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Cherenkov radiation from relativistic channeled particles

E.I. Fiks[1], O.V. Bogdanov[1], Yu. L. Pivovarov[1]

[1]Tomsk Polytechnic University, Tomsk, Russia

Motivation

The practical applications – e.g. Cherenkov counters are well-known. But they are valid when the trajectory is strictly straight-line and the charged particle moves with a constant velocity – in this case the radiation direction is fixed at the Cherenkov angle.

These conditions can be destroyed due to several reasons:

- **Multiple scattering** (electrons) in a radiator - trajectories are not straight-line [1]
- **Slowing down** (Relativistic Heavy Ions - RHI) – trajectory remains straight-line, but the velocity decrease due to ionization energy loss [2-4]
- **Channeling in a crystal**: trajectory is characterized by longitudinal motion along crystallographic planes with relativistic velocity and by periodic motion in transverse direction

Goal of our work: study in detail influence of the planar channeling effect on the Cherenkov radiation (ChR) spectral and angular distributions.

Earlier works in this field:

[1] K.G. Dedrick // Phys. Rev. 87 (1952) p. 891.
[2] E. I. Fiks, O. V. Bogdanov, Y. L. Pivovarov // Journal of Experimental and Theoretical Physics Vol. 115 No. 3 (2012) p. 392.
[3] E. I. Fiks, O. V. Bogdanov, Y. L. Pivovarov, H. Geissel, C. Scheidenberger // Nucl. Instr. Meth. Phys. Res. B. Vol. 309 (2013) p. 146.
[4] E. I. Fiks, O. V. Bogdanov, Y. L. Pivovarov, H. Geissel, C. Scheidenberger // J. Ruzicka Nucl. Instr. Meth. Phys. Res. B. Vol. 314 (2013) p. 51.
[5] V. G. Baryshevskii (1982)
[6] V. Dolgikh, E. Vyatkin // RPJ (1988) p. 137.
[7] D. Popov, E. Rozum, S. Uglov et al. // Phys. Stat. Sol. (1984, 1987)
[8] S. Bellucci and V. A. Maisheev // J. Phys.: Condens. Matter 18 (2006) p. S2083.
[9] V. L. Ginzburg: Theoretical Physics and Astrophysics . - Pergamon Press; 1st edition (February 1979)
[10] V. M. Grichine // Radiation Physics and Chemistry 67 (2003) p. 93.

Motion equation

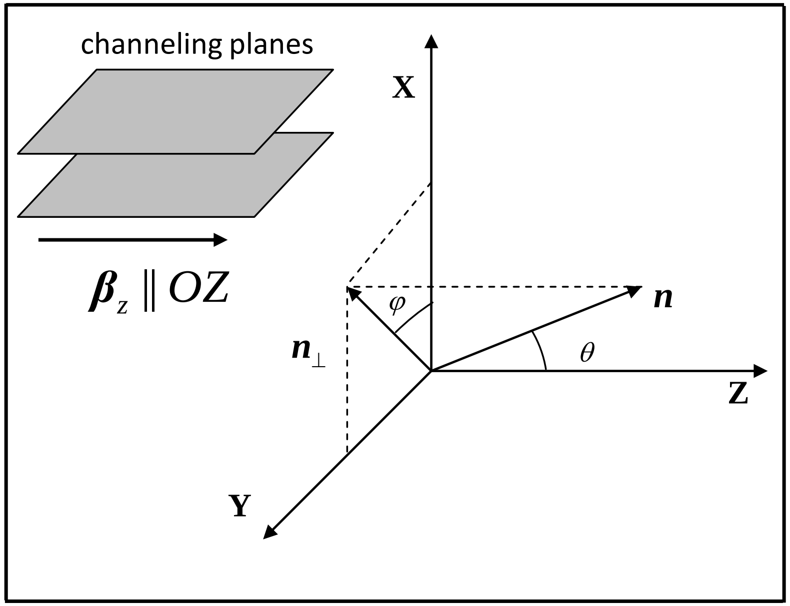
$$\mathbf{n} = \{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$$

$$\mathbf{r}(t) = \{x(t), 0, z(t)\} \qquad \mathbf{v}(t) = \{v_x(t), 0, v_z(t)\}$$

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m}\sqrt{1-\frac{\mathrm{v}^2}{\mathrm{c}^2}}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v},\mathbf{B}] - \frac{\mathbf{v}}{\mathrm{c}^2}(\mathbf{vE})\right\}$$

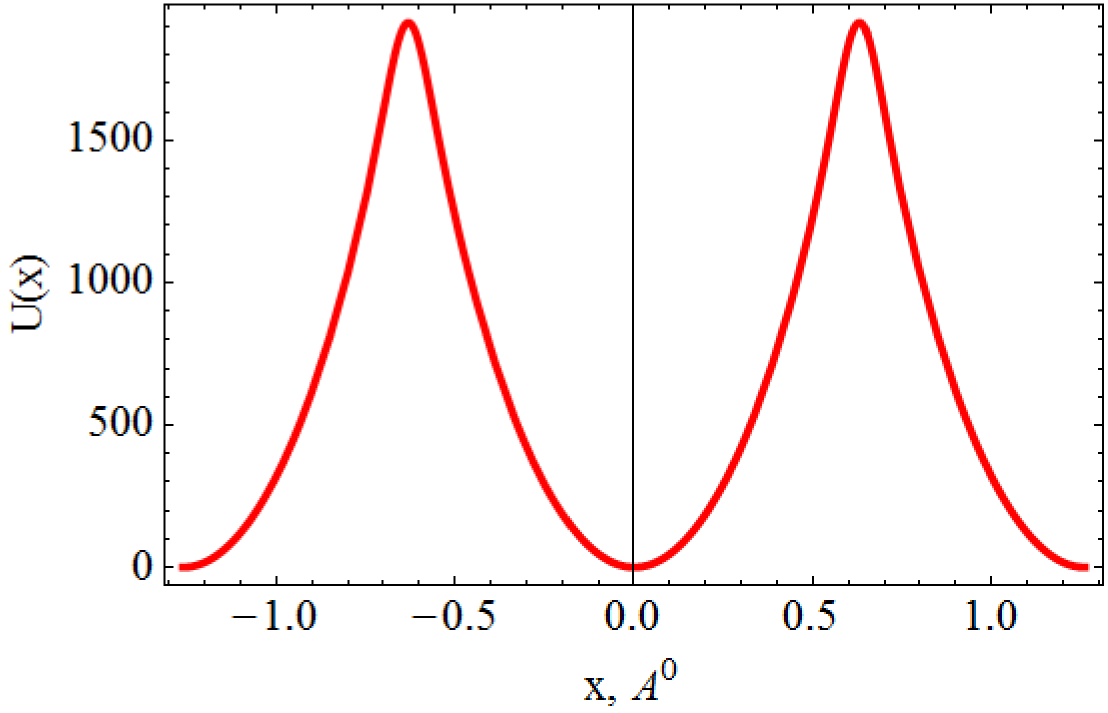
$$\mathbf{B} = 0 \qquad E_z = 0 \qquad (eZ)E_x = -\frac{\partial U}{\partial x}$$

$$\begin{cases} \dfrac{d\mathrm{v}_x}{dt} = -\dfrac{\partial U}{\partial x}\dfrac{1}{m\gamma\gamma_x^2} \\ \dfrac{d\mathrm{v}_z}{dt} = \dfrac{\partial U}{\partial x}\dfrac{1}{m\gamma}\dfrac{\mathrm{v}_z\mathrm{v}_x}{\mathrm{c}^2} \end{cases}$$

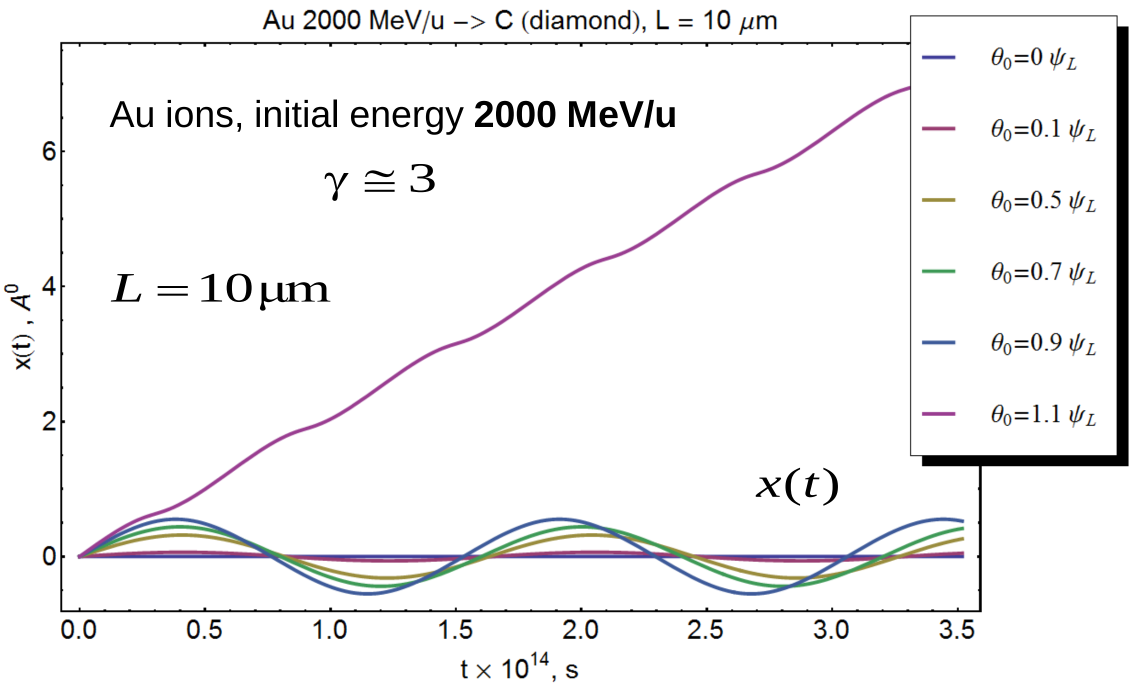


Crystal potential

Au ions in a Diamond crystal: planar channeling



The plane potential U(x) is calculated using the approximation of the atomic form factor (see e.g. [Kh. Chouffani, PhD Thesis (Washington, D.C., 1995)])

