New Frontiers in Theoretical Physics

XXXIV Convegno Nazionale di Fisica Teorica

Holographic graphene bilayers

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Overview

- Graphene $\longrightarrow$ conformal system of massless fermions in 2+1-dim interacting through electromagnetic forces
  - $\alpha_{\text{graphene}} = \dfrac{e^2}{4\pi\hbar c}\dfrac{c}{v_F} \sim \dfrac{300}{137} = 2.2$
- AdS/CFT $\longrightarrow$ D3/probe D5
- Dual theory $\longrightarrow$ $\mathcal{N}=4$ SYM at large 't Hooft coupling λ coupled to fundamental hypermultiplets along a 2+1-dim defect
- We study the D3/probe D5-$\overline{\text{D5}}$ system as an holographic model of a graphene bilayer
- The effects of an external magnetic field and of the introduction of a charge density are examined
- Two channels for chiral symmetry breaking
  - intra-layer condensate
  - inter-layer condensate

D3/probe D5-$\overline{\text{D5}}$

- Stack of N D3-branes $\longrightarrow$ $\mathrm{AdS}_5 \times S^5$ background

$$ds^2 = \frac{dr^2}{r^2} + r^2\left(-dt^2 + dx^2 + dy^2 + dz^2\right) + d\psi^2 + \sin^2\psi d^2\Omega_2 + \cos^2\psi d^2\tilde{\Omega}_2$$

where $d^2\Omega_2 = \sin\theta d\theta d\phi$ and $d^2\tilde{\Omega}_2 = \sin\tilde{\theta} d\tilde{\theta} d\tilde{\phi}$

- Embed N_5 D5 and $\overline{\text{D5}}$ probes in this background ($N_5 \ll N$)

- DBI + WZ actions

$$S = T_5 N_5 \left[-\int d^6\sigma \sqrt{-\det(g + 2\pi\alpha' F)} + 2\pi\alpha' \int C^{(4)} \wedge F\right]$$

D5-$\overline{\text{D5}}$ embedding

- Worldvolume coordinates and ansatz for the embedding of the D5-$\overline{\text{D5}}$

	t	x	y	z	r	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
D3	×	×	×	×						
D5/$\overline{\text{D5}}$	×	×	×	$z(r)$	×	$\psi(r)$	×	×		

- Induced metric on the D-branes worldvolume

$$ds^2 = \frac{dr^2}{r^2}\left(1 + (r^2 z')^2 + (r\psi')^2\right) + r^2\left(-dt^2 + dx^2 + dy^2\right) + \sin^2\psi d^2\Omega_2$$

- Charge density and external magnetic field $\longrightarrow$ D5 world-volume gauge fields (in the $a_r = 0$ gauge)

$$\frac{2\pi}{\sqrt{\lambda}}F = a_0'(r)dr \wedge dt + b dx \wedge dy$$

$$b = \frac{2\pi}{\sqrt{\lambda}}B \qquad a_0 = \frac{2\pi}{\sqrt{\lambda}}A_0$$

DBI action

- DBI action for N_5 D5 ($\overline{\text{D5}}$)

$$S = \mathcal{N}_5 \int dr \sin^2\psi \sqrt{r^4 + b^2} \sqrt{1 + (r\psi')^2 + (r^2 z')^2 - (a_0')^2}$$

where $\mathcal{N}_5 = \dfrac{\sqrt{\lambda} N N_5}{2\pi^3} V_{2+1}$

- $a_0(r)$ and $z(r)$ are cyclic variables $\longrightarrow$ their canonical momenta are constants

$$Q = -\frac{\delta \mathscr{L}}{\delta a_0'} \equiv \frac{2\pi \mathcal{N}_5}{\sqrt{\lambda}} \rho \qquad \rho = \frac{\sin^2\psi \sqrt{r^4 + b^2}\, a_0'}{\sqrt{1 + (r\psi')^2 + (r^2 z')^2 - (a_0')^2}}$$

$$\Pi_z = \frac{\delta \mathscr{L}}{\delta z'} \equiv \mathcal{N}_5 f \qquad f = \frac{\sin^2\psi \sqrt{r^4 + b^2}\, r^4 z'}{\sqrt{1 + (r\psi')^2 + (r^2 z')^2 - (a_0')^2}}$$

