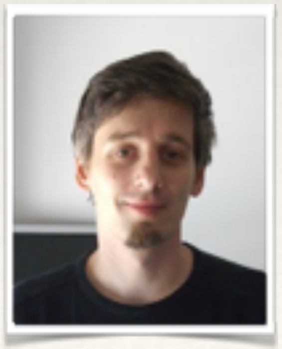
Mario Collura presents…

Interaction Quench in one-dimensional Bose Gas

Relaxation to the GGE and steady-state entanglement properties

…based on Phys. Rev. A 89, 013609 (2014) and J. Stat. Mech. P01009 (2014)

Joint work with



and

Márton Kormos   Pasquale Calabrese

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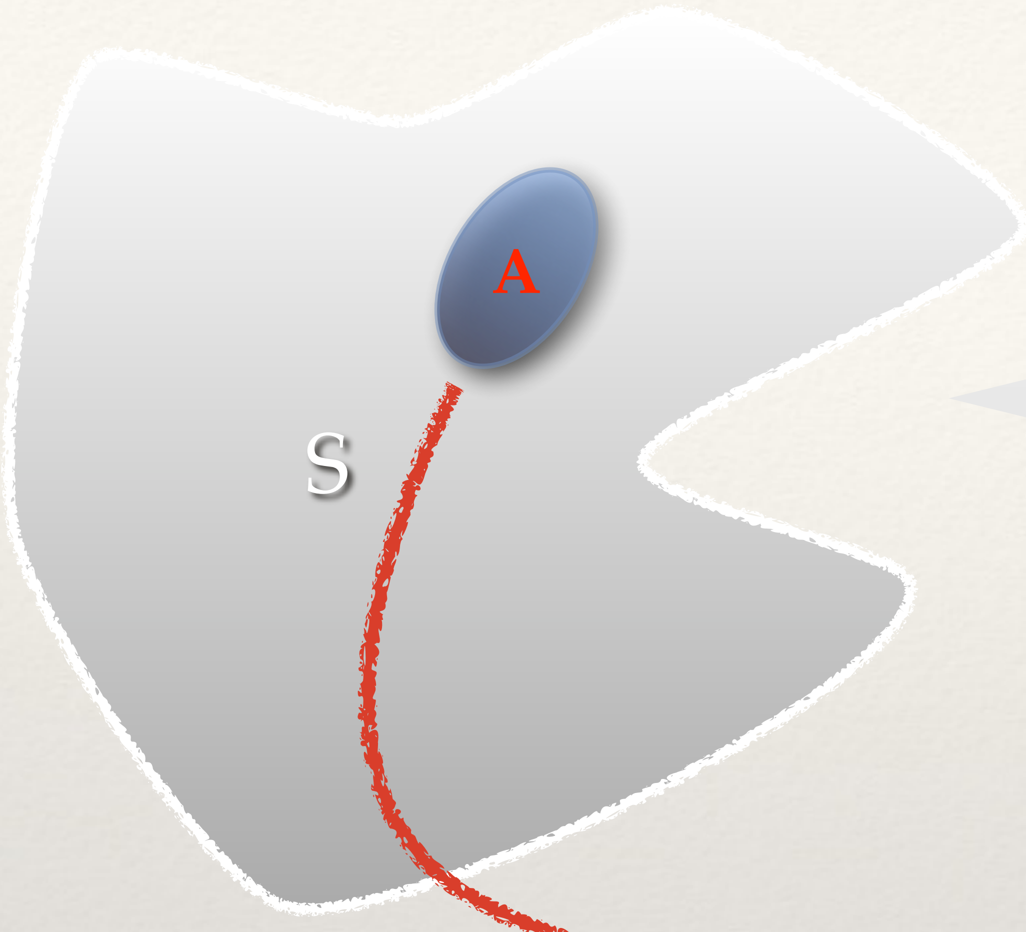
The Equilibration principle



