N=1 and N=2 pure supergravities on a manifold with boundary

Laura Andrianopoli

Politecnico di Torino

New Frontiers in Theoretical Physics

Cortona, 28-31 May, 2014

Based on L. A., R. D'Auria:
arXiv:1405.2010

Plan of the seminar

1. Geometric approach to the boundary problem in asymptotically AdS_4 Supergravity
2. The pure $N = 1$ theory
3. The pure $N = 2$ theory
4. Conclusions and Oulooks

Boundary problem in gravity/supergravity

Long standing issue:

- ◇ Gibbons-Hawking, ('77) (Path int. approach to quantum gravity: Dirichlet boundary conditions required on fields)
- ◇ Horava-Witten ('96) (11D SUGRA/$[S_1/Z_2] \Leftrightarrow E_8 \times E_8$ het. string. Couplings fixed to cancel anomalies on $\partial\mathcal{M}$)
- ◇ AdS/CFT ('97, ...) (Bulk fields (metric) diverge at $\partial\mathcal{M}$
  - $\hookrightarrow$ cured by inclusion of counterterms at the boundary (Holographic renormalization))

General lesson: For $\partial\mathcal{M} \neq 0$, bulk theory needs to be supplemented by boundary terms!

Geometric approach at the gravity level

(Aros, Contreras, Olea, Troncoso, Zanelli ('99); Olea ('05, ...))
Diffeomorphism invariance of the bulk Einstein lagrangian + Λ is broken in the presence of a boundary
⇒ Restored by adding a topological term (Gauss-Bonnet):

$$\mathcal{L}_{GB} = R^{ab} \wedge R^{cd} \epsilon_{abcd} = d\left(\omega^{ab} \wedge \mathcal{R}^{cd} - \omega^{a}{}_{\ell} \wedge \omega^{\ell b} \wedge \omega^{cd}\right) \epsilon_{abcd}$$

The expansion of $\mathcal{L}_{GB}$ in the radial coordinate ⊥ to boundary:

- regularizes action and conserved charges
- reproduces holographic renormalization counterterms

Supergravity case

A systematic way to face the boundary problem in supergravity:
$\Rightarrow$ Geometric Approach to Superspace (in 4D, $\mathcal{M}_{4|4N}$)

- theory in terms of superfields $\mu^{\mathcal{A}}(x,\theta) \in \mathcal{M}_{4|4N}$
- $\mathcal{S} = \int_{\mathcal{M}_4} \mathcal{L}[\mu]$; $\mathcal{M}_4(x,\theta) \subset \mathcal{M}_{4|4N}$, bos.; $\mathcal{L}$ 4-form in $\mathcal{M}_{4|4N}$
- In this setting, SuSy transformations are diffeomorphisms in θ-directions of superspace $\mathcal{M}_{4|4N}(x,\theta)$:

$$SUSY: \quad \mathcal{M}_4(x,\theta) \to \mathcal{M}_4(x,\theta+\delta\theta)$$

$\Rightarrow$ can be described in terms of Lie derivatives ℓ_ϵ:

$$\ell_\epsilon = \iota_\epsilon \, d + d \, \iota_\epsilon$$

$\epsilon(x,\theta)$ SuSy parameter, ι_ϵ contraction op.: $\iota_\epsilon(V^a) = 0$, $\iota_\epsilon(\psi) = \epsilon$

Supergravity case

In particular, the SuSy transf. of the lagrangian 4-form is:

$$\delta_\epsilon \mathcal{L} = \ell_\epsilon \mathcal{L} = \iota_\epsilon \, (d\mathcal{L}) + d \, \iota_\epsilon(\mathcal{L})$$

Necessary (non-trivial) condition for SuSy inv: $\iota_\epsilon \, (d\mathcal{L}) = 0$
$\rightsquigarrow$ assumed true for bulk-supergravity lagrangians $\mathcal{L}_{bulk}$.
$\Rightarrow$ SuSy inv. of action $\delta_\epsilon \mathcal{S} = 0$ also requires:

$$\delta_\epsilon \mathcal{S} = \int_{\mathcal{M}_4} d \, \iota_\epsilon(\mathcal{L}) = \int_{\partial\mathcal{M}_4} \iota_\epsilon(\mathcal{L}) = 0 \quad \Rightarrow \quad \iota_\epsilon(\mathcal{L})|_{\partial\mathcal{M}_4} = d\phi$$

In general not satisfied by $\mathcal{L}_{bulk}$ if $\partial\mathcal{M}_4 \neq 0$!
$\Rightarrow$ SuSy invariance requires to add boundary terms

$$\mathcal{L}_{bulk} \to \mathcal{L}_{bulk} + \mathcal{L}_{bdy}$$

Pure N=1, with cosmological constant $\Lambda = -12e^2$

The bulk lagrangian is:

$$\mathcal{L}_{bulk} = -\frac{1}{4}\mathcal{R}^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} - \bar{\psi}\gamma_5\gamma_a \wedge \rho \wedge V^a +$$

$$-\mathrm{i}e\bar{\psi}\gamma_5\gamma_{ab} \wedge \psi \wedge V^a \wedge V^b - \frac{1}{2}e^2 V^a \wedge V^b \wedge V^c \wedge V^d \epsilon_{abcd}\,.$$