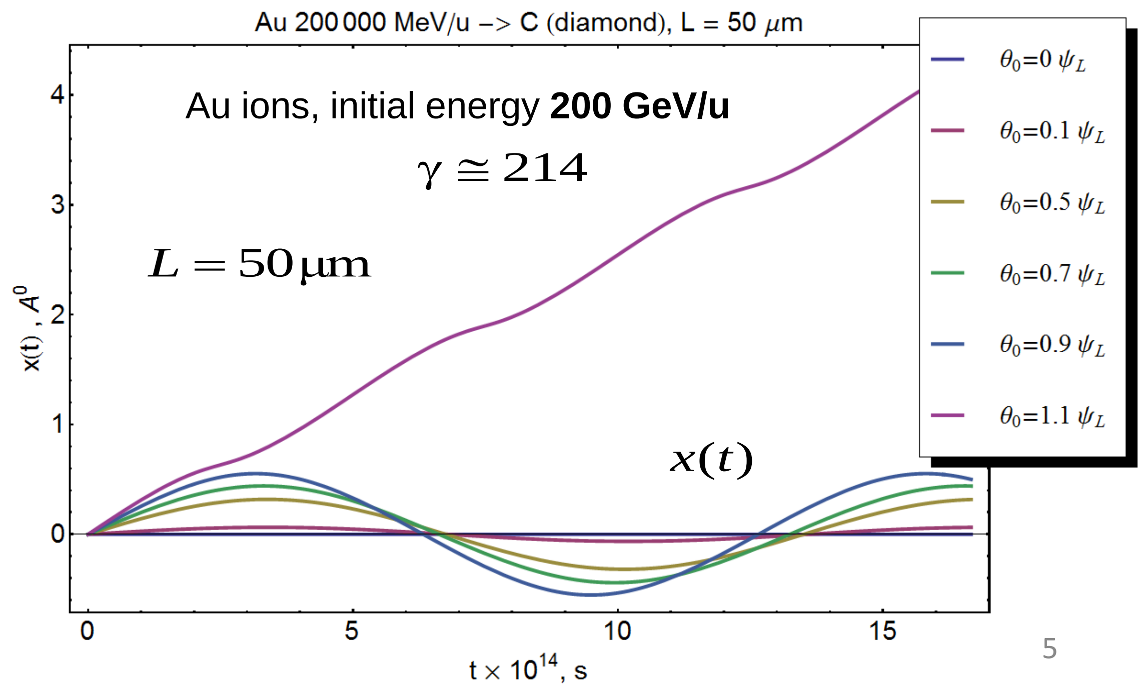


Au ions ->
diamond crystal
(110) planar channeling

Typical trajectories

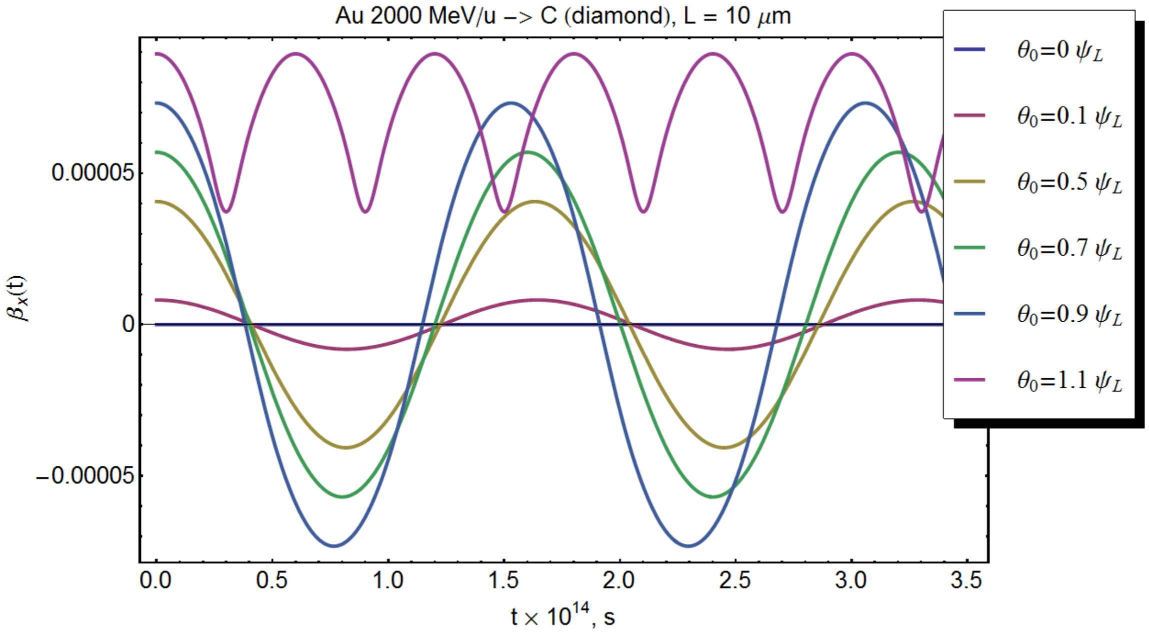
incidence point $x_0 = 0$

$$\begin{cases} \dfrac{dv_x}{dt} = -\dfrac{\partial U}{\partial x}\dfrac{1}{m\gamma\gamma_x^2} \\ \dfrac{dv_z}{dt} = \dfrac{\partial U}{\partial x}\dfrac{1}{m\gamma}\dfrac{v_z v_x}{c^2} \end{cases}$$



θ_0 - incidence angle

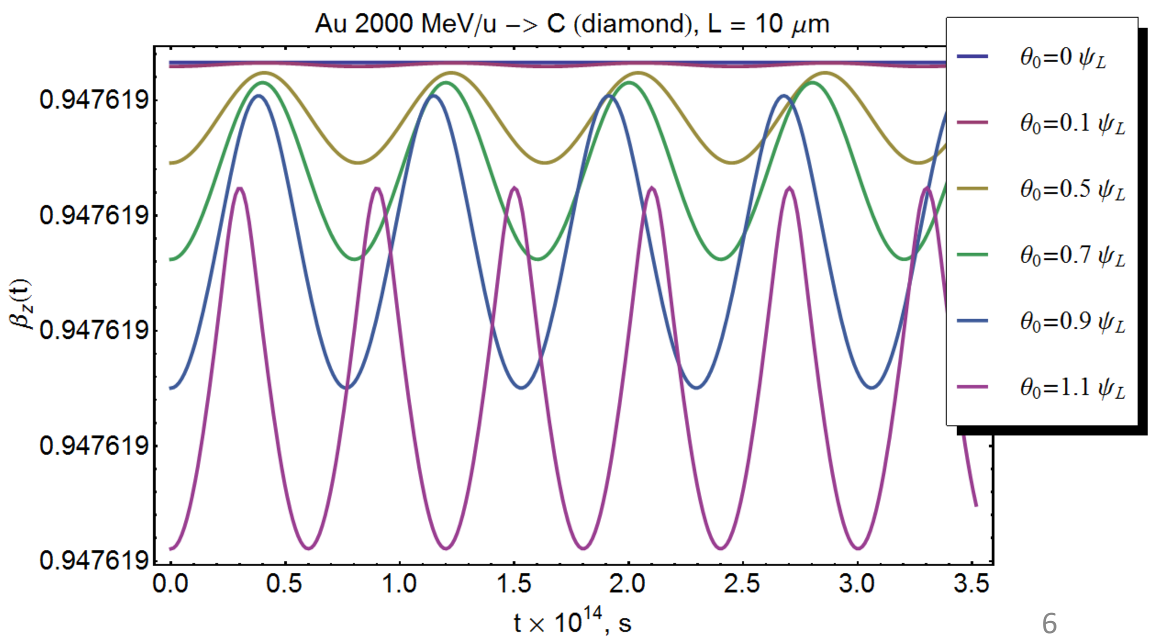
ψ_L - Lindhard angle

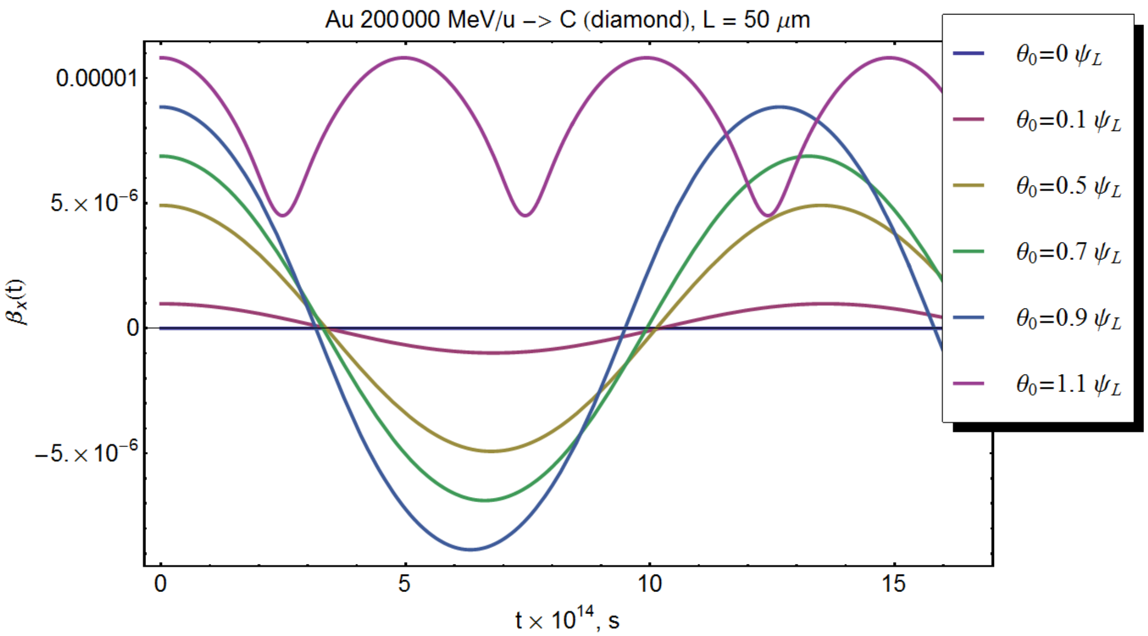


Au ions,
initial energy **2000 MeV/u**
diamond crystal **10 μm**

$\beta_x(t)$

$\beta_z(t)$

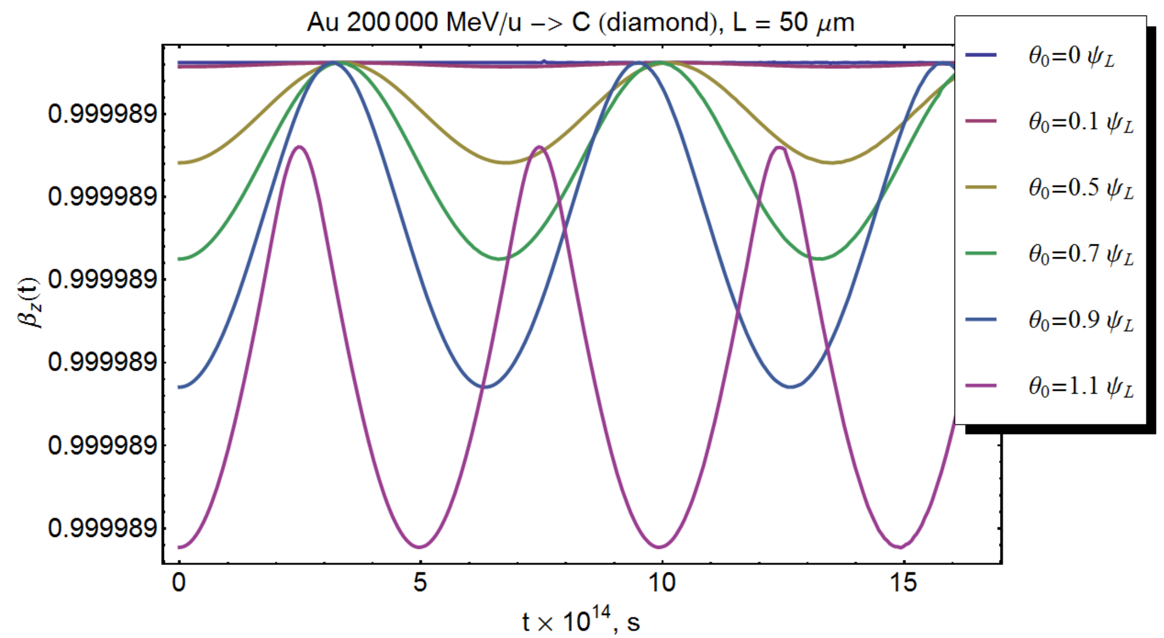




Au ions,
initial energy **200 GeV/u**
diamond crystal **50 μm**

$\beta_x(t)$

$\beta_z(t)$



Cherenkov radiation spectral-angular distributions

$$\frac{d\varepsilon}{d\Omega d\omega} = \frac{e^2 Z^2 \omega^2}{4\pi^2 c^3}\sqrt{\varepsilon}\left|\int_0^\tau [\mathbf{n}[\mathbf{n}\mathbf{v}]] e^{i(\omega t - \mathbf{k}\mathbf{r})} dt\right|^2$$

$$\mathbf{n} = \{\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta\}$$

$$\mathbf{k} = \frac{\omega}{c}\sqrt{\varepsilon}\,\mathbf{n} \qquad \mathbf{r} = \{x(t),\ 0,\ z(t)\}$$

