AdS₆ solutions of type II supergravity

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New frontiers of Theoretical Physics

Cortona, May 30th 2014

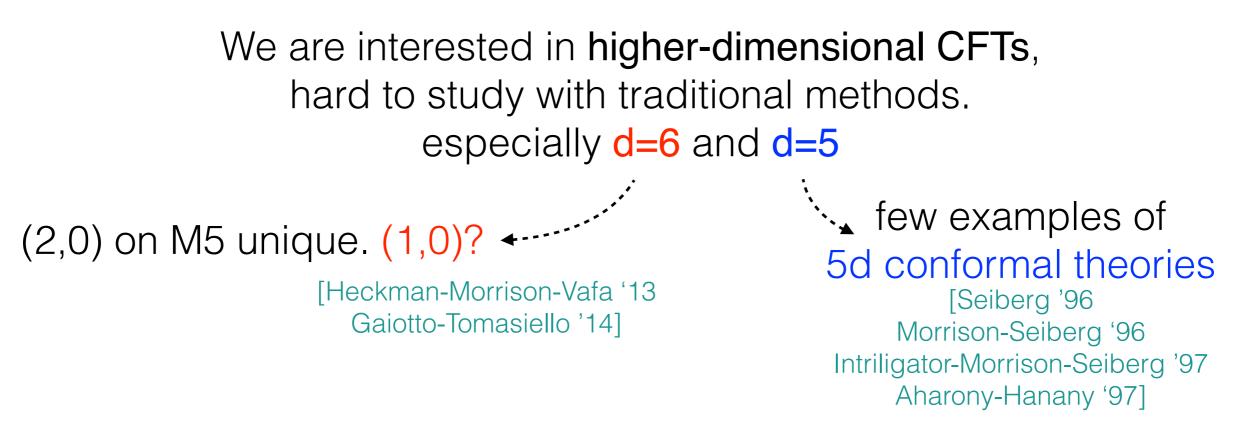




Motivation

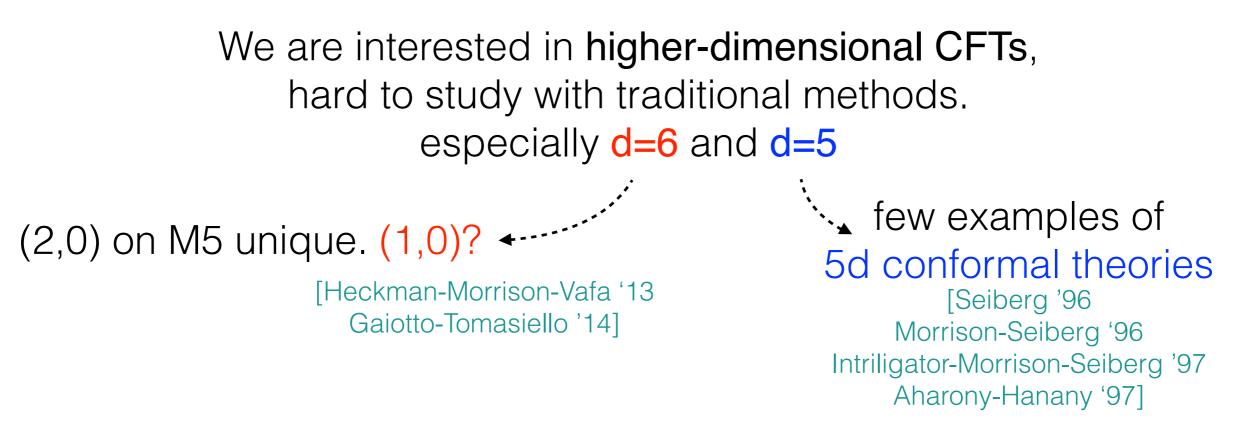
We are interested in higher-dimensional CFTs, hard to study with traditional methods. especially d=6 and d=5 (2,0) on M5 unique. (1,0)? ••••••• [Heckman-Morrison-Vafa '13 Gaiotto-Tomasiello '14] few examples of 5d conformal theories [Seiberg '96 Morrison-Seiberg '97 Aharony-Hanany '97]

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We are going to attack the problem **holographically:**

