Nonlocal infrared modifications of gravity and dark energy

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Cortona 2014

the general idea: modify GR in the infrared using non-local terms

- motivation: explaining DE

  IR modification $\longrightarrow$ mass term?

- (local) massive gravity: Fierz-Pauli, dRGT, bigravity
  - significant progresses (ghost-free), still open issues

  see talk by Hassan

- our approach: mass term as coefficient of non-local terms some sources of inspiration:

- $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$ is equivalent to

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{m_\gamma^2}{\Box}\right)F^{\mu\nu}$$

(Dvali 2006)

duality between locality and gauge-invariance for massive theories

- degravitation $$\left(1 - \frac{m^2}{\Box}\right)G_{\mu\nu} = 8\pi G T_{\mu\nu}$$ (Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

we can introduce a mass parameter without breaking the gauge-invariance of the theory

different possible implementations of the idea

- $$G_{\mu\nu} - m^2(\Box^{-1}G_{\mu\nu})^T = 8\pi G T_{\mu\nu}$$

(M. Jaccard,MM, E. Mitsou 2013)

however, instabilities in the cosmological evolution

(S.Foffa,MM, E. Mitsou 2013)

- $$G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$$

(MM 2013)

nice cosmological properties (w_{DE}=-1.04).

- last twist

$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

(MM and M.Mancarella 2014)

- Conceptual aspects
  - effective classical theory vs fundamental nonlocal theories
  - absence of ghosts
  - degrees of freedom
  - no vDVZ discontinuity

MM 2013;
S. Foffa, MM and E. Mitsou 2013

A. Kehagias and MM, 2014

- Cosmological consequences
  - background evolution. Prediction for w_{DE}

MM 2013; MM and M.Mancarella 2014

  - cosmological perturbations and comparison with data

Y. Dirian, S. Foffa, N. Khosravi, M. Kunz, MM 1403.6068

Non-local QFT or classical effective equations?

- we have $\Box_{\rm ret}^{-1}$ directly in the EoM (rather than in the solution). This EoM cannot come from the variation of a Lagrangian. E.g.

$$\frac{\delta}{\delta\phi(x)}\int dx'\phi(x')(\Box^{-1}\phi)(x') = \frac{\delta}{\delta\phi(x)}\int dx'dx''\phi(x')G(x',x'')\phi(x'')$$
$$= \int dx'[G(x,x')+G(x',x)]\phi(x')$$

- we can repalce $\Box^{-1}\to\Box_{\rm ret}^{-1}$ after the variation, as a formal trick to get the EoM from a Lagrangian.

Deser-Waldron 2007,
Barvinski 2012

However, any connection to the QFT described by this Lagrangian is lost.

EoMs involving $\Box_{\rm ret}^{-1}$ emerge from a classical or a quantum averaging of a more fundamental (local) QFT

- classically, when separating long and short wavelength and integrating out the short wave-length

  (e.g cosmological perturbation theory, or GWs)

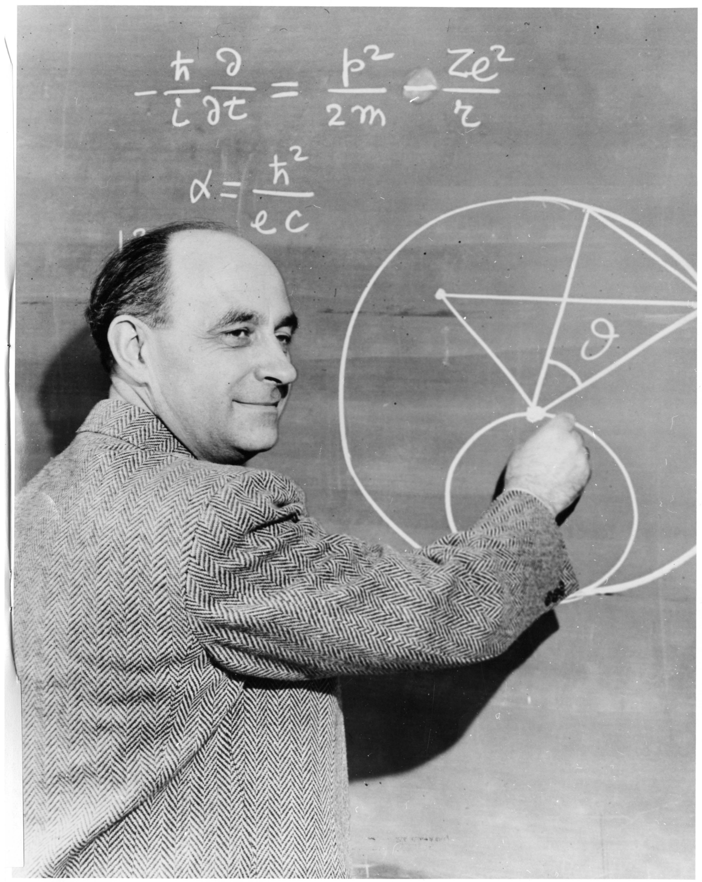
- in QFT, when computing the effective action that includes the effect of radiative corrections. This provides effective non-local field eqs for $\langle 0|\hat{\phi}|0\rangle, \langle 0|\hat{g}_{\mu\nu}|0\rangle$

