Bimetric Theories: Ghost-Free Spin-2 Interactions

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Collaborators:

- SFH, A. Schmidt-May, M. von Strauss

  arXiv:1109.3230 (with J. Enander, E. Mörtsell), 1203.5283, 1204.5202, 1208:1515, 1208:1797, 1212:4525, 1303.6940, 1406.xxxx (to appear)

- SFH, R. A. Rosen,

  arXiv:1103.6055, 1106.3344, 1109.3515 (with Schmidt-May), 1109.3230, 1111.2070

Outline of the talk

Motivation

Generic massive & interacting spin-2 fields

The Ghost Problem

Ghost-free bimetric theory

Bimetric Theory & Conformal Gravity

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Motivation

- New physics beyond the Standard Model (SM) and General Relativity (GR):

  Dark matter, Dark energy (the cosmological constant problem), Matter-Antimatter asymmetry, Neutrinos, Inflation, Quantum gravity, $\cdots$

- New physics needs new tools (theoretical models):

  Supersymmetry, String Theory, Something yet unknown ?

- Spin-2 physics as a tool for new physics:

  Modifies gravity, orthogonal to other approaches

The Spin Inventory

- **Spin $0, 1/2, 1$:** Theories exist for massless, massive fields. Interactions are known. Building blocks of SM.

  Ex: $$\sqrt{|\det g|}(F_{\mu\nu}F^{\mu\nu} - m^2\, g^{\mu\nu}A_\mu A_\nu)$$

- **Spin 2:** Theory for a single, massless field $g_{\mu\nu}$:

  Einstein-Hilbert action: $$\sqrt{|\det g|}\, R(g)$$

- Are there consistent theories for massive or multiple spin-2 fields? Neutral? Charged? Higher Spins?

New theories beyond SM & GR, but the ghost problem!

Outline of the talk

Linear massive spin-2 fields

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Massless spin-2:

$$\textit{Non-linear}: \qquad R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 0$$

$$\textit{Linear}: \qquad \mathcal{E}^{\rho\sigma}_{\mu\nu}\, h_{\rho\sigma} \equiv \Box h_{\mu\nu} + \cdots = 0$$

2 polarization modes ($\pm 2,\ \pm \not{1},\ \not{0}$)

Massive spin-2:

$$\textit{Linear}: \qquad \mathcal{E}^{\rho\sigma}_{\mu\nu}\, h_{\rho\sigma} + m^2_{\rm FP}\left(h_{\mu\nu} - a\, \eta_{\mu\nu} h^{\rho}_{\rho}\right) = 0$$

Absence of ghost $\Rightarrow$ a=1 *[Fierz, Fierz, Pauli, 1939]*

5 polarization modes ($\pm 2,\ \pm 1,\ 0$)

Non-linear extension: The Boulware-Deser Ghost (1972)

Nonlinear massive spin-2 fields

- "Massive gravity" (needs a new field $f_{\mu\nu}(x)$)

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 \, V(f^{-1}g) \right], \qquad V \sim f^{\mu\rho} \, f^{\nu\sigma} \, g_{\mu\nu} \, g_{\rho\sigma} + \cdots$$

- Interacting spin-2 fields (dynamical $g_{\mu\nu}$ and $f_{\mu\nu}$)

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 \, V(f^{-1}g) \right] + \mathcal{L}(f, \nabla f)(?)$$

Bimetric: $\mathcal{L}(f, \nabla f) = m_f^2 \sqrt{-f} \, R_f (?)$

[Isham-Salam-Strathdee, 1971]

Generically, both contain a nonlinear $\mathcal{GHOST}$

[Boulware-Deser, 1972]

Outline of the talk

The Ghost Problem

Ghost: A field with negative kinetic energy

Example:

$$\mathcal{L} = T - V = (\partial_t \phi)^2 \cdots \qquad (\textit{healthy})$$

But

$$\mathcal{L} = T - V = -(\partial_t \phi)^2 \cdots \qquad (\textit{ghost})$$

Consequences:

- Negative probabilities, violation of unitarity
- Instability: unlimited energy transfer from ghost to other fields

Ghost in Spin-2 Theories:

Generic massive gravity:

- Linear: 5 modes
- Non-linear: 5 + 1 (ghost)

Generic bimetric theory:

- Linear: 5 (massive) + 2 (massless) modes
- Non-linear: 7 + 1 (ghost)

Do ghost-free massive gravity & bimetric theories exist?

