Multiple scattering in channeling

Multiple scattering by crystal nuclei and electrons → particle dechanneling

In short crystals dechanneling occurs only for particles with large amplitudes in channels mainly due to scattering by nuclei

MS angle for channeled particles depends on their position in channels

$$\frac{\Delta \bar{\theta}^2}{\Delta z}(x) = P(x) \frac{\Delta \bar{\theta}^2}{\Delta z}_R$$

P(x) – local density of nuclei in channel

MS distribution is Gaussian in first approximation

One of the first estimate of MS angle was given by Rossi

$$\frac{\Delta \bar{\theta}^2}{\Delta z}_R = \left(\frac{E_s}{pv}\right)^2 \frac{1}{L_R} \quad , \quad E_s = 21\, MeV$$

PDG suggests fit of Moliere distribution for projected angle

$$\theta_o = \frac{13.6\, MeV}{pv} \sqrt{L/L_R}\, (1 + 0.038 \ln(L/L_R))$$

Simulation of channeling in bent crystals

In our model CRYD (Crystal deflector) particle trajectories in BC
are found by numeric solution of motion equation in effective planar potential

$$U_{eff}(x, R) = U(x) + \frac{pv}{R} x$$

Moliere approach for atomic potential is used

$$V(r) = \frac{Ze^2}{r} \sum_{i=1}^{3} \alpha_i \exp(-\beta_i r / a)$$

It gives analytical form for planar potential U(x). Electron density $\rho(x)$ derives from
Poisson equation. They are averaged by the thermal vibrations of atoms

Change of transverse velocity of a particle due to MS is calculated after the passage
in effective potential a small distance $\Delta z << \lambda$

Rossi expression are used for MS by crystal nuclei

Experimental results obtained with SPS CERN beams of protons
have been compared with simulation by this model in many papers

Agreement was sufficiently good

About multiple scattering of high energy protons in crystal deflectors

by A.M. Taratin and W. Scandale

Purposes:

1. Better estimation for Gaussian part of MS - $\dfrac{\Delta\bar{\theta}^2}{\Delta z}\Big|_R$

2. Suggestion of a fast generator for the whole distribution
of multiple scattering including its tail

Methods:

1. Simulation of particle trajectories in binary collision model
for amorphous crystal orientation

2. Straight simulation of MS as the sequences of random collisions with atoms
Impact paramters are uniformly distributed on the symmetry cell
for a given crystallography direction

Simulation of MS in binary collision model (BCOL)

Entrance points are randomly and uniformly distributed on the symmetry cell

Beam direction (θ_o, φ_o) is far from <111> Si axis and main crystal planes
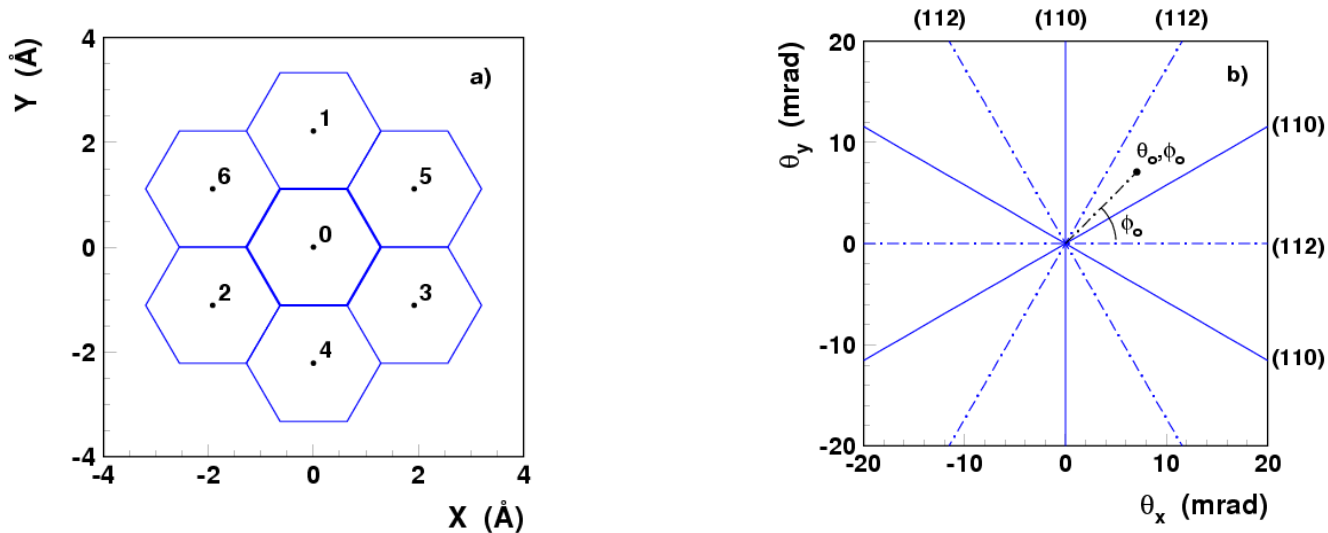
Scattering by the atom with Moliere potential in momentum approximation

