# Resonant detector for multiple-qp Hall spectroscopy


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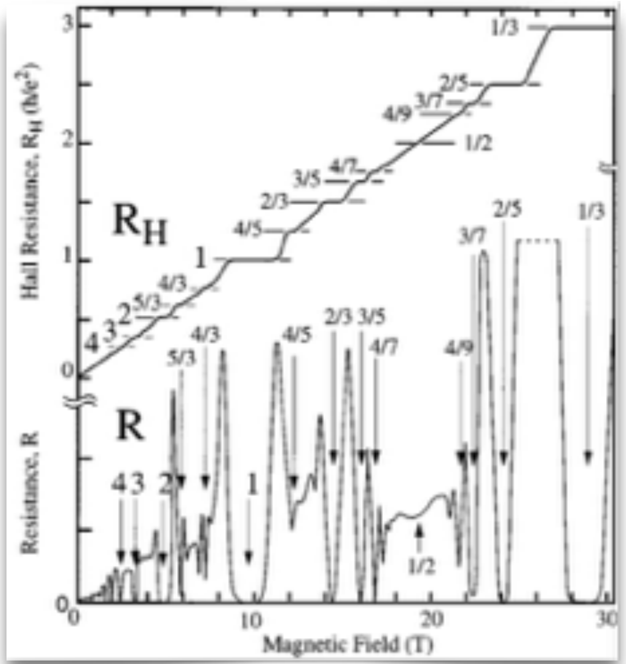
HybridNanoDev_project

FUTURO IN RICERCA 2012

CIG-COHEAT

HORIZON 2020

# FQHE: edge states & qps



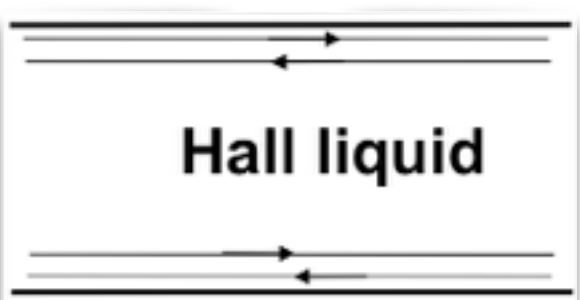

- Topological protected edge states
- Fractional statistics & charges
  Laughlin PRL'83
- Chiral edge states with gapless modes
  Wen PRB90,*Halperin PRB 82, Buttiker PRB 88, Beenakker PRL 9*

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$\sigma_{xx} = 0$$

- Laughlin sequence $\nu = \frac{1}{2np+1} = 1, \frac{1}{3}, \frac{1}{5}, \ldots$
- Jain sequence $\nu = \frac{p}{2np+1} = \frac{2}{5}, \frac{2}{3}, \cdots$

Jain PRL'89, Wen & Zee PRB'92, Kane & Fisher PRB'95




Hierarchical models

# Edge states & Multiple-qp

- Chiral Luttinger liquids

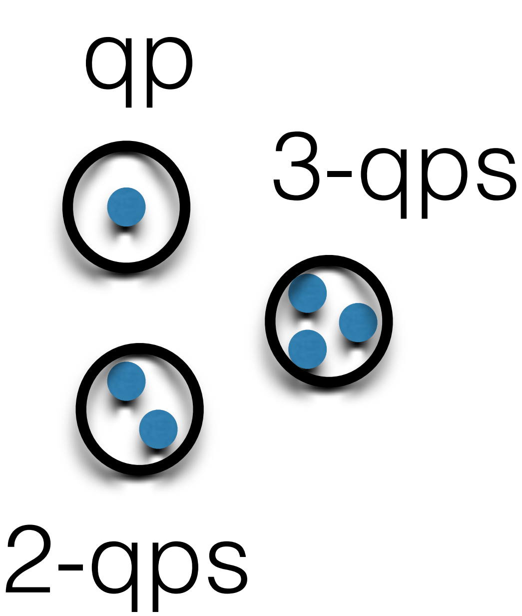
$$\mathcal{L} = \frac{1}{4\pi}(K_{ij}\partial_x\phi_i\partial_t\phi_j + V_{ij}\partial_x\phi_i\partial_x\phi_j + 2\epsilon^{\mu\nu}t_j\partial_\mu\phi_j A_\nu)$$

- Multiple-qps excitations $\Psi_l(x) \propto e^{l^T \cdot K \cdot \phi}$

- Filling factor $\nu = t^T \cdot K^{-1} \cdot t$

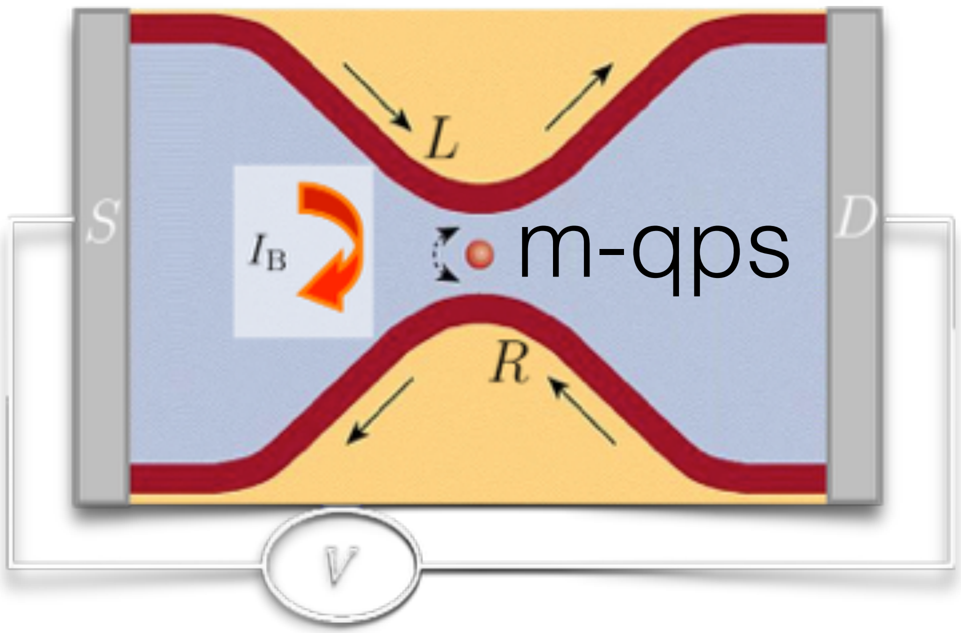
- Fractional charges $q_l = \frac{1}{2\pi}\, l^T \cdot K^{-1} \cdot t = me^*$

- Fractional statistics $\theta_l = 2\pi\, l^T \cdot K^{-1} \cdot l$




$$\Psi_l(x)\Psi_l(y) = \Psi_l(y)\Psi_l(x)e^{-i\theta_l \mathrm{sgn}(x-y)}$$

# QPC:Current & Noise




- Weak backscattering current

$$I = \nu \frac{e^2}{h} V - I_B \qquad I_B \ll I$$

- Power-law signatures in the scaling dimension $\Delta_m$

$$G_B^{(m)} \propto T^{2\Delta_m - 2} \qquad I_B^{(m)} \propto V^{2\Delta_m - 1}$$