Complication: Since the ghost shows up nonlinearly, its absence needs to be established nonlinearly

Construction of ghost-free massive gravity

- Development of "Decoupling limit" analysis

  [Creminelli, Nicolis, Papucci, Trincherini, (hep-th/0505147)]

- Ghost-free massive gravity proposal

  [de Rham, Gabadadze, (1007.0443); de Rham, Gabadadze, Tolley, (1011.1232)]

$$V_{dRGT}\left(\sqrt{g^{-1}\,\eta}\right)\,,$$

- Proof of absence of ghost, generalization to $V(\sqrt{g^{-1}\,f})$

  [SFH, Rosen (1106.3344, 1111.2070)]

  [SFH, Rosen, Schmidt-May (1109.3230)]

- Interacting spin-2 fields g & f (Bimetric theory)

  [SFH, Rosen (1109.3515)]

- Multiple spin-2 fields

  [Hinterbichler, Rosen (1203.5783)]

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Ghost-free bimetric theory

A dynamical theory for spin-2 fields $g_{\mu\nu}$ & $f_{\mu\nu}$

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 \, V(g^{-1} f) \right] + \mathcal{L}(f, \nabla f)$$

- What is $V(g^{-1}f)$?
- what is $\mathcal{L}(f, \nabla f)$?
- Proof of absence of ghost
- What are the implications of the theory?

Digression: Elementary symmetric polynomials of $\mathbb{X}$ with eigenvalues $\lambda_1, \cdots, \lambda_4$:

$$e_0(\mathbb{X}) = 1, \qquad e_1(\mathbb{X}) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4,$$
$$e_2(\mathbb{X}) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4,$$
$$e_3(\mathbb{X}) = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4,$$
$$e_4(\mathbb{X}) = \lambda_1\lambda_2\lambda_3\lambda_4 = \det \mathbb{X}.$$

$$e_0(\mathbb{X}) = 1, \qquad e_1(\mathbb{X}) = [\mathbb{X}],$$
$$e_2(\mathbb{X}) = \tfrac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]),$$
$$e_3(\mathbb{X}) = \tfrac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]),$$
$$e_4(\mathbb{X}) = \tfrac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]),$$
$$e_k(\mathbb{X}) = 0 \quad \text{for} \quad k > 4,$$

$$[\mathbb{X}] = \mathrm{Tr}(\mathbb{X}), \qquad e_n(\mathbb{X}) \sim (\mathbb{X})^n$$

- The $e_n(\mathbb{X})$'s and $\det(\mathbb{1} + \mathbb{X})$:

$$\det(\mathbb{1} + \mathbb{X}) = \sum\nolimits_{n=0}^{4} e_n(\mathbb{X})$$

- Introduce "deformed determinant" :

$$\widehat{\det}(\mathbb{1} + \mathbb{X}) = \sum\nolimits_{n=0}^{4} \beta_n \, e_n(\mathbb{X})$$

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- Observation:

$$V\left(\sqrt{g^{-1}f}\right) = \sum_{n=0}^{4} \beta_n \, e_n\left(\sqrt{g^{-1}f}\right)$$

Ghost-free “bi-metric” theory

Ghost-free combination of *kinetic* and *potential* terms for g & f:

$$\mathcal{L} = m_g^2 \sqrt{-g} R_g - 2m^4 \sqrt{-g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) + m_f^2 \sqrt{-f} \, R_f$$

[SFH, Rosen (1109.3515,1111.2070)]

Symmetry under $f \leftrightarrow g$,

$$\sqrt{-g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) = \sqrt{-f} \sum_{n=0}^{4} \beta_{4-n} \, e_n(\sqrt{f^{-1} g})$$

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Hamiltonian analysis: 7 nonlinear propagating modes, no ghost!

$$C(\gamma,\pi) = 0\,, \quad C_2(\gamma,\pi) = \frac{d}{dt}C(x) = \{H, C\} = 0$$

Ghost-free Matter couplings

Similar to GR minimal couplings:

$$\mathcal{L}_g(g,\phi) + \mathcal{L}_f(f,\tilde{\phi})$$

Convention:

