Fermions, Wigs and Attractors

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28 maggio 2014

Based on arXiv: 1309.8021 - 1403.5097

In collaboration with P.A. Grassi, A. Marrani, A. Mezzalira and W. Sabra

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- In general, AdS/CFT works for supergravity *i.e.* for a theory with fermionic dofs.
  - New internal (Grassmannian) dofs for the fluid
  - New conserved charges
  - New contributions to "classical" thermodynamic variables
- Extend the construction to all supergravity fields
  - New contributions to BH conserved charges (?)
  - New solutions-generating technique for supergravity

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Attractor mechanism

For an extremal BH in matter-coupled supergravities

In approaching the Event Horizon, the moduli completely lose memory of the initial data, and take values dependent only on the electric/magnetic charges of the BH:

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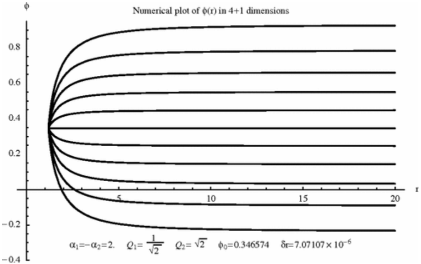

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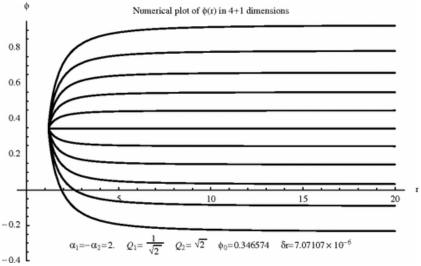

Regardless of the initial conditions, the Horizon values depend ONLY on the charges, but nevertheless the evolution remains DETERMINISTIC!

Example

$\mathcal{N}=2,\ D=4$ Axion-Dilaton-Einstein-Maxwell Sugra coupled to a gauge multiplet: $\{\ g_{\mu\nu}, A_\mu\ ;\ A'_\mu, \phi\ \}$

$$S=\int d^4x\sqrt{-g}\,[\ R-2\partial^\mu\phi\partial_\mu\phi-\frac{1}{2}e^{-2\phi}\,(\ F^{\mu\nu}F_{\mu\nu}+F'^{\mu\nu}F'_{\mu\nu}\ )\ ]$$

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Note that as susy parameters we use the “anti-Killing spinors”.

Killing Spinor

Space are endorsed with both isometries and superisometries, the latter generated by Killing spinors:

- Computation of the Killing Spinor ϵ:

$$\left(\partial_M + \frac{1}{4}\,\omega_M^{AB}\big|_{\text{empty}}\,\Gamma_{AB} + \frac{\Lambda}{2}\,e_M^A\big|_{\text{empty}}\,\Gamma_A\right)\epsilon = 0$$

- ϵ: For example in AdS_3 2 $\mathbb{C}$ fermionic components $\longrightarrow$ 4 real dof's

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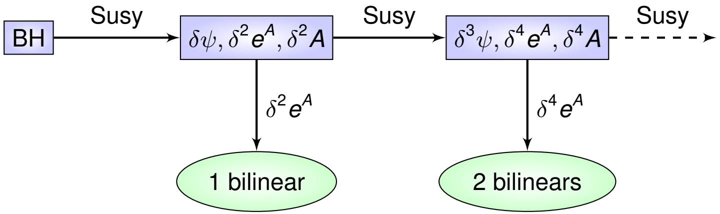

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- Development of algorithms to compute, order by order (Wig)
- Implementation of algorithms in `Mathematica` code

LGCG, P. A. Grassi and A. Mezzalira – hep-th/1207.0686

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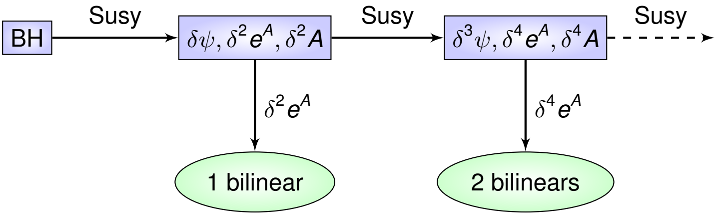

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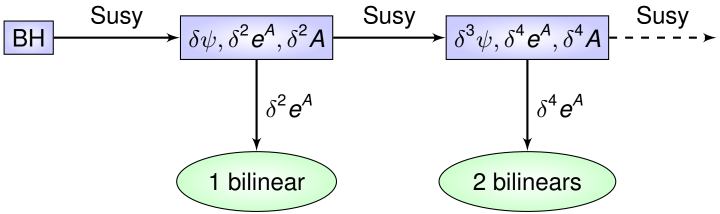

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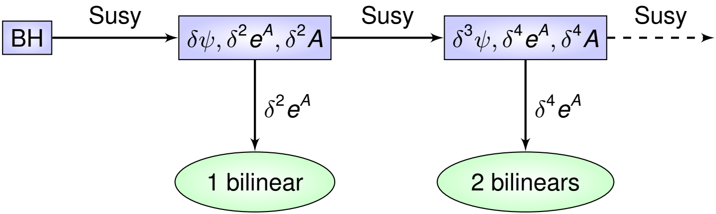

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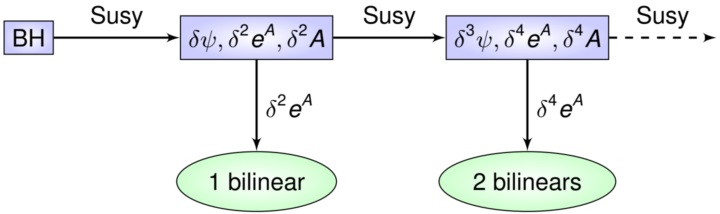