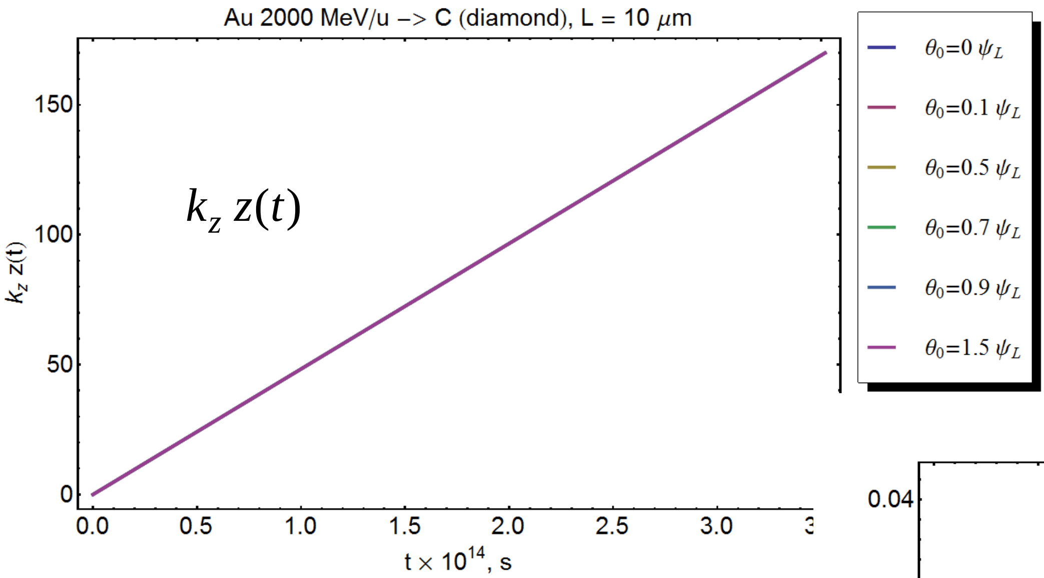
$$\mathbf{v} = \{v_x,\ 0,\ v_z\} = \{x'(t),\ 0,\ z'(t)\}$$

Assuming $k_x x(t) << 1$ one obtains (see e.g. [V.M.Grichine. Rad.Phys. and Chem. 67 (2003) 93-103])

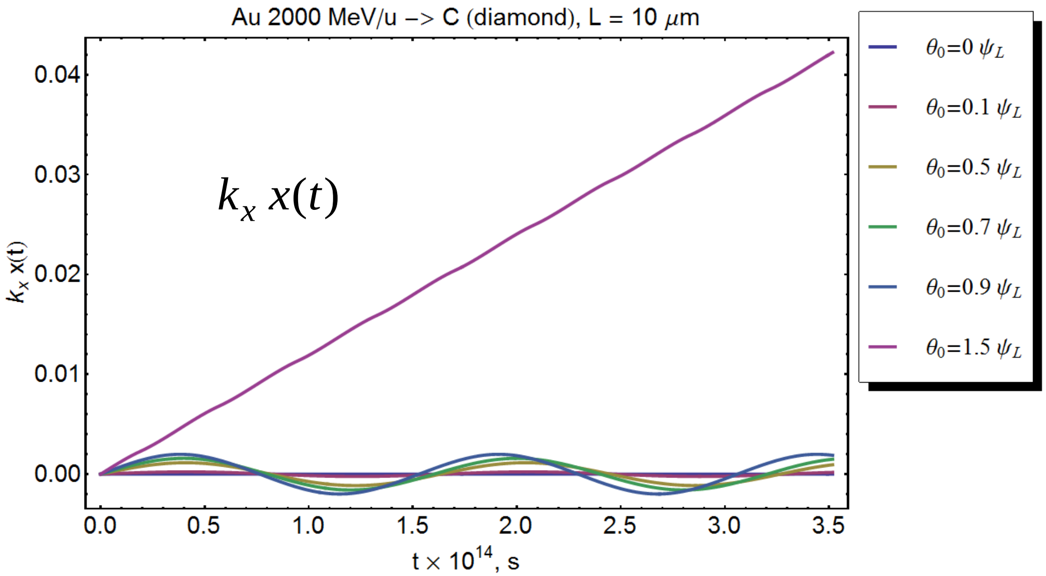
$$e^{i(\omega t - \mathbf{k}\mathbf{r})} = e^{i(\omega t - k_x x(t) - k_z z(t))} = e^{-i k_x x(t)} e^{i(\omega t - k_z z(t))} = (1 - i k_x x(t)) e^{i(\omega t - k_z z(t))}$$

$$\mathbf{v} = \mathbf{v}_x + \mathbf{v}_z = \{v_x,\ 0,\ 0\} + \{0,\ 0,\ v_z\}$$

$$\begin{aligned}
[\mathbf{n}[\mathbf{n}\mathbf{v}]] e^{i(\omega t - \mathbf{k}\mathbf{r})} &= \left([\mathbf{n}[\mathbf{n}\mathbf{v}_x]] + [\mathbf{n}[\mathbf{n}\mathbf{v}_z]]\right)(1 - i k_x x(t)) e^{i(\omega t - k_z z(t))} = \\
&= [\mathbf{n}[\mathbf{n}\mathbf{v}_z]] e^{i(\omega t - k_z z(t))} - i k_x x(t) [\mathbf{n}[\mathbf{n}\mathbf{v}_z]] e^{i(\omega t - k_z z(t))} + \\
&+ [\mathbf{n}[\mathbf{n}\mathbf{v}_x]] e^{i(\omega t - k_z z(t))} - i k_x x(t) [\mathbf{n}[\mathbf{n}\mathbf{v}_x]] e^{i(\omega t - k_z z(t))}
\end{aligned}$$



Au ions,
initial energy **2000 MeV/u**
diamond crystal **10 μm**



$$[\mathbf{n}[\mathbf{n}\mathbf{v}]]e^{i(\omega t-\mathbf{kr})} = \left([\mathbf{n}[\mathbf{n}\mathbf{v}_x]]+[\mathbf{n}[\mathbf{n}\mathbf{v}_z]]\right)(1-i\,k_x x(t))e^{i(\omega t-k_z z(t))} =$$

$$= [\mathbf{n}[\mathbf{n}\mathbf{v}_z]]e^{i(\omega t-k_z z(t))} - i\,k_x x(t)[\mathbf{n}[\mathbf{n}\mathbf{v}_z]]e^{i(\omega t-k_z z(t))} +$$

$$+ [\mathbf{n}[\mathbf{n}\mathbf{v}_x]]e^{i(\omega t-k_z z(t))} - i\,k_x x(t)[\mathbf{n}[\mathbf{n}\mathbf{v}_x]]e^{i(\omega t-k_z z(t))}$$

Cherenkov radiation spectral-angular distributions: channeling correction

$$\frac{d\varepsilon}{d\Omega d\omega} = \frac{e^2 Z^2 \omega^2}{4\pi^2 c^3} \sqrt{\varepsilon} \left| \mathbf{E}_\omega \right|^2$$

$$\mathbf{E}_\omega = \int_0^\tau [\mathbf{n}[\mathbf{n}\mathbf{v}]] e^{i(\omega t - \mathbf{kr})} dt = \mathbf{E}_{TF} + \mathbf{E}_{Channeling}$$

$$\mathbf{E}_{TF} = \int_0^\tau [\mathbf{n}[\mathbf{n}\mathbf{v}_Z]] e^{i(\omega t - k_z z(t))} dt \qquad \mathbf{E}_{Channeling} = \int_0^\tau (-i k_x x(t)) [\mathbf{n}[\mathbf{n}\mathbf{v}_Z]] e^{i(\omega t - k_z z(t))} dt$$

$$\frac{d\varepsilon}{d\Omega d\omega} = \frac{e^2 Z^2 \omega^2}{4\pi^2 c^3} \sqrt{\varepsilon} \left| \mathbf{E}_{TF} + \mathbf{E}_{Channeling} \right|^2 \cong$$

$$\cong \frac{e^2 Z^2 \omega^2}{4\pi^2 c^3} \sqrt{\varepsilon} \left(\left| \mathbf{E}_{TF} \right|^2 + 2 \operatorname{Re}(\mathbf{E}_{TF}, \mathbf{E}_{Channeling}{}^*) \right)$$

Cherenkov radiation spectral-angular distributions: channeling correction

$$\frac{d\varepsilon}{d\Omega d\omega} = \left.\frac{d\varepsilon}{d\Omega d\omega}\right|_{TF} + \left.\frac{d\varepsilon}{d\Omega d\omega}\right|_{Channeling}$$

standard Tamm-Frank distribution:

$$\left.\frac{d\varepsilon}{d\Omega d\omega}\right|_{TF} = \frac{e^2 Z^2 \omega^2}{4\pi^2 c^3}\sqrt{\varepsilon}\left|\int_0^\tau \left[\mathbf{n}\left[\mathbf{n}\mathbf{v}_z\right]\right] e^{i(\omega t - k_z z(t))} dt\right|^2$$

correction that appears in the case of planar channeling:

$$\left.\frac{d\varepsilon}{d\Omega d\omega}\right|_{Channeling} =$$

$$= \frac{e^2 Z^2 \omega^2}{2\pi^2 c^3}\sqrt{\varepsilon}\,\mathrm{Re}\left(\int_0^\tau \left[\mathbf{n}\left[\mathbf{n}\mathbf{v}_z\right]\right] e^{i(\omega t - k_z z(t))} dt, \left(\int_0^\tau (-i k_x x(t)) \left[\mathbf{n}\left[\mathbf{n}\mathbf{v}_z\right]\right] e^{i(\omega t - k_z z(t))} dt\right)^*\right)$$

Cherenkov radiation angle

Standard Tamm-Frank distribution (constant velocity): $\mathbf{r} = \{0,\ 0,\ z(t)\}$ $\mathbf{v} = \{0, 0, v_z\}, \quad v_z = const$

