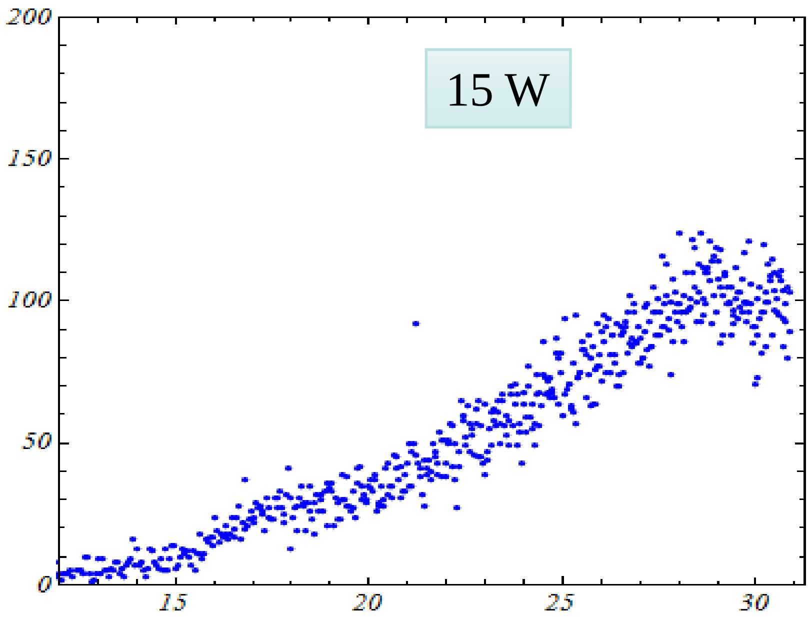
Numerical examples



Numerical examples

Number of Photons/sec





Energy KeV

Conclusions

The presentation is devoted to the investigation of bremsstrahlung from electrons with the energy 20 MeV in a crystal with complex base in the presence of deformation field with an arbitrary profile. The latter can be induced, for example, by the acoustic wave. The presence of the deformation can serve as an additional mechanism for the control of angular-frequency characteristics of the radiated photon. In comparison with undistorted crystal, the coherent part of the cross-section contains an additional summation over the inverse lattice vector of the one-dimensional supperlattice induced by the deformation field.

The role of the coherent effects in the cross-section of bremsstrahlung is essential when the electron enters into the crystal at small angles with respect to crystallographic axis. In this case the main contribution to the coherent part of the cross-section comes from the crystallographic planes perpendicular to the chosen axis. The numerical calculations of the cross-section were carried out for a quartz single crystal with the Moliere parameterization of the screened atomic potentials and for the deformation field, induced by transverse acoustic wave of the S-type with gigahertz frequency. The calculations show that, with dependence of the values for the problem parameters, the presence of the deformation can either decrease or increase the cross-section of bremsstrahlung.

Conclusions

Radiation peak is observed in the spectral distribution in the energy range 0-1 KeV, the behavior of which coincides with the incoherent bremsstrahlung and is described by the Bethe-Heitler law. The form of the distribution does not depend on the orientation of the sample with respect to the direction of the propagation of the formed radiation and on the direction of the incident electron beam.

For the first time, the coherent bremsstrahlung is observed from relativistic electrons on a quartz single crystal plate of X-cut for short wavelengths of X-ray radiation. The intensity of coherent bremsstrahlung is experimentally investigated in dependence of the amplitude for an acoustic field. With the initial increase of the amplitude the intensity of the radiation line increases and then, started from some value, it decreases

Conclusions

In dependence of the working parameters, new bands and peaks are observed in the spectral distribution and separate peaks are suppressed. On the base of coherent bremsstrahlung in presence of external acoustic fields, a method is proposed for the generation of monochromatic radiation in the range of short wavelengths of X-ray radiation with controllable working parameters in the space and time.

Theory is constructed for the intensity of spectral components and for the spectral distribution in dependence of the acoustic field parameters. New bands and peaks appear in the spectral distribution and some peaks are suppressed. These results qualitatively agree with the experimental results

Thank you!

Until a New Meeting in Armenia at Meghri 2015