On periodically driven AdS/CFT

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Based on arXiv: 1308.2132 with S. B. Gudnason, S. Elitzur and E. Rabinovici

Introduction

Advances in experimental studies of cold atom systems motivates the study of thermally isolated systems which are driven or quenched by an external force

This can be modeled with a time-dependent Hamiltonian

$$H = H_0 + \lambda H(t)$$

It remains challenging to study such problems in the regime of strong coupling.

AdS/CFT provides a framework where strongly coupled field theory can be studied using a weakly coupled gravity dual.

On the other side, periodically driven systems may teach us about the bulk gravity dual

Setting

I'll consider the case where the boundary theory is perturbed in a way which is periodic in time:

$$H = H_0 + \lambda\,\delta H \cos \omega t$$

and I'll start with a thermal state

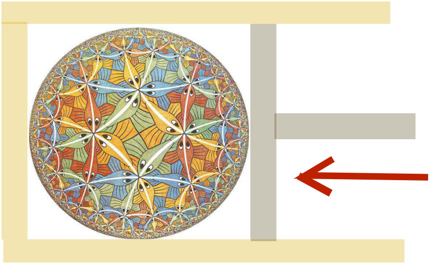
The unperturbed Hamiltonian belongs to a CFT, and the perturbation

$$\delta H = \int d^{d-1}x \mathcal{O}_\Delta$$

where $\mathcal{O}_\Delta$ is a relevant scalar operator with dimension Δ

Setting

I'll consider $d = 4$ CFT with relevant deformation with $1 < \Delta < 4$ (above unitarity bound).



Gravity dual: AdS_5 with non-trivial boundary condition for the scalar

The bulk dual

The dual is gravitational theory with a negative cosmological constant:

$$S = \frac{1}{16\pi G_N} \int d^5x \sqrt{-g} \left(R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 \right) .$$

(mass units defined from AdS curvature set to 1).

$1/G_N$ is proportional to the $(\text{Weyl})^2$ term in the trace anomaly. The operator with dimension Δ is dual to a scalar field with mass:

$$m^2 = \Delta(\Delta - d)$$

In the case of the $\mathcal{N} = 4$ in theory in $d = 4$ one can consider the case of $\Delta = 2$ (mass for scalar fields) or $\Delta = 3$ (mass for fermions), and $G_N = \pi/(2N_c^2)$.

The driving force

The unperturbed metric is the AdS black brane:

$$ds^2 = \frac{-(1-\rho^4)d\tau^2 - 2d\rho\, d\tau + d\vec{X}^2}{\rho^2}\,.$$

The scalar field nearby the boundary can be expanded as:

$$\tilde{\phi}_1 = \sum_{j=0}^{\infty} \rho^{\Delta_- + j} a_{\Delta_- + j} + \rho^{\Delta_+ + j} a_{\Delta_+ + j}\,.$$

Taking $\mathcal{O}_{\Delta_+}$ as a source $(\omega_T = \omega/(\pi T))$:

$$a_{\Delta_-} = \cos \omega_T \tau\,, \qquad a_{\Delta_+} = Re(\chi(\omega_T) e^{-i\omega_T \tau})\,.$$

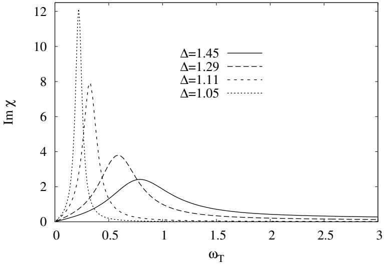
For $m^2 < 2$, one can take also $\mathcal{O}_{\Delta_-}$ as a source:

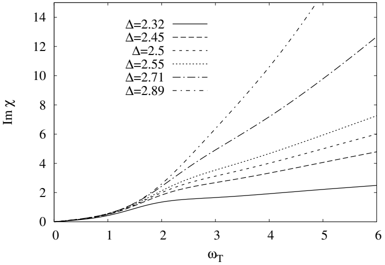
$$a_{\Delta_+} = \cos \omega_T \tau\,, \qquad a_{\Delta_-} = Re(\chi(\omega_T) e^{-i\omega_T \tau})\,.$$

Ingoing boundary condition at horizon: $\chi(\omega_T)$ is the retarded one-point function.