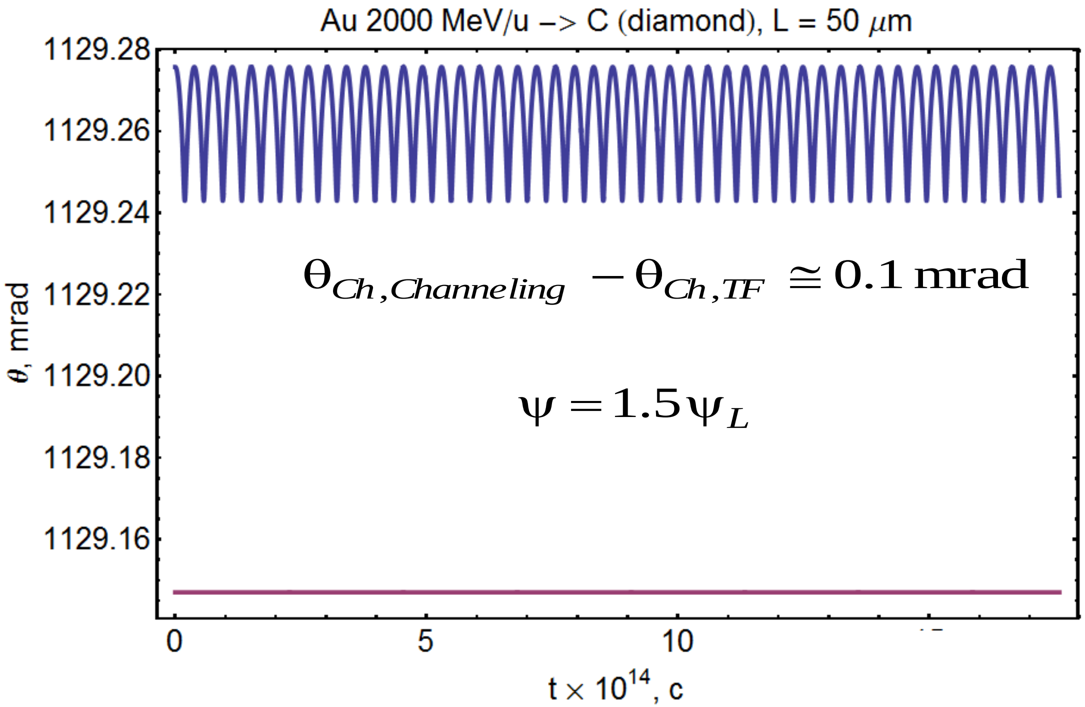
$$\left.\frac{d\varepsilon}{d\Omega d\omega}\right|_{TF} = \frac{e^2 Z^2 \omega^2}{4\pi^2 c^3} \sqrt{\varepsilon} \left| \int_0^{\tau} [\mathbf{n}[\mathbf{n}\mathbf{v}]] e^{i(\omega t - \mathbf{k}\mathbf{r})} dt \right|^2 \Rightarrow \theta_{Ch,TF} = \arccos\left(\frac{1}{\beta_z \sqrt{\varepsilon}} \right)$$

Planar channeling case: $\mathbf{r} = \{x(t),\ 0,\ z(t)\}$ $\mathbf{v} = \{v_x,\ 0,\ v_z\} = \{x'(t),\ 0,\ z'(t)\}$

$$\left.\frac{d\varepsilon}{d\Omega d\omega}\right|_{TF} = \frac{e^2 Z^2}{4\pi^2 c} \left| \int_{-\infty}^{+\infty} \frac{[\mathbf{n}[(\mathbf{n} - \boldsymbol{\beta})\boldsymbol{\beta}']]}{(1 - \sqrt{\varepsilon}\,\boldsymbol{\beta}\mathbf{n})^2} e^{i(\omega t - \mathbf{k}\mathbf{r})} dt \right|^2 \Rightarrow \begin{cases} 1 - \sqrt{\varepsilon}\,\boldsymbol{\beta}\mathbf{n} = 0, \\ \mathbf{n} = \{\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta\} \end{cases}$$

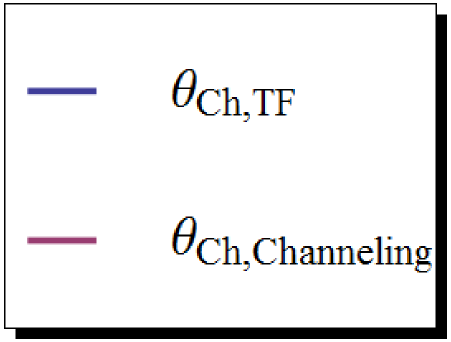
$$\theta_{Ch,Channeling} = \arccos\left(\frac{4\beta_z\sqrt{\varepsilon} - \sqrt{16\,\beta_z{}^2\varepsilon - 4(\beta_x{}^2\varepsilon - 2\beta_z{}^2\varepsilon - \beta_x{}^2\varepsilon\cos(2\varphi))(\beta_x{}^2\varepsilon + 2\beta_z{}^2\varepsilon + \beta_x{}^2\varepsilon\cos(2\varphi))}}{2(\beta_x{}^2\varepsilon + 2\beta_z{}^2\varepsilon + \beta_x{}^2\varepsilon\cos(2\varphi))} \right)$$