We will pave the way for a full classification of **N = 1 supersymmetric AdS₆ vacua in type IIB**: classification problem reduced to two PDEs

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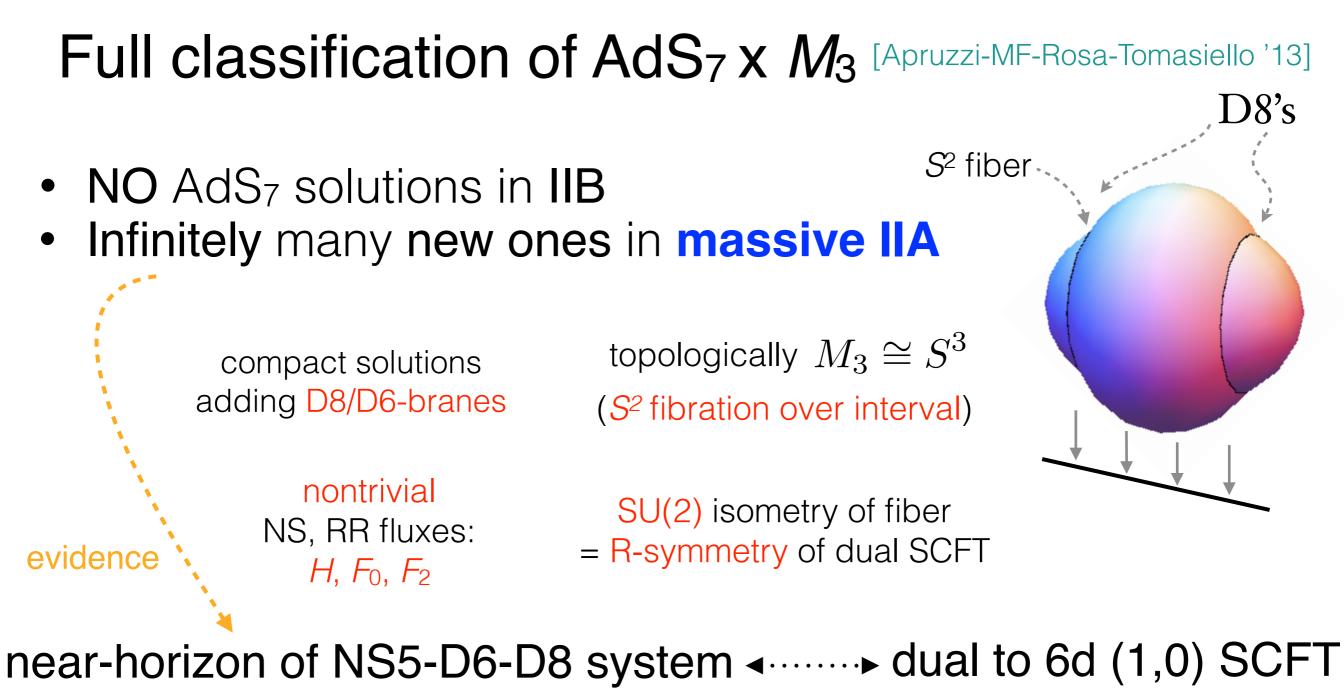
- NO AdS7 solutions in IIB
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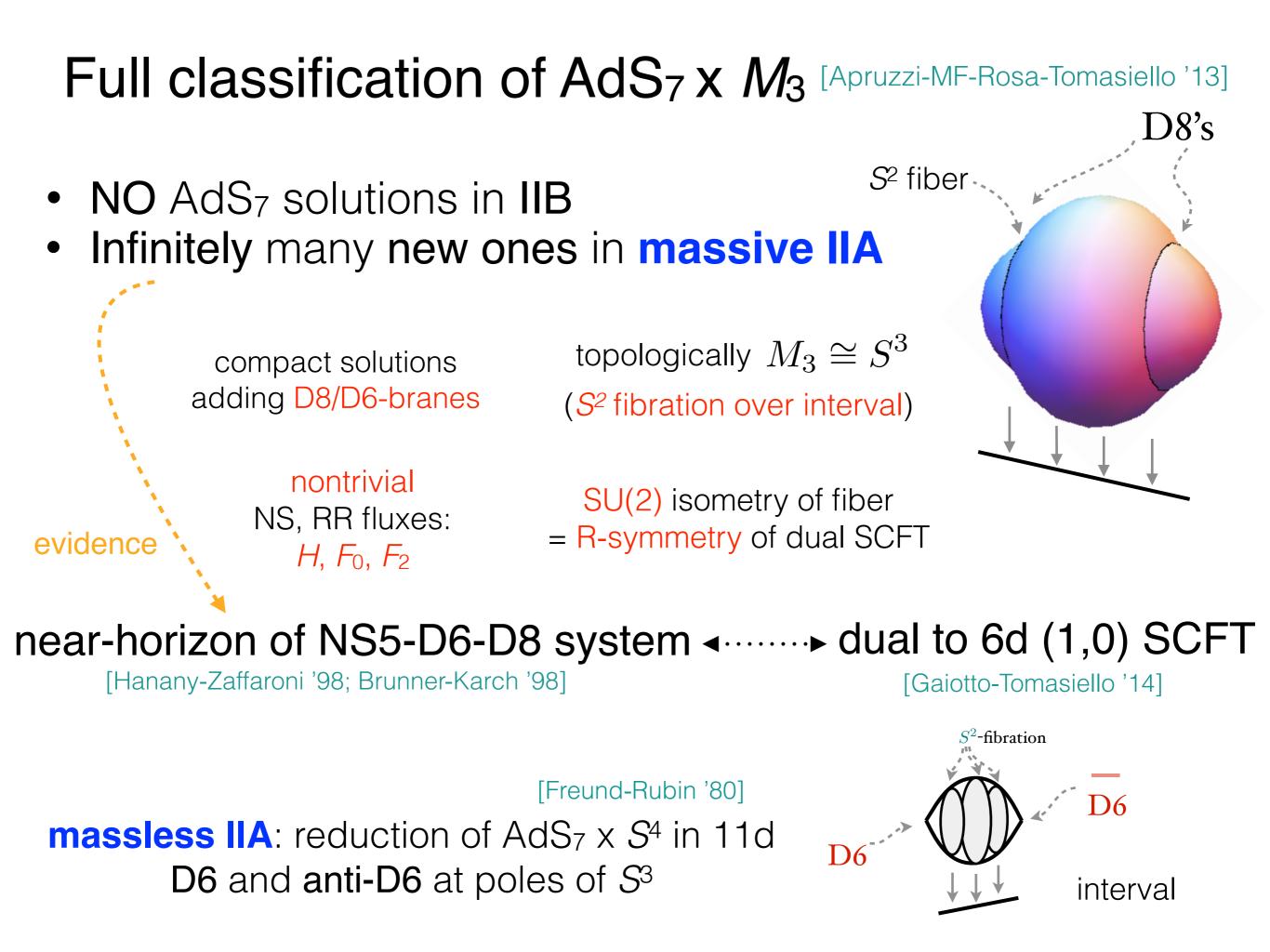
SU(2) isometry of fiber = R-symmetry of dual SCFT