Work done on the system

The imaginary part of $\chi(\omega_T)$ gives the amount of work done on the system in a cycle.



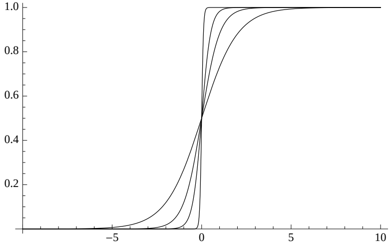


At large ω the work done per cycle scales as $\omega^{2\Delta-d}$.
For $\Delta < 2$ the work per cycle has a peak at ω_0. $\omega_0 \propto (\Delta-1)^{1/2}$ for Δ nearby the unitarity bound

Work in the case of a quench

A quench corresponds to an abrupt change in the coupling from λ_0 to λ_f

$$\lambda(t) = \lambda_0 \frac{1 + \tanh t/t_0}{2}$$



Work in the case of a quench

The time scale t_0 acts as UV cutoff for the Fourier modes of the quench:

$$\mathcal{F}(\lambda(t)) = \frac{i\sqrt{\pi}}{2\sqrt{2}} \frac{t_0}{\sinh(\pi t_0 \omega/2)}$$

For $\Delta > 2$ the energy diverges as:

$$\int^{1/t_0} \frac{\mathrm{Im}\, G_R(\omega)}{\omega} d\omega \propto \int^{1/t_0} \frac{\omega^(2\Delta - 4)}{\omega} d\omega \propto \frac{1}{t_0^{2\Delta-4}}$$

Buchel, Myers, Van Niekerk, Lehner, arXiv:1307.4740, 1302.2924, 1206.6785

Horizons and entropy

Gauge theory entropy can be reconstructed from the Bekenstein-Hawking entropy of a horizon

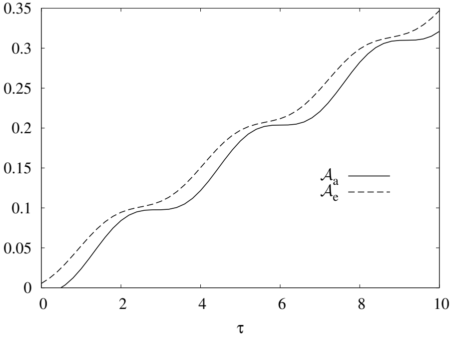
Area of horizon (projected on the boundary along in-falling null geodesic) should be identified with entropy density in the dual field theory

This is unambiguous and well-established in the time-independent case; in the out-of-equilibrium case the situation is more sutble

Event or apparent horizons give different notions of entropy

Horizons and entropy

Metric backreaction was computed at leading order; the resulting growth is:

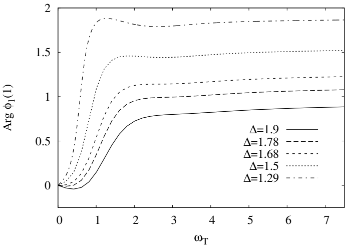


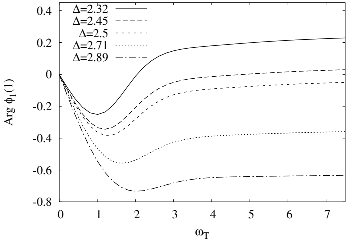
In this case, apparent and event horizon are rather similar; the event horizon area is larger than the apparent horizon one

Horizons and entropy

Equilibration is rather efficient: the entropy growth in a cycle is the same as the one given by the equation of state of the undeformed CFT, for the corresponding increase in internal energy.

