Dual description of Sigma Models on strongly curved supermanifolds

Alessandra Cagnazzo

DESY theory group

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Plan of the talk

1. Introduction
2. Sigma Model
3. The Dual Model

Sigma models on coset spaces are very important integrable models. The study of these becomes difficult if we go in the strong curvature regime. For the study of this regime are widely used dualities (strong/weak coupling).

Sigma models on coset spaces are very important integrable models. The study of these becomes difficult if we go in the strong curvature regime. For the study of this regime are widely used dualities (strong/weak coupling).

Duality to explore:

Sigma Model on $S^{2S+1|2S}$

?

$OSP(2S+2|2S)$ Gross-Neveu model

Candu, Saleur '08

$S = 0$ was studied, we will see it for $S = 1$.

Sigma Model on $S^{2S+1|2S}$

We have a sigma model on a Coset $\frac{G}{H}$ with isometry group G.

$$\mathbb{S}^{2S+1|2S} = \frac{OSP(2S+2|2S)}{OSP(2S+1|2S)}$$

$$\mathcal{L} = g_{ij}\bar{\partial}\phi^j\partial\phi^i \qquad \phi : \Sigma \to \frac{G}{H}$$

$$J(z,\bar{z}) \qquad \bar{J}(z,\bar{z})$$

R $\rightarrow$ Radius of the sphere. We want to analyze the low lying spectrum the sigma model on $\mathbb{S}^{3|2}$

Fields

At level $(h, \bar{h})$ in the spectrum we find the field:

$$\Phi_\Lambda(z, \bar{z}) = V_{\Lambda\lambda}\, f((h-n)\partial, nJ, (\bar{h}-m)\bar{\partial}, m\bar{J})^\lambda\, (z, \bar{z}) \tag{1}$$

Φ is an $OSP(4|2)$ multiplets, J transform in $OSP(3|2)$ multiplets.

Multiplicity?

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Multiplicity? $\rightarrow$ Can be found with a bit of representation theory. It is given by the multiplicity of the projective cover of λ in the decomposition of the projective cover of Λ

One loop anomalous dimension

We want to include also the information coming from the one loop anomalous dimension so we need some information from group theory. For the Sigma Model we know the one loop expression for the anomalous dimension.

$$\delta = \frac{1}{2R^2}\big(\underbrace{C_\Lambda}_{OSP(4|2)} + \underbrace{C_\mu + C_{\bar{\mu}}}_{OSP(3|2)} \big)$$

Candu, Mitev, Schomerus '13

a bit of language...

Focus on Supergroups. These can be viewed as fermionic extensions of ordinary Lie groups.

$$\mathbf{g} = \mathbf{g}_0 + \mathbf{g}_1$$

One difference is that in a supergroup we have existence of reducible but indecomposable representation.
Finite dim. rep.:

$$\text{Typical (long) } \mathcal{L}_\mu = \mathcal{P}_\mu$$
$$\text{Atypical (short) } \mathcal{L}_\mu \neq \mathcal{P}_\mu$$

$\mathcal{L}_\mu$ is the irrep. and $\mathcal{P}_\mu$ is a projective cover

$OSP(4|2)$

Now we focus on $S = 1$

$$[\underbrace{j_1}_{Sp(2)}, \underbrace{j_2\ , j_3}_{SO(4)}]$$

Highest weight representation, with j_1, j_2 and j_3 referring to the highest weight of the bosonic subalgebra. Casimir:

$$C = -4j_1(j_1 - 1) + 2j_2(j_2 + 1) + 2j_3(j_3 + 1)$$

For an atypical representation:

$$C = k^2 \qquad \text{with } k \text{ integer}$$

so the atypical rep. can be labeled by two integers

$$\Lambda_{k,l} \qquad \Gamma_k = \{\Lambda_{k,l}, l \in \mathbb{Z}\}$$

Van der Jeugt

$OSP(3|2)$

$$[\underbrace{q}_{Sp(2)}, \underbrace{p}_{SO(3)}]$$

All the $OSP(4|2)$ representation can be embedded in $OSP(3|2)$, it is not hard to find the decomposition of the $OSP(4|2)$ in $OSP(3|2)$ reps just by diagonal embedding. This will be relevant in the SM analysis.

$$C = (p + 2q)(p - 2q + 1)$$

In this case all the atypical representations have Casimir zero.