$\iota_\epsilon(\mathcal{L}_{bulk})|_{\partial\mathcal{M}_4} \neq d\phi \Rightarrow \delta_\epsilon \mathcal{S}_{bulk} \neq 0\,.$ Possible boundary terms:

$$\begin{aligned}\mathcal{L}_{bdy} &= d\left[\alpha\left(\omega^{ab} \wedge \mathcal{R}^{cd} - \omega^a{}_\ell \wedge \omega^{\ell b} \wedge \omega^{cd}\right)\epsilon_{abcd} + \beta\bar{\psi} \wedge \gamma^5\rho\right] \\ &= \alpha\mathcal{R}^{ab} \wedge \mathcal{R}^{cd}\epsilon_{abcd} + \beta\left(\bar{\rho}\gamma_5\rho + \frac{1}{4}\mathcal{R}^{ab}\bar{\psi}\gamma_5\gamma_{ab}\psi\right)\end{aligned}$$

$$\mathcal{L}_{full} = \mathcal{L}_{bulk} + \mathcal{L}_{bdy}$$

Note that the field eq.s on $\mathcal{M}_{4|4}$ have boundary contributions:

$$\begin{cases} \frac{\delta \mathcal{L}_{full}}{\delta \omega^{ab}} = 0 & \Rightarrow \quad \mathcal{R}^{ab}|_{\partial\mathcal{M}} = \frac{1}{8\alpha}\left(V^a V^b + \frac{\mathrm{i}}{2}\beta\bar{\psi}\gamma^{ab}\psi\right)_{\partial\mathcal{M}} \\ \frac{\delta \mathcal{L}_{full}}{\delta \bar{\psi}} = 0 & \Rightarrow \quad \rho|_{\partial\mathcal{M}} = \frac{1}{2\beta}\left(\gamma_a \psi V^a\right)_{\partial\mathcal{M}} \end{cases} \tag{1}$$

Supercurvatures on $\partial\mathcal{M}_4$ dynamically fixed to constant values!
Upon use of (1) we find:

$$\iota_\epsilon(\mathcal{L}_{full})|_{\partial\mathcal{M}} = 0 \quad \Leftrightarrow \quad \frac{\beta}{16\alpha} + \frac{1}{2\beta} = 2\mathrm{i}\, e$$

that is

$$\beta = 16\mathrm{i}\, e\alpha\,(1+k)\,, \quad k^2 = 1 + \frac{1}{32\, e^2\alpha}\,; \qquad (\beta \neq 0 \Rightarrow k \neq -1).$$

Extra rel. among $\alpha, \beta, e \equiv 0$, due to $\gamma_{ab}\psi\bar{\psi}\gamma^{ab}\psi = 0$ (N=1 Fierz).

For $\alpha = -\frac{1}{32\,e^2}$, $\beta = -\frac{\mathrm{i}}{2e}$ $(k = 0)$, $\mathcal{L}_{full}$ becomes:

$$\mathcal{L}_{full} = -\frac{1}{32e^2}\hat{R}^{ab} \wedge \hat{R}^{cd}\epsilon_{abcd} - \frac{\mathrm{i}}{2e}\hat{\bar{\rho}}\gamma_5\hat{\rho}$$

in terms of the $OSp(1|4)$-covariant field-strengths:

$$\begin{cases} \hat{R}^{ab} = \mathcal{R}^{ab} + 4e^2 V^a \wedge V^b + e\bar{\psi}\gamma^{ab}\psi \\ \hat{\rho} = \rho - \mathrm{i}e\gamma_a\psi \wedge V^a \end{cases} . \qquad (2)$$

This is in fact nothing but the Mac Dowell–Mansouri action!

N.B.: In terms of (2), the field eq.s on $\partial\mathcal{M}_4$ are:

$\hat{R}^{ab}|_{\partial\mathcal{M}_4} = 0\,, \quad \hat{\rho}|_{\partial\mathcal{M}_4} = 0$ SuSy extension of Olea results!

But $N = 1$ Sugra also allows $k \neq 0$

Pure N=2, with cosmological constant $\Lambda = -\frac{3}{2}P^2$

$$\mathcal{L}_{bulk} = -\frac{1}{4}\mathcal{R}^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} + (\bar{\psi}^A \gamma_a \wedge \rho_A - \bar{\psi}_A \gamma_a \wedge \rho^A) \wedge V^a +$$
$$\left[\left(\theta \tilde{F}_{ab} + \frac{1}{2}\epsilon_{abcd}\tilde{F}^{cd}\right) F - \frac{1}{24}\left(\tilde{F}_{\ell m}\tilde{F}^{\ell m} - \frac{\theta}{2}\epsilon_{pqrs}\tilde{F}^{pq}\tilde{F}^{rs}\right) V^c V^d \epsilon_{abcd}\right] V^a V^b$$
$$-L^0\left[F - \frac{L^0}{2}\left(\bar{\psi}^A\psi^B\epsilon_{AB} + \bar{\psi}_A\psi_B\epsilon^{AB}\right)\right]\left[(\theta - \mathrm{i})\bar{\psi}^C\psi^D\epsilon_{CD} + h.c.\right]$$
$$+\mathrm{i}\left(S_{AB}\bar{\psi}^A\gamma_{ab} \wedge \psi^B - h.c.\right) V^a \wedge V^b - \frac{1}{16}P^2 V^a \wedge V^b \wedge V^c \wedge V^d \epsilon_{abcd}\,,$$