- Current noise signatures: charge measurement

$$S(\omega = 0) = \int_{-\infty}^{+\infty} \langle \{\delta I_B(t), \delta I_B(0)\}_+ \rangle \qquad \delta I_B = I_B - \langle I_B \rangle$$

$$S^{(m)} = I_B^{(m)} \coth\left(\frac{m e^* V}{2 k_B T}\right)$$

$k_B T \gg m e^* V$: $S^{(m)} \approx 2 k_B T G_B$

$k_B T \ll m e^* V$: $S^{(m)} \approx m e^* I_B^{(m)}$

# Multiple-qp evidences

- Fractional charges: single-qps evidences

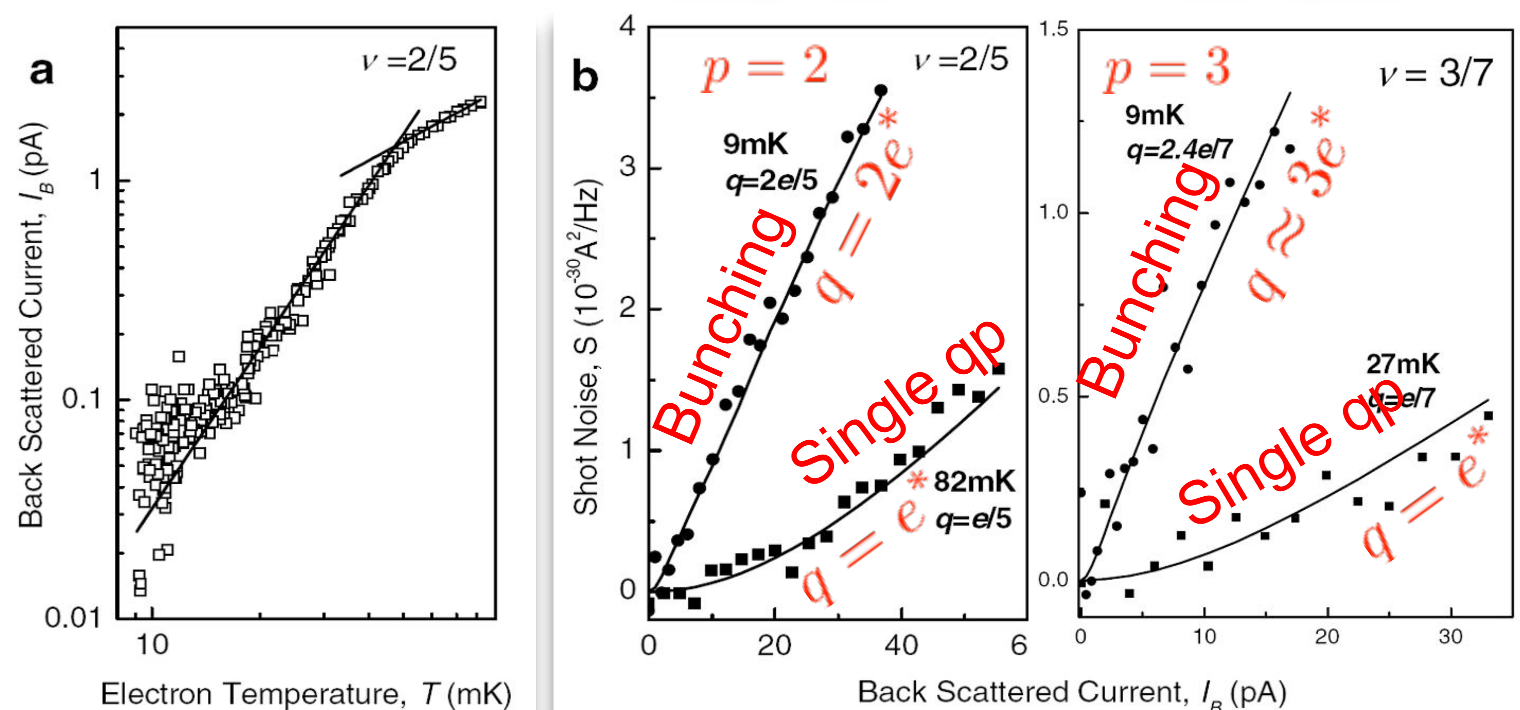
Theory:Kane & Fisher PRL 94, Fendley, Ludwig & Saleur PRL 95

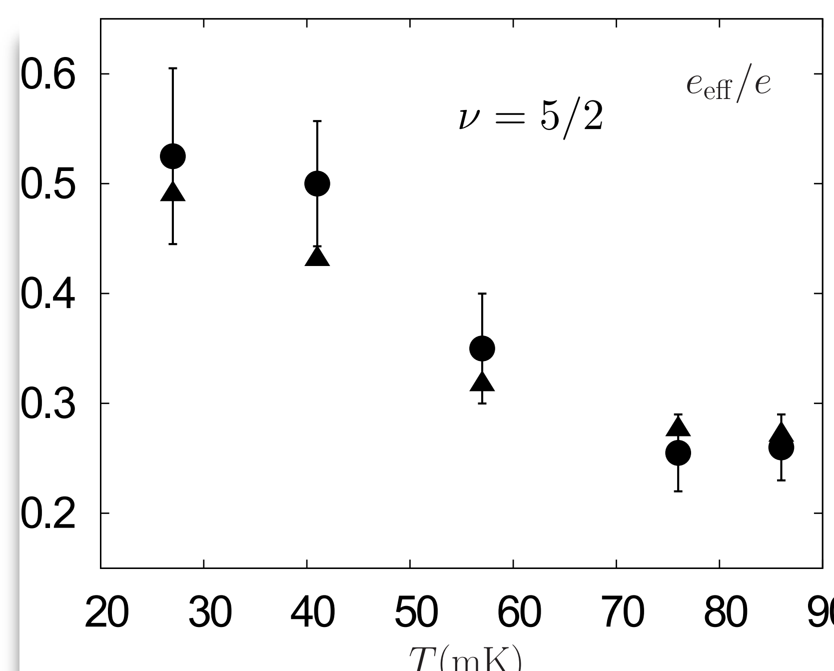
Exp:De-Picciotto… Nature 97,Saminadayar…. PRL’97,Reznikov… Nature’99

Robert B. Laughlin, Horst L. Störmer and Daniel C. Tsui
"for their discovery of a new form of quantum fluid with fractionally charged excitations"

- Multiple-qp. evidences

Chung…PRL03, Bid PRL03,Dolev….