$$[0, 0] \qquad [q, 2q - 1]$$

Van der Jeugt

Spectrum

- $(h=0,\bar{h}=0)$: $[0,0]$ that gives $OSP(3|2)$-multiplets $\Lambda_{k,0}$, Spherical Harmonics, $\delta^{(1)}=0$
- $(h=1,\bar{h}=0)$: $[\frac{1}{2},0]$ gives $\Lambda_{0,1}$, Noether Currents, $\delta^{(1)}=0$
- $(h=1,\bar{h}=1)$: This sector is very interesting since is sensitive to the Equations of Motion of the Sigma Model

$$\Gamma_{[\frac{1}{2},0]\times[\frac{1}{2},0]} \cong \Lambda_{0,0}+2\Lambda_{0,1}+\Lambda_{0,2}+\Lambda_{1,0}+\sum_{l=2}^{\infty}(3\Lambda_{l,0}+\Lambda_{l,1}+\Lambda_{l,-1})+\text{typicals}$$

$$\delta^{(1)}=\frac{l^2}{R^2}$$

Gross-Neveu model

$$S_0 = \frac{1}{2\pi}\int d^2z\left[< \Psi, \bar{\partial}\Psi > + < \bar{\Psi}, \partial\bar{\Psi} >\right]$$

$\Psi = (\psi_1, \dots, \psi_{2S+2}, \beta_1, \dots, \beta_S, \gamma_1, \dots, \gamma_S)$
WZW model on

$$G = \frac{G \times G}{G} \qquad \text{with } \; G = OSP(2S+2|2S)$$

has $G \times G$ isometry and can be deformed in various ways, that may or may not preserve the isometry.
We want to consider a G preserving deformation

$$S_{int} = \frac{g}{\pi}\int d^2z J^a(z)\eta_{a,b}\bar{J}^b(\bar{z})$$

We want to study the example $S = 1$

Duality

We want to compare the spectra of the two theories (GN or WZW and SM).

$$g = \frac{1 - R^2}{1 + R^2} \qquad (g \to -1) \leftrightarrow (R \to \infty)$$
$$(g \to 0) \leftrightarrow (R \to 1) \tag{2}$$

the WZW model is weakly coupled at $g = 0$ an the SM can be perturbatively treated at $R = \infty$. To interpolate the two sides we sill need to know the anomalous dimension for the operators in the two sides.

Anomalous dimension

The states in the WZW spectrum will be given by the fusion of left and right states.

$$(h, \bar{h}) \;\rightarrow\; \text{conformal dimension}$$

$$h - h_0 = \bar{h} - \bar{h}_0 = \delta \;\rightarrow\; \text{anomalous dimension}$$

$$\delta = \frac{g\ C_\Lambda}{2(1-g^2)} - \frac{g}{2(1+g)}(C_L + C_R)$$

This form for the anomalous dimension is valid if the diagonal representation is atypical.

Candu, Mitev, Schomerus '12

DUALITY TEST

$(h, \bar{h})$=(0,0) Spherical harmonics

$$\Lambda_{k,0} \qquad \text{in WZW}$$

- they first appear at $h = \frac{k^2}{2}$
- at level $(\frac{k^2}{2}, \frac{k^2}{2})$ from $\Lambda_{k,0}^2$ we get $\Lambda_{2k,0}$, if we consider the anomalous dimension at $R \to \infty$:

$$\delta = -\frac{C_\Lambda}{8} = -\frac{k^2}{2} \;\; \Rightarrow \;\; (0,0)$$

We find spherical harmonics on supersphere.

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- (1,0)(0,1) ⇒ MATCH in the block of the zero! (Currents)
- (1,1) ⇒ MATCH in the block of the zero! (EoM)

Instabilities

Both in the SM and int he WZW model we see instabilities.
In fact for $R \to \infty$ in the WZW model and, already at the first loop, for $R \to 1$ in the Sigma Model there are infinitely many modes that go to $-\infty$. This means that on the way from strong to weak coupling there are infinitely many modes that becomes relevant.
WZW model ($g \to -1$):

$$\begin{aligned} \delta &= -\frac{C_\Lambda}{8} & C_\Lambda &= 2(C_L + C_R) \\ \delta &= \infty & C_\Lambda &< 2(C_L + C_R) \\ \delta &= -\infty & C_\Lambda &> 2(C_L + C_R) \end{aligned}$$

S. Ryu,C. Mudry, A. W. W. Ludwig, and A. Furusaki 2010

Conclusions

- We have studied a proposed duality that would allow us to study sigma models on backgrounds with strong curvature.
- We have seen some examples supporting this duality, comparing directly the spectra for the first few levels, for some 1/2 BPS operators.
- There is the issue of stability issue that needs to be fixed in order to be able to compare the spectra of the two theories.
- It would be of great interest to study similarly the non compact case (AdS).

Thank you

GATIS
Gauge Theory as an Integrable System