$$\theta(r) = \frac{2Ze^2}{pva} \sum_{i=1}^{3} \alpha_i \beta_i K_1(\beta_i r / a)$$

θ_o=10 mrad, φ_o=45° is amorphous orientation for 400 GeV/c protons

Some illustrations of BCOL results for orientations close to <111> axis
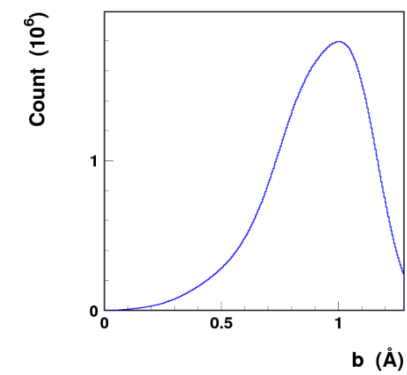
Distributions: angular   space   impact

θ_o=0.1 µrad

θ_o=5 µrad
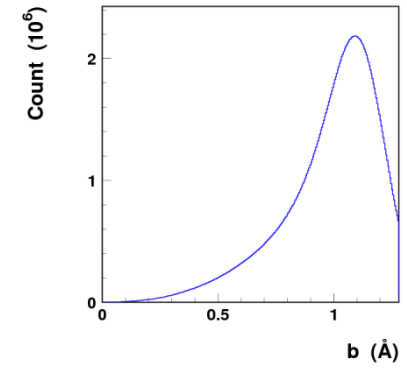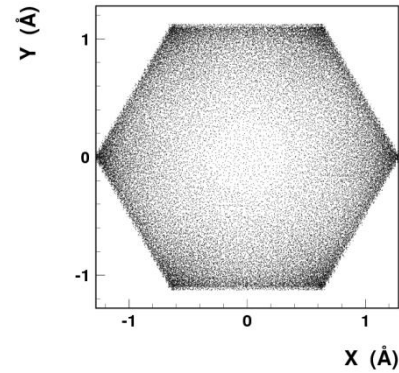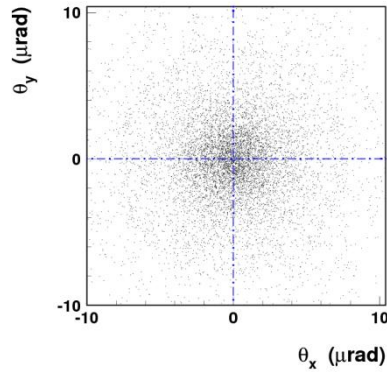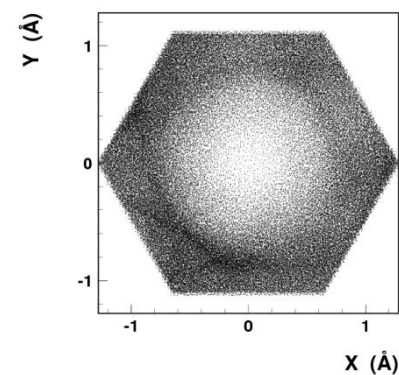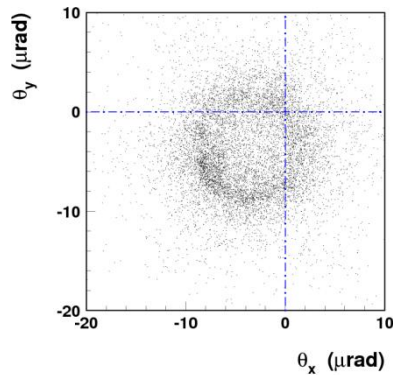
θ_o=0.5 mrad
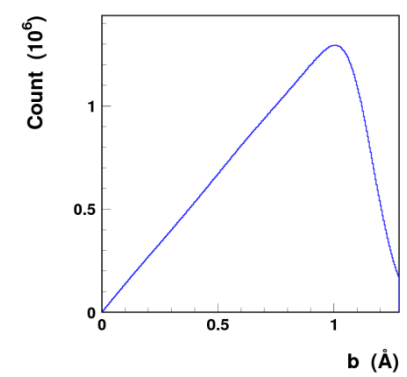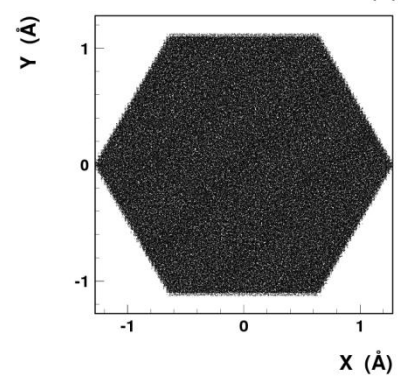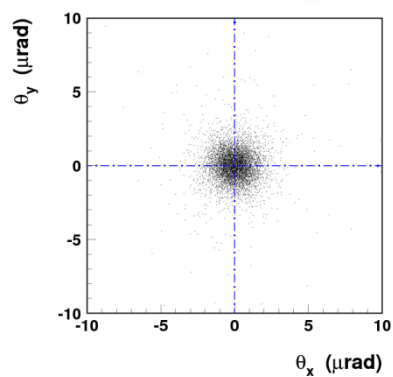
BCOL results for amorphous orientations in <111>Si

