Quantum quench from a tensor thermal state: free and interacting integrable systems

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arXiv:1404.1319 [cond-mat.quant-gas], M. Collura, G.M.

Summary

1. Quantum Quench and Motivations
2. Transport Properties for free fermions in a D-dimensional space
3. Integrable systems: preliminary results

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Quantum Quench

- we select an initial state (in our case the state is a thermal tensor state $\rho_{\mathcal{L}} \otimes \rho_{\mathcal{R}}$)

$$\rho_0 = |\psi_0\rangle\langle\psi_0|$$

- $H_0 \to H$: change of a parameter, change of the geometry of the problem...
- Unitary evolution

$$\rho(t) = e^{-iHt}\rho_0 e^{iHt}$$

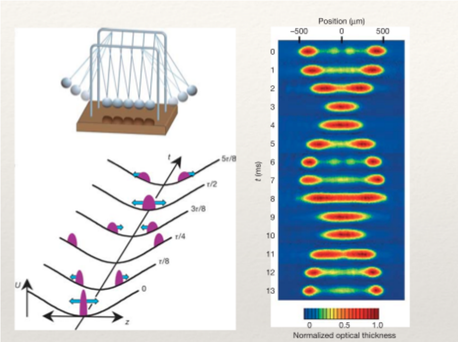
- $|\psi(t)\rangle = \sum_n e^{-iE_n t}|n\rangle\langle n|\psi_0\rangle$, the importance of the overlaps
- technical difficult: the double sum in EV of an observable $\mathcal{O}$

$$\langle\psi(t)|\mathcal{O}|\psi(t)\rangle = \sum_n \sum_m e^{-i(E_n - E_m)t}\langle m|\mathcal{O}|n\rangle\langle n|\psi_0\rangle\langle m|\psi_0\rangle$$

- we don't solve exactly the dynamics, but we can compute the expectation value of observables in the limit $t \to \infty$
- it's possible to obtain many results for integrable system, question also integrable systems equilibrates to a thermal state?
- for example see Calabrese, Caux, Essler, Mussardo and collaborators

Experimental Evidence

- Quantum Newton's Cradle (T. Kinoshita, T. Wenger, D. S. Weiss, Nature 440,900 (2006))



- Lieb-Liniger model (hard-core bosons) → equilibrates to a non thermal state
- Integrable models do not thermalize
- Ultracold Atoms → test: integrable models, lattice gauge theory, high-Tc superconductors etc...

GGE and NESS states

- GGE conjecture: integrable systems does not relax to a thermal state, but the equilibrium is described by a Generalized Gibbs Ensemble M. Rigol et all., Phys. Rev. Lett. 98, 050405 (2007)
- I need to described the equilibrium states with all the local conserved charges of the theory
- Attention!!! If I use all the conserved charges we have a tautology

$$[I_n, I_m] = 0$$

$$\rho_{GGE} = \frac{1}{Z_{GGE}} \exp(-\sum_n \lambda_n I_n), \qquad Tr\, I_n \rho_{GGE} = \langle I_n \rangle_0$$

- other stationary state NESS $\rightarrow$ effects of the boundaries and different limit in the thermodynamic limit
- $L \rightarrow \infty$ and $t \rightarrow \infty$: GGE $\frac{t}{L} \gg 1$, NESS $\frac{t}{L} \ll 1$

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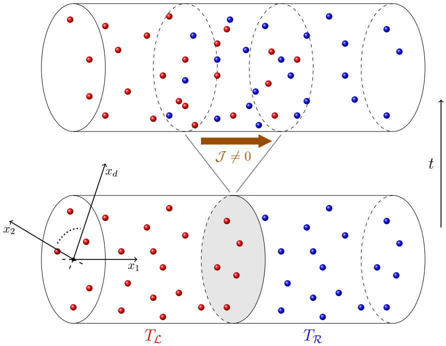
Framework

- spatial domain $\mathcal{V} \equiv [-L/2, L/2] \otimes \mathcal{V}_{d-1}$.



- Why study free fermions in D-dimension? Compare with (Doyon et al., 2013), coming from Holographic results (strong coupling problems)

$$\vartheta_{rel} = a \left(\frac{T_{\mathcal{L}}^{d+1} - T_{\mathcal{R}}^{d+1}}{u_{\mathcal{L}} + u_{\mathcal{L}}} \right)$$

Observables

- we study the time evolution of the two-point correlation function with the continuity equation

$$\mathcal{J}(x_1,t) = -\int_{-\infty}^{x_1} dz\,\partial_t n(z,t), \qquad \mathcal{J}(x_1,t) \equiv 2\,\mathrm{Im}\left[L^{1-d}\int_{\mathcal{V}_{d-1}}\prod_{i=2}^{d} dx_i \langle \hat{\Psi}^\dagger(x)\partial_{x_1}\hat{\Psi}(x)\rangle_t\right]$$

$$\vartheta(x_1,t) = -\int_{-\infty}^{x_1} dz\,\partial_t \mathcal{E}(z,t)$$

$$\vartheta(x_1,t) \equiv 2\,\mathrm{Im}\left[L^{1-d}\int_{\mathcal{V}_{d-1}}\prod_{i=2}^{d} dx_i \langle [\partial_{x_1}\hat{\Psi}^\dagger(x)]\mathcal{H}(x)\hat{\Psi}(x)\rangle_t\right]$$