Full classification of AdS₇ x M₃ [Apruzzi-MF-Rosa-Tomasiello '13] **8**'s S^2 fiber • NO AdS₇ solutions in IIB Infinitely many new ones in massive IIA topologically $M_3 \cong S^3$ compact solutions adding D8/D6-branes (S² fibration over interval) nontrivial SU(2) isometry of fiber NS, RR fluxes: = R-symmetry of dual SCFT H, F_0, F_2

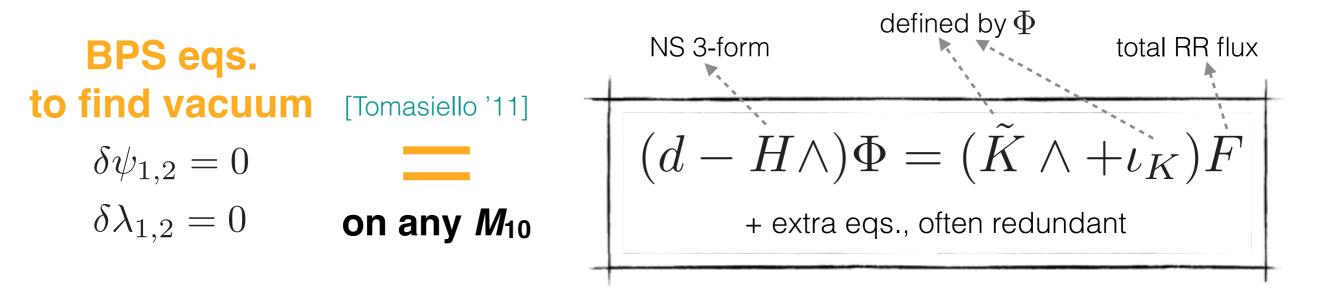


[Hanany-Zaffaroni '98; Brunner-Karch '98]

[Gaiotto-Tomasiello '14]

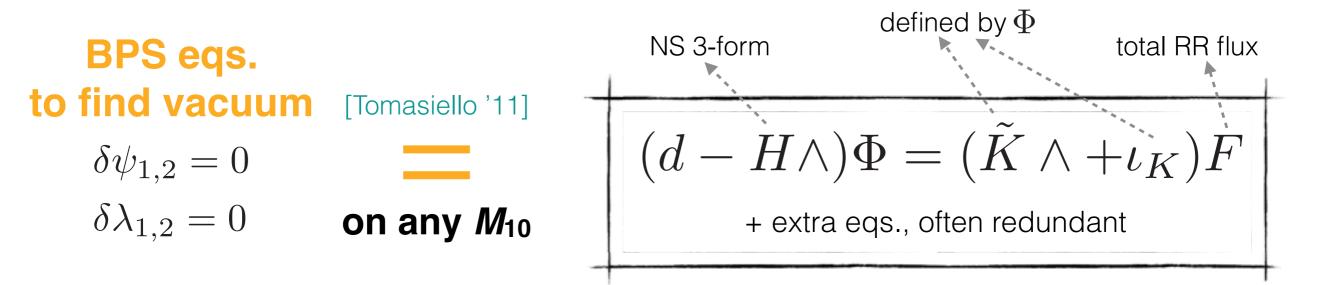


10d susy parameters $\epsilon_{1,2}$ define one **G-structure** Φ on $T\oplus T^*$



gives system of differential eqs. for forms on internal space

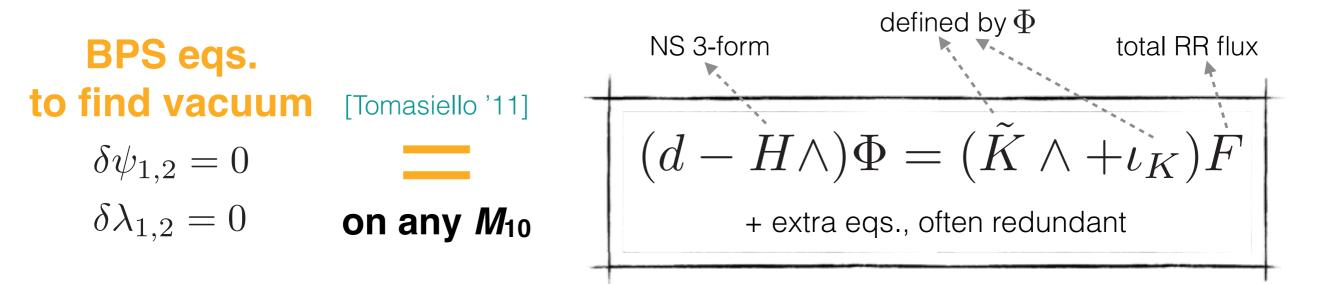
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AdS₇ x M_3 Id. x Id. structure \supset Vielbein AdS₆ x M_4