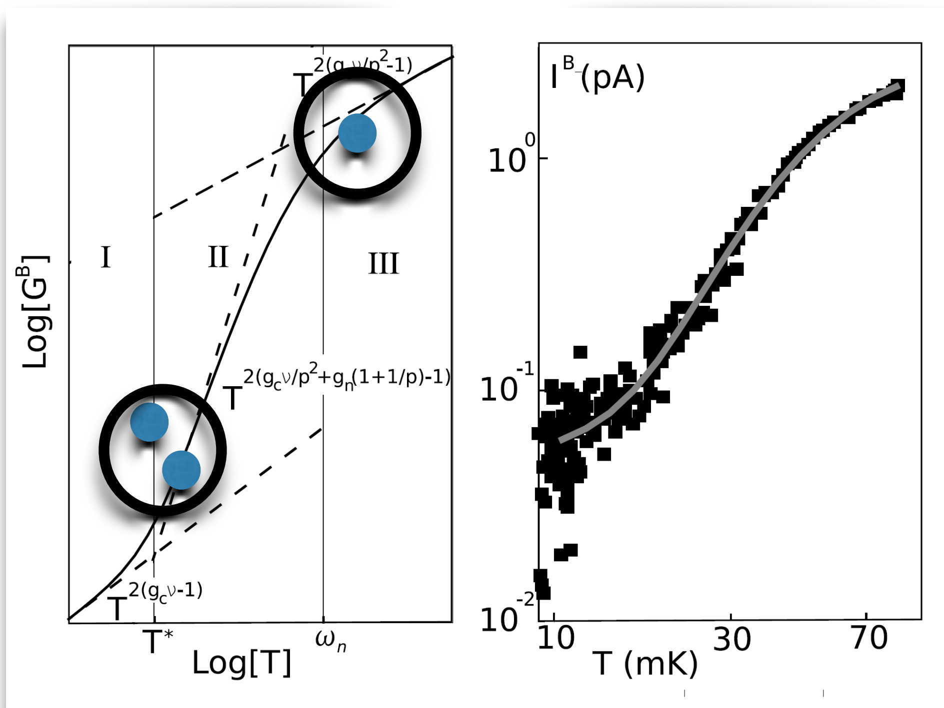


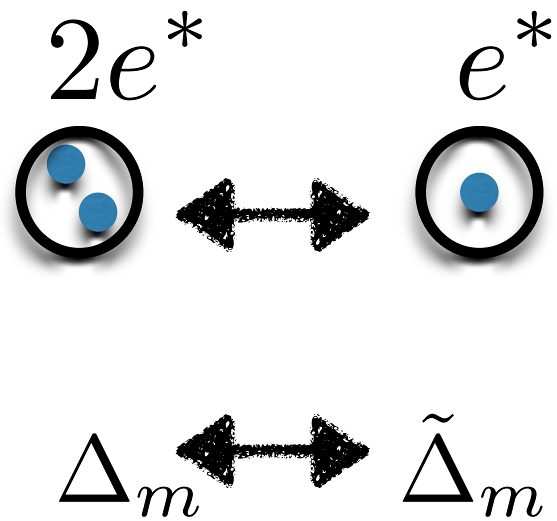



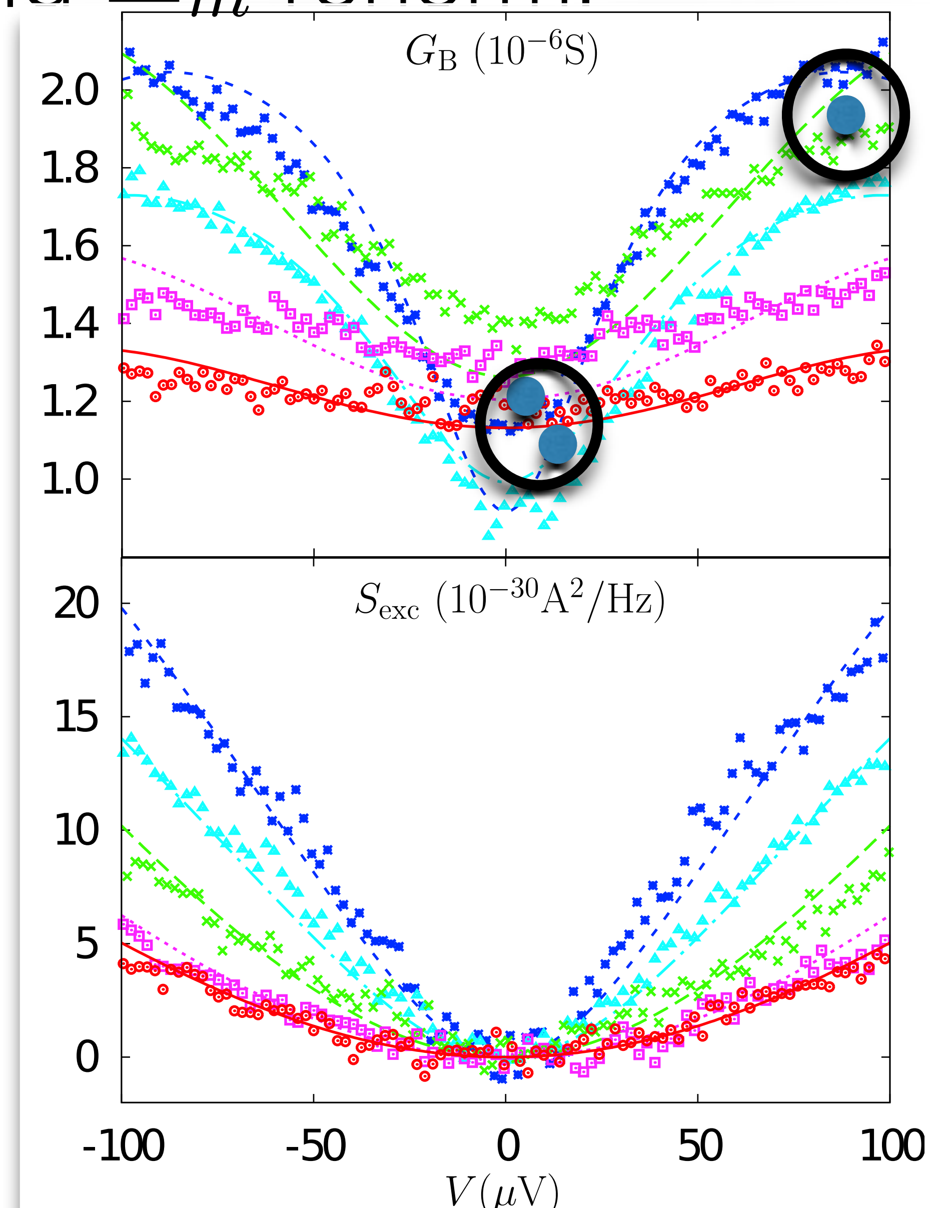

M. Heiblum (2/5,3/7,2/3,5/2,..),Willet(5/2),Yacoby (2/3),…

# Theoretical explanations

- Single-qp and multiple-qp crossover and $\Delta_m$ renorm.




D. Ferraro, A. B, M. Merlo, N. Magnoli, M. Sassetti PRL 08
D. Ferraro, A. B., N. Magnoli, M. Sassetti, NJP10
D. Ferraro, A. B., N. Magnoli, M. Sassetti, PRB10
M. Carrega, D. Ferraro, A. B., N. Magnoli, M. Sassetti PRL11
M. Carrega, D. Ferraro, A. B., N. Magnoli, M. Sassetti, NJP12
A. B., D. Ferraro, M. Carrega, N. Magnoli, M. Sassetti NJP12

Good agreement with many observations,
simple & coherent explanations

Theorist

Exp
?

# Why not at finite frequency ?

- Josephson resonances $\omega_m = me^* V/\hbar$

  Blanter&Buettiker Phys.Rep.00, Rogovin&ScalapinoAnn. Phys 74

- Rich theoretical tools & interesting non-equilibrium phys.

  Chamon..PRB95; Chamon..PRB96;Dolcini..PRB05; Bena..PRB06;Bena..PRB07

- Interesting questions: how to measure it?

  Lesovik..JETP97;Gavish U..PRB00;Gavish U.. arXiv:0211646; Bednorz.. PRL13; Aguado..PRL00;