- the in-in matrix elements satisfy non-local and retarded equations

Jordan 1986, Calzetta-Hu 1987

Our general question: which effective nonlocal theories give a meaningful cosmology?

- top-down approach: find the correct fundamental theory (massive gravity, bimetric theory,...?)
- bottom-up: find first the correct effective theory



- e.g Standard Model vs Fermi theory
  - start from the fundamental YM theory
  - or understand which terms correctly describe weak interaction at low energies

$$\text{e.g. } (\bar\psi\psi)^2,\ (\bar\psi\gamma_5\psi)^2,\ (\bar\psi\gamma_\mu\psi)^2,\ \ldots [\bar\psi\gamma_\mu(1-\gamma_5)\psi]^2,$$

- So, we interpret our non-local eqs as a classical, effective equation, derived from a more fundamental local theory by a classical or quantum averaging
- any problem of quantum vacuum stability can only be addressed in this fundamental theory
- the theory $S_{\rm NL}=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R-m^2R\frac{1}{\Box^2}R\right]$

  could be the truncation of the correct effective theory
- the theory $G_{\mu\nu}-m^2(g_{\mu\nu}\Box^{-1}R)^T=8\pi G T_{\mu\nu}$

  could be an example of resummation
- our general question: which effective nonlocal theories give a meaningful cosmology?

Absence of vDVZ discontinuity and of a strong coupling regime

A. Kehagias and MM 2014

- write the eqs of motion of the non-local theory in spherical symmetry: U(r), S(r), plus

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- for mr <<1: low-mass expansion

- for $r \gg r_S$: Newtonian limit (perturbation over Minowski)

- match the solutions for $r_S \ll r \ll m^{-1}$ (this fixes all coefficients)

- result: for r>>r_s

$$A(r) = 1 - \frac{r_S}{r}\left[1 + \frac{1}{3}(1 - \cos mr)\right]$$

$$B(r) = 1 + \frac{r_S}{r}\left[1 - \frac{1}{3}(1 - \cos mr - mr \sin mr)\right]$$

for r_s<<r<< m^{-1}: $A(r) \simeq 1 - \frac{r_S}{r}\left(1 + \frac{m^2 r^2}{6}\right)$

the limit $m \to 0$ is smooth !

By comparison, in massive gravity the same computation gives

$$A(r) = 1 - \frac{4}{3}\frac{r_S}{r}\left(1 - \frac{r_S}{12 m^4 r^5}\right)$$

vDVZ discontinuity

breakdown of linearity below $r_V=(r_s/m^4)^{1/5}$

Cosmological consequences

- define $U = -\Box^{-1} R\,, \quad S = -\Box^{-1} U$

- in FRW we have 3 variables: H(t), U(t), W(t)=H^2(t)S(t).