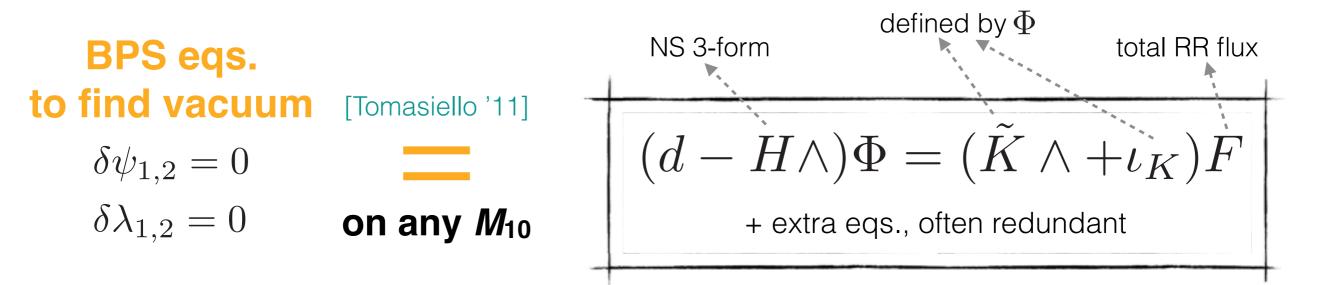
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original example: AdS₄ X Mink₄ X M_6 [Graña-Minasian-Petrini-Tomasiello '05]

SU(3) x SU(3) structure (nice differential equations)

$$d_{H}(\Psi_{-} - \Psi_{+})^{0} - 2(\Phi_{-} + \Phi_{+})^{0} = 0$$

$$d_{H}(\Phi_{-} - \Phi_{+})^{\alpha} - 3(\Psi_{-} + \Psi_{+})^{\alpha} = 0$$

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$$\|\eta^{1}\|^{2} = \|\eta^{2}\|^{2} = e^{A}$$

A = warping F = total RR flux $d_H = d - H \wedge$

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 $\Phi_+ \Psi_\pm$: **SU(2)-covariant** differential forms on M_4 0 SU(2)-singlet *up to factors of dilaton and warping $\alpha = 1, 2, 3$ SU(2)-triplet

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$$\eta^{1,2} \text{ 4d part of susy parameters}$$

$$Clifford map = \begin{bmatrix} \gamma^{i_{1}...i_{k}} \\ \partial \\ dx^{i_{1}} \wedge ... \wedge dx^{i_{k}} \end{bmatrix} \qquad SU(2) \text{ rotates supercharges} \begin{pmatrix} \eta_{\pm} \\ \eta_{\pm}^{c} \end{pmatrix}$$

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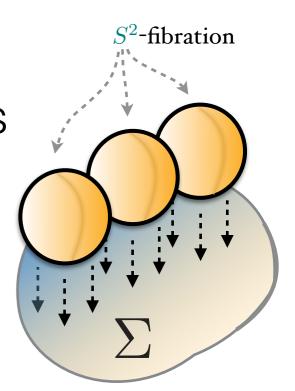
one Vielbein $\{e_i\}$ and 4 angles α , x, $S^2(\theta_1, \theta_2)$ generalized structure on M_4 M_4 is 4d • **Parametrizing** the forms, we can solve the systems one Vielbein $\{e_i\}$ and 4 angles α , x, $S^2(\theta_1, \theta_2)$

Determine explicitly metric on M₄ and fluxes

generalized structure on M₄

 M_4 = S^2 fibration over a 2d space $\Sigma(\alpha, x)$

nontrivial H, F₁, F₃



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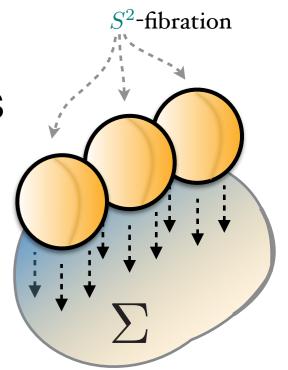
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- M_4 has **SU(2) isometry** = **R-symmetry** of dual 5d CFT (N = 1 5d algebra unique. R-symmetry unique)
- All **Bianchi** (and EoM) for fluxes automatically satisfied (in AdS7, modify Bianchi to include D8/D6-brane sources)

So far: the pure spinor formalism has allowed us to solve (almost all) BPS equations, but... 2 differential equations still need to be solved: highly nonlinear PDEs

We reduced the classification problem to finding solutions to 2 PDEs^{*}

PDEs govern geometry of $\Sigma(\alpha,x)$

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solution depends on 2 functions at the boundary of \sum : well-formed system