# Symmetrized or non-symmetrized ?

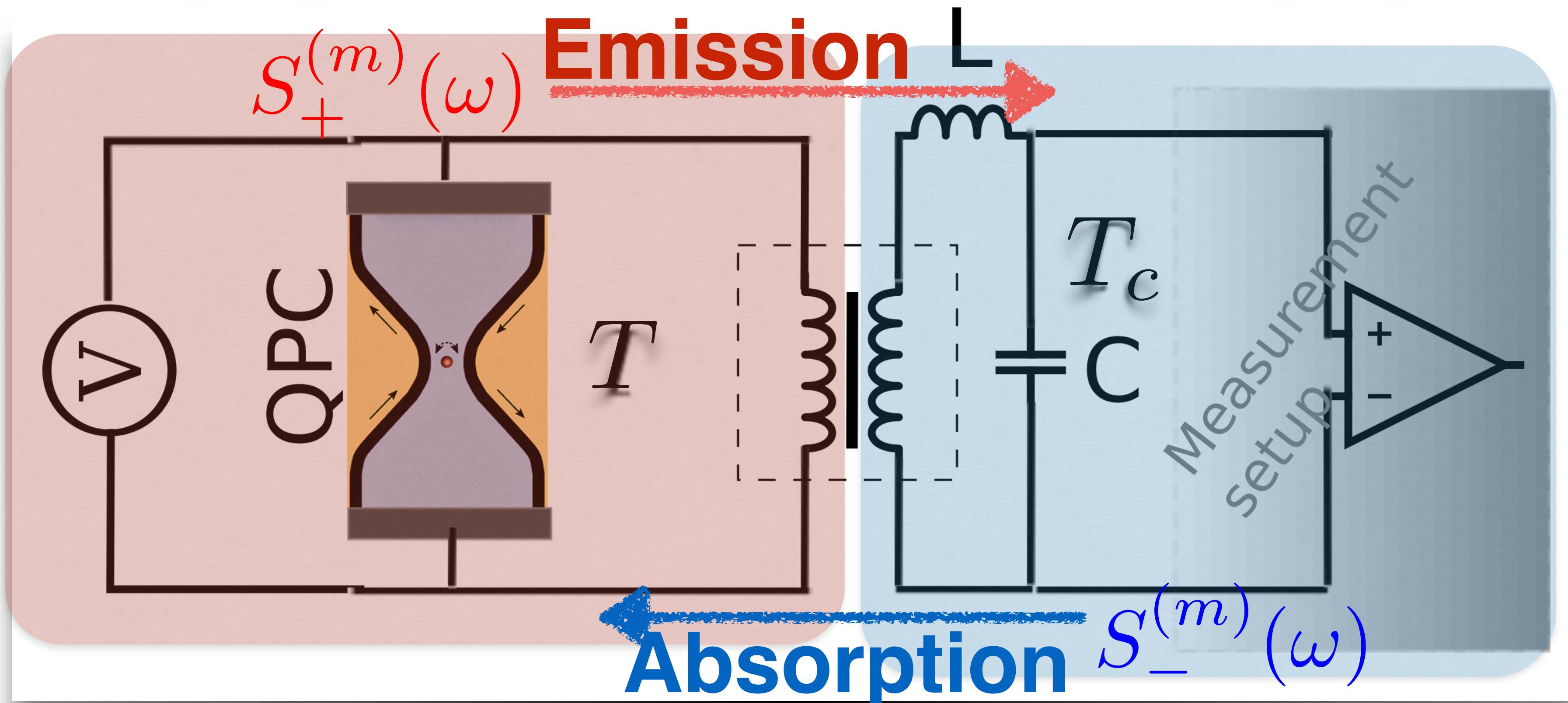
- Symmetrized noise (Landau docet) $[I(t), I(t')] \neq 0$

$$S^{(m)}(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t} \langle \{\delta I_B(t), \delta I_B(0)\}_+ \rangle = \sum_{i=\pm} S_i^{(m)}(\omega)$$

- Non-symmetrized (Emission/absorption from QPC)
  Aguado PRL00

$$S_{+/-}^{(m)}(\omega) = \int_{-\infty}^{+\infty} e^{\pm i\omega t} \langle \delta I_B^{(m)}(t) \delta I_B^{(m)}(0) \rangle$$

# Finite frequency detection




Resonant

$\omega = \sqrt{1/LC}$

Cold detector

$T_c \ll T$

Hot detector

$T_c \gg T$

- Impedance matched resonant detection scheme

Lesovik G B and Loosen R JETP 65 295 (1997); Gavish U,….arXiv:0211646

- Output power proportional to variation of LC energy $\delta\langle x^2 \rangle$

$$S_{meas}^{(m)}(\omega) = K \left\{ S_+^{(m)}(\omega) + n_B(\omega) \left[ S_+^{(m)}(\omega) - S_-^{(m)}(\omega) \right] \right\}$$

$$n_B(\omega) = \frac{1}{e^{\omega/T_C} - 1} \qquad K = \left( \frac{\alpha}{2L} \right)^2 \frac{1}{2\eta} \ll 1$$

$-\omega \, \Re e\left[ G_{ac}^{(m)}(\omega) \right]$

# Noise properties in QPC-LC

- Detector quantum limit (Cold detector) $k_B T_c \ll \omega$

$$S^{(m)}_{meas}(\omega) \approx K S^{(m)}_{+}(\omega) + \mathcal{O}(e^{-\hbar\omega/k_B T_c})$$

- Absorbitive QPC limit (Hot detector) $k_B T_c \gg \omega$

$$S^{(m)}_{meas}(\omega) \approx K \left\{ S^{(m)}_{+}(\omega) - k_B T_c \Re e \left[ G^{(m)}_{ac}(\omega) \right] \right\}$$

- Is it measurable? $\omega_0 = e^* V/\hbar$

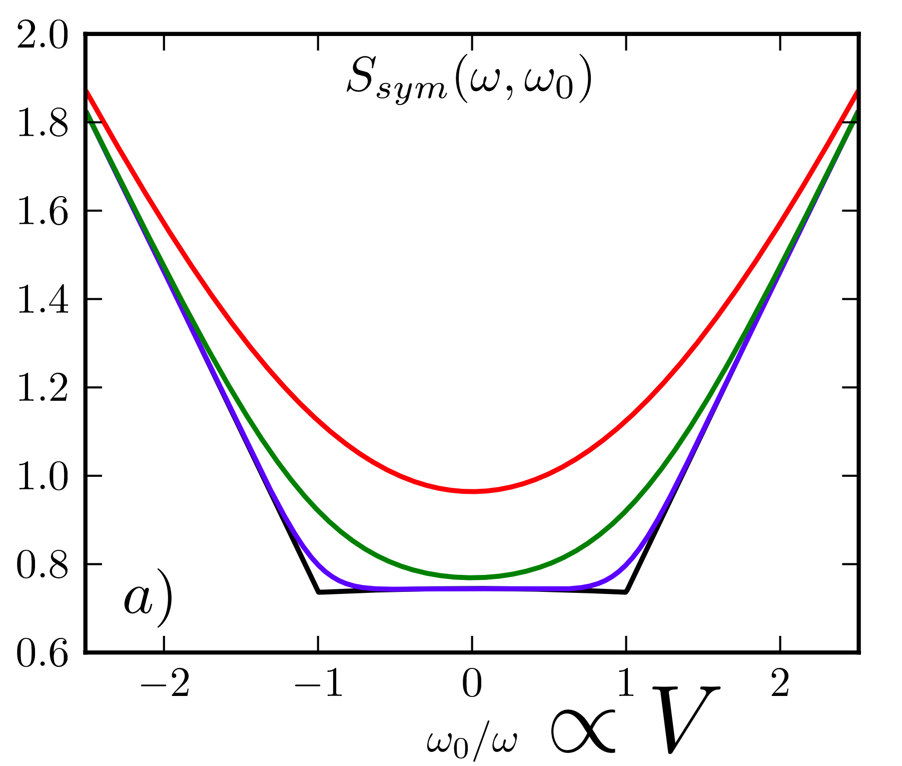
- $S_{meas} \equiv S_{ex} \qquad T = T_c$

- Lowest order in the tunnelling $|t_m|^2$(purely additive)

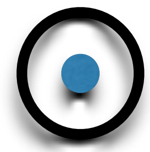
$$S_{sym}(\omega) = \sum_m S^{(m)}_{sym}(\omega) \qquad S_{meas}(\omega) = \sum_m S^{(m)}_{meas}(\omega)$$