  define x=ln a(t), h(x)=H(x)/H_0

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(U, U', W, W')$$
$$U'' + (3+\zeta)U' = 6(2+\zeta)$$
$$W'' + 3(1-\zeta)W' - 2(\zeta' + 3\zeta - \zeta^2)W = U$$

$$\gamma = m^2/(9H_0^2) \qquad \zeta = h'/h$$

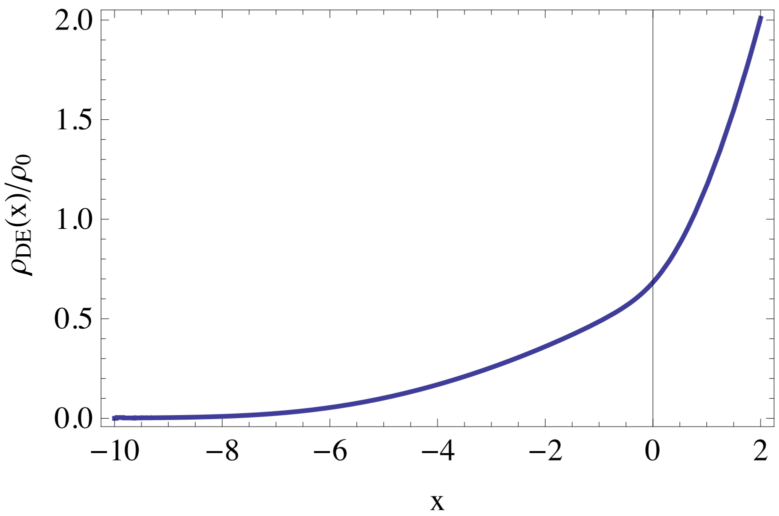
- there is an effective DE term, with

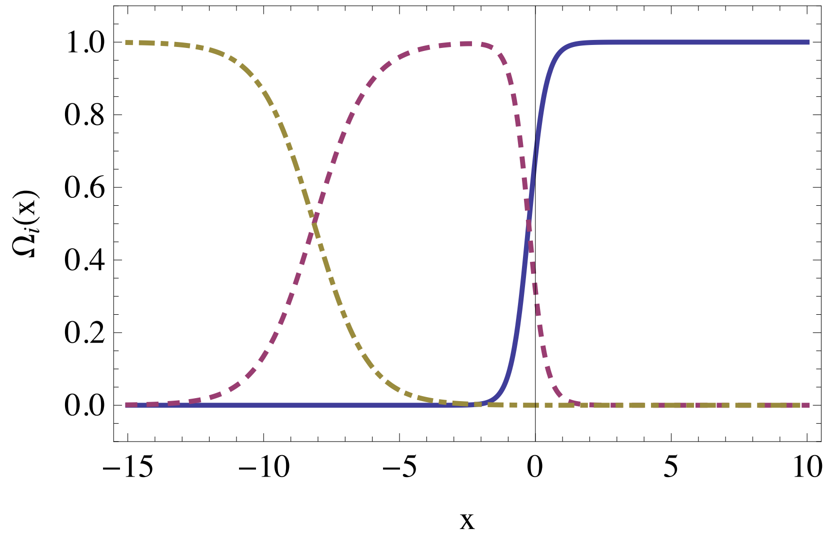
$$\rho_{\rm DE}(x) = \rho_0 \gamma Y(x) \qquad \rho_0 = 3H_0^2/(8\pi G)$$

- define w_{DE} from $\dot{\rho}_{\rm DE} + 3(1 + w_{\rm DE}) H \rho_{\rm DE} = 0$

- the model has the same number of parameters as ΛCDM, with $\Omega_\Lambda \leftrightarrow \gamma$.