- to extract the analytic results we use a semiclassical approximation $\rightarrow$ ballistic hypothesis $\rightarrow$ time evolution following the trajectory of the particles

$$n(x,t) = \frac{1}{2}\sum_{\sigma=\pm1}\int dp \int dx_0\, n_0(x_0,p)\delta(x - x_0 - \sigma pt/m).$$

Analytic Results for non relativistic case

- dispersion relation $\epsilon(k) = \frac{k^2}{2m}$

$$\mathcal{J}_{NESS} = \frac{m^{\frac{d-1}{2}}}{(2\pi)^{\frac{d+1}{2}}} \left[\frac{-\mathrm{Li}_{\frac{d+1}{2}}\left(-e^{\beta_{\mathcal{L}}\mu_{\mathcal{L}}}\right)}{\beta_{\mathcal{L}}^{\frac{d+1}{2}}} - \frac{-\mathrm{Li}_{\frac{d+1}{2}}\left(-e^{\beta_{\mathcal{R}}\mu_{\mathcal{R}}}\right)}{\beta_{\mathcal{R}}^{\frac{d+1}{2}}} \right]$$

$$\vartheta_{NESS} = \frac{d+1}{2} \frac{m^{\frac{d-1}{2}}}{(2\pi)^{\frac{d+1}{2}}} \left[\frac{-\mathrm{Li}_{\frac{d+3}{2}}\left(-e^{\beta_{\mathcal{L}}\mu_{\mathcal{L}}}\right)}{\beta_{\mathcal{L}}^{\frac{d+3}{2}}} - \frac{-\mathrm{Li}_{\frac{d+3}{2}}\left(-e^{\beta_{\mathcal{R}}\mu_{\mathcal{R}}}\right)}{\beta_{\mathcal{R}}^{\frac{d+3}{2}}} \right]$$

- Low temperature limit

$$\mathcal{J}_{NESS} = \frac{m^{\frac{d-1}{2}}}{(2\pi)^{\frac{d+1}{2}}} \left[\frac{\mu_{\mathcal{L}}^{\frac{d+1}{2}} - \mu_{\mathcal{R}}^{\frac{d+1}{2}}}{\Gamma(\frac{d+3}{2})} + \frac{\pi^2}{6\,\Gamma(\frac{d-1}{2})} \left(\frac{\mu_{\mathcal{L}}^{\frac{d-3}{2}}}{\beta_{\mathcal{L}}^2} - \frac{\mu_{\mathcal{R}}^{\frac{d-3}{2}}}{\beta_{\mathcal{R}}^2} \right) \right]$$

$$\vartheta_{NESS} = \frac{(d+1)m^{\frac{d-1}{2}}}{2(2\pi)^{\frac{d+1}{2}}} \left[\frac{\mu_{\mathcal{L}}^{\frac{d+3}{2}} - \mu_{\mathcal{R}}^{\frac{d+3}{2}}}{\Gamma(\frac{d+5}{2})} + \frac{\pi^2}{6\,\Gamma(\frac{d+1}{2})} \left(\frac{\mu_{\mathcal{L}}^{\frac{d-1}{2}}}{\beta_{\mathcal{L}}^2} - \frac{\mu_{\mathcal{R}}^{\frac{d-1}{2}}}{\beta_{\mathcal{R}}^2} \right) \right]$$

- behaviour independent by the spatial dimensions of the system and for 1D we recover conformal behaviour

Analytic Results for massless relativistic case

- different dispersion relation

$$\epsilon(k) = v_F|k|,$$

$$\mathcal{J}_{rel} = \frac{(2^{d-1}-1)\Gamma(d)\zeta(d)}{(2\pi)^d\Gamma(\frac{d+1}{2})v_F^{d-1}}\left(\frac{1}{\beta_{\mathcal{L}}^d} - \frac{1}{\beta_{\mathcal{R}}^d}\right)$$

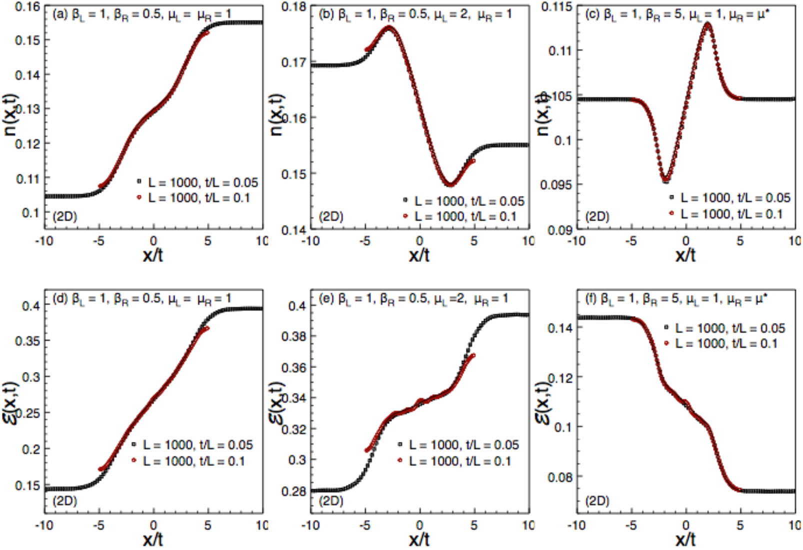
$$\vartheta_{rel} = \frac{(2^d-1)\Gamma(d+1)\zeta(d+1)}{2(2\pi)^d\Gamma(\frac{d+1}{2})v_F^{d-1}}\left(\frac{1}{\beta_{\mathcal{L}}^{d+1}} - \frac{1}{\beta_{\mathcal{R}}^{d+1}}\right)$$