For θ_o=10 mrad and φ_o=45°
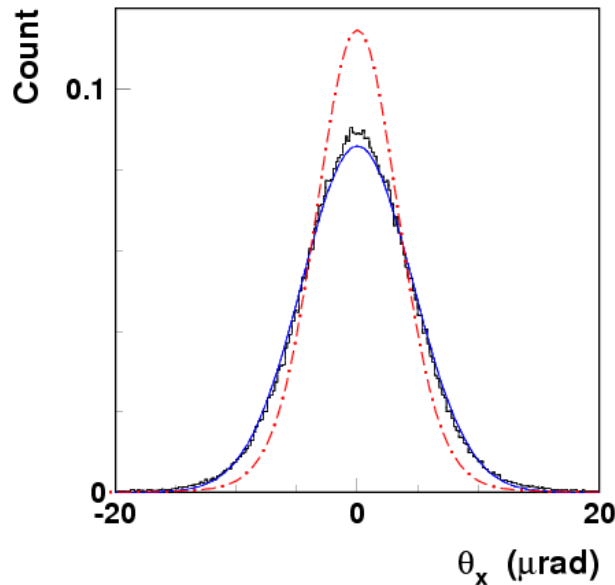
Gaussian fit

BCOL distribution → σ_s=4.56 μrad

Moliere distribution → σ_m=3.83 μrad

With taking into account MS by the crystal electrons → σ_s=4.72 μrad
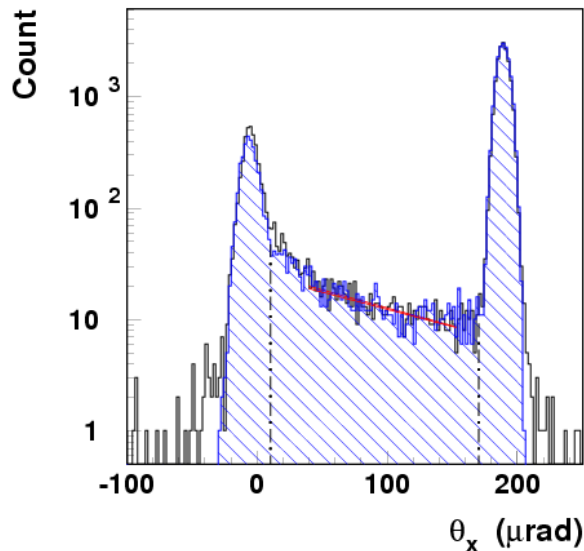
From our experimental data → σ_{ex}=4.6 ÷ 4.9 μrad