(using EDS machinery)

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[Brandhuber-Oz '99]

in massive IIA, there exists only one solution*:

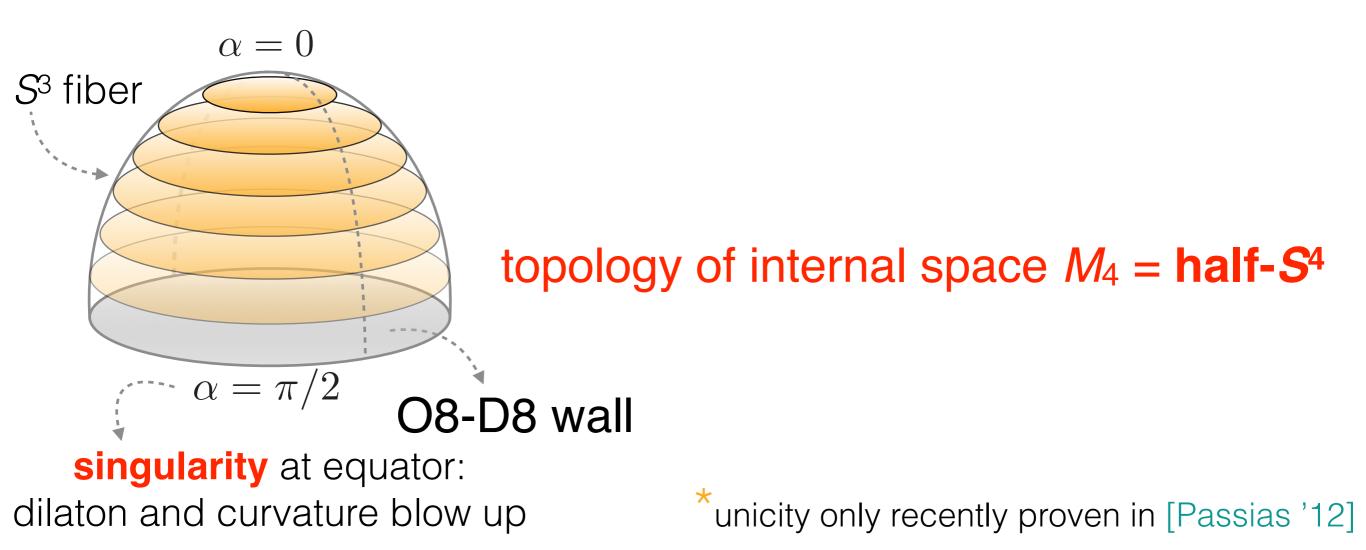
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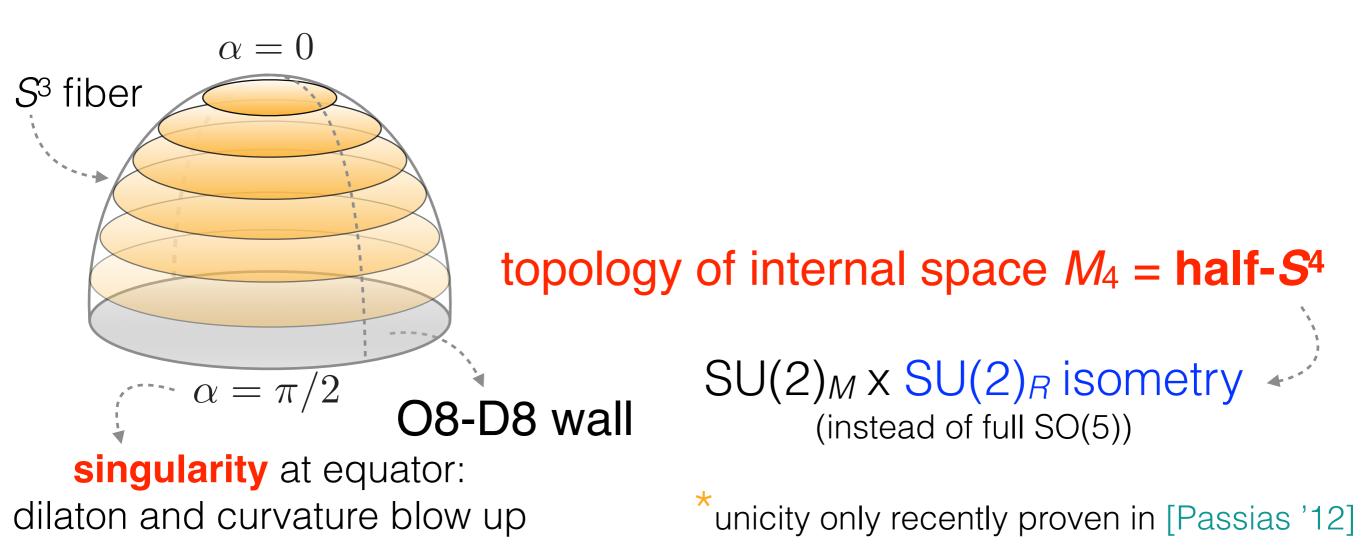
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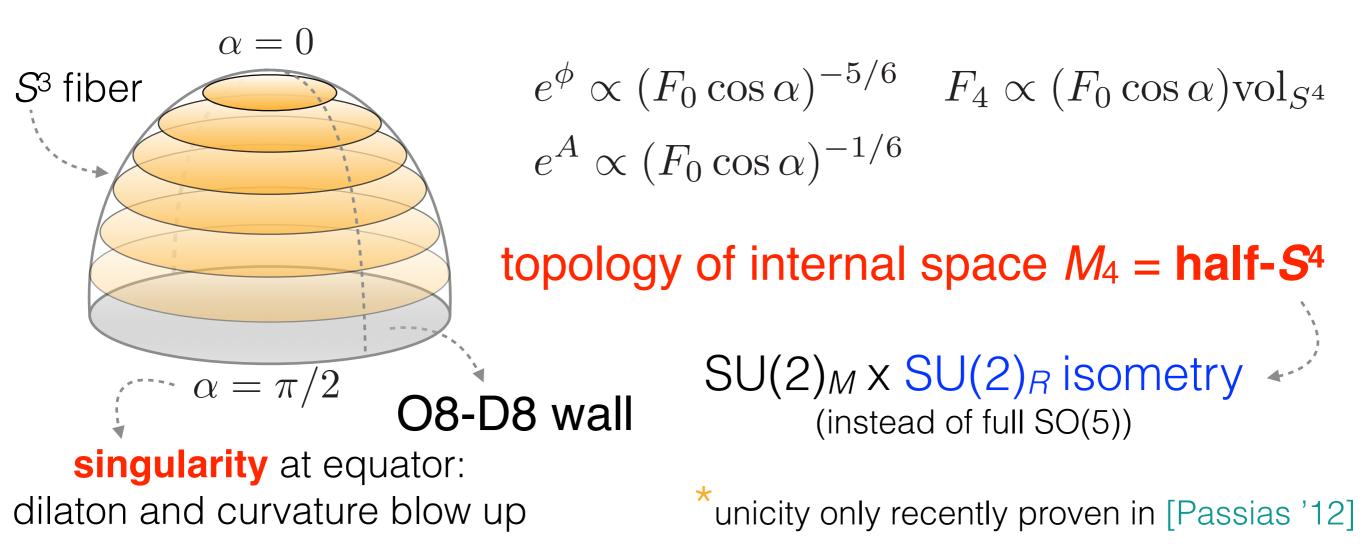
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easily recovered from our formalism: \blacksquare we read off the physical fields from the forms $\Phi_{\pm} \ \Psi_{\pm}$



Abelian T-dual of Brandhuber-Oz

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In our language, it's an exact solution to the 2 PDEs which does not depend on the coordinate *x*

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smeared O7-D7

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smeared NS5

- inherits singularity at equator $\alpha = \frac{\pi}{2}$ of **dual** half-S⁴
- **new** singularity at $\alpha = 0$

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Via a simple Ansatz, we recover it as solution to our PDEs

(same singularities as abelian T-dual vacuum)

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To-do list:

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- find most general solution to PDEs
- modify Bianchi to include D-brane sources
- establish correspondence between new IIB backgrounds and 5d SCFT's which admit a gravity dual