- In 1D we recover the conformal behaviour consistent with (Bernard, Doyon, 2012)

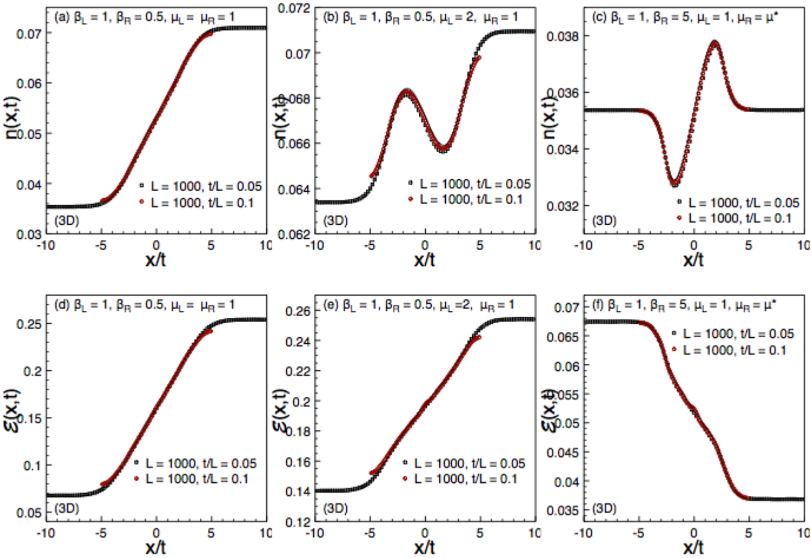
$$\vartheta_{rel}|_{d=1} = \frac{\pi}{24}\left(\frac{1}{\beta_{\mathcal{L}}^2} - \frac{1}{\beta_{\mathcal{R}}^2}\right)$$

- this result agrees with (De Luca, Viti, Bernard, Doyon, 2013, Ising chain) and (Collura, Karevski, 2014)
- no presence of left and right velocities $\rightarrow$ mean free path is ∞, then we don't have *shockwaves*

Numerical Results: 2D



Numerical Results: 3D



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Work in progress about transport properties in integrable system

- we use the GTBA (Caux, Essler2013) to obtain the NESS

$$\mathcal{O}(t) = \sum_{\{\lambda\}} \sum_{\{\mu\}} e^{-S^*_{\{\lambda\}} - S_{\{\mu\}}} e^{i(\omega_{\{\lambda\}} - \omega_{\{\mu\}})t} \langle \{\lambda\} | \mathcal{O} | \{\mu\} \rangle.$$

- we go in the continuum limit

$$\mathcal{O}(t) = \int \mathcal{D}[\rho] e^{S^{YY}_\rho} \sum_{\{\lambda\}} \left(e^{-S^*_{\{\lambda\}} - S_\rho} e^{i(\omega_{\{\lambda\}} - \omega_\rho)} \frac{\langle \{\lambda\} | \mathcal{O} | \rho \rangle}{2} + \lambda \leftrightarrow \rho \right).$$

- from the normalization we obtain a new free energy $\mathcal{F}_\rho = 2\mathrm{Re} S_\rho - S^{YY}_\rho$

$$\frac{\partial \mathcal{F}_\rho}{\partial \rho}|_{\rho_s} = 0.$$

- this equation is coupled with $\rho(\lambda) + \rho^h(\lambda) = \frac{1}{2\pi} + \int_{-\infty}^{+\infty} d\lambda' K(\lambda - \lambda')\rho(\lambda')$

$$\lim_{t \to \infty} \langle \mathcal{O}(t) \rangle = \langle \rho_s | \mathcal{O} | \rho_s \rangle.$$

- the question is : in the case of the problem of the two temperature $|\rho_s\rangle$ is the NESS or the GGE or into the overlaps are present both the two states? Work in progress

Conclusions

Results

- we study the transport properties of a free Fermi gas in D-dimensions
- we find a behaviour proportional to $T_{\mathcal{L}}^2 - T_{\mathcal{R}}^2$ in any dimensions
- only in 1D we recover the conformal behaviour for massive and massless cases, but this is probably an accident

Work in Progress

- we will use the GTBA to study the transport properties of integrable interacting systems
- Ising, XX and Lieb-Liniger model are under investigations
- we will find NESS, GGE or both?

THANKS