Difference of phase between source and entropy:



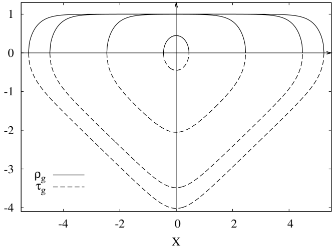


Source and entropy production are in phase just in the adiabatic limit $\omega_T \to 0$

Geodesics and two-point functions

Space-like geodesics are related to two point equal-time (Wightman) functions of operators with large dimensions $\Delta_p \gg 1$:

$$\langle \mathcal{O}_{\Delta_p}(t,\vec{x})\mathcal{O}_{\Delta_p}(t,\vec{x}')\rangle_{\mathrm{ren}} \propto e^{-\Delta_p \mathcal{L}_{0,R}} ,$$



They probe a time in the past which is proportional to the length of the distance $L = |\vec{x} - \vec{x}'|$

Geodesics and two-point functions

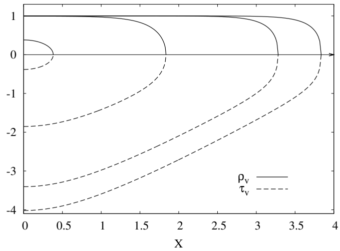
The delay in the thermalization of correlation functions at the leading order in the perturbation is linear in the distance L (in the regime $L \gg T^{-1}$) and it is independent of the operator dimension Δ of the perturbation:

$$\tau_{d,g} \approx -L/(2\sqrt{2})\,.$$

Thermalization time becomes longer with the scale L in some universal way.

Entanglement entropy and extremal surfaces

Entanglement entropy of a region of space $\mathcal{B}$ can be computed considering the extremal surface which ends on the boundary $\partial\mathcal{B}$ (Ryu-Takayanagi)



The delay in thermalization of entanglement entropy of a spherical region is again linear in the size of the region L, in the regime $L \gg T^{-1}$:

$$\tau_{d,e} \approx -\sqrt{\frac{3}{2}}\frac{L}{4}\,.$$

Energy fluctuations in thermally isolated driven systems

In a thermal bath, Gibbs measure give energy fluctuations:

$$\sigma^2_{E,eq} = T^2 C_v \, .$$

Assume that one starts with a thermal bath, and then one performs non-adiabatic work from the outside.

The work done per cycle $A = \langle W \rangle$ its variance $A = \langle W^2 \rangle - \langle W \rangle^2$ are related by fluctuation-dissipation relations:

$$\beta B = 2A \, .$$

arXiv:1102.1735, Bunin, D'Alessio, Kafri and Polkovnikov

Two regimes in energy fluctuations

Depending on the details of the cyclic process, the resulting energy distribution is different from the Gibbs ensemble.

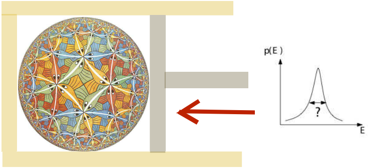
Entropy: $S(E) = E^\gamma$

Work per cycle as function of the energy: $W = E^s$

Depending on $\eta = 2(s-1) + \gamma$:

- For $\eta < 0$, the variance $\sigma^2/\sigma^2_{E,eq}$ of the energy approaches to a constant after a long number of cycles
- For $\eta > 0$, the variance $\sigma^2/\sigma^2_{E,eq} \propto E^\eta$

Driven CFT and energy fluctuations



Entropy: $S(E) = E^{\gamma}$ with $\gamma = (d-1)/d$

Work per cycle as function of the energy: $W = E^s$, with $s = \frac{2\Delta - d - 1}{d}$, in the almost adiabatic limit $\omega \ll T$

$$\eta = \frac{4\Delta - 3d - 3}{d}$$

There is a transition in the energy fluctuations for $\Delta > 3(d+1)/4$

$\Delta > 15/4 = 3.75$ in four dimensions; $\Delta > 3$ in three dimensions

Energy fluctuations and the bulk

All the observables that we computed (entropy density, two-point functions and entanglement entropy) behaved smoothly nearby $\Delta = 15/4$

Question: How to detect the transition in energy fluctuations which should happen in correspondence of such value of Δ ?

Conjecture: It should be related to Hawking radiation leaking out of the boundary due to coupling to the external source

Conclusion

We computed the work performed by a periodic relevant perturbation of dimension Δ on a four dimensional theory with AdS_5 dual, as a function $\omega_T = \omega/(\pi T)$

We inspected the leading order backreaction on the metric and a few observables: entropy density, two-point functions and entanglement entropy. All this observables behave in a smooth way as a function of the dimension of the periodic perturbation Δ

General results in statistical mechanics suggest a transition in the behaviour of energy fluctuations for $\Delta > 3.75$; it would be interesting to detect this in the bulk dual