Initial State: $H_0|\Psi_0\rangle = E_0|\Psi_0\rangle$

Quantum Quench: $H_0 \longrightarrow H$

Evolved State: $|\Psi(t)\rangle = \mathrm{e}^{-iHt}|\Psi_0\rangle$

The whole system is always in a pure state!
Perhaps we should look at a subsystem!

$$\hat{\rho}_A(t) = \mathrm{Tr}_{\bar{A}}\hat{\rho}(t)$$

Let's suppose… $\lim_{t\to\infty} \hat{\rho}_A(t) = \hat{\rho}_A^s,\ \exists\, \hat{\rho}^s \in \mathcal{H}_S \,:\, \hat{\rho}_A^s = \mathrm{Tr}_{\bar{A}}\hat{\rho}^s,\ \forall A \subset S$

$$\forall\, \hat{O}_A \in \mathcal{H}_A,\ \lim_{t\to\infty} \mathrm{Tr}\{\hat{O}_A\hat{\rho}(t)\} = \mathrm{Tr}\{\hat{O}_A\hat{\rho}^s\}$$

The GGE conjecture

Many experiments have shown a substantial difference between **integrable** and **not-integrable** systems:

NOT-INTEGRABLE: Equilibrate to a thermal state.

INTEGRABLE: Does not thermalize!

It was proposed that the equilibrium state is described by a ***Generalized Gibbs Ensemble****, i.e.* ***one needs to take into account all conserved quantities*** [M. Rigol et all. '07].

Let the GGE be…

- Maximize the Entropy (similarly to the Gibbs ensemble construction)
- Taking into account a **maximal set** of **conserved charges in involution**: $[\hat{I}_n,\ \hat{I}_m] = 0$
- A **charge** should be **local**, i.e. it must be written as an **integral of a local density**:

$$\hat{J} = \int dx\, \hat{\mathcal{J}}(x)\, [\text{local}],\ \ \hat{K} = \iint dxdy\, \hat{\mathcal{K}}(x,y)\, [\text{non local}]$$

$$\hat{\rho}_{GGE} = Z_{GGE}^{-1} \exp\left\{ -\sum_n \lambda_n \hat{I}_n \right\}$$

The Lagrange multipliers are fixed by the initial condition

$$\mathrm{Tr}\{\hat{I}_n \hat{\rho}_{GGE}\} = \langle \hat{I}_n \rangle_0$$

Inspecting the GGE

1) Quenches in a quadratic theory

- The linear mapping between pre- and post-quench field operators makes the work easy and often possible to analytically solve the time-evolution of local observables.

[P. Calabrese, J. Cardy '06; M. A. Cazalilla '06; M. Cramer et all. '08; P. Calabrese, F. Essler, M. Fagotti '11; F. Essler, S. Evangelisti, M. Fagotti '12;MC, S. Sotiriadis, and P. Calabrese, '13]

2) Quenches in integrable interacting systems

- The stationary properties can be deduced using more involved techniques (usually starting from a specific initial state):

  Quench Action Method (or GTBA) *[J.-S. Caux, F. Essler '13; J. De Nardis et all. '14]*

  Quantum Transfer Matrix Approach *[M. Fagotti, F. Essler '13; B. Pozsgay '13]*
- The time-evolution of local observable is accessible via numerical techniques (t-DMRG, t-iTEBD, etc.) *[M. Fagotti, MC, F. Essler, P. Calabrese '14]*
- GTBA vs GGE *[B. Wouters et all. '14; B. Pozsgay et all. '14; G. Goldstein, N. Andrei '14]*

Is there anything in between (1) and (2) !?!?!?