Cherenkov angle

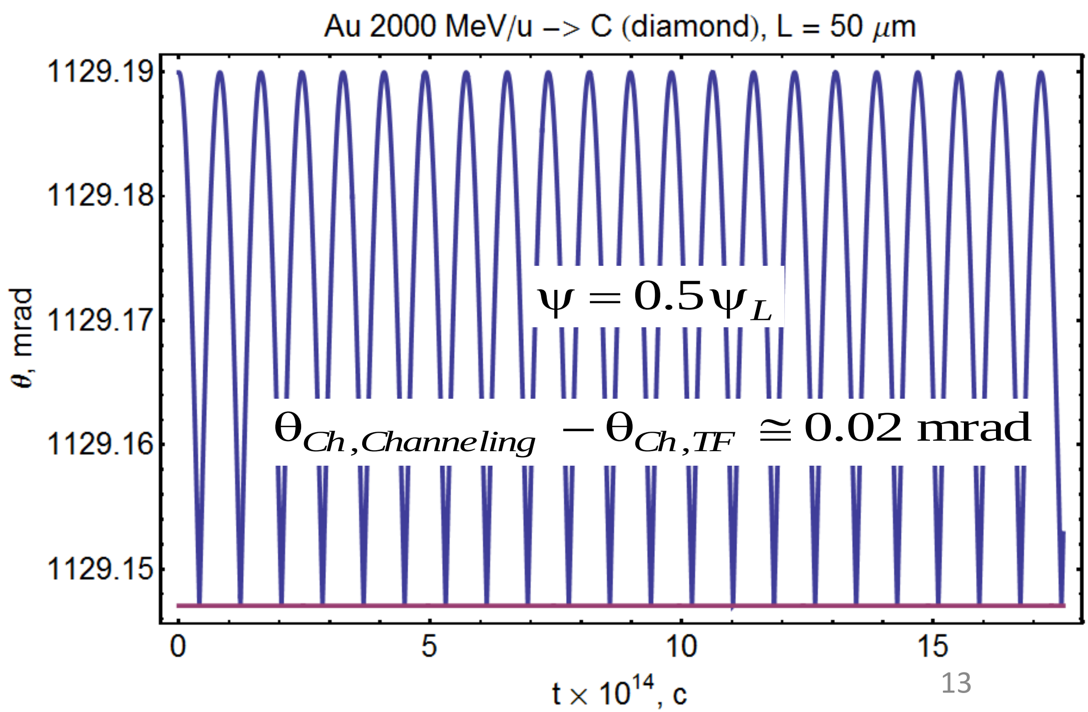


above-barrier motion

Au ions,
initial energy **2000 MeV/u**
diamond crystal **50 μm**

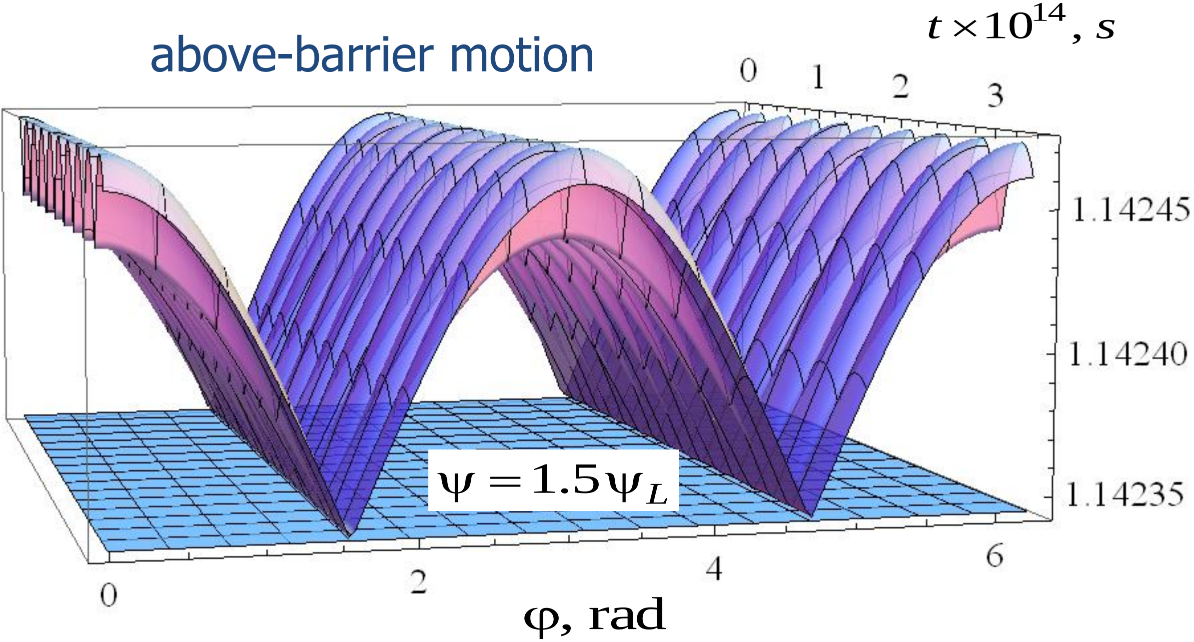


under-barrier motion

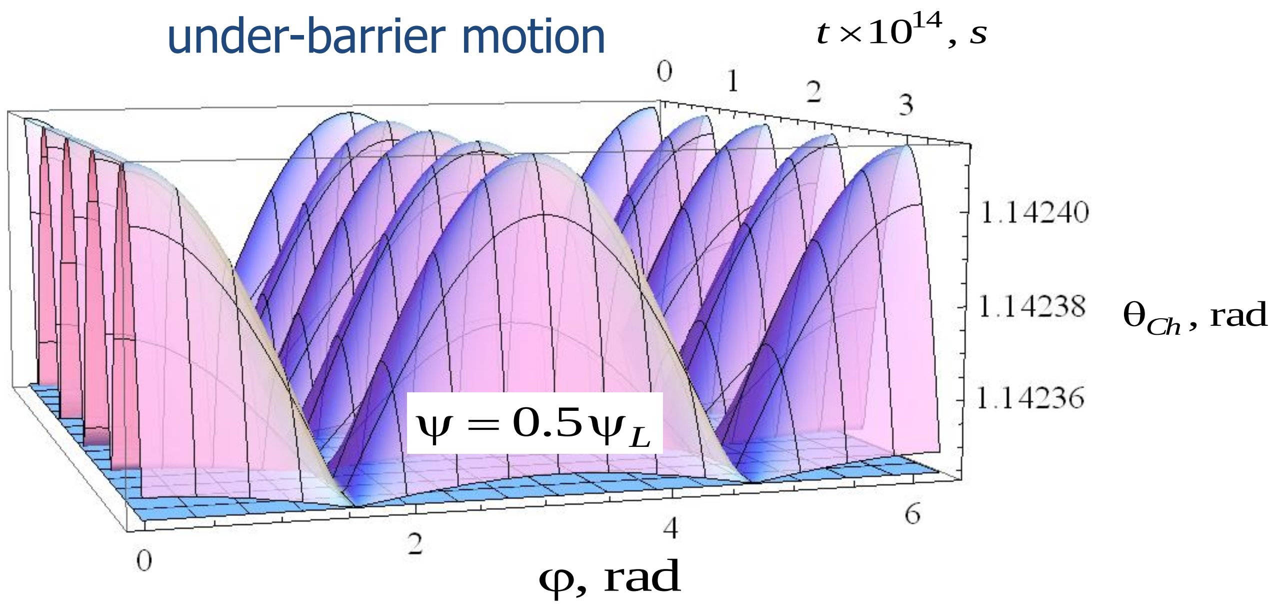


Cherenkov angle φ - dependence

above-barrier motion



under-barrier motion

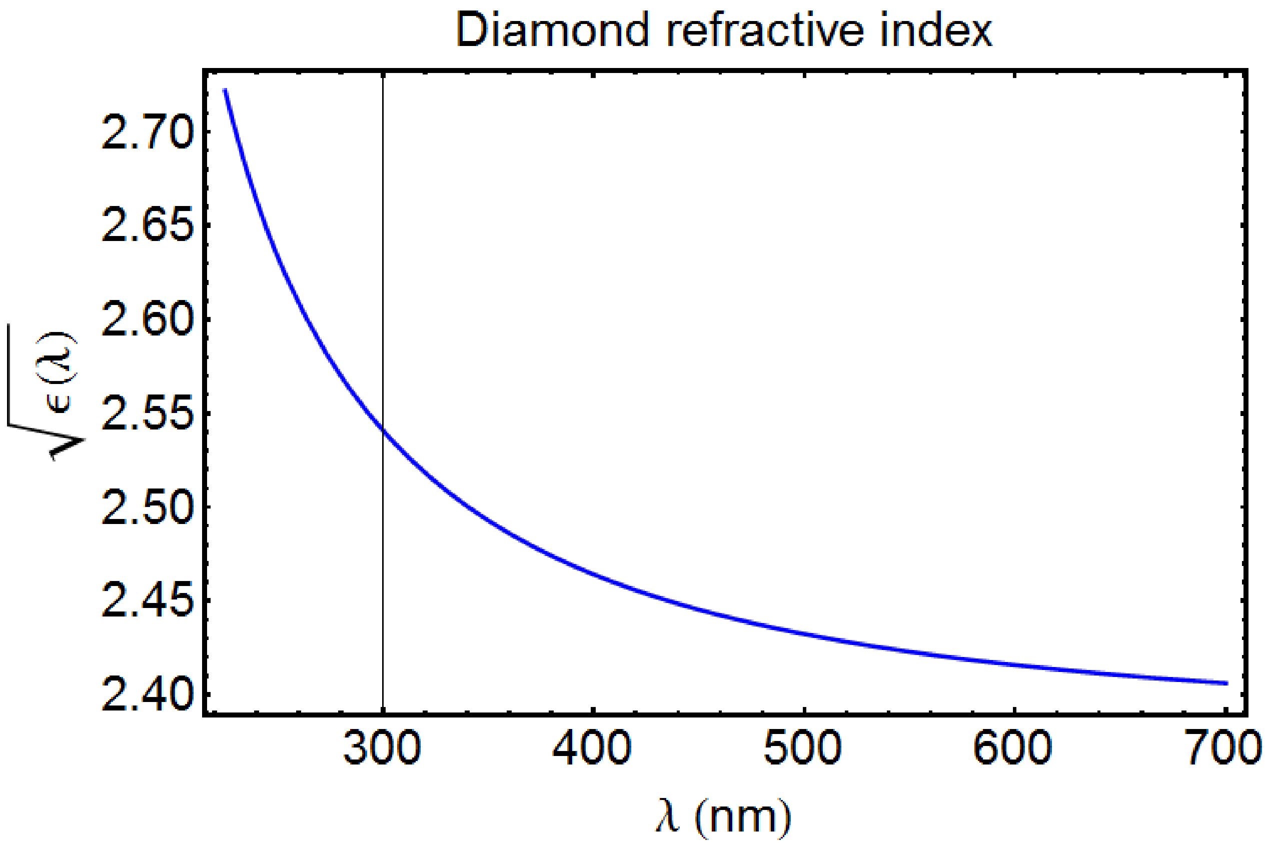


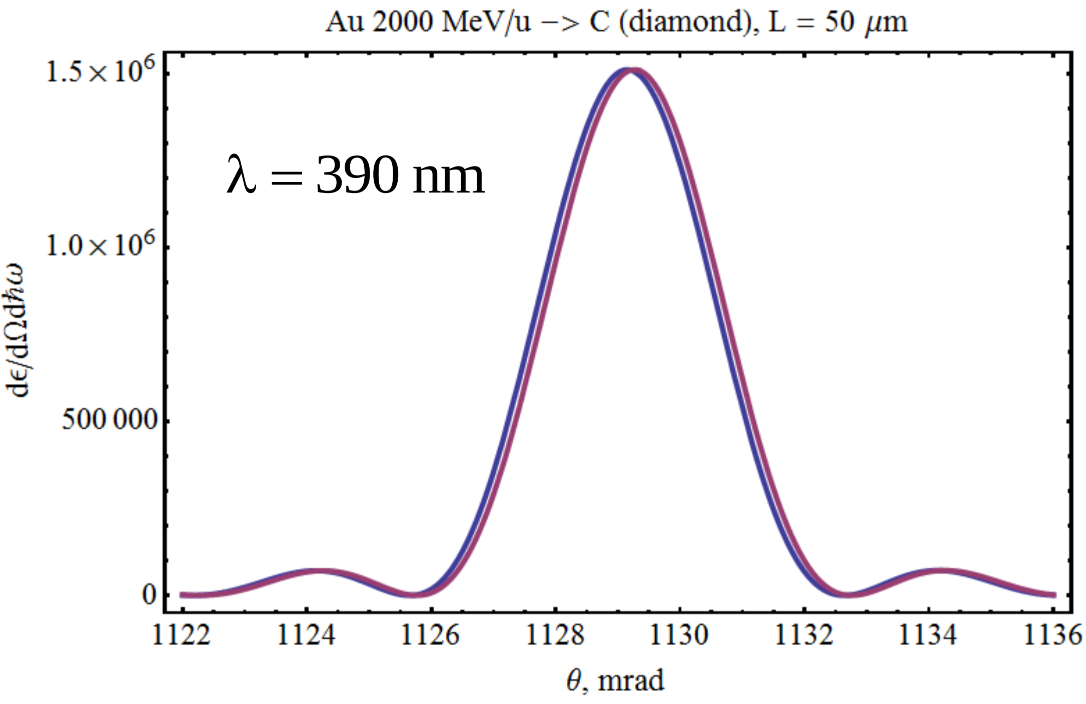
Au ions,
initial energy **2000 MeV/u**
diamond crystal **10 μm**

Diamond refractive index

$$\varepsilon(\lambda) - 1 = \frac{4.3356\,\lambda^2}{\lambda^2 - 0.1060^2} + \frac{0.3306\,\lambda^2}{\lambda^2 - 0.1750^2}$$

[http://refractiveiindex.info]

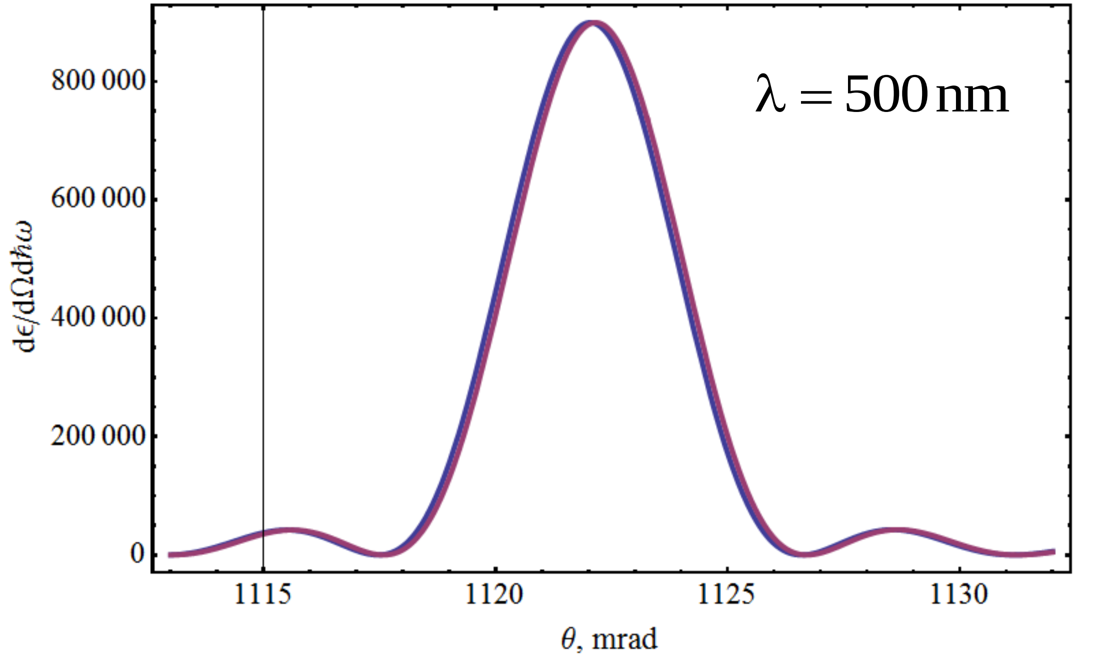




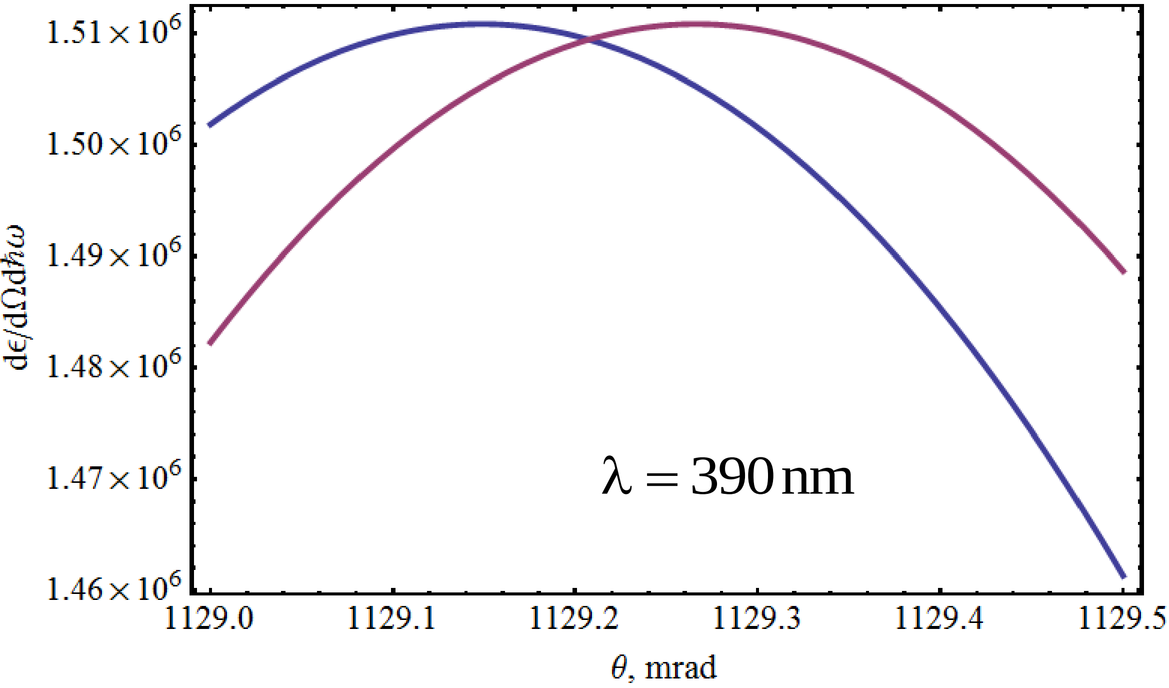
Cherenkov radiation angular distributions (above-barrier motion)