  - $\rho =$ charge density on the D5 ($\overline{\text{D5}}$)

Equations of motion

Solving for $a_0'(r)$ and $z'(r)$ in terms of ρ and f we get

$$a_0' = \frac{\rho r^2 \sqrt{1 + r^2 \psi'^2}}{\sqrt{r^4 \left(b^2 + r^4\right) \sin^4 \psi + \rho^2 r^4 - f^2}}$$

$$z' = \frac{f \sqrt{1 + r^2 \psi'^2}}{r^2 \sqrt{r^4 \left(b^2 + r^4\right) \sin^4 \psi + \rho^2 r^4 - f^2}}$$

The EoM for $\psi(r)$ is

$$\frac{r\psi'' + \psi'}{1 + r^2 \psi'^2} - \frac{\psi' \left(f^2 + \rho^2 r^4 + r^4 \left(b^2 + 3r^4\right) \sin^4 \psi\right) - 2r^3 \left(b^2 + r^4\right) \sin^3 \psi \cos \psi}{f^2 - \rho^2 r^4 - r^4 \left(b^2 + r^4\right) \sin^4 \psi} = 0$$

Note: the magnetic field b can be rescaled to 1 by rescaling $r \to \sqrt{b} r$, $f \to b^2 f$, $\rho \to b\, \rho$

Asymptotic behaviour

Asymptotic behaviour at $r \to \infty$ for the embedding functions $z(r)$, $\psi(r)$ and the gauge field $a_0(r)$

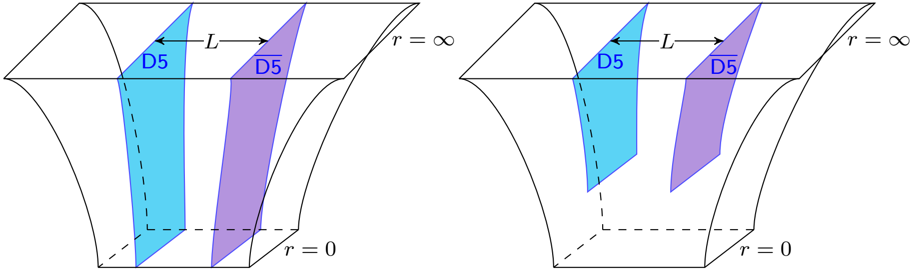
- $$z(r) \underset{r\to\infty}{\simeq} \pm\frac{L}{2} - \frac{f}{5r^5} + \dots \qquad \text{(for D5/}\overline{\text{D5}}\text{)}$$
  - L = separation between the D5 and the $\overline{\text{D5}}$
  - $f \propto$ expectation value for the inter-layer chiral condensate
- $$\psi(r) \underset{r\to\infty}{\simeq} \frac{\pi}{2} + \frac{m}{r} + \frac{c}{r^2} + \dots$$
  - $m \propto$ mass term for the fermions $\longrightarrow$ we consider solution with $m = 0$
  - $c \propto$ expectation value for the intra-layer chiral condensate
- $$a_0(r) \underset{r\to\infty}{\simeq} \mu - \frac{\rho}{r} + \dots$$
  - μ = chemical potential

Unconnected solutions

Eq. for $z(r) \longrightarrow z' = \dfrac{f\sqrt{1+r^2\psi'^2}}{r^2\sqrt{r^4\left(b^2+r^4\right)\sin^4\psi+\rho^2 r^4-f^2}}$

If $f=0$ $\longrightarrow$ the solution is trivial $\longrightarrow$ $z=\pm L/2$ (for D5/$\overline{\text{D5}}$)

Unconnected solution



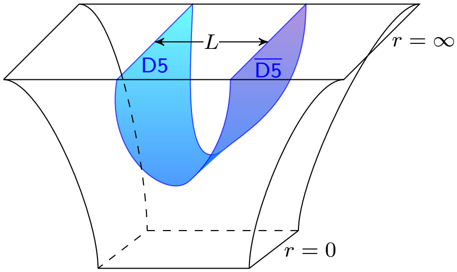
"Black hole" embedding Minkowski embedding