Statement of the problem

Interaction quench in the Lieb-Liniger (LL) model *[V. Gritsev, T. Rostunov, E. Demler '10]*

LL Hamiltonian:

$$H = \int_0^L dx \left[\partial_x \hat{\phi}^\dagger(x) \partial_x \hat{\phi}(x) + c\, \hat{\phi}^\dagger(x) \hat{\phi}^\dagger(x) \hat{\phi}(x) \hat{\phi}(x)\right]$$

QUENCH

c = 0

FREE BOSONS

$$\hat{\phi}(x) = \frac{1}{\sqrt{L}} \sum_q \mathrm{e}^{iqx} \hat{\xi}_q$$

$$|\psi_0(N)\rangle = \frac{1}{\sqrt{N!}} \hat{\xi}_0^N |0\rangle$$

$$N, L \to \infty,\ n = N/L$$

c = ∞

HARD-CORE BOSONS

$$H = \int dx\, \partial_x \hat{\Phi}^\dagger(x) \partial_x \hat{\Phi}(x)$$

$$x \neq y \quad [\hat{\Phi}(x), \hat{\Phi}(y)] = 0, \quad [\hat{\Phi}(x), \hat{\Phi}^\dagger(y)] = 0$$

$$[\hat{\Phi}(x)]^2 = [\hat{\Phi}^\dagger(x)]^2 = 0$$

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FREE FERMIONS

$$H = \int dx\, \partial_x \hat{\Psi}^\dagger(x)\, \partial_x \hat{\Psi}(x)$$

$$\{\hat{\Psi}(x), \hat{\Psi}(y)\} = 0$$

$$\{\hat{\Psi}(x), \hat{\Psi}^\dagger(y)\} = \delta(x-y)$$

Easily diagonalized!

$$\hat{\phi}(x) \longrightarrow \hat{\Phi}(x) = \hat{P}_x\, \hat{\phi}(x)\, \hat{P}_x \longleftrightarrow \hat{\Psi}(x) = \exp\left\{ i\pi \int_0^x dz\, \hat{\Phi}^\dagger(z) \hat{\Phi}(z) \right\} \hat{\Phi}(x)$$

n.b.: $\hat{P}_x \equiv |0\rangle\langle 0|_x + |1\rangle\langle 1|_x$

Correlation functions

(1) The **Fermionic Two-Point Function**:

$$\langle \hat{\Psi}^\dagger(x)\hat{\Psi}(y)\rangle = \sum_{j=0}^{\infty} \frac{(-2)^j}{j!}\int_x^y dz_1 \cdots \int_x^y dz_j \, \langle \hat{\Phi}^\dagger(x)\hat{\Phi}^\dagger(z_1)\cdots\hat{\Phi}^\dagger(z_j)\hat{\Phi}(z_j)\cdots\hat{\Phi}(z_1)\hat{\Phi}(y)\rangle$$

We can treat the hard-core boson fields as they were canonical bosonic fields…

$$\langle \hat{\phi}^\dagger(x)\hat{\phi}^\dagger(z_1)\cdots\hat{\phi}^\dagger(z_j)\hat{\phi}(z_j)\cdots\hat{\phi}(z_1)\hat{\phi}(y)\rangle = \frac{1}{L^{j+1}}\frac{N!}{(N-j-1)!}$$

TRANSLATIONAL INVARIANCE IMPLIES TIME INDEPENDENCE

$$\langle \hat{\Psi}^\dagger(x)\hat{\Psi}(y)\rangle = n\mathrm{e}^{-2n|x-y|}, \quad n(k) \equiv \langle \hat{n}(k)\rangle = \frac{4n^2}{k^2+4n^2}$$

THE GGE IS DIAGONAL IN TERM OF $n(k)$

(2) The **Dynamical Density-Density Correlation Function**:

$$\langle \hat{\rho}(x_1,t_1)\hat{\rho}(x_2,t_2)\rangle = \frac{1}{L^2}\sum_{k_1,k_2,k_3,k_4} \mathrm{e}^{-i(k_1-k_2)x_1-i(k_3-k_4)x_2}\mathrm{e}^{i(k_1^2-k_2^2)t_1}\mathrm{e}^{i(k_3^2-k_4^2)t_2}\langle \hat{\eta}^\dagger_{k_1}\hat{\eta}_{k_2}\hat{\eta}^\dagger_{k_3}\hat{\eta}_{k_4}\rangle$$

$$\langle \hat{\eta}^\dagger_{k_1}\hat{\eta}_{k_2}\hat{\eta}^\dagger_{k_3}\hat{\eta}_{k_4}\rangle = \frac{1}{L^2}\int_0^L dz_1dz_2dz_3dz_4\mathrm{e}^{i(k_1z_1-k_2z_2+k_3z_3-k_4z_4)}\langle \hat{\Psi}^\dagger(z_1)\hat{\Psi}(z_2)\hat{\Psi}^\dagger(z_3)\hat{\Psi}(z_4)\rangle$$