$g_{\mu\nu}$: the gravitational metric, ϕ: observed matter. Then,

$$m_g \sim M_P \ \text{(Planck mass)}$$

Equations of motion:

$$m_g^2 \left[R_{\mu\nu}(g) - \tfrac{1}{2} g_{\mu\nu} R(g)\right] + V^g_{\mu\nu}(g,f) = T^g_{\mu\nu}(g,\phi)$$
$$m_f^2 \left[R_{\mu\nu}(f) - \tfrac{1}{2} f_{\mu\nu} R(f)\right] + V^f_{\mu\nu}(g,f) = T^f_{\mu\nu}(f,\tilde{\phi})$$

Matter equations of motion are unchanged (geodesic motion)

Physical content (Mass spectrum)

The theory has 7 propagating modes. Consider

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}\,, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + \delta f_{\mu\nu}$$

Well defined mass spectrum (FP masses) exists for $\bar{f} = c^2 \bar{g}$,

Linear modes:

Physical content (Mass spectrum)

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$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}\,, \qquad f_{\mu\nu} = \bar{f}_{\mu\nu} + \delta f_{\mu\nu}$$

Well defined mass spectrum (FP masses) exists for $\bar{f} = c^2 \bar{g}$,

Linear modes:

Massless spin-2 (2): $\delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \frac{m_f^2}{m_g^2}\,\delta f_{\mu\nu}\right)$

Massive spin-2 (5): $\delta M_{\mu\nu} = \left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right)$,

- $m_{\rm FP}^2$, c^2 and Λ are given in terms of the $\beta_0\,, \cdots \beta_4$.
- $g_{\mu\nu}\,,\ f_{\mu\nu}$ are mixtures of *massless* and *massive* modes

[SFH, Schmidt-May, von Strauss 1208:1515, 1212:4525]

Limits of bimetric theory (m_g vs m_f)

dRGT Massive gravity limit (fixed $f_{\mu\nu}$):

$$m_g = M_P\,, \qquad m_f^2 \to \infty$$

Most of the recent work has been done for this theory.

The General Relativity limit:

$$m_g = M_P\,, \qquad m_f \to 0$$

More natural and phenomenologically viable.

Bimetric as “gravity + a massive spin-2 field”

- Gravity:
  $g_{\mu\nu}$ coupled to observed matter (not purely massless)

- Massive spin-2 field:
  $M_{\mu\nu} = g_{\mu\rho}\left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\nu} - c\, g_{\mu\nu}$ coupled to gravity

Bimetric as "gravity + a massive spin-2 field"

- Gravity:
  $g_{\mu\nu}$ coupled to observed matter (not purely massless)
- Massive spin-2 field:
  $M_{\mu\nu} = g_{\mu\rho}\left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\nu} - c\, g_{\mu\nu}$ coupled to gravity
- For $M_P = m_g >> m_f$: $g_{\mu\nu}$ is mostly massless!

In this setup, the *long range* of gravitational force is correlated with gravity being the *weakest* force in the spin-2 sector

What next?

Observational considerations:

Viability of applications to cosmology and gravity, classical solutions, cosmological perturbations, gravitationsl waves, etc. (Old motivations may not be met)

[See talk by Luigi Pilo]

Theoretical considerations:

Deeper understanding of structure, origin of masses, dynamics, generalizations, the charged case, limitations, etc.
Relation to “partial masslessness” and Conformal gravity

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Digression I: Conformal gravity

Higher Derivative gravity:

$$S^{\mathrm{HD}}_{(2)}[g] = m_g^2 \int d^4x \sqrt{g} \left[\Lambda + c_1 R(g) - \frac{c_2}{m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) \right]$$

7 modes: massless spin-2 + massive spin-2 (ghost) *[Stelle (1977)]*

Set: $\Lambda = 0, c_1 = 0$ $\quad\Rightarrow\quad$ Conformal gravity

Digression I: Conformal gravity

Conformal Gravity:

$$S^{\mathrm{CG}}[g] = c \int d^4x \sqrt{g} \left[R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right] = c \int d^4x \sqrt{g} W^2 \,,$$

(similar to the Yang-Mills action)

Invariance: $g_{\mu\nu} \to e^{\phi} g_{\mu\nu}$

EoM (Bach tensor):

$$B_{\mu\nu} \equiv -\nabla^2 P_{\mu\nu} - \nabla_\mu \nabla \nu P \cdots = 0$$

$$P_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{6} g_{\mu\nu} R$$

Spectrum: 6 modes: 2 (massless spin-2) + 4 ghosts

[Riegert (1984), Maldacena (2011)]

Digression II: Partially massless FP theory

Back to the Fierz-Pauli equation:

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\, h_{\rho\sigma} - \Lambda\left(h_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu} h^{\rho}_{\rho}\right) + \frac{m^2_{\rm FP}}{2}\left(h_{\mu\nu} - \bar{g}_{\mu\nu} h^{\rho}_{\rho}\right) = 0$$

$\bar{g}_{\mu\nu}$: dS/Einstein background.

