Correlators with excited twist operators and plain operators

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Foreword

- This talk builds