Dynamical density-density correlation function

$$\langle \hat{\rho}(x_1,t_1)\hat{\rho}(x_2,t_2)\rangle = n^2 + F_0(\Delta x,\Delta t)F_1(\Delta x,\Delta t) - |F_1(\Delta x,\Delta t)|^2 + |F_2(\Delta x, t_1+t_2)|^2$$

$$F_0(x,t) = \int \frac{dk}{2\pi} \mathrm{e}^{-ikx+ik^2t} = \frac{1+\mathrm{sgn}(t)i}{2\sqrt{2\pi|t|}}\mathrm{e}^{-i\frac{x^2}{4t}}$$

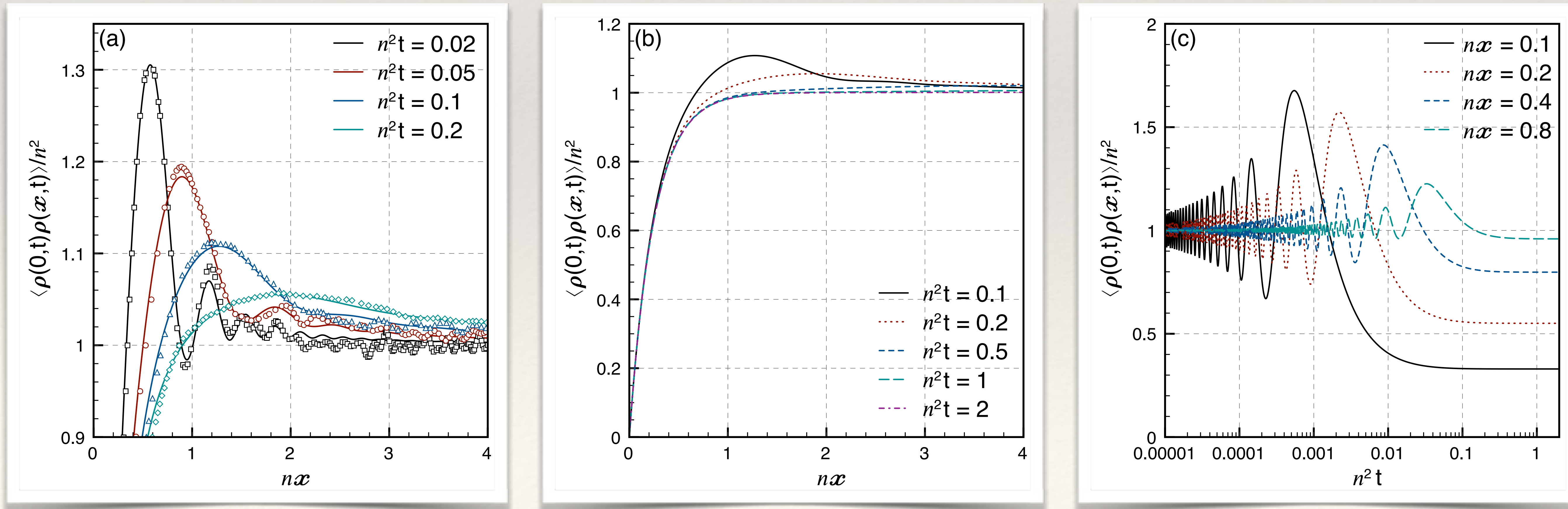
$$F_1(x,t) = \int \frac{dk}{2\pi} \mathrm{e}^{ikx-ik^2t} n(k)$$

$$F_2(x,t) = \frac{1}{2n}\int \frac{dk}{2\pi} \mathrm{e}^{ikx+ik^2t} kn(k)$$

For $t_1, t_2 \to \infty$, $F_2 \to 0$

The density-density correlation function is described by the GGE.