Connected solutions

If $f \neq 0$ the solution for $z(r)$ is

$$z(r) = f \int_{r_0}^{r} d\tilde{r} \frac{\sqrt{1 + \tilde{r}^2 \psi'(\tilde{r})^2}}{\tilde{r}^2 \sqrt{\tilde{r}^4 \left(b^2 + \tilde{r}^4\right) \sin^4 \psi(\tilde{r}) + \rho^2 \tilde{r}^4 - f^2}}$$

r_0 such that $r_0^4 \left(b^2 + r_0^4\right) \sin^4 \psi(r_0) + \rho^2 r_0^4 - f^2 = 0$

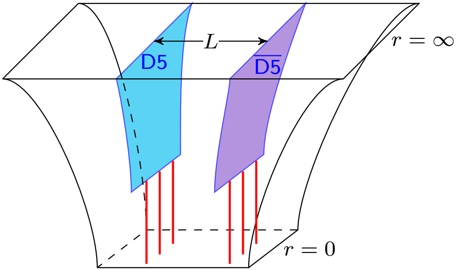


Minkowski embedding

- D-brane worldvolume confined in the region $r \geq r_0$
- in order to have a sensible solution we have to glue smoothly the D5/$\overline{\text{D5}}$ solutions at $r = r_0$ $\longrightarrow$ connected solution
- $f_{\text{D5}} = -f_{\overline{\text{D5}}}$ and $\rho_{\text{D5}} = -\rho_{\overline{\text{D5}}}$ $\longleftrightarrow$ D5-$\overline{\text{D5}}$ system is neutral

Minkowski vs. BH embeddings

- $(f = 0, c \neq 0)$-solutions can in principle be either BH or Mink. embeddings
- In practice if $\rho \neq 0$ only BH embeddings are allowed
- Mink. embeddings $\longrightarrow$ D-brane pinches off at $r = \bar{r}$ where $\psi(\bar{r}) = 0$
- If $\rho \neq 0 \longrightarrow a_0'$ is singular at $\bar{r} \longrightarrow$ there must be charge sources $\longrightarrow$ F-strings suspended between the D5 and the Poincaré horizon $(r = 0)$
- $T_{\text{F1}} > T_{\text{D5}} \longrightarrow$ strings pull the D5 to $r = 0 \longrightarrow$ BH embed.
  [Kobayashi et al. hep-th/0611099]
- For unconnected solutions $(f = 0)$ Mink. embeddings are allowed only if $\rho = 0$



D-brane separation and chemical potential

- Separation between the D5 and the $\overline{\text{D5}}$ for the connected solution $(f \neq 0)$

$$L = 2\int_{r_0}^{\infty} dr\, z'(r) = 2f\int_{r_0}^{\infty} dr\, \frac{\sqrt{1+r^2\psi'^2}}{r^2\sqrt{r^4\left(b^2+r^4\right)\sin^4\psi + \rho^2 r^4 - f^2}}$$

- Chemical potential

$$\mu = \int_{r_0}^{\infty} a_0'(r)\, dr = \rho\int_{r_0}^{\infty} dr\, \frac{r^2\sqrt{1+r^2\psi'^2}}{\sqrt{r^4\left(b^2+r^4\right)\sin^4\psi + \rho^2 r^4 - f^2}}$$

where r_0 is the solution of $r_0^4\left(b^2+r_0^4\right)\sin^4\psi(r_0) + \rho^2 r_0^4 - f^2 = 0$
if $f = 0 \longrightarrow r_0 = 0$

D-brane separation and chemical potential

For the constant solution $\psi = \pi/2$ the integrals can be done analitically

- The turning point r_0 of the connected solution is

$$r_0 = \frac{\sqrt[4]{\sqrt{\left(b^2+\rho^2\right)^2+4f^2}-b^2-\rho^2}}{\sqrt[4]{2}}$$

- The separation between the branes for the connected solution is

$$L = \frac{f\sqrt{\pi}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{7}{4};-\frac{f^2}{{r_0}^8}\right)}{2{r_0}^5\Gamma\left(\frac{7}{4}\right)}$$