Higuchi Bound:

$$m^2_{FP} = \tfrac{2}{3}\Lambda$$

New gauge symmetry:

$$\Delta h_{\mu\nu} = (\nabla_\mu \nabla_\nu + \tfrac{\Lambda}{3})\xi(x)$$

Gives 5-1=4 propagating modes *[Deser, Waldron, $\cdots$ (1983-2012)]*

Non-linear extension? Relation to Conformal Gravity?

Curvature expansion of bimetric equations

[SFH, Schmidt-May, von Strauss, 1303:6940]

$$m_g^2 \left[R_{\mu\nu}(g) - \tfrac{1}{2} g_{\mu\nu} R(g)\right] + V^g_{\mu\nu}(g,f) = 0 \qquad (1)$$

$$m_f^2 \left[R_{\mu\nu}(f) - \tfrac{1}{2} f_{\mu\nu} R(f)\right] + V^f_{\mu\nu}(g,f) = 0 \qquad (2)$$

From (1), ($P_{\mu\nu} = R_{\mu\nu} - \frac{1}{6} g_{\mu\nu} R$),

$$f_{\mu\nu} = a\, g_{\mu\nu} + \frac{b}{m^2} P^{\mu}_{\ \nu} - \frac{c}{m^4} \text{``}P^2\text{''} \cdots$$

Substitute in (2),

$$A\, g_{\mu\nu} + \frac{B}{m^2}\left[R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R\right] + \frac{C}{m^4} B_{\mu\nu} + \frac{D}{m^4} \text{``}P^2\text{''} + \frac{\text{``}P^3\text{''}}{m^6} + \cdots = 0$$

Can $A = B = D = 0$? Yes!

A ghost-free generalization of conformal gravity?

When parameters $\beta_0, \cdots \beta_4$ satisfy ($\alpha = m_f/m_g$),

$$\alpha^4 \beta_0 = 3\alpha^2 \beta_2 = \beta_4 \,, \qquad \beta_1 = \beta_3 = 0$$

the bimetric EoM's imply

$$B_{\mu\nu}(g) + \mathcal{O}(R^3(g)/m^2) = 0$$

Invariance in low curvature limit:

$$\Delta g_{\mu\nu} = \phi g_{\mu\nu} + \cdots$$

But now the parent bimetric theory is ghost-free!

Symmetry at higher orders? Always 6 propagating modes (rather then 7 modes) ?

(Note: No massive gravity limit $\alpha \to \infty$!)

Extended Weyl invariance

Explicit symmetry to order ∂^6/m^6 (for $m_g = m_f$):

$$\Delta g_{\mu\nu} = \phi g_{\mu\nu} - \hat{m}^{-2}(P_{\mu\nu}\phi + \nabla_\mu \nabla_\nu \phi) + \hat{m}^{-4}(\partial^4) + \hat{m}^{-6}(\partial^6) + \cdots$$
$$\Delta f_{\mu\nu} = \phi f_{\mu\nu} - \hat{m}^{-2}(P'_{\mu\nu}\phi + \nabla'_\mu \nabla'_\nu \phi) + \hat{m}^{-4}(\partial^4) + \hat{m}^{-6}(\partial^6) + \cdots$$

[SFH, Schmidt-May, von Strauss, 1406:xxxx]

PM symmetry in dS/Einstein backgrounds $\bar{f} = c^2 \bar{g}$,

$$\Delta M_{\mu\nu} = (\nabla_\mu \nabla_\nu + \tfrac{\Lambda}{3})\xi(x)$$
$$\Delta G_{\mu\nu} = 0$$

The specific bimetric theory (or an appropriate generalization) could provide a nonlinear realization Partial Masslessness (away from Einstein backgrounds) as well as a ghost-free extension of Conformal Gravity (more work needed).

Discussion

- Ghost-free spin-2 interactions
- Relevance to CG and PM theories.
- Relation between metric and vielbein formulations
- Derivative interactions, changed spin-2 fields
- Causal structure of theories with 2 “metrics”, related consistency issues.
- Origin of the interactions (a la Higgs)
- General results on dynamics and viability

Thank you!