*n.b.: **symbols** are from V. Gritsev, T. Rostunov, E. Demler, J. Stat. Mech. (2010) P05012.*



Stationary Entanglement Entropies

The **GGE** density matrix is diagonal in the momentum modes, **Wick's theorem is restored** and all multi-point correlators can be determined in terms of the **fermionic two-point function**

$$C(x,y) = n\mathrm{e}^{-2n|x-y|}$$

Rényi Entropies & Reduced Correlation Matrix

$$S_A^{(\alpha)} = \frac{1}{1-\alpha}\ln \mathrm{Tr}\hat{\rho}_{\mathrm{A}}^{\alpha} = \frac{1}{1-\alpha}\mathrm{Tr}\ln[\mathbb{C}_A^{\alpha} + (1-\mathbb{C}_A)^{\alpha}]$$

$$\mathbb{C}_A^k(x,y) \equiv \int_A dz_1 \dots dz_{k-1} C(x,z_1)C(z_1,z_2)\dots C(z_{k-1},y)$$

Spectrum of the Reduced Correlation Matrix

$$\int_0^{\ell} dy\, n\mathrm{e}^{-2n|x-y|} v_m(y) = \lambda_m v_m(y)$$

The integral equation can be recast as the 2°-order differential equation [where $\omega_m^2 \equiv 4n^2(1/\lambda_m - 1)$]

$$\partial_x^2 v_m(x) = -\omega_m^2 v_m(x)$$

[p.s.: a similar approach as been used for a different kernel by V. Eisler and I. Peschel in J. Stat. Mech. P04028 (2013)]

The eigenvalues are determined by the boundary conditions

$$\lambda_m = \frac{1}{1+\Omega_m^2}$$

$$\left\{\tan(n\ell\Omega) = -\Omega,\ \tan(n\ell\Omega) = \frac{1}{\Omega}\right\}$$

Stationary Entanglement Entropies

For large $n\ell$, Ω_m becomes a continuum variable in [0,∞] with density of roots

$$\sigma(\Omega) \approx \frac{2n\ell}{\pi}\left(1 + \frac{1}{n\ell}\frac{1}{1+\Omega^2}\right)$$

From which we obtain analytically the **leading** and **sub-leading** terms of the **Rényi Entropies**

$$S_A^{(\alpha)} = \int_0^\infty d\Omega\, \sigma(\Omega) e_\alpha\left(\frac{1}{1+\Omega^2}\right)$$

$$e_\alpha(\lambda) \equiv \frac{1}{1-\alpha}\ln[\lambda^\alpha + (1-\lambda)^\alpha]$$