PDG for RMS deflection → θ_o=4.17 μrad

Planar potential model CRYD with using BCOL value for σ_s

Mean square deflection per unit length : $\Delta\overline{\theta}^2 / \Delta z_R = \sigma_s^2 / L$

Deflection angle distribution of 400 GeV/c protons

For channeling in (110) Si, the bend angle α=189 μrad

Experiment at the SPS CERN

CRYD simulation

Channeling efficiency : 77% (ex) and 80% (sim)

Dechanneled fraction : 6.4% (ex) and 5.9% (sim)

Straight simulation of multiple scattering in Si crystal

MS for a particle is the sequence of single scatterings with atoms

Impact parameters in scatterings are randomly and uniformly distributed
on the crystal symmetry cell for a given direction

<111> Si cell   <100> Si cell
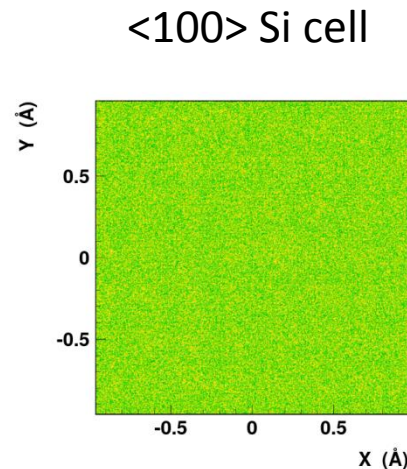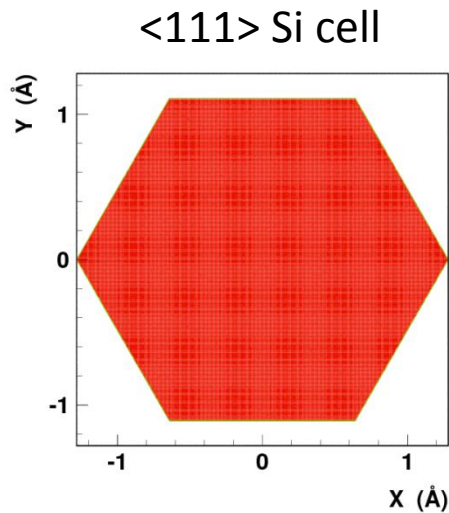
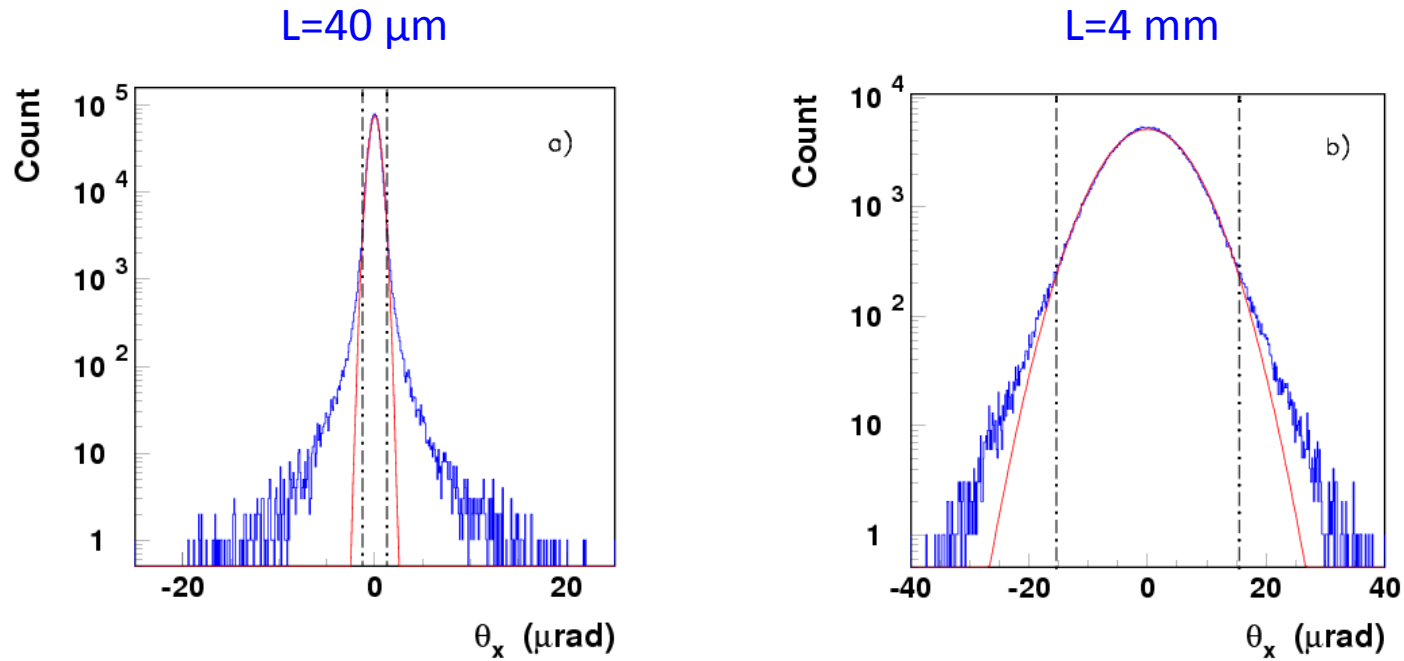
Number of scatterings along L