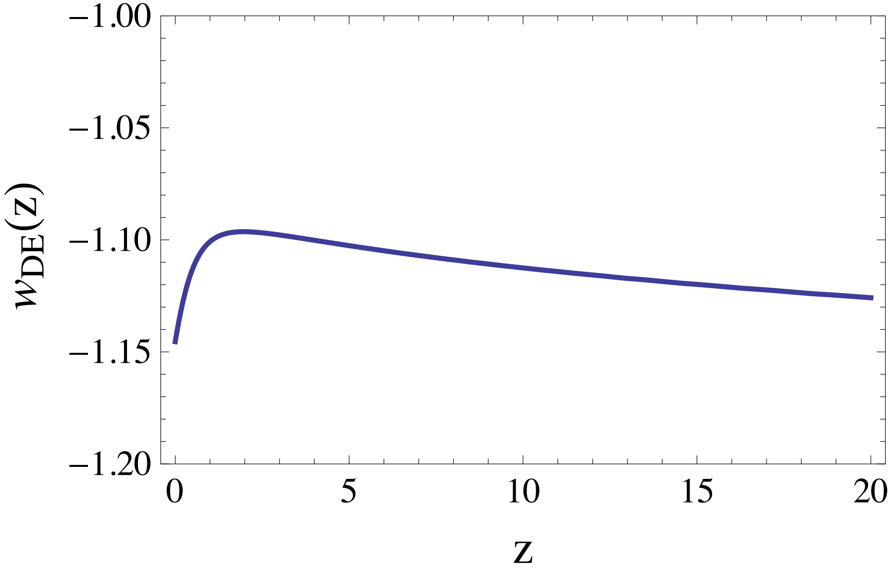
- results:





- Fixing γ = 0.0089.. ($m=0.28\ H_0$) we reproduce $\Omega_{DE}=0.68$

- having fixed γ we get a pure prediction for the EOS:



fit $w(a)=w_0+(1-a)\,w_a$

in the region $0< z <1.6$

$w_0= -1.144, \quad w_a=0.084$

on the phantom side !

general consequence of $\dot{\rho}_{\mathrm{DE}} + 3(1+w_{\mathrm{DE}})H\rho_{\mathrm{DE}} = 0$

together with $\rho>0$ and $d\rho/dt>0$

Cosmological perturbations

Y. Dirian, S. Foffa, N. Khosravi, M. Kunz, MM
1403.6068

- well-behaved?

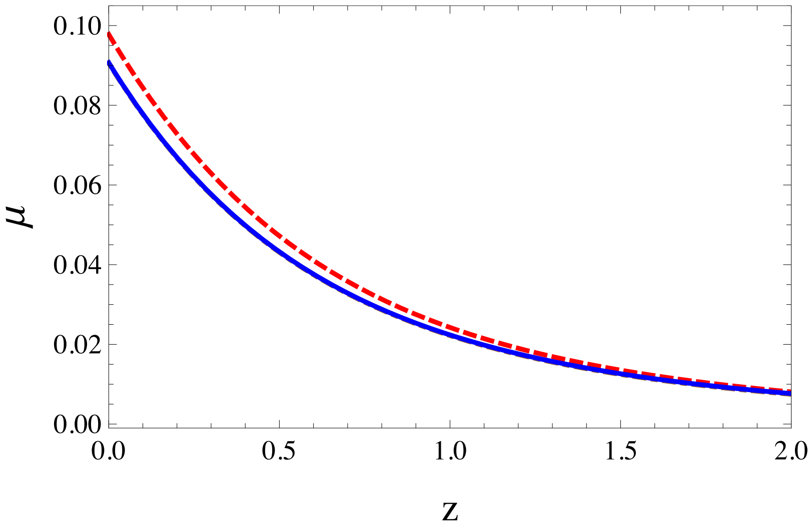
- consistent with structure formation?
  - Deser-Woodard nonlocal model ruled out at the 8σ level by the comparison with structure formation