ψ_A, ψ^A chiral components; $F = dA + L^0(\bar{\psi}^A\psi^B\epsilon_{AB} + \bar{\psi}_A\psi_B\epsilon^{AB})$;
$S_{AB} = \frac{1}{2\sqrt{2}}P\delta_{AB}$

(From general N=2 SUGRA, setting to zero the matter multiplets still keeping a FI term P: ($L^\Lambda \to L^0 = \frac{1}{\sqrt{2}}$, $\mathcal{N}_{\Lambda\Sigma} \to \mathcal{N}_{00} = \theta - \mathrm{i}$; $\mathrm{i}P_\Lambda^{x=2}\sigma_{AB}^{x=2} \to P\delta_{AB}$)

If $\partial\mathcal{M} \neq 0$, SuSy inv. requires $\mathcal{L}_{bulk} \to \mathcal{L}_{bulk} + \mathcal{L}_{bdy} = \mathcal{L}_{full}$:

$$
\begin{aligned}
\mathcal{L}_{bdy} &= d\Big\{\alpha(\omega^{ab}\wedge\mathcal{R}^{cd} - \omega^{a}{}_{\ell}\wedge\omega^{\ell b}\wedge\omega^{cd})\epsilon_{abcd} + \\
&\quad +\beta S_{AB}\bar{\psi}^{A}\rho^{B} + \bar{\beta}\bar{S}^{AB}\bar{\psi}_{A}\rho_{B} + \gamma A\mathcal{F}\Big\} \\
&= \alpha\mathcal{R}^{ab}\wedge\mathcal{R}^{cd}\epsilon_{abcd} + \beta S_{AB}\bar{\rho}^{A}\rho^{B} + \bar{\beta}\bar{S}^{AB}\bar{\rho}_{A}\rho_{B} + \gamma\mathcal{F}\wedge\mathcal{F} + \\
&\quad +\frac{1}{4}\mathcal{R}^{ab}\left(\beta S_{AB}\bar{\psi}^{A}\gamma_{ab}\psi^{B} + \bar{\beta}\bar{S}^{AB}\bar{\psi}_{A}\gamma_{ab}\psi_{B}\right) + \\
&\quad +\frac{\mathrm{i}}{2}\mathcal{F}\left(\beta S_{AB}P^{B}{}_{C}\bar{\psi}^{A}\psi^{C} - \bar{\beta}\bar{S}^{AB}P_{B}{}^{C}\bar{\psi}_{A}\psi_{C}\right)
\end{aligned}
$$

and $\iota_\epsilon\left(\mathcal{L}_{full}\right)_{\partial\mathcal{M}} = 0$ requires:

$$
\alpha = -\frac{1}{4P^2}\,, \quad \beta = \mathrm{i}\frac{4}{P^2} = -\bar{\beta}\,, \quad \gamma = -\frac{1}{2}\theta
$$

Note:

- N=2 SuSy inv. fixes all the coefficients (equiv. to $k = 0$ case in $N = 1$)
- $\gamma = -\frac{1}{2}\theta$ such as to exactly cancel the (topological) θ-term
- In terms of $OSp(2|4)$-covariant hatted field-strengths:

$$\mathcal{L}_{full}^{(space-time)} = -\frac{1}{4P^2}\hat{R}^{ab}\wedge\hat{R}^{cd}\epsilon_{abcd} + \frac{1}{2}F\wedge {}^*F + \\ + \frac{4}{P^2}\mathrm{i}\left(S_{AB}\hat{\bar{\rho}}^A\wedge\hat{\rho}^B - \bar{S}^{AB}\hat{\bar{\rho}}_A\wedge\hat{\rho}_B\right)$$

corresponding to "$N = 2$ Mac-Dowell-Mansouri"!

Conclusions

For asymptotically AdS_4 SUGRA:

- If $\partial\mathcal{M}_4 \neq 0$ $\mathcal{L}_{SUGRA}$ should include boundary contributions
- $N = 2$ SUSY completely constrains $\mathcal{L}_{bdy}$: It is topological, but does not allow θ-term for graviphoton
- The supercurvatures on $\partial\mathcal{M}_4$ are dynamically fixed, not the fields (alternative to Gibbons-Hawking)
  - The $OSp(2|4)$-covariant supercurvatures all vanish on $\partial\mathcal{M}$

Outlooks

- What expression the supersymmetric $\mathcal{L}_{bdy}$ has in 3D intrinsic coordinates? Counterterms for holographic renormalization?
- How does the $\Lambda \to 0$ case work?
- What happens for higher N theories? And for matter coupled theories? They should allow for vacua more general than AdS_4.
- How all this extends to higher dimensional supergravities?