$$N_L = L/d$$

d – interatomic distance along S

Resulted X-deflection in N scatterings is the sum

$$\theta_x(N) = \sum_{n=1}^{N_L} \theta(r_n) x_n / r_n \quad ,$$

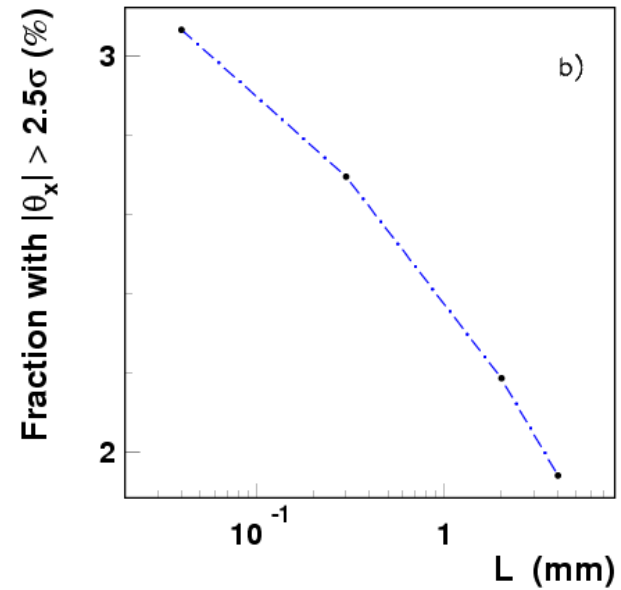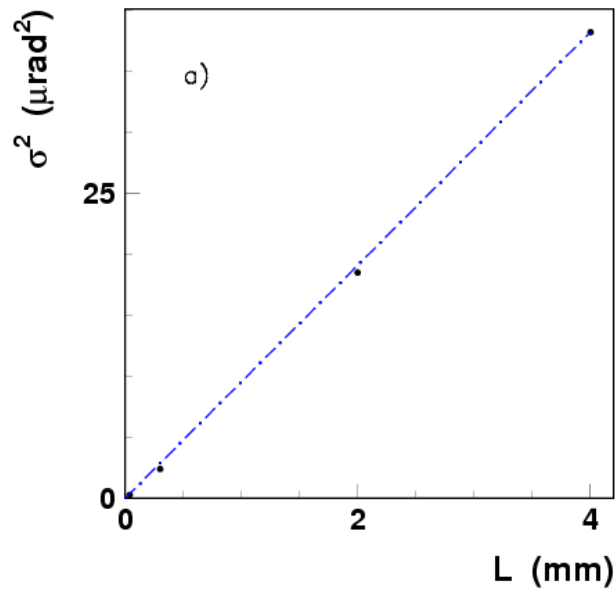$$r_n = (x_n^2 + y_n^2)^{1/2} \quad , \quad N_L = L/d_a \quad ,$$