Dodelson and Park 1310.4329

- Bayesian model comparison with ΛCDM

- the perturbations are well-behaved and differ from ΛCDM at a few percent level

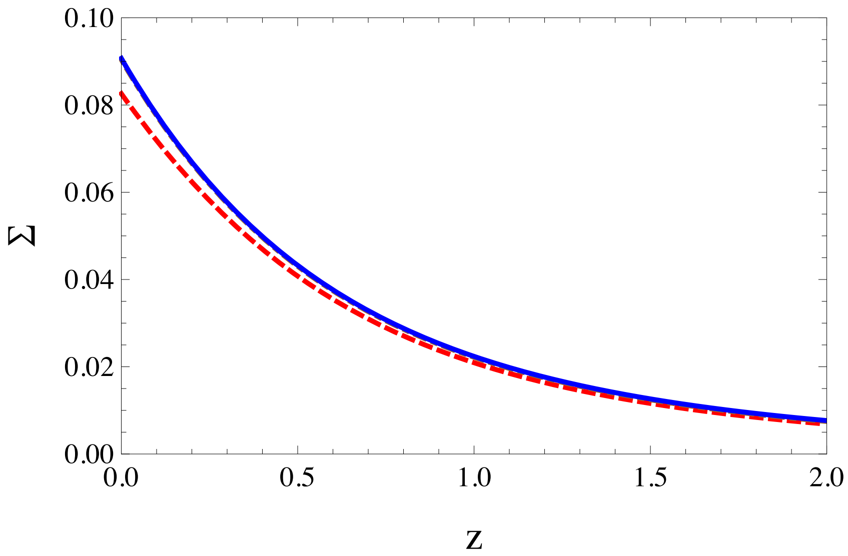
$$\begin{aligned}\Psi &= [1+\mu(a;k)]\Psi_{\rm GR} \\ \Psi-\Phi &= [1+\Sigma(a;k)](\Psi-\Phi)_{\rm GR}\end{aligned}$$



- deviations at z=0.5 of order 4%

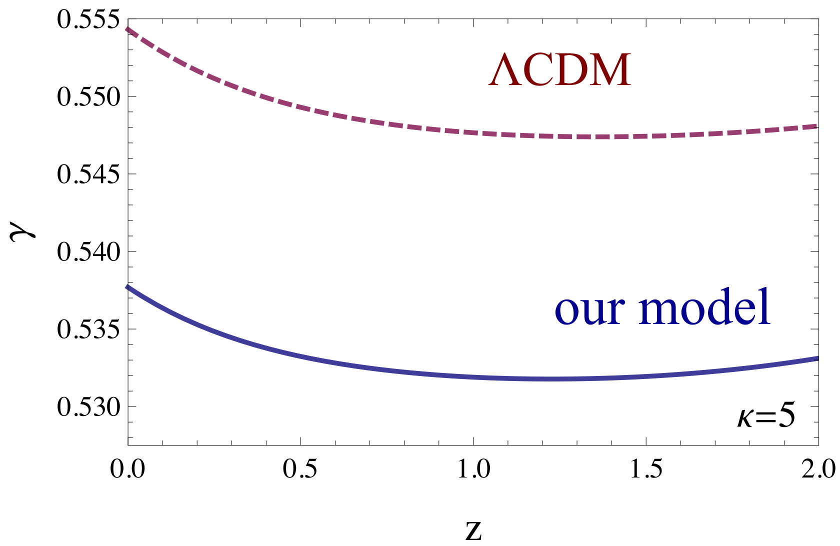
- consistent with data: CFHTLenS gives ΔΨ/Ψ=0.05±0.25 (Simpson et al 1212.3339)

Lensing: again deviations at 4% level



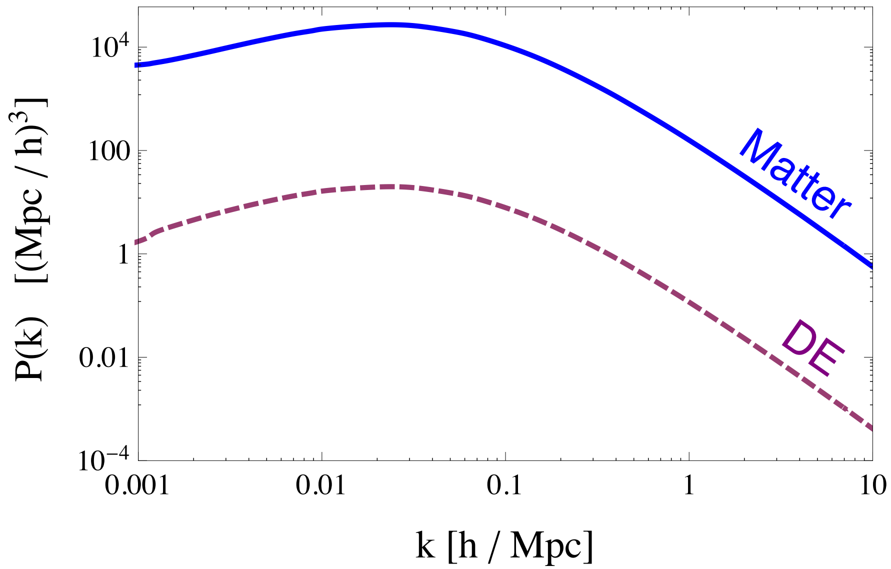
growth index:

$$\frac{d\log\delta_M(a;k)}{d\ln a} = [\Omega_M]^{\gamma(z;k)}$$

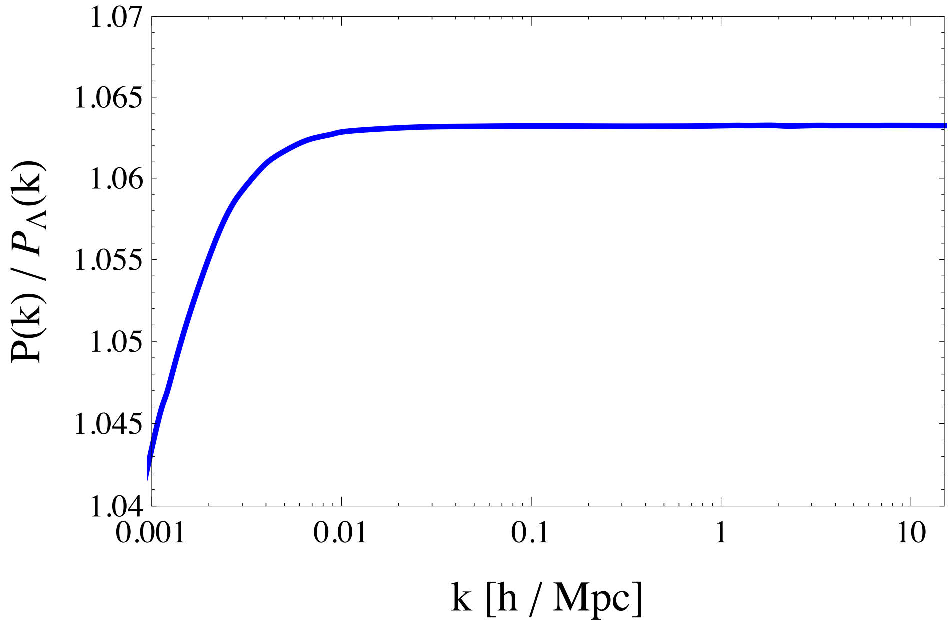


- linear power spectrum

DE clusters but its linear power spectrum is small compared to that of matter



matter power spectrum compared to ΛCDM



Comparison with ΛCDM

- A caveat: this is not wCDM!

- for the model $G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$ (MM 2013)

  the perturbations have been recently computed and compared them to CMB, BAO, SNIa and growth rate data

Nesseris and Tsujikawa 1402.4613

- If $h_0>0.70$ the data strongly support this nonlocal model over ΛCDM
- If $0.67<h_0<0.70$ the two models are statistically comparable

(however, CMB studied using the shift parameter, rather than a full Boltzmann code)

- for the model

$$S_{\mathrm{NL}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

we find that

- structure formation: statistically equivalent to ΛCDM with present data

- SNIa: fit to the JLA data gives equivalent χ^2

- CMB: full Boltzmann code analysis under way

Conclusions

- we have an interesting IR modification of GR
- and testable predictions
  - w(0)=-1.14 + a full prediction for w(z)
    - DES Δw=0.03 (stage IV+Planck Δw=0.01)
    - EUCLID Δw=0.01
  - $\mu(a) = \mu_s a^s \rightarrow \boldsymbol{\mu_s = 0.09, s = 2}$
    - Forecast for EUCLID, $\Delta\mu$=0.01
  - Σ(z): lensing deviations at a few %
  - $\gamma = \mathbf{0.53}$

Thank you!

