Electromagnetic response of strongly coupled plasmas

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based on arXiv:1404.4048 with D. Forcella and D. Musso

Introduction and Motivations

How media with electrically movable components can respond to EM field
$\longrightarrow$ Linear response of a system to small external perturbation

- interaction between EM field and quasi–normal modes $\longrightarrow$ various propagation and dissipation channels
- emergence of exotic EM effects : Negative Refraction (NR) and Additional Light Waves (ALW)
- theorized in 1968 (Vaselago), experimentally realized in 2000 (Smith et al) $\longrightarrow$ meta–materials
- string theory approach $\longrightarrow$ NR and ALW are features of charged fluids with hydrodynamic description

Amariti, Forcella, Mariotti, Policastro [2010/2011]

Outline

Analysis beyond hydrodynamics of electromagnetic (EM) wave modes in strongly coupled systems with spatial dispersion

model : transverse EM modes for globally neutral plasma

tool : gauge/gravity correspondence

main aim : study the presence of NR and ALW

bonus : some hints for plasma phenomenology ?

Electromagnetic waves in presence of spatial dispersion

Response to EM field of medium with spatial dispersion via permittivity tensor

$$D_i = \epsilon_{ij}(\omega\,,\,\vec{q})\;E_j$$

symmetries $\longrightarrow$ separation into longitudinal and transverse part

$$\epsilon_{ij}(\omega,\vec{q}) = \epsilon_T(\omega,\vec{q})\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) + \epsilon_L(\omega,\vec{q})\frac{q_i q_j}{q^2}$$

Maxwell equations $\longrightarrow$ dispersion relations

$$\epsilon_T(\omega,\vec{q}) \;= \frac{q^2}{\omega^2} \qquad\qquad \epsilon_L(\omega,\vec{q}) \;= 0$$

$\epsilon_{T,L}$: expressed by current-current correlator function

$$\epsilon_{T,L}(\omega,q) = 1 - 4\pi e^2\,\frac{\mathcal{G}_{T,L}(\omega,q)}{\omega^2}$$

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Dispersion relations for transverse and longitudinal EM modes

Solutions $\vec{q} = \vec{q}(\omega)$ $\longrightarrow$ transverse EM wave modes propagating in the medium

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Gauge/gravity correspondence

strongly coupled CFT
in $4d$ Minkowski
at finite temperature

$\longleftarrow$ dictionary $\longrightarrow$

classical gravity
in $5d$ asymptotically
AdS–black hole

- Universal behaviour of strongly coupled theory with energy momentum tensor $T_{\mu\nu}$ and $U(1)$ conserved current J_μ from Einstein–Maxwell with metric g_{mn} and gauge field A_m

$$S = -\frac{N^2}{32\pi^2}\int d^5x\sqrt{-g}\,(R-2\Lambda) + \frac{N^2}{16\pi^2}\int d^5x\,\sqrt{-g}\,\frac{1}{4}F_{mn}F^{mn}$$

- current–current correlator function for globally neutral media is obtained from the fluctuations of $5d$ photon A_m on a uncharged black hole background

$$ds^2 = \frac{\pi^2 T^2}{u}\left[-f(u)dt^2 + dx_i dx^i\right] + \frac{1}{4u^2 f(u)}du^2 \qquad \text{with } f(u) = 1 - u^2$$

Vector transverse fluctuations

- gauge fixing $A_u = 0 \oplus$ symmetries $\longrightarrow$ boundary directions $\mu \oplus$ propagating along z

$$k^\mu = (\omega, 0, 0, q) \qquad\qquad A_\alpha(u, \omega, q)\, dx^\alpha = \phi(u, \omega, q)\, dx$$

- transverse component EoM

$$\phi'' - \frac{2u}{1-u^2}\phi' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2(1-u^2)}{u(1-u^2)^2}\phi = 0$$

with $\mathfrak{w} = \dfrac{\omega}{2\pi T}$ and $\mathfrak{q} = \dfrac{q}{2\pi T}$ $\longrightarrow$ no explicit dependence from temperature!