MS model for 450 GeV/c protons in <111> Si

L=40 µm

L=4 mm

The distribution tails are non-Gaussian
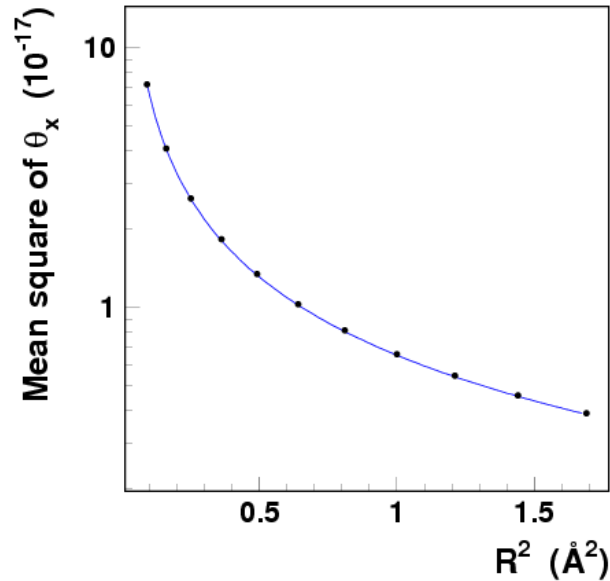Their value and length depend on the crystal length

Linearity of σ^2 is improved for large length of a target

Tail fraction of MS distribution is considerably larger than for Gaussian (1.24%)
Besides MS tail is considerably longer

MS is the same for different crystal orientations

Mean square deflection in a single collision is inversely proportional to collision area
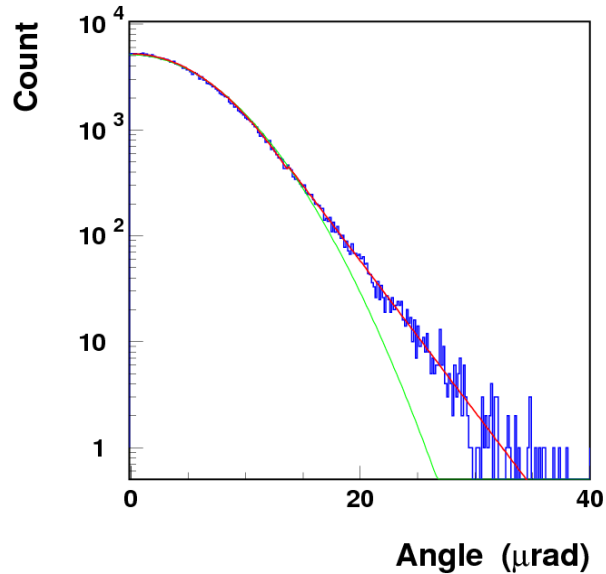
$$MSD_1 \sim 1/S$$

But collision area $S \sim 1/d \rightarrow MSD_1 \sim d$

Number of collisions $N_L \sim 1/d$ and $MSD_1 \sim d$

Therefore, multiple scattering does not depend on d, it is the same for any direction
$$MSD_N = N*MSD_1 \neq f(d)$$

Generator of MS for 4 mm long Si crystal

Non-Gaussian tail for $\theta_x > 2.2\sigma$ is well fitted by a single exponent

MS generator is composed by two functions :

for $\theta_x < 2.2\sigma$, $\quad f_1(\theta_x) = a \, \exp(-\theta_x^2 / 2\sigma^2)$

for $\theta_x > 2.2\sigma$, $\quad f_2(\theta_x) = \exp(b - c \, \theta_x)$

Conclusions

1. RMS deflection of protons due to MS in Si crystal for its amorphous orientation σ_s
obtained by BCOL simulation is in good agreement with our experiment
and 15% larger than from Moliere theory

