Scattering amplitudes in $\mathcal{N} = 2$ SuperConformal QCD

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Cortona, May 28^{th}

Work in progress

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Outline

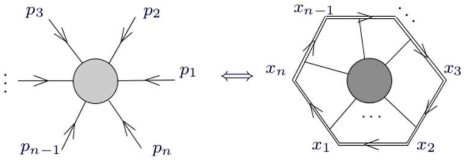
1. Why scattering amplitudes in $\mathcal{N} = 2$ SQCD?
2. $\mathcal{N} = 2$ SQCD: action and known results
3. one–loop and two–loop scattering amplitudes
4. conclusions and outlook

Why $\mathcal{N} = 2$ SQCD?

Remarkable properties of planar scattering amplitudes in $\mathcal{N} = 4$ SYM:

- superamplitudes are invariant under:

superconformal group + dual superconformal group



MHV scattering amplitudes $\longleftrightarrow$ Wilson loop duality

[Drummond, Korchemsky, Sokatchev, '07]

Remarkable properties of planar scattering amplitude in $\mathcal{N} = 4$ SYM:

- no UV divergences
- IR divergences: $\mathcal{A}^l \sim \frac{1}{\epsilon^{2l}}(\mathit{finite} + \mathcal{O}(\epsilon))$

  IR divergences + finite part of four and five–point amplitudes are completely fixed by exponentiation (BDS ansatz) [Bern, Dixon, Smirnov,'05]

$$\mathcal{M}^{BDS}(\epsilon) = \exp\left[\sum_{l\ loops} \lambda^l \left(f^{(l)}(\epsilon)\,\mathcal{M}^{(l)}(l\,\epsilon) + C^{(l)}\right)\right]$$

  key ingredients: dual conformal invariance and integrability

- maximum transcendentality principle [Kotikov, Lipatov, '01]

  all terms $\sim \frac{\epsilon^k}{\epsilon^{2l}}$ have degree of transcendentality k

On the other side, what happens in QCD?

- IR structure well understood: exponentiation of IR divergences
  [Catani, '98; Sterman, Tejeda-Yeomans, '03; Becher, Neubert, '09]
- the maximal transcendental piece of QCD cusp anoumalous dimension is the same of $\mathcal{N} = 4$ SYM [Kotikov, Lipatov, Onishchenko, Velizhanin, '04]

What happens in theories with less than maximal amount of supersymmetry?

ABJM shares some beautiful structure with $N = 4$ SYM (dual conformal invariance, exponentiation , WL duality, transcendentality...)

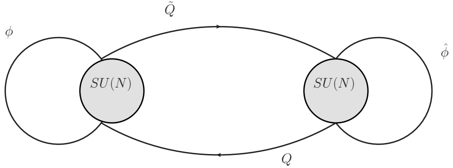
N=2 SCQCD ...still to explore!

Interpolating theory

$\mathcal{N} = 2$ SQCD belongs to a family of interpolating theories $SU(N) \times SU(N)$:

[Eguchi, Hori, Ito, Yang, '96]

$$
\begin{aligned}
S = & \int \mathrm{d}^4x\, \mathrm{d}^2\theta \left[\frac{1}{g^2} \mathrm{Tr}(W^\alpha W_\alpha) + \frac{1}{\hat{g}^2} \mathrm{Tr}(\hat{W}^\alpha \hat{W}_\alpha) \right] \\
& + \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathrm{Tr} \left[e^{-gV} \bar{\phi} e^{gV} \phi + e^{-\hat{g}\hat{V}} \bar{\hat{\phi}} e^{\hat{g}\hat{V}} \hat{\phi} + \bar{Q}^I e^{gV} Q_I e^{-\hat{g}\hat{V}} + \bar{\tilde{Q}}_I e^{\hat{g}\hat{V}} \tilde{Q}^I e^{-gV} \right] \\
& + i \int \mathrm{d}^4x\, \mathrm{d}^2\theta \left[g\, \mathrm{Tr}(\tilde{Q}^I \phi Q_I) - \hat{g}\, \mathrm{Tr}(Q_I \hat{\phi} \tilde{Q}^I) \right] - i \int \mathrm{d}^4x\, \mathrm{d}^2\bar{\theta} \left[g\, \mathrm{Tr}(\bar{Q}^I \bar{\phi} \bar{\tilde{Q}}_I) - \hat{g}\, \mathrm{Tr}(\bar{\tilde{Q}}_I \hat{\bar{\phi}} \bar{Q}^I) \right]
\end{aligned}
$$



- at $g = \hat{g}$: $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM
- at $\hat{g} = 0$: $\mathcal{N} = 2$ SQCD