- boundary conditions for horizon $u = 1$ (outgoing) and boundary $u = 0$ $\longrightarrow$ solution propagating from the horizon

From ϕ to correlator

Prescription: Correlation function obtained by functionally differentiate the on-shell bulk action with respect to the boundary value of ϕ

- Near boundary expansion $\phi = \phi_0 + u\,\phi_1 + u\ln(u)\,\tilde{\phi}_1 + \ldots$
- presence of UV–divergences $\longrightarrow$ renormalization procedure

$$S_{\rm ren} = \frac{(NT)^2}{16}\int \frac{d\omega d^3q}{(2\pi)^4}\,\phi_0\left[\phi_1 - \phi_0(\mathfrak{w}^2 - \mathfrak{q}^2)\frac{c}{2}\right] ,$$

- Hence, the retarded correlation function for transverse current is

$$G^{(c)}(\mathfrak{w},\mathfrak{q}) = -\frac{(NT)^2}{16}\left[\frac{\phi_1}{\phi_0} - c(\mathfrak{w}^2 - \mathfrak{q}^2)\right]$$

- $\phi_0 = \phi_0(\mathfrak{w},\mathfrak{q})$ and $\phi_1 = \phi_1(\mathfrak{w},\mathfrak{q})$ from numerical algorithms

EM wave modes

Wave equation $\longrightarrow$ solve for $\mathfrak{q} = \mathfrak{q}(\mathfrak{w})$

$$\frac{\phi_1(\mathfrak{w},\mathfrak{q})}{\phi_0(\mathfrak{w},\mathfrak{q})} - (\mathfrak{w}^2 - \mathfrak{q}^2)(c - \frac{16\pi}{e^2 N^2}) = 0$$

Wave modes interpretation

$$e^{-i(\omega t - qz)} = e^{-\mathrm{Im}[q]z} \, e^{-i[\omega t - \mathrm{Re}[q]z]}$$

- Passive medium: damped wave in direction of propagation $\longrightarrow \mathrm{Im}\,[q] > 0$ $\longrightarrow$ direction of energy flux
- Phase velocity $\longrightarrow$ sign of $\mathrm{Re}\,[q]$

$$\frac{\mathrm{Re}\,[q]}{\mathrm{Im}\,[q]} < 0 \longrightarrow \text{negative refraction}$$

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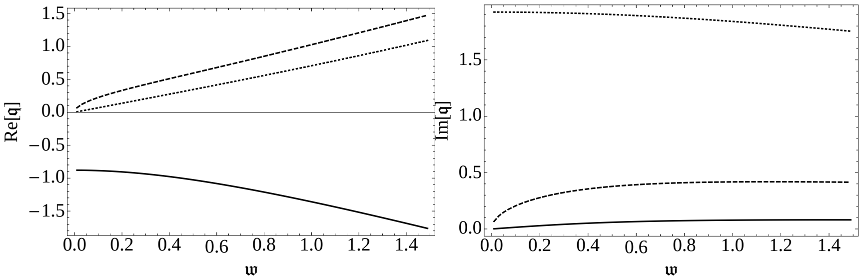
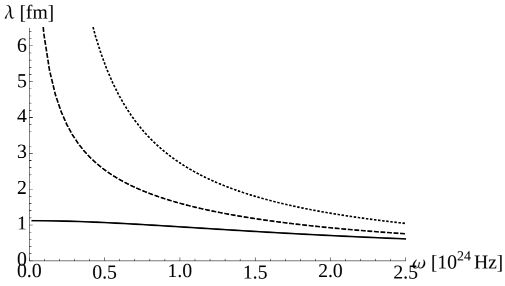
ALW and NR



- different EM modes $\longrightarrow$ Additional Light Waves
- presence of low dissipative EM mode with Negative Refraction

Phenomenological application: QGP at RHIC or LHC

- QGP in temperature–dominated regime
- wavelength $\lambda = \dfrac{\hbar c}{T\ \mathrm{Re}[\mathfrak{q}]}$
- typical temperature for QGP in LHC and RHIC: $\sim 200 MeV$



NR mode: wavelength $\sim$ typical dimensions of QGP sample

Results and Open Issues

Analysis

- ✓ EM linear response of strongly coupled plasma beyond hydrodynamic limit
- ✓ study of EM transverse modes $\longrightarrow$ emergence of exotic features (NR and ALW)
- ✓ key ingredient: spatial dispersion
- ✓ study of EM longitudinal mode $\longrightarrow$ only ALW
- ✓ first look for applications to QGP

Future directions

- × Relation NR and quasi–normal modes
- × More deep phenomenological analysis
- × Extension to charged case