2. Accuracy of CRYD simulation in Gaussian approach for MS with σ_s from BCOL
is better than 5% for channeling efficiency and about 10% for dechanneled fraction

3. Straight simulation of MS as the sequences of random collisions
with atoms gives RMS deflection close to the value obtained by BCOL

4. Collision number is different for different orientations at the same crystal length
However, MS is the same

5. Fast generator of multiple scattering was realized
for AM crystal orientations

CRYD model test for Volume Reflection orientation

Experiment on nuclear probability measurement at the SPS CERN

R=10.26 m

$\theta_{VR}+3\sigma$
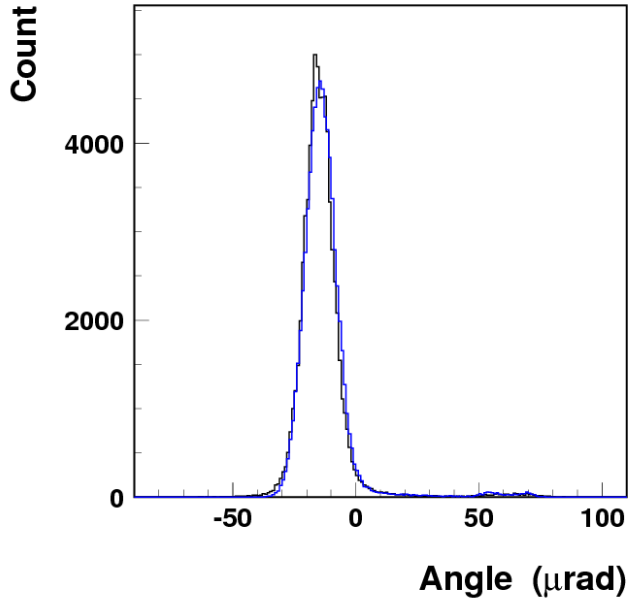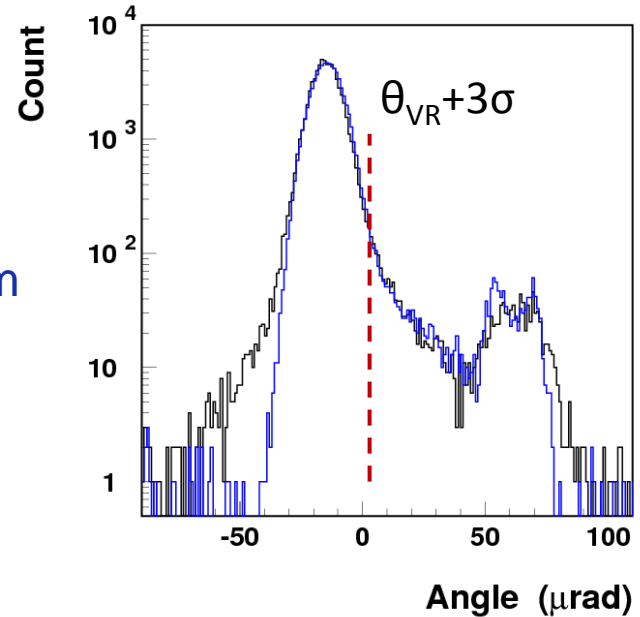
VR angle θ_{VR} :   15 μrad (ex) and 14.26 μrad (sim)

VR inefficiency ε : 3.07% (ex) and 2.99% (sim)

CRYD model results for STF45 crystal

Note: simulation performed with Gaussian incident beam

Experimental value presented with torsion correction for crystal
Therefore, angular distribution of considered beam fraction is not Gaussian

Channeling efficiency with angular cut ±10 µrad: 54% (ex) and 58.4% (sim)

with angular cut ±5 µrad: 68.9%% (ex) and 70.7% (sim)

Dechanneling length with angular cut ±10 µrad: 1.28 mm (ex) and 1.3 mm (sim)

with angular cut ±5 µrad: 1.41 mm (ex) and 1.42 mm (sim)

Volume reflection angle :   14.03 µrad (ex) and 14.8 µrad (sim)

RMS of VR deflection angle :   8.03 µrad (ex) and 6.6 µrad (sim)

RMS of VR outgoing angle :   8.3 µrad (sim)