Au ions **2000 MeV/u** ->
diamond crystal **50 μm**
(110) planar channeling

incidence angle $\psi = 1.5\,\psi_L$

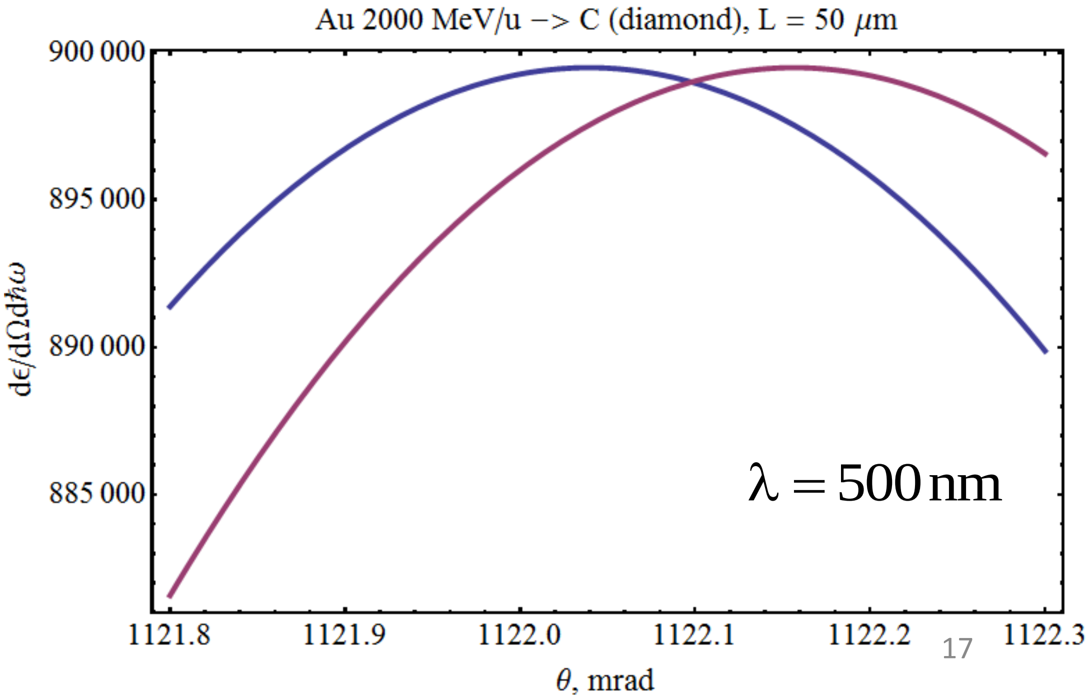


Cherenkov radiation angular distributions (above-barrier motion)

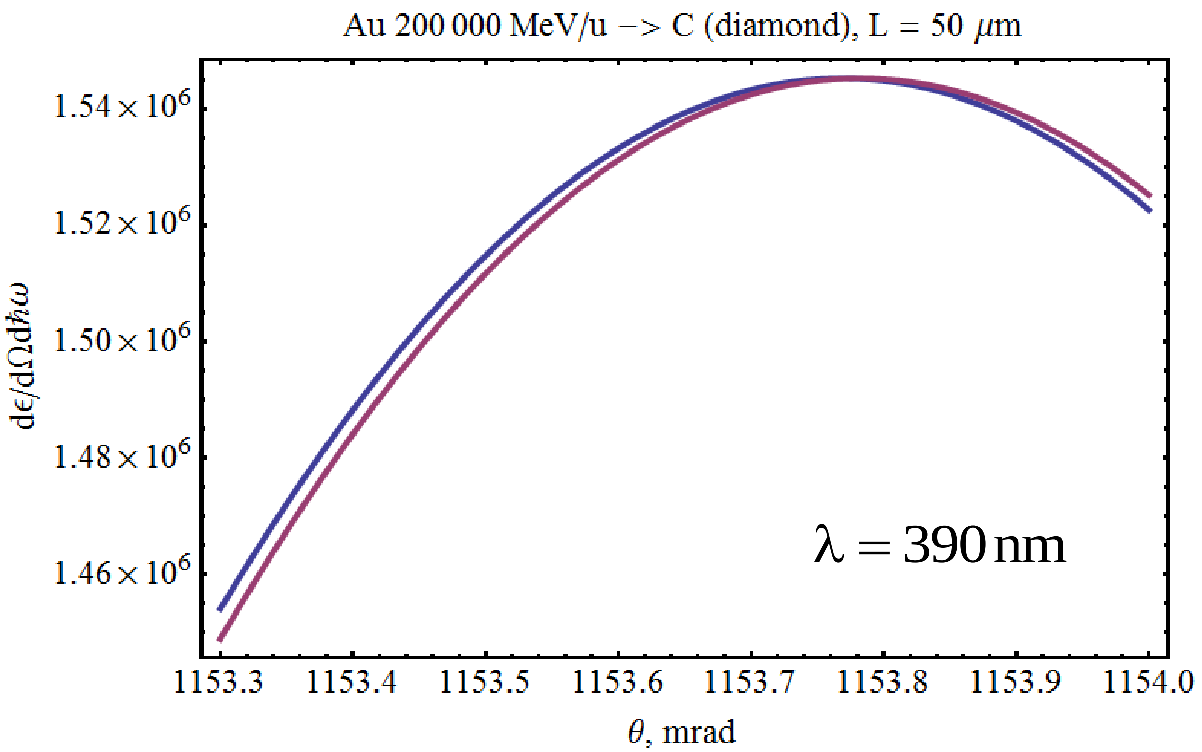


Au ions **2000 MeV/u** -> diamond crystal **50 μm** **(110)** planar channeling

incidence angle $\psi = 1.5\,\psi_L$



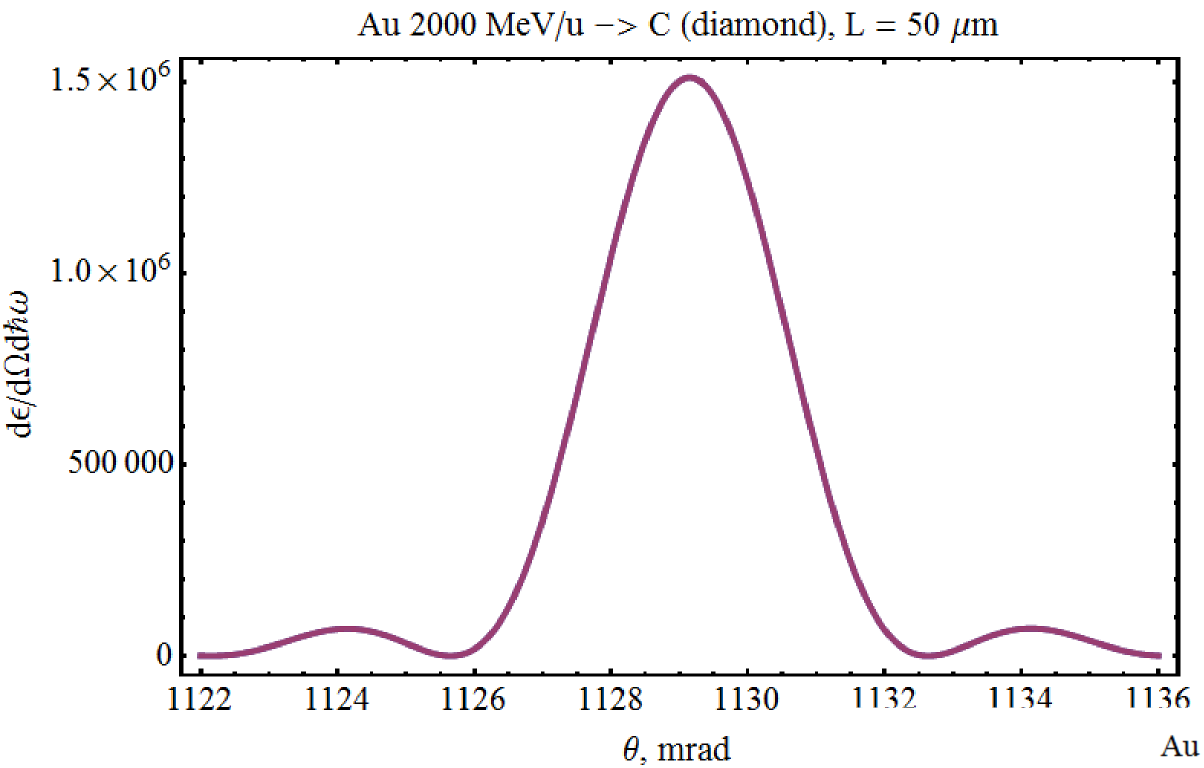
Cherenkov radiation angular distributions (above-barrier motion)



Au ions **200 GeV/u** -> diamond crystal **50 μm** **(110)** planar channeling

incidence angle $\psi = 1.5\,\psi_L$

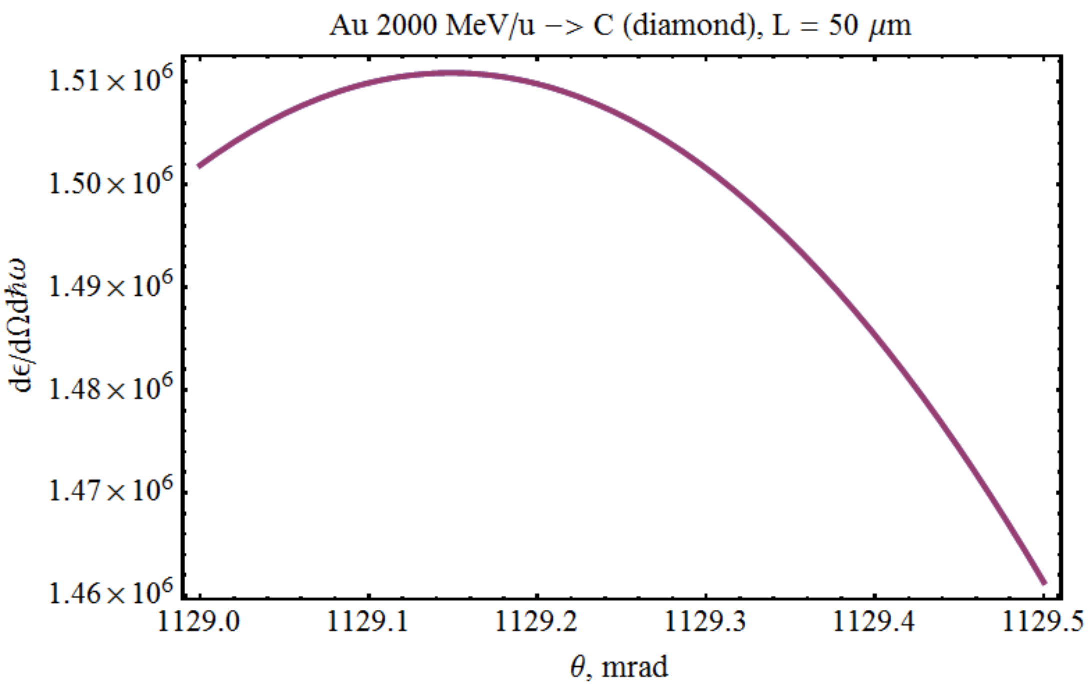
Cherenkov radiation angular distributions (under-barrier motion)