$\mathcal{N} = 2$ SQCD

Action written in terms of $\mathcal{N} = 1$ superfields

$$S = \frac{1}{g^2} \int d^4x \, d^2\theta \, \mathrm{Tr}(W^\alpha W_\alpha) + \int d^4x \, d^4\theta \, \mathrm{Tr}(e^{-gV} \, \bar{\Phi} \, e^{gV} \, \Phi)$$
$$+ \int d^4x \, d^4\theta \, \left\{ \bar{Q}^I \, e^{gV} \, Q_I + \tilde{Q}^I \, e^{-gV} \, \bar{\tilde{Q}}_I \right\}$$
$$+ ig \int d^4x \, d^2\theta \, \tilde{Q}^I \, \Phi \, Q_I - ig \int d^4x \, d^2\bar{\theta} \, \bar{Q}^I \, \bar{\Phi} \, \bar{\tilde{Q}}_I$$

$$SU(N_c) \times U(N_f) \times SU(2)_R \times U(1)_r$$

$$N_f = 2N_c$$

	field	$SU(N_c)$	$U(N_f)$
N=2 vector multiplet	V	Adj	1
	Φ	Adj	1
N=2 hypermultiplet	Q	$\square$	$\square$
	$\tilde{Q}$	$\bar{\square}$	$\bar{\square}$

Known results I: AdS/CFT correspondence

Any gauge theory at large N is expected to be dual to a string theory

$g=\hat{g}$: $\mathbb{Z}_2$ orbifold of $\mathcal{N}=4$ SYM in planar limit

$\downarrow$

dual to a type II B superstring on $AdS_5 \times S^5/\mathbb{Z}_2$

[Kachru, Silverstein, '98]

$\hat{g}=0$: $\mathcal{N}=2$ SQCD in the Veneziano limit,
i.e $N_c, N_f \to \infty$, N_f/N_c is fixed

$\downarrow$

dual to a non critical string background with seven dimensions [Gadde, Pomoni, Rastelli, 09]

Known results II: integrability

It is known that $\mathcal{N}=4$ SYM is completely integrable, is it the same for $\mathcal{N}=2$ SQCD?

- despite some intriguing hints of integrability, the theory fails to be integrable at two loops [Gadde, Liendo, Rastelli, Yan,'12]
- speculation about integrability in the subsector which involves only N=2 vector multiplet [Gadde, Liendo, Rastelli, Yan,'12]
- the presence of scattering amplitude/WL duality suggests the presence of integrability. The lightlike Wilson loop was computed up to three loops, and it was found that up to two loops it coincides with $\mathcal{N}=4$ [Andree, Young, '10]...

  ... but what happens for scattering amplitudes?

Classification of amplitudes

We computed massless scattering amplitudes at one loop and at two loops in the planar Veneziano limit, using N=1 superspace Feynman diagrams, with chiral superfields Φ and Q as external particles.

Classification of processes according to the color representation:

- Adjoint sector $\Phi\bar{\Phi}\Phi\bar{\Phi}$
- Mixed sector $Q\bar{Q}\Phi\bar{\Phi}$
- Fundamental sector $Q\tilde{Q}\bar{\tilde{Q}}\bar{Q}$

$\rightarrow$ all these amplitudes are MHV: the ordering doesn't matter!

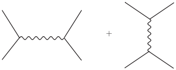
How to compute an amplitude?

For each process:

1. draw all possible super Feynman diagrams, using super Feynman rules
2. for each diagram perform D algebra in order to get a local integral in superspace: $\int \mathrm{d}^4 p\, \mathrm{d}^4\theta \dots \int \mathrm{d}^d k \dots$
3. do a projection using $\int \mathrm{d}^4 p\, \mathrm{d}^4\theta \dots = \int \mathrm{d}^4 p\, \bar{D}^2 D^2 \dots$, in order to select the lowest components of superfields
4. simplify the loop integrals $\int \mathrm{d}^d k \dots$, using on shell symmetries and expand the results in terms of master integrals
5. combine all the contributions and expand the result in terms of poles of dimensional regularization parameter $\epsilon = 2 - d/2$

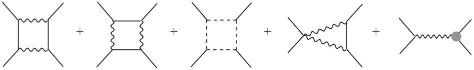
Adjoint sector: $\Phi\ \bar{\Phi}\ \Phi\ \bar{\Phi}$

$\mathcal{A}^{(0)} =$

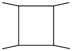


$= -g^2 \dfrac{(s+t)^2}{st}$

$\mathcal{A}^{(1)} =$



$= g^4 N\, (s+t)^2$



So the one–loop reduced amplitude $\mathcal{A}^{(1)}/\mathcal{A}^{(0)}$ is:

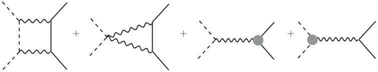
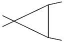
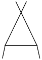
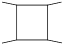
$$\mathcal{M}^{(1)} = \frac{2\, g^2 N_c}{(4\pi^2)} \left\{ -\frac{1}{\epsilon^2}\left(\frac{\mu}{s}\right)^{\epsilon} - \frac{1}{\epsilon^2}\left(\frac{\mu}{t}\right)^{\epsilon} + \frac{2}{3}\pi^2 + \frac{1}{2}\ln^2\frac{t}{s} \right\}$$