- Keldysh formalism blow up in Fermi's rule: rate $\mathbf{\Gamma}^{(m)}(E)$

# Non-interacting result




$\nu = 1$

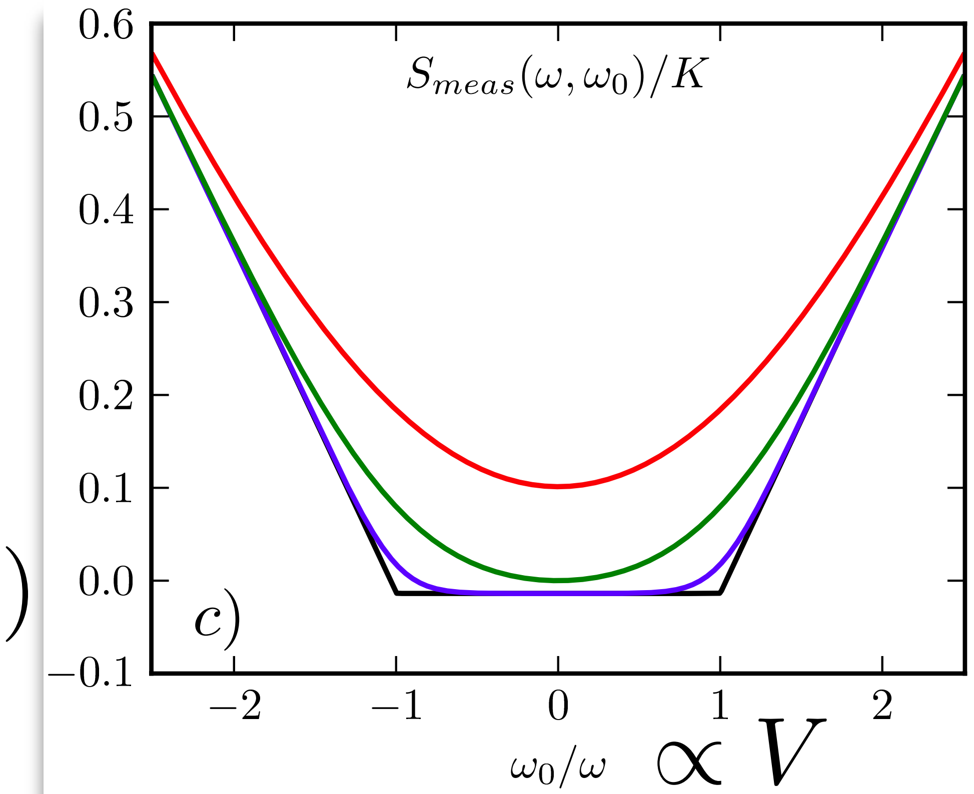


Electron

$T_c = 15\text{mK}$

$\omega = 7.9\text{GHz}(60\text{mK})$

$\omega_c = 660\text{GHz}(5\text{K})$




$T = 0.1, 5, 15, 30[\text{mK}]$

$$S_{sym}(\omega,\omega_0) = 2\frac{\tilde{S}_0}{\omega_c}\left[\theta(\omega_0-\omega)\omega_0 + \theta(\omega-\omega_0)\omega\right] \qquad \tilde{S}_0 = \frac{e^2}{2}\frac{|t_1|^2}{2\pi\alpha^2}\frac{1}{\omega_c}$$

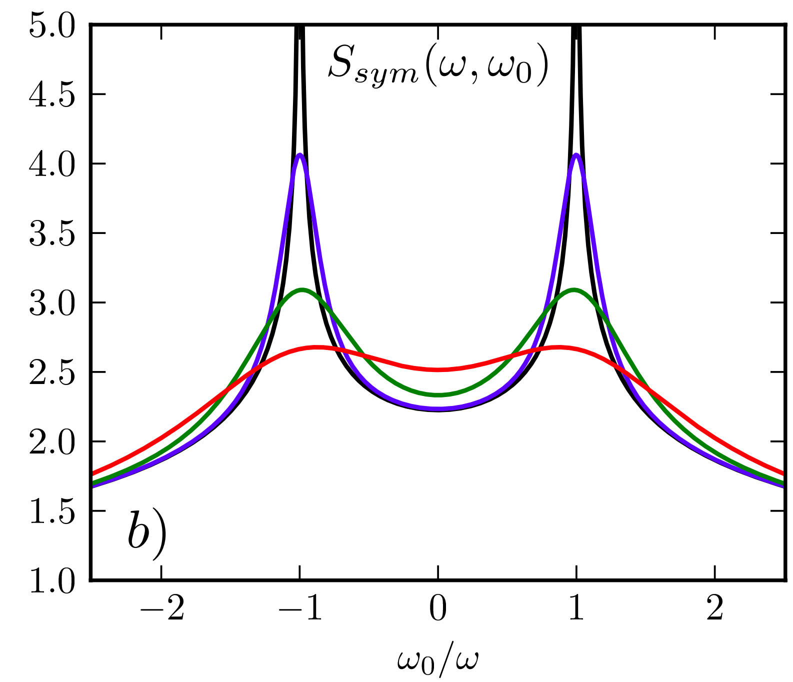
$$S_{meas}(\omega,\omega_0) \approx KS_+(\omega,\omega_0) = \frac{K}{2}\left(S_{sym}(\omega,\omega_0) - 2\tilde{S}_0\frac{\omega}{\omega_c}\right) \qquad k_B T_c \ll \omega$$

$$\mathbf{\Gamma}^{(1)}(E) \propto \theta(E)E$$

Lesovik G B and Loosen R JETP 65 295 (1997)

# Interacting case: Laughlin

$$\nu = 1/3$$




$$e^* = \frac{e}{3}$$

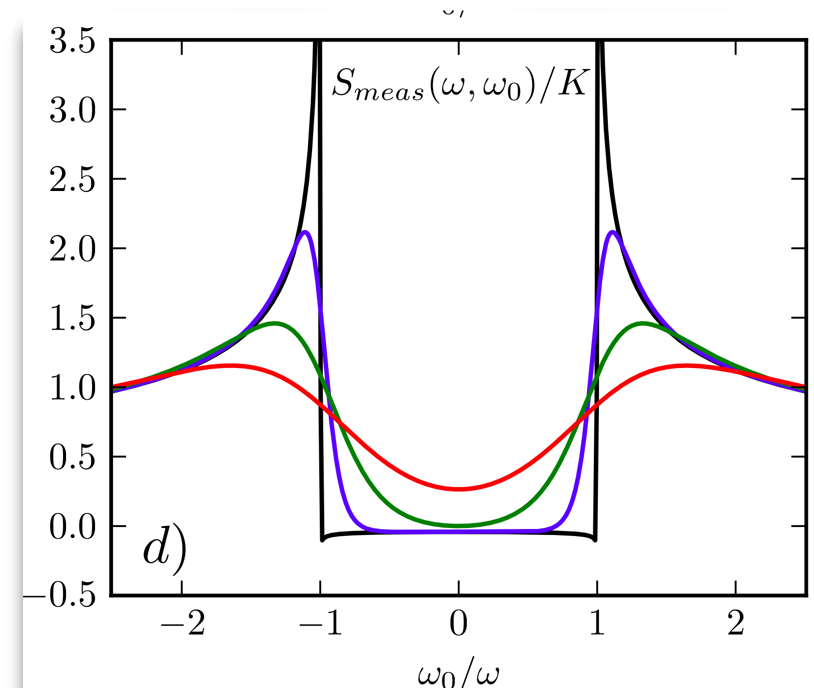
Single-qp

$$T_c = 15\text{mK}$$

$$\omega = 7.9\text{GHz}(60\text{mK})$$

$$\omega_c = 660\text{GHz}(5\text{K})$$

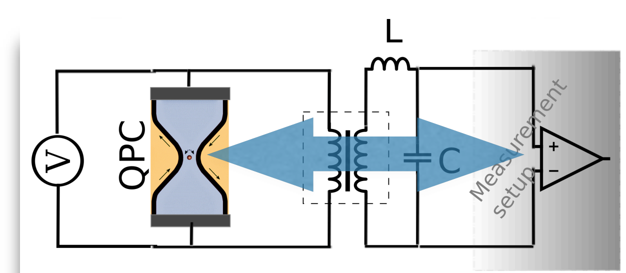
$$T = 0.1, {\color{blue}5}, {\color{green}15}, {\color{red}30}[\text{mK}]$$




$$S_{sym}(\omega,\omega_0) \approx |\omega - \omega_0|^{4\Delta^{(1)}_{1/3}-1}$$

Chamon, Freed & Wen PRB95,PRB96

- Detector quatum limit $k_B T_c \ll \omega$
- QPC Shot noise $k_B T \ll \omega_0$