- The chemical potential is

$$\mu = \frac{\rho\sqrt{\pi}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{f^2}{{r_0}^8}\right)}{r_0\Gamma\left(\frac{3}{4}\right)}$$

Solutions

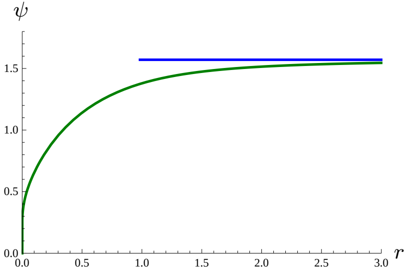
- We must look for non-trivial (*i.e.* non-constant) solutions for ψ
- EoM for ψ is a non-linear ODE
- Numerical method to find solutions imposing the suitable asymptotic condition

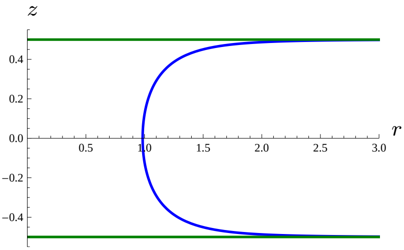
$$\psi(r) \underset{r\to\infty}{\simeq} \frac{\pi}{2} + \frac{c}{r^2} + \dots$$

- We used a shooting technique
- $(f \neq 0,\ c \neq 0)$-solutions seem not to exist
  - states of mixed inter/intra-layer condensation do not occur
- The other types of solutions are instead allowed
  - $f = 0,\ c = 0\ (z = \pm L/2,\ \psi = \pi/2) \longrightarrow$ chiral symm.
  - $f = 0,\ c \neq 0 \longrightarrow$ intra
  - $f \neq 0,\ c = 0 \longrightarrow$ inter

Plot of solutions

- Example of plots of non-trivial solutions with $\sqrt{b}L \simeq 2$ and $\mu/\sqrt{b} \simeq 1.7$
  - $f = 0$, $c \neq 0 \longrightarrow$ intra
  - $f \neq 0$, $c = 0 \longrightarrow$ inter

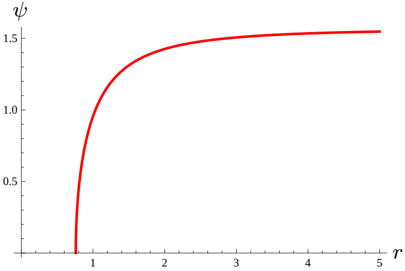




Solutions with zero charge density

- We are interested in solutions at fixed L and μ
- Eq. for a_0 is $\longrightarrow$ $a_0' = \dfrac{\rho r^2 \sqrt{1 + r^2 \psi'^2}}{\sqrt{r^4 \left(b^2 + r^4\right) \sin^4 \psi + \rho^2 r^4 - f^2}}$
- It has a trivial solution $\longrightarrow$ $a_0 = \text{const}$ when $\rho = 0$
- Other solutions with $\rho = 0$ and $a_0 = \mu$
- Among these the only relevant one $\longrightarrow$ Minkowski embedding with $f = 0$ and $c \neq 0$

[Evans,Kim 1311.0149]



Free energy

Which configuration is favored?

- Compare the free energies of the different solutions at the same L and μ
- The right quantity to define the free energy is the action evaluated on solutions $\longrightarrow \mathcal{F}[L,\mu] = S[\psi, z, a_0]$

$$\delta\mathcal{F} = \int_0^\infty dr \left(\delta\psi \frac{\partial\mathcal{L}}{\partial\psi'} + \delta a_0 \frac{\partial\mathcal{L}}{\partial a_0'} + \delta z \frac{\partial\mathcal{L}}{\partial z'}\right)' = -\rho\delta\mu + f\delta L$$