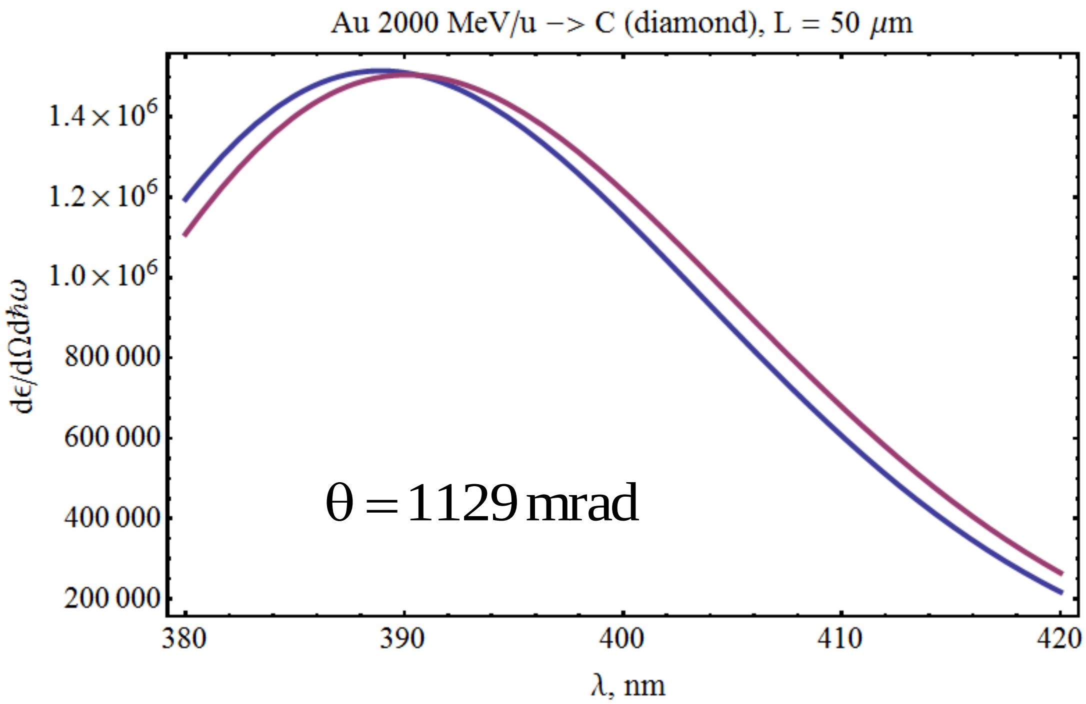
Au ions **2000 MeV/u** -> diamond crystal **50 μm** **(110)** planar channeling

incidence angle $\psi = 0.5\,\psi_L$

$\lambda = 390\,\mathrm{nm}$

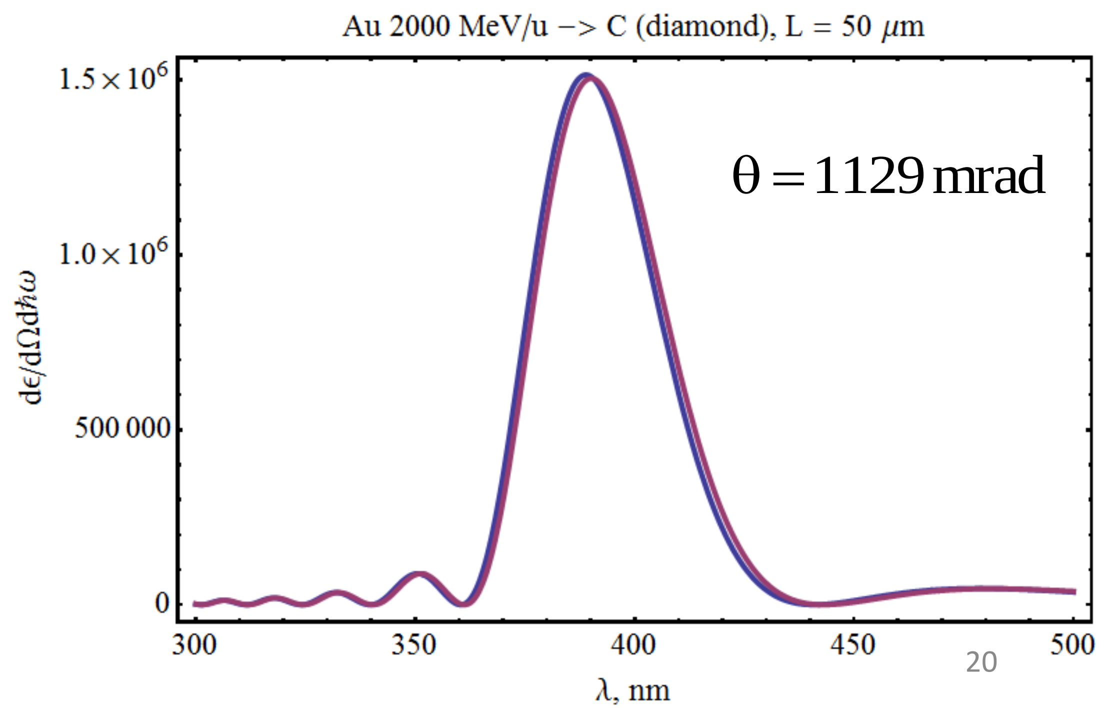


Cherenkov radiation spectral distributions (above-barrier motion)

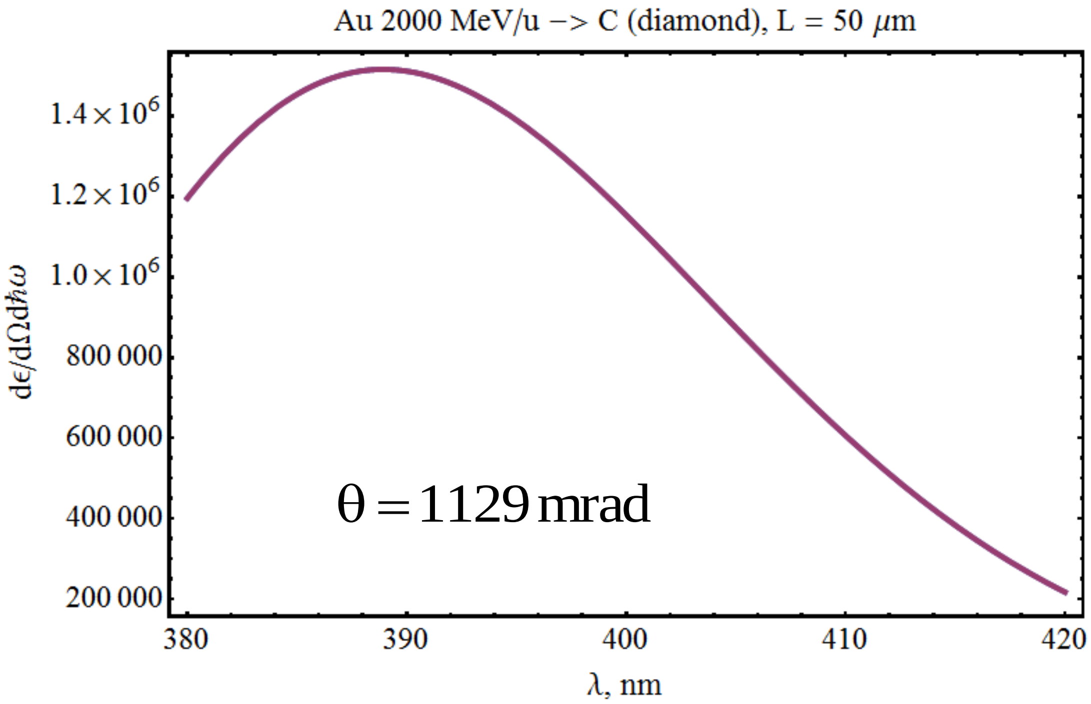


Au ions **2000 MeV/u** ->
diamond crystal **50 μm**
(110) planar channeling

incidence angle $\psi = 1.5\,\psi_L$

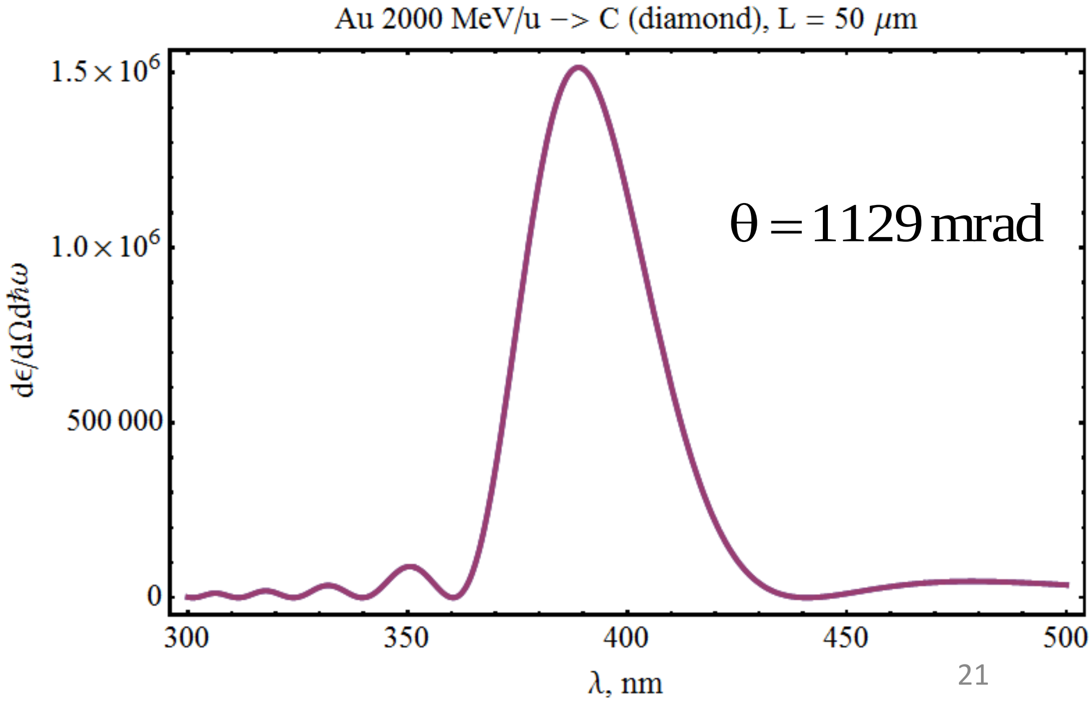


Cherenkov radiation spectral distributions (under-barrier motion)