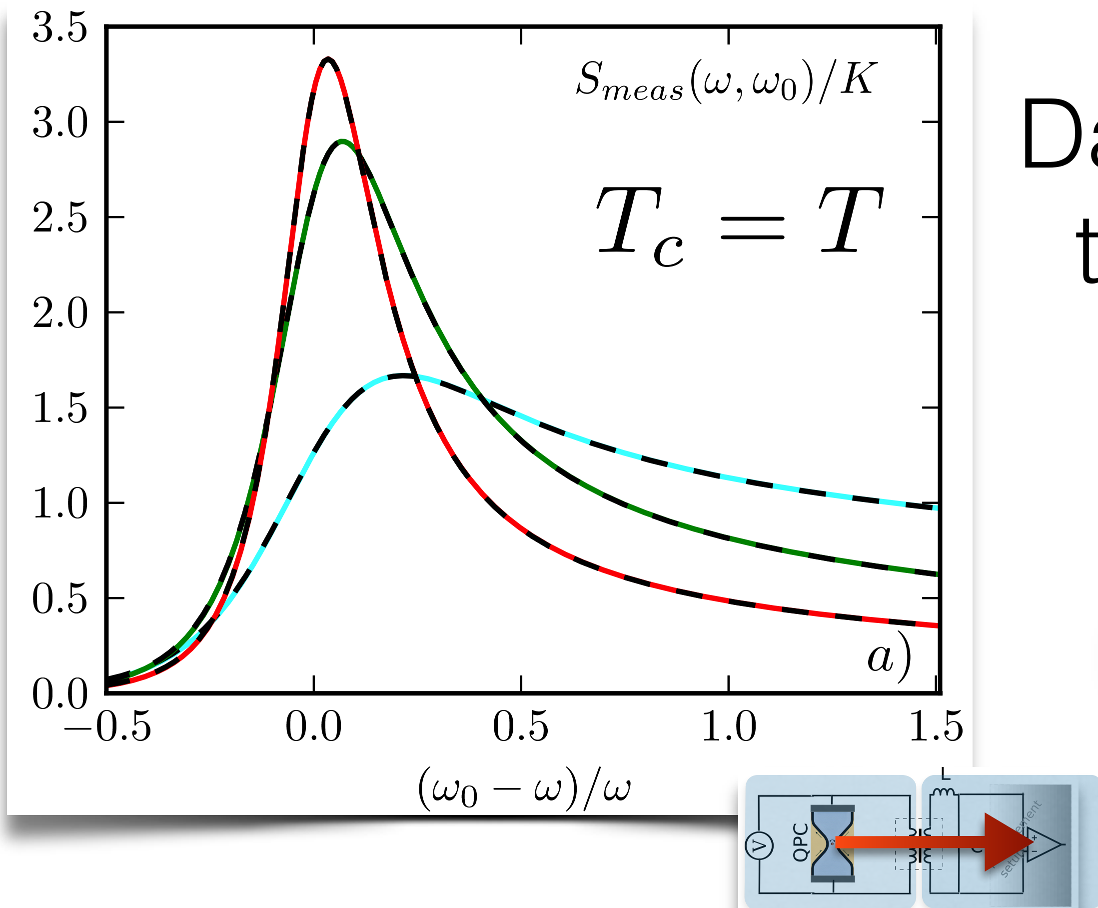



$$S^{(m)}_{meas}(\omega,\omega_0) \approx S^{(m)}_{+}(\omega) \approx K\frac{(me^*)^2}{2}\mathbf{\Gamma}^{(m)}(-\omega + m\omega_0) \qquad \omega \sim \omega_0 \qquad m = 1$$

$S^{(m)}_{meas}(\omega,\omega_0)$ returns directly the rates.......