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$\mathcal{N}=2,\ D=4$ Minimally Coupled MESGT

Field Content

Bosons

$e^a_\mu \quad A^\Lambda_\mu \quad z^i$

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Where

$$G^{i-}_{\mu\nu} = -g^{i\bar{j}}\bar{f}^\Gamma_{\bar{j}}\,(\text{Im}\mathcal{N})_{\Gamma\Lambda}\,\tilde{F}^{\Lambda-}_{\mu\nu}$$

Special Kähler Geometry

Scalar (complex) fields coordinatize a complex Kähler manifold

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The manifold is also special since there exist a C_{ijk} satisfying

$$R_{\bar{i}j\bar{l}k} = -g_{j\bar{l}}g_{k\bar{i}} - g_{k\bar{l}}g_{j\bar{i}} + g^{t\bar{s}}\bar{C}_{\bar{i}\bar{l}\bar{s}}C_{tkj}$$

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What you have to keep in mind:

$$(\mathrm{Im}\mathcal{N})_{\Gamma\Lambda} f_i^{\Lambda} L^{\Gamma} = 0 \qquad (\mathrm{Im}\mathcal{N})_{\Gamma\Lambda} f_i^{\Lambda} \bar{L}^{\Gamma} \neq 0$$

Axion-Dilaton Model for DE-Black Holes

In this model

$$K = -\ln\left[2\left(z+\bar{z}\right)\right] \qquad \mathcal{N}_{\Gamma\Lambda} = -i\,\mathrm{diag}\left(z, 1/z\right)$$

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In this model

$$K=-\ln\left[2\left(z+\bar{z}\right)\right] \qquad \mathcal{N}_{\Gamma\Lambda}=-i\,\mathrm{diag}\left(z,1/z\right)$$

At zeroth order, you get (Attractor Mechanism!)

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but at fourth order the ids. in the precedent slide implies...

$$\delta^{(4)}z=\frac{M^4}{(M+r)^4}\frac{p^0q_0-p^1q_1}{\left(p^0+iq_1\right)^2\left(p^0-iq_1\right)\left(q_0+ip^1\right)}\mathcal{Q}\sin^2\phi\sin^2\theta$$

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(...not so attractive, is it?). Note that a purely electric (magnetic) configuration

leaves the scalar field unchanged.

$\mathcal{N}=2$ $D=5$ Minimally Coupled MESGT

Field Content

Bosons

e_μ^a A_μ^I ϕ^i

Fermions

ψ_μ^i λ^{xi}

$\mathcal{N} = 2$ $D = 5$ Minimally Coupled MESGT

Field Content

Bosons

$e_\mu^a \quad A_\mu^I \quad \phi^i$

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$\psi_\mu^i \quad \lambda^{xi}$

Supersymmetry transformations

$$\delta e_\mu^a = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu \ ,$$
$$\delta A_\mu^I = -\frac{1}{2}\bar{\epsilon}\gamma_\mu\lambda^x h_x^I$$
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$$\delta\psi_\mu^i = \nabla_\mu \epsilon^i + \text{stuff}\ldots \ ,$$
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In 5D the (real) scalars coordinatize a Real Special Kähler manifold. This time we will need just

$$\partial_\mu h^I = 0 \Rightarrow \partial_\mu\phi^x = 0 \qquad h_{Ix}F_{\mu\nu}^I = 0$$

Universal result

Using the two ids of Real Geometry we get

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$$\delta^{(4)}\phi^x = \mathcal{A}^\mu \partial_\mu \phi^x + \mathcal{B}^{\mu\nu} h_{Ix} F^I_{\mu\nu} + \{\ldots\} = 0$$

where $\mathcal{A}$ and $\mathcal{B}$ are cumbersome expressions and $\{\ldots\}$ are terms which goes to zero on the chosen background.

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So in 5*D* the Attractor Mechanism *is really* attractive! Ok but... Why?

The Attractor Mechanism is sensitive to the dyonicity of the solution.

In 5*D* no dyonic solutions are present so, the AM is unchanged at all orders.

Interpretation

The wig generates fermionic corrections (in the forms of bilinears) to bosonic objects, such as the metric and the gauge field. What are them?

- No classical counterpart
- Generated through supersymmetry

- Quantize the fermionic zero mode
- Compute quantistic vev for the various operator

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But: very difficult for gravity! Use monopoles instead

(work in progress . . .)

Results and Open Issues

Susy Fluid-dynamics

- Wig computation
- Dual Fluid: no dissipative corrections $\oplus$ presence of new dof
- Analysis of Energy Momentum Tensor

1209.4100 - 1302.5060

- Wigs for AdS_3, AdS_4 and AdS_5 BH

1207.0686 - 1209.4100

Other models

- AdS_5: 1^{st} order correction to Euler equations $\oplus$ Fermionic Corrections to AdS_3 dual Fluid

1105.4706 - 1302.5060

Minimally coupled Sugra

- Modification of AM in $\mathcal{N}=2$ $D=4$
- Modification of AM in $\mathcal{N}=2$ $D=5$

1309.0821 - 1403.5097

- To Do:
  - "Twin wigs" (wigs of twin-supergravities)
  - Wigs of Monopoles in $\mathcal{N}=2$ SYM and interpretation

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