Au ions **2000 MeV/u** -> diamond crystal **50 μm** (110) planar channeling

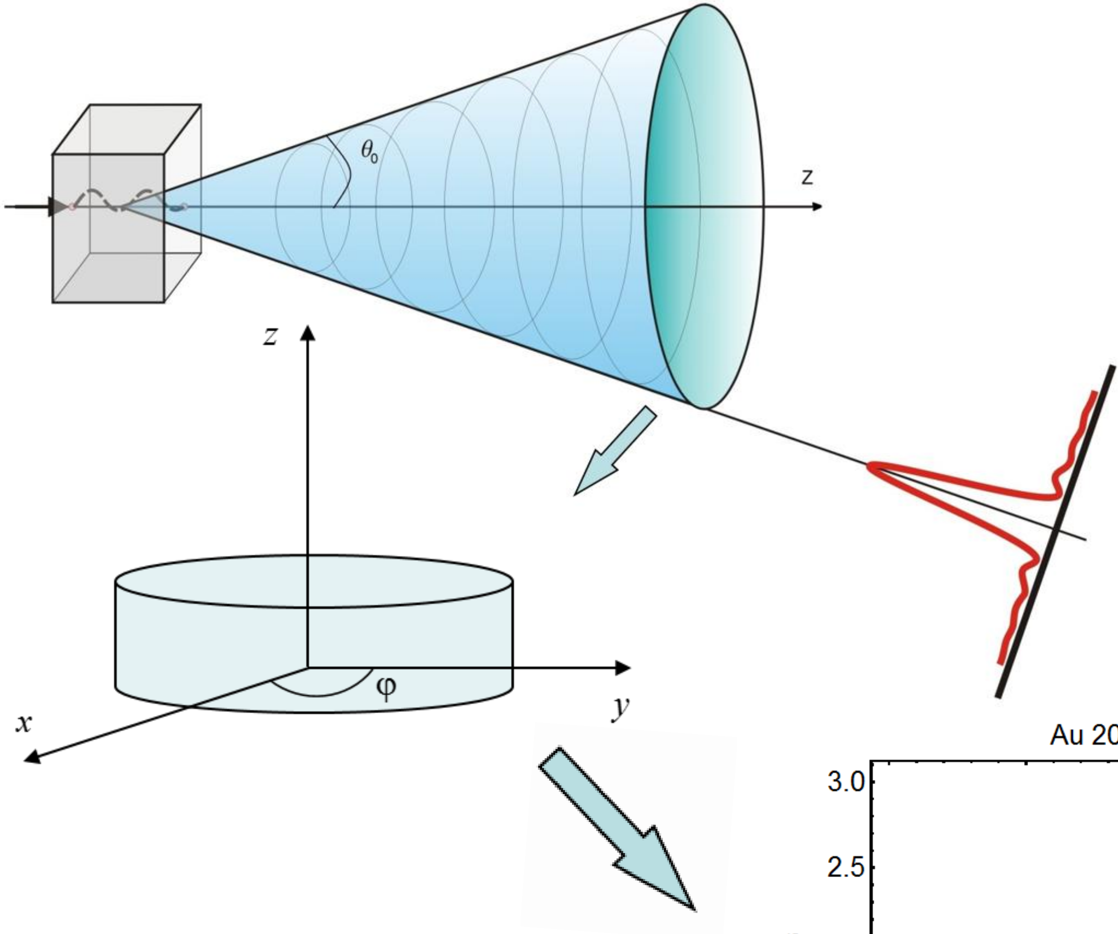
incidence angle $\psi = 0.5\,\psi_L$

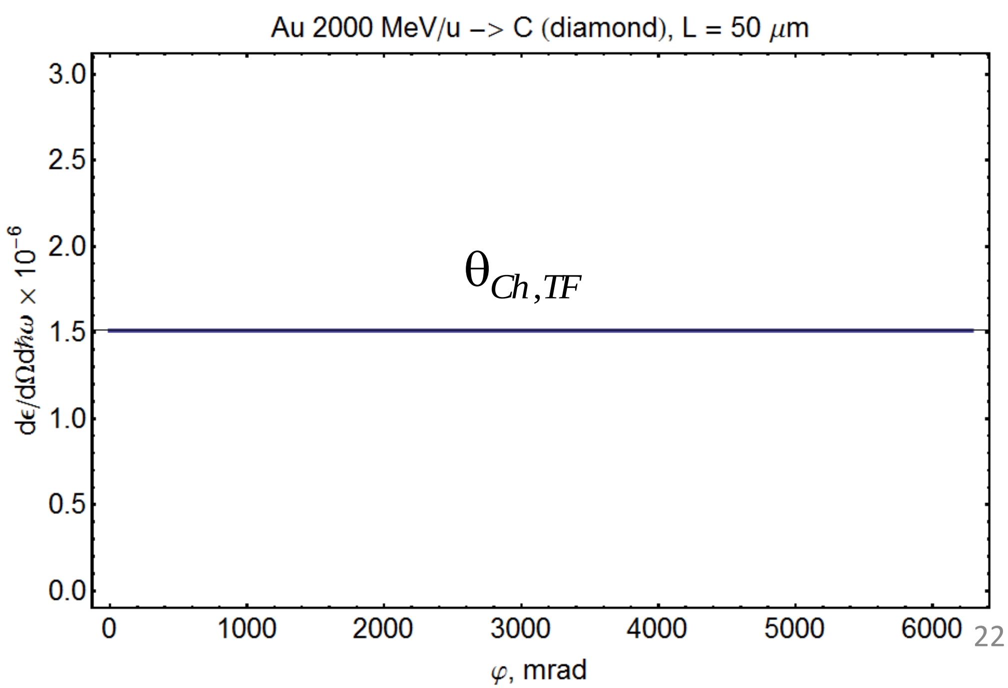


φ - dependence (rectilinear motion)

$$\mathbf{v} = \{0,0,v_z\}, \quad v_z = const$$

Au ions **2000 MeV/u** -> diamond crystal **50 µm** (110) planar channeling

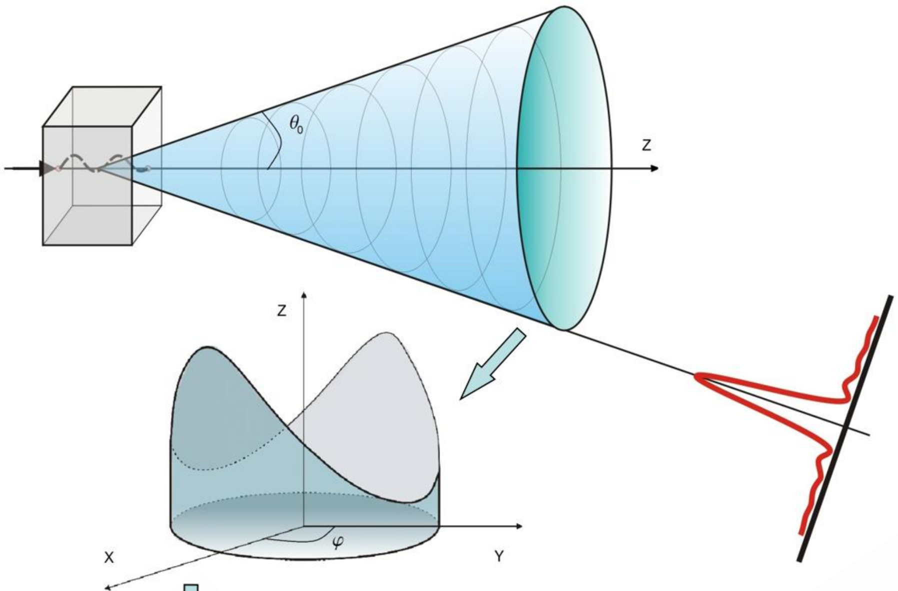




φ - dependence (channeling condition)

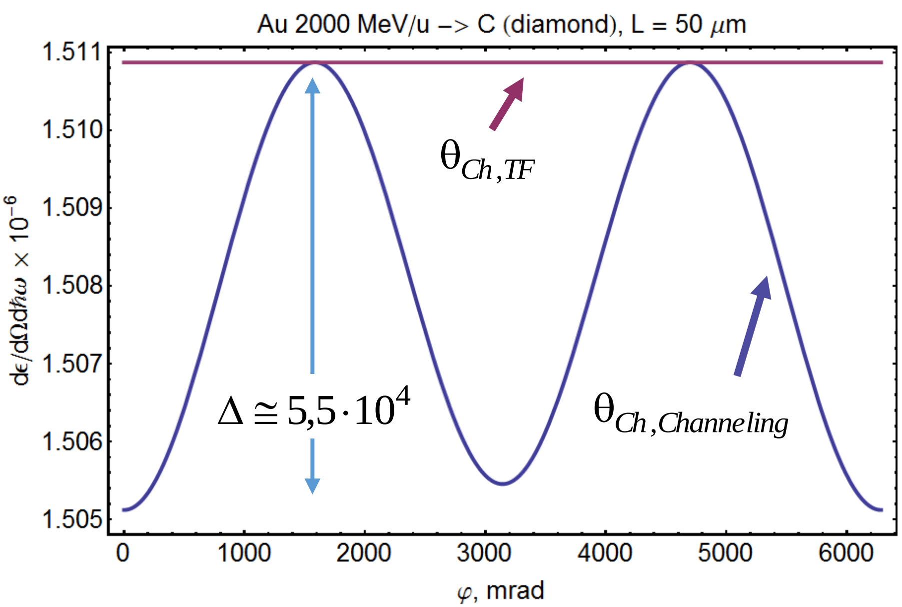
$$\mathbf{v} = \{v_x, 0, v_z\}$$

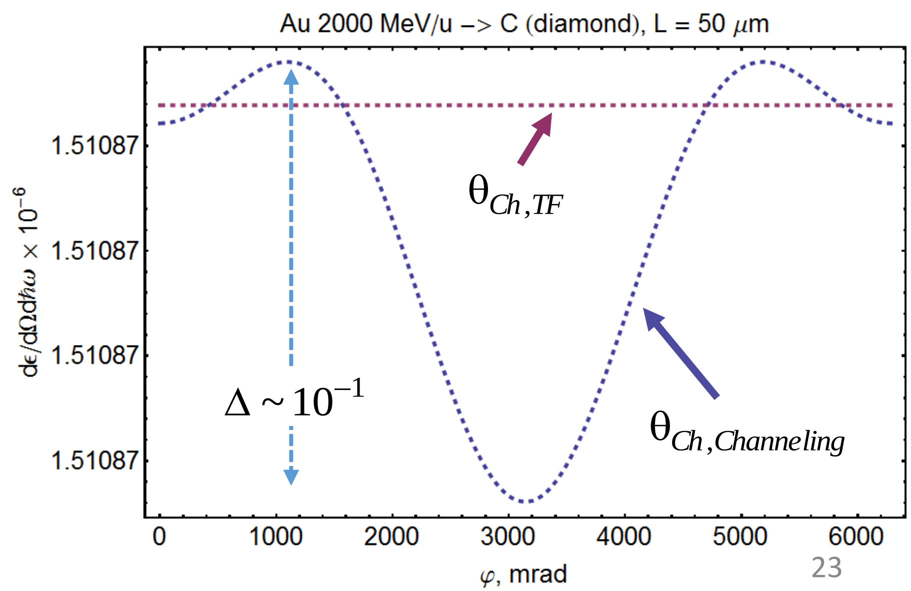
Au ions **2000 MeV/u** -> diamond crystal **50 μm** (110) planar channeling



above-barrier motion $\psi = 1.5\,\psi_L$

under-barrier motion $\psi = 0.5\,\psi_L$





Conclusion

- Cherenkov radiation from channeled RHI in a diamond crystal was studied.
- Correction that appears in the case of planar channeling depends on the RHI trajectory in the crystal.
- Spectral and angular distributions of the ChR from channeled RHI were calculated in the optical region and it was shown that transverse RHI motion leads to the shift of the Cherenkov radiation angle compared to the standard Tamm-Frank theory.
- In the case of channeling ChR intensity depends on the azimuthal angle.

These effects manifest itself more brilliant in the case of the above-barrier RHI motion.