# Rate detection

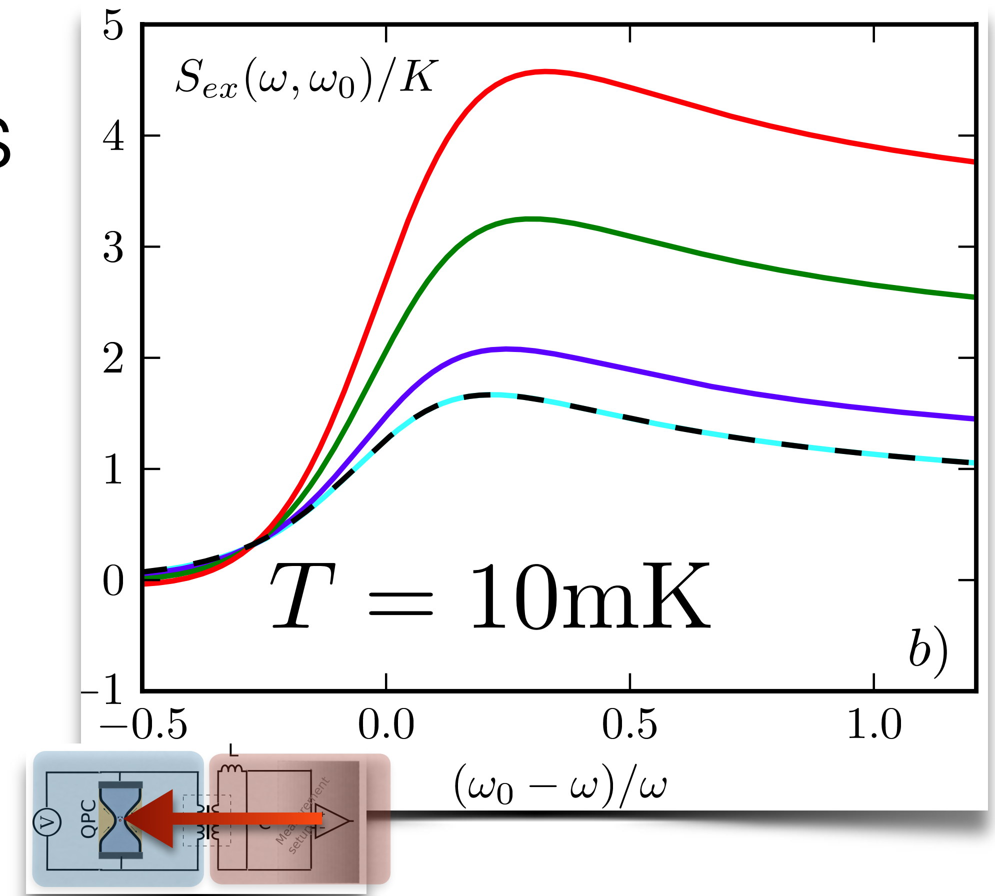
$\nu = 1/3, 1/5, 1/7$




Dashed lines theoretical rates

$$\Delta_{\nu}^{(1)} = \frac{\nu}{2}$$

$T_c = 10, 30, 60, 90$ mK

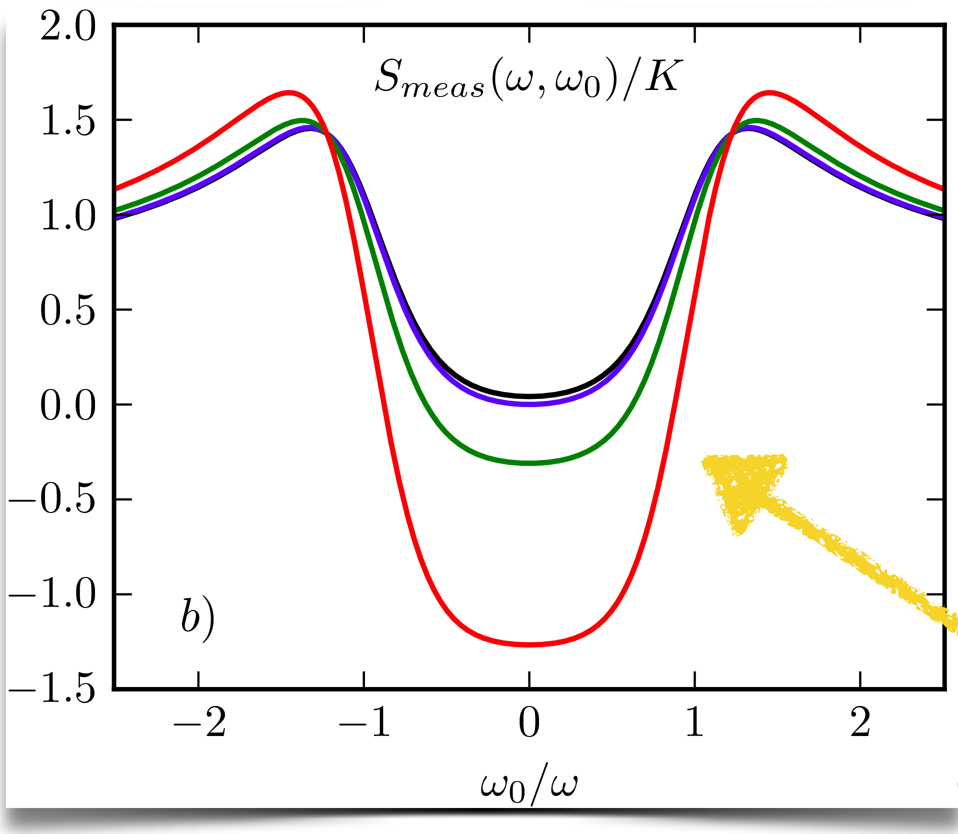



It is possible to extract the scaling dimensions without requiring an extended window in frequency and bias simplifying the experimental requirements

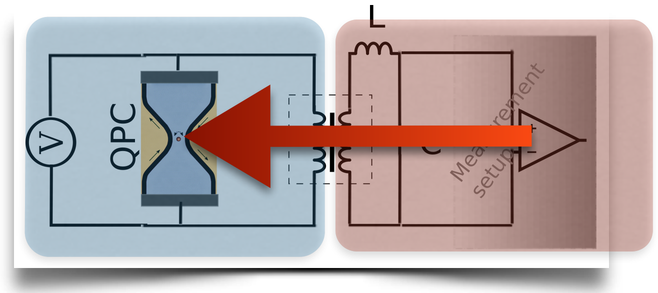
Note that $S_{meas} \equiv S_{ex}$ $T = T_c$

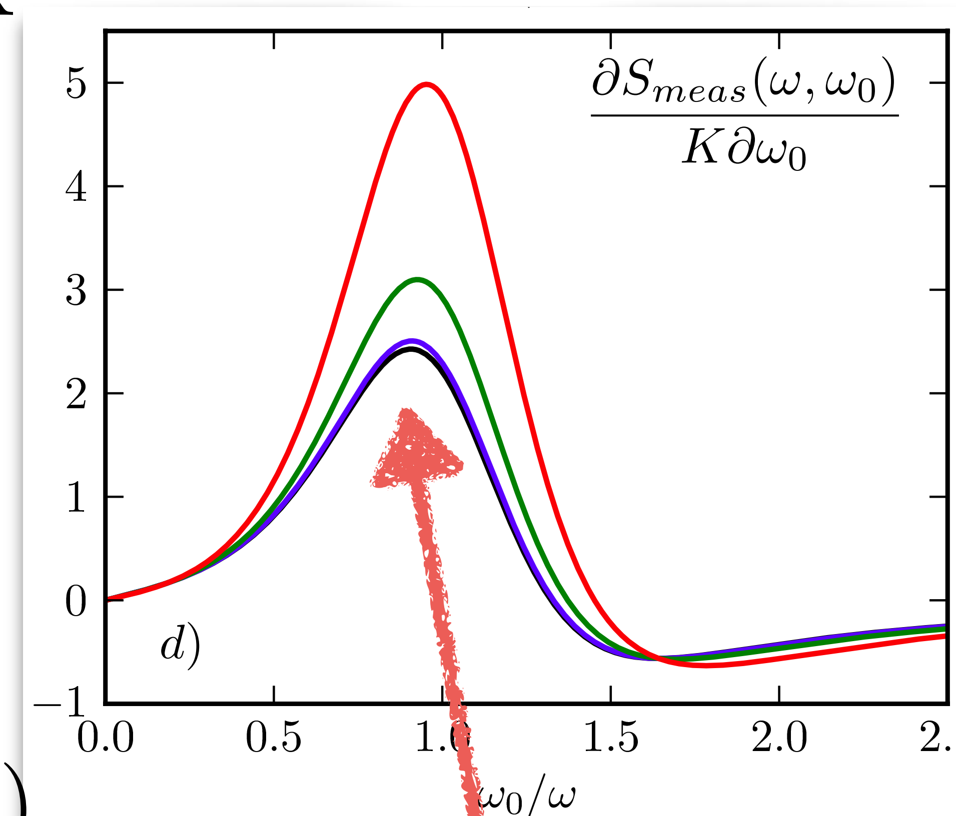
# Hotter is better?