Degrees of freedom

$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

- define $U = -\Box^{-1}R\,, \quad S = -\Box^{-1}U$
- the eqs. $\Box U = -R, \quad \Box S = -U$

  do not describe radiative d.o.f !

$$-\Box^{-1}R = U_{\rm hom}(x) - \int d^4x'\,\sqrt{-g(x')}\,G(x;x')R(x')$$

The homogeneous solution is fixed by the definition of i.e. by the def of the non-local theory.

It is not a free Klein-Gordon field !

- linearize the eqs of motion. Scalar sector:

$$h_{00} = 2\Psi, \quad h_{0i} = 0, \quad h_{ij} = 2\Phi\delta_{ij}$$

$$\begin{aligned}
\boldsymbol{\nabla}^2 \left[\Phi - (m^2/6)S\right] &= -4\pi G\rho \\
\Phi - \Psi - (m^2/3)S &= -8\pi G\sigma \\
(\Box + m^2)U &= -8\pi G(\rho - 3P) \\
\Box S &= -U
\end{aligned}$$

Φ and Ψ remain non-radiative!

In contrast, in massive gravity with FP mass term $(\Box - m^2)\Phi = 0$

and with generic mass there is a $(\Box\Phi)^2$ in the action (ghost)

U and S are non-radiative despite the KG operator.

No radiative d.o.f. in the scalar sector !

- beyond the scalar sector: linearizing the eq of motion

$$\mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \tfrac{d-1}{d} m^2 P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma} = -16\pi G\, T^{\mu\nu}$$

$$P^{\mu\nu} \equiv \eta^{\mu\nu} - \tfrac{\partial^\mu \partial^\nu}{\Box}$$

the corresponding matter-matter interaction is

$$\tilde{T}_{\mu\nu}(-k) \frac{1}{2k^2} \left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right) \tilde{T}_{\rho\sigma}(k)$$

$$+\frac{1}{6}\tilde{T}(-k)\left(\frac{1}{k^2} - \frac{1}{k^2 - m^2}\right)\tilde{T}(k)$$

- no vDVZ discontinuity!

- For m=O(H_0), solar system test easily passed. Corrections are O(m^2/k^2) =10^{-30} for k=(1 a.u)$^{-1}$.

- massless graviton + extra contribution to $\tilde{T}_{\mu\nu}(-k)\tilde{D}^{\mu\nu\rho\sigma}(k)\tilde{T}_{\rho\sigma}(k)$

$$\frac{1}{d(d-1)}\tilde{T}(-k)\left[-\frac{i}{k^2}-\frac{i}{(-k^2+m^2)}\right]\tilde{T}(k)$$

these are the contribution of U and S and do not correspond to a radiative dof. In a quantum treatment there are no creation/ annihilation operators associated to them

A fake ghost in massless GR

$$S^{(2)}_{\mathrm{EH}} = \frac{1}{2}\int d^{d+1}x\, h_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma}$$

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + \frac{1}{2}(\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu) + \frac{1}{d}\eta_{\mu\nu}s$$

$$S^{(2)}_{\mathrm{EH}} = \frac{1}{2}\int d^{d+1}x\, \left[h^{\mathrm{TT}}_{\mu\nu}\Box(h^{\mu\nu})^{\mathrm{TT}} - \frac{d-1}{d}\, s\Box s\right]$$

$$S_{\mathrm{int}} = \frac{\kappa}{2}\int d^{d+1}x\, h_{\mu\nu}T^{\mu\nu} = \frac{\kappa}{2}\int d^{d+1}x\, \left[h^{\mathrm{TT}}_{\mu\nu}(T^{\mu\nu})^{\mathrm{TT}} + \frac{1}{d}sT\right]$$

$$\Box h^{\mathrm{TT}}_{\mu\nu} = -\frac{\kappa}{2}T^{\mathrm{TT}}_{\mu\nu}\,, \qquad \Box s = \frac{\kappa}{2(d-1)}T$$