Same result as in N=4 SYM, agree with previous work [Glover, Khoze, Williams, '08]

✓ dual conformal invariance
✓ uniform transcendentality weight

Mixed sector: $Q\ \bar{Q}\ \Phi\ \bar{\Phi}$

 $= -g^2 \, \frac{s+t}{s}$

 $= g^4 \, N_c \Big(-t$  $+ t$  $+ \frac{2t^2 + st}{2}$  $\Big)$

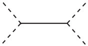
So the one–loop reduced amplitude is:

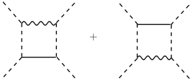
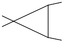
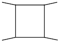
$$\mathcal{M}^{(1)} = \frac{g^2 N_c}{(4\pi^2)} \left\{ -\frac{2}{\epsilon^2}\left(\frac{\mu}{t}\right)^{\epsilon} - \frac{1}{\epsilon^2}\left(\frac{\mu}{s}\right)^{\epsilon} + \frac{3}{4}\pi^2 + \frac{1}{2}\ln^2\frac{t}{s} - \frac{t}{u}\left[\frac{\pi^2}{2} + \frac{1}{2}\ln^2\frac{t}{s}\right]\right\}$$

✗ dual conformal invariance

✓ uniform transcendentality weight

Fundamental sector: $Q\ \tilde{Q}\ \bar{\tilde{Q}}\ \bar{Q}$

 $= -g^2$

 $= g^4\, N_c \Big(-2s$  $+\, st$  $\Big)$

So the one–loop reduced amplitude is:

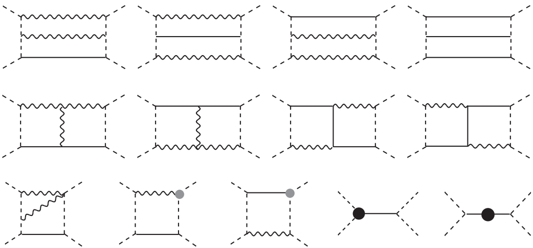
$$\mathcal{M}^{(1)} = \frac{2\, g^2 N_c}{(4\pi^2)} \left\{ -\frac{1}{\epsilon^2}\left(\frac{\mu}{t}\right)^{\epsilon} + \frac{7}{12}\pi^2 + \frac{1}{2}\ln^2\frac{t}{s} \right\}$$

✗ dual conformal invariance

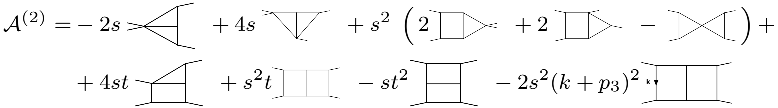
✓ uniform transcendentality weight

Two–loop amplitude

Diagrams contributing to the two-loop amplitude $Q\ \tilde{Q}\ \bar{\tilde{Q}}\ \bar{Q}$

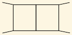
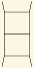
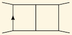
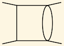
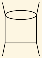


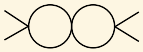
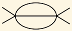
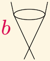
Combining all the contributions

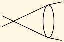
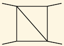


Two–loop amplitude

...in terms of master integrals:

$\mathcal{A}^{(2)} = s^2\ t$  $-\ s\ t^2$  $-\ 2\ s^2$  $-\ \frac{12\ a\ s^2}{t}$  $-\ 24\ a\ t$ 

$+\ 4\ a^2$  $-\ \frac{6\ c\ (s+5t)}{st}$  $-\ \frac{6\ c\ (s+2t)}{t^2}$  $-\ 12\ b$ 

$+\ \frac{2\ b\ (5t-3s)}{t}$  $+\ \frac{6\ (s+t)\ (s+2t)}{t}$ 

$$a = -\frac{1-2\epsilon}{2\epsilon} \qquad b = \frac{(1-2\epsilon)(2-6\epsilon)}{(2\epsilon)^2} \qquad c = \frac{(1-2\epsilon)(2-6\epsilon)(4-6\epsilon)}{(-2\epsilon)^3}$$

✓ uniform transcendentality weight: every Feynman diagram respects the maximum transcendentaly principle

Conclusions and outlook

The study of scattering amplitudes in $\mathcal{N} = 2$ SQCD can be useful to understand the origin of the beautiful structures of $\mathcal{N} = 4$ SYM

✓ One–loop amplitude was computed in *all sectors*
✓ Two–loop amplitude was computed in the *fundamental sector*

It would be interesting to go further:

- study the two–loop amplitude in the adjoint and mixed sectors
- study the three-loop amplitude, in order to test scattering amplitudes/Wilson loop duality in a less symmetric framework

Thanks!