$T_c = 5, 15, 30, 60$ mK




$\nu = 1/3$




$T_c = 15\text{mK}$

$\omega = 7.9\text{GHz}(60\text{mK})$

$\omega_c = 660\text{GHz}(5\text{K})$

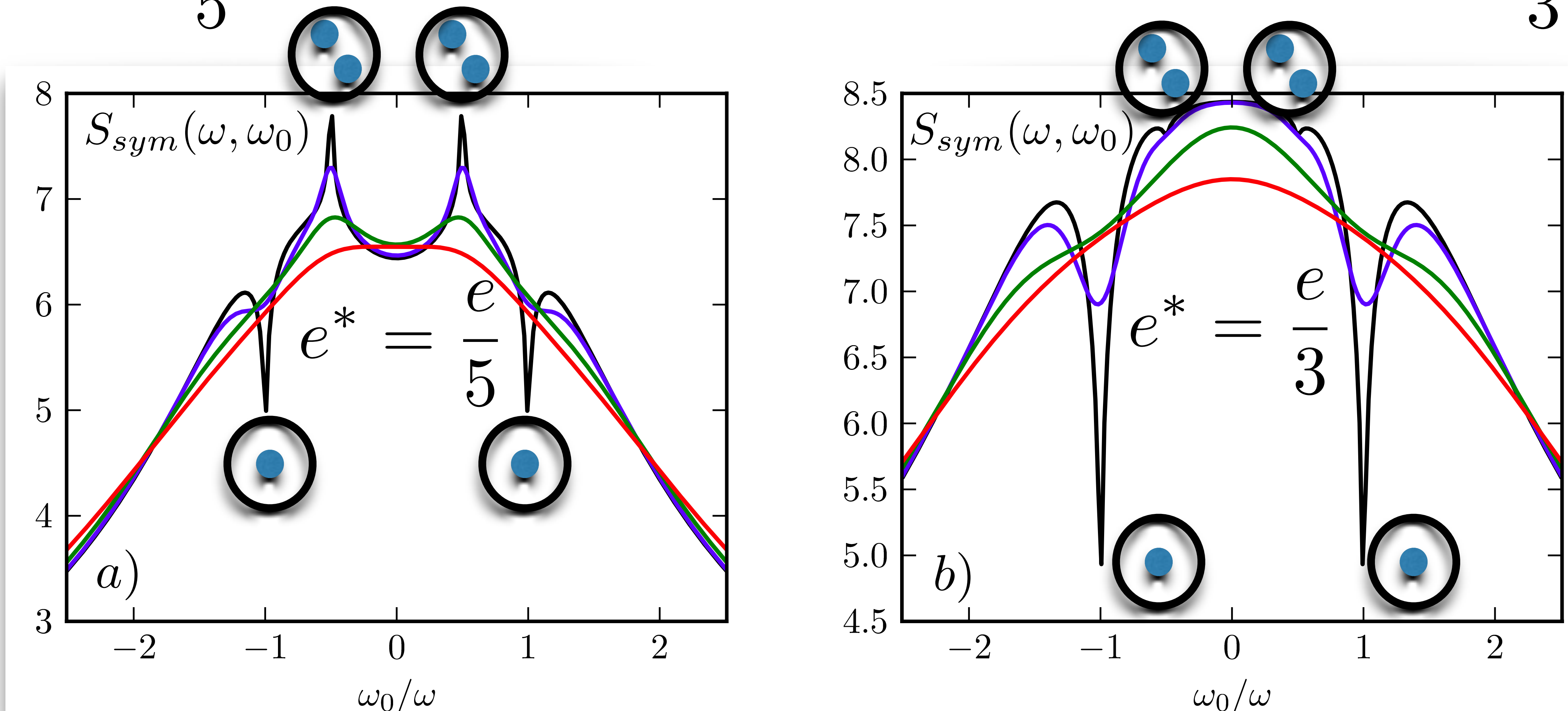
The QPC cannot excite detector modes so it behaves absorbing energy

The QPC can excite detector

The combined effect is an enhancement of jump/peak

# Resolving m-qp scalings? $S_{sym}$

$\nu = \frac{2}{5}$ $\quad T = 0.1, 5, 15, 30 [\mathrm{mK}]$ $\quad \nu = \frac{2}{3}$



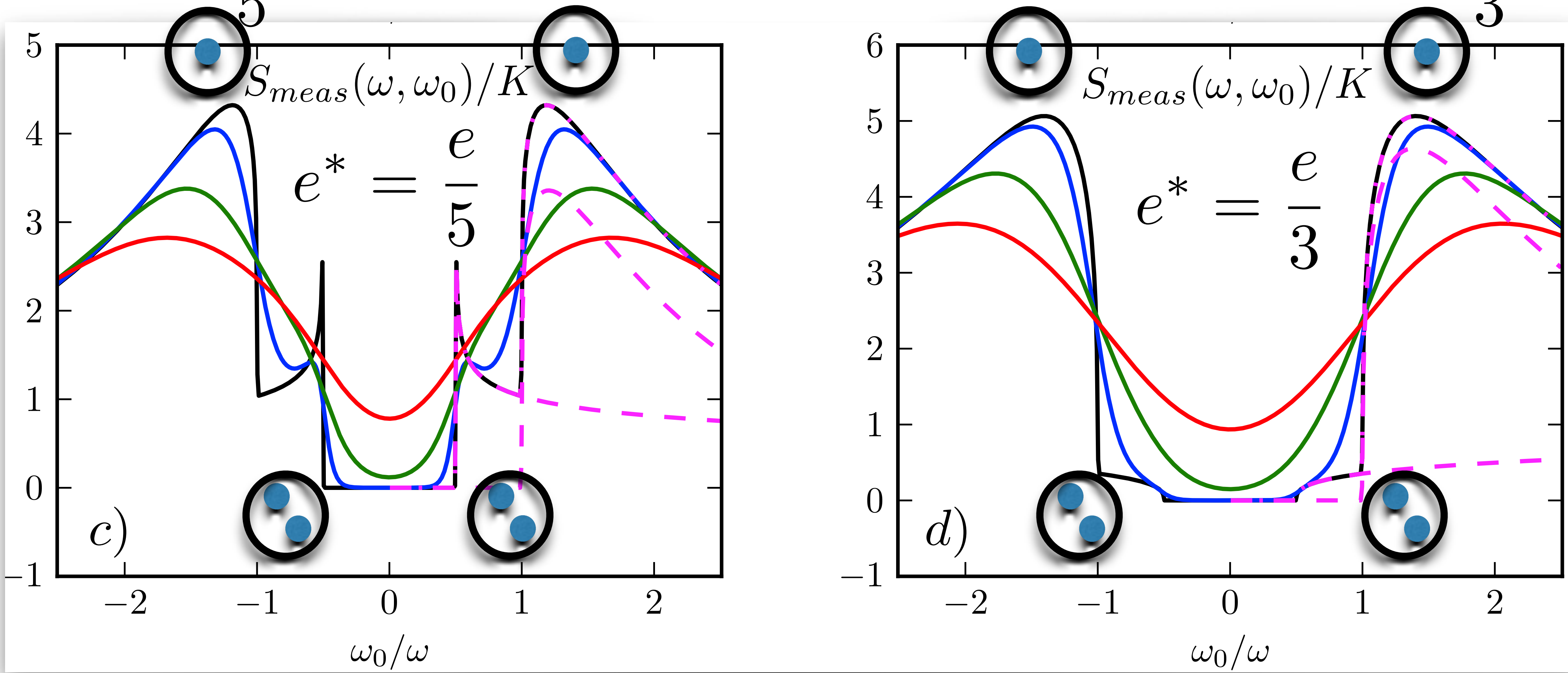

$$S^{(m)}_{sym}(\omega, \omega_0) \approx |\omega - \omega_0|^{4\Delta^{(m)}_\nu - 1}$$

Chamon, Freed & Wen PRB95,PRB96

- $\omega \approx m\omega_0$ Josephson resonances
- Peaks ($\Delta^{(m)}_\nu < 1/4$) or dips ($\Delta^{(m)}_\nu > 1/4$)
- Thermal effect spoil the signatures

# Multiple-qp spectroscopy: $S_{meas}$

$\nu = \frac{2}{5}$ $T = 0.1, 5, 15, 30 [\mathrm{mK}]$ $\nu = \frac{2}{3}$




Note that $S_{meas} \equiv S_{ex}$ $T = T_c$

$$S_{meas}(\omega, \omega_0) \approx \alpha_1 \mathbf{\Gamma}^{(1)}(\omega_0 - \omega) + \alpha_2 \mathbf{\Gamma}^{(2)}(2\omega_0 - \omega)$$

- Rates are directly fitted: scaling dimensions at finite T
- Multiple-qps are observed in different window

# Conclusion

- QPC+LC resonator is a powerful tool
- f.f. noise resolve the presence of multiple qps
- Multiple-qp spectroscopy can be done at realistic T
- Information on qps by analysing bias behaviour
- Changing detector temperature increases the sensibility
- Validate composite edge model theories
- This techniques can be used in other systems

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