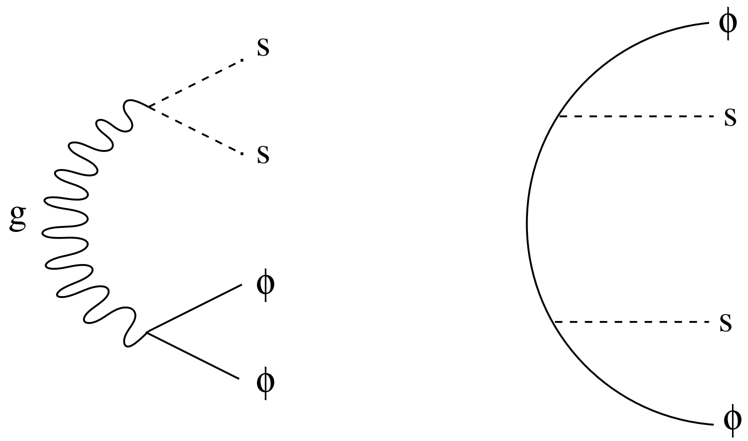
It looks as if there are many more propagating d.o.f

Furthermore s seems a ghost !

S. F. Hassan, R. A. Rosen, and A. Schmidt-May 2012

- the contribution of s is not canceled by the helicity-0 component of $h_{\mu\nu}^{TT}$!

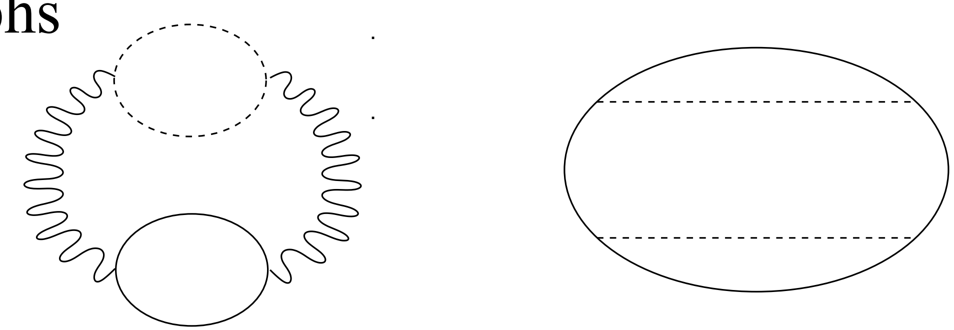
Evident if we look at vac $\rightarrow$ ss$\varphi\varphi$ diagrams



(a) (b)

More subtle in vac to vac graphs

$$-\frac{i}{k^2-i\epsilon}+\frac{i}{k^2+i\epsilon}$$


(a) (b)

S. Foffa, MM and E. Mitsou 2013

- the origin of the problem is that s is a non-local function of $h_{\mu\nu}$:

$$s = \left(\eta^{\mu\nu} - \frac{1}{\Box} \partial^\mu \partial^\nu \right) h_{\mu\nu} = P^{\mu\nu} h_{\mu\nu}$$

- example: $\boldsymbol{\nabla}^2 \phi = \rho$

$$\tilde{\phi} \equiv \Box^{-1} \phi \qquad\qquad \Box \tilde{\phi} = \boldsymbol{\nabla}^{-2} \rho \equiv \tilde{\rho}$$

it looks as if we have generated a dynamical dof!

However, the solution of the homogeneous eq are spurious!

the same happens for s: s is non-radiative, and we must discard the solutions of the homogeneous eq $\Box s = 0$

- at the quantum level, no annihilation/creation operators associated to it; s cannot be put on the external lines (otherwise, the vacuum in GR would decay!)

- the same happens in our non-local theory. The extra term in

$$\mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 (P^{\mu\nu} h_{\mu\nu})^2$$
$$= \frac{1}{2} \left[h_{\mu\nu}^{\mathrm{TT}} \Box (h^{\mu\nu})^{\mathrm{TT}} - \frac{d-1}{d} s(\Box + m^2) s \right]$$

is just a mass term for s ! However, it remains a non-radiative field, as in GR, and we must discard the plane-wave solutions of

$$(\Box + m^2) s = \tfrac{\kappa}{2(d-1)} T \,,$$

again, no propagating dof associated to s, and no issue of quantum vacuum decay !