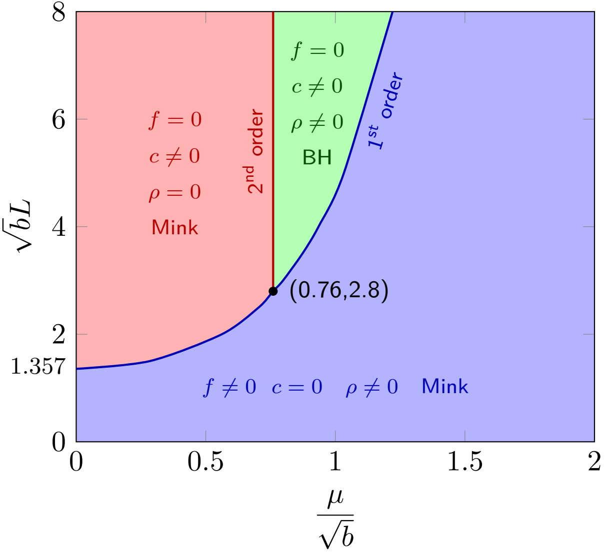
$$\mathcal{F}[L,\mu] = \mathcal{N}_5 \int_{r_0}^\infty dr \, \frac{r^2\left(b^2+r^4\right)\sin^4\psi\sqrt{1+r^2\psi'^2}}{\sqrt{r^4\left(b^2+r^4\right)\sin^4\psi + \rho^2 r^4 - f^2}}$$

- $\mathcal{F} \longleftrightarrow$ implicit function of L and μ

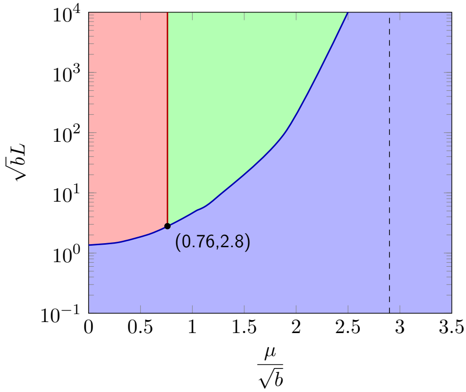
Free Energy

- The free energy of each solution is UV divergent
- Regularization $\longrightarrow$ subtracting to the free energy of each solution that of the trivial $(f = 0, c = 0; \rho \neq 0)$-solution (with the same μ)
- We use the regularized free energy to study the dominant configuration at fixed values of L and μ
- We construct the phase diagram working on a series of constant L slices

Phase diagram

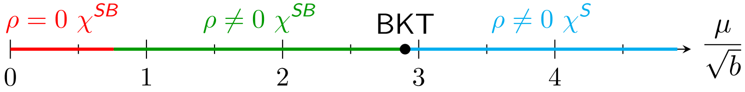


Large L limit



- For $L \to \infty$ we recover the known results for a single layer

[Evans,Gebauer,Kim,Magou 1003.2694; Jensen,Karch,Son,Thompson 1002.3159]



Conclusions

D3/probe D5-$\overline{\text{D5}}$ system as an holographic model of a graphene bilayer

- Two channels for chiral symmetry breaking $\longrightarrow$ intra/inter-layer condensates
- Inter-layer condensate is possible only for overall neutral system
- No pahse with both inter- and intra-layer condensates
- Study of the phase diagram $\left(\mu/\sqrt{b}, \sqrt{b}L\right)$
- For two layers at a finite distance with an external magnetic field and a chemical potential $\longrightarrow$ chiral symmetry is always broken
- Three relevant phases $\longrightarrow$ intra $\rho = 0$, intra $\rho \neq 0$, inter

Outlook

This work can be extended in several directions:

- The temperature can be taken into account
- Study of non-neutral system $(\rho_{\text{D5}} + \rho_{\overline{\text{D5}}} \neq 0)$
- We can use a different holographic model for bilayer semi-metal $\longrightarrow$ D3/probe D7-$\overline{\text{D7}}$

Extra slides

Classification of the solutions

Scheme of the possible types of solutions

	$f = 0$	$f \neq 0$
$c = 0$	unconnected, $\psi = \pi/2$ BH, chiral symm.	connected, $\psi = \pi/2$ Mink, inter
$c \neq 0$	unconnected, ψ not constant BH/Mink, intra	connected, ψ not constant Mink, intra/inter