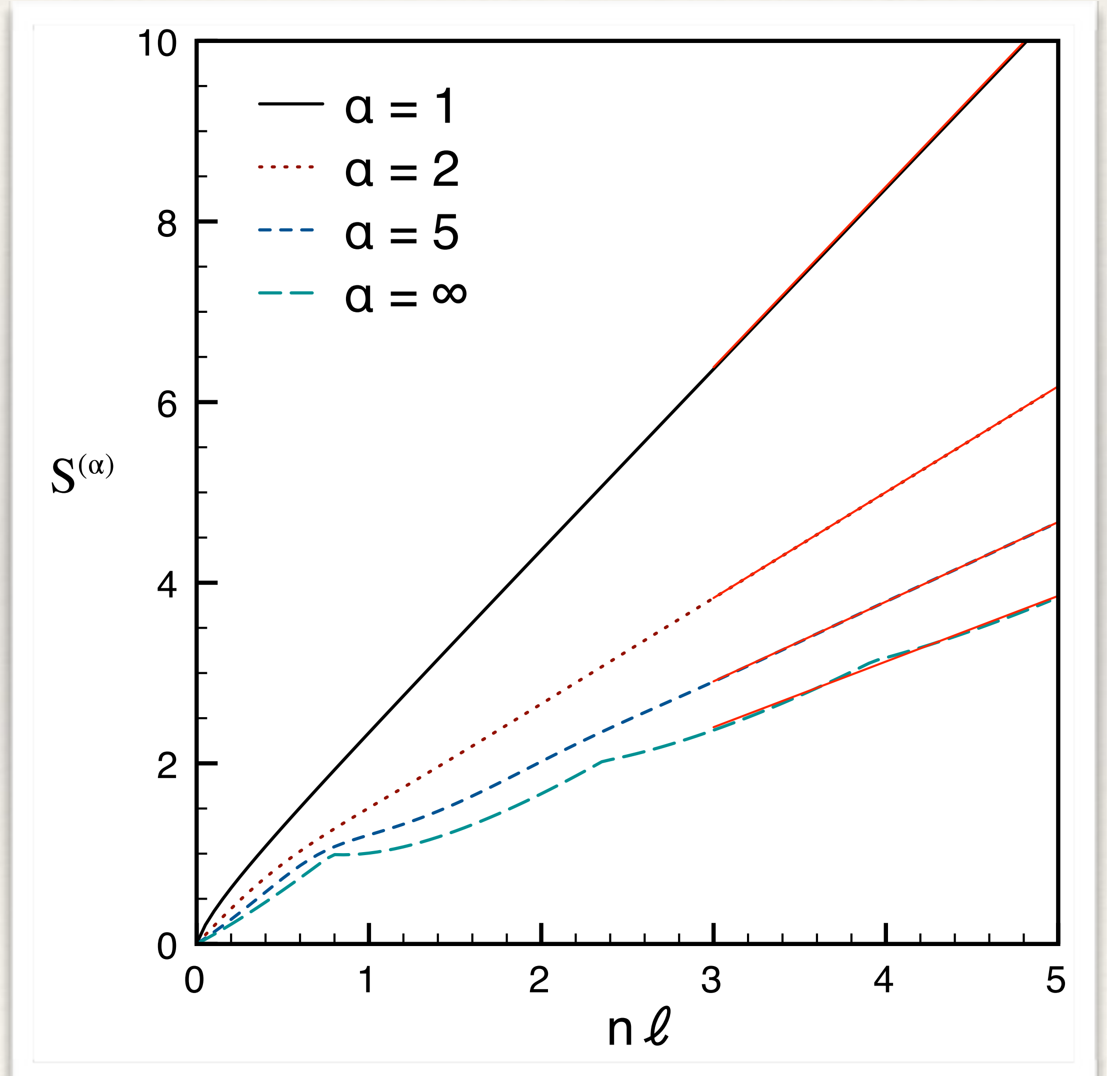
e.g.:

$$S_A^{(1)} = 2n\ell + (2\ln 2 - 1) + \mathcal{O}(\mathrm{e}^{-4n\ell})$$

$$S_A^{(2)} = (4 - 2\sqrt{2})n\ell + \ln(24 - 16\sqrt{2}) + \mathcal{O}(\mathrm{e}^{-4n\ell})$$

$$S_A^{(3)} = n\ell + \ln(4/3) + \mathcal{O}(\mathrm{e}^{-4n\ell})$$

$$S_A^{(\infty)} = (2 - 4/\pi)n\ell + (2\ln 2 - 4C/\pi) + \mathcal{O}(\mathrm{e}^{-4n\ell})$$



Conclusions

- *We studied the **non-equilibrium dynamics of the Lieb-Liniger model after an interaction quench from $c = 0$ to $c = \infty$.***
- *We **analytically** obtained the **dynamical density-density correlation function**.*
- *The **GGE** properly describe the **large-time limit** of the **density-density correlators**.*
- *Using the **full spectrum** of the **reduced two-point fermionic function** we evaluated the **stationary Rényi Entropies.***
- *We **analytically** extract the **leading** and **sub-leading** contribution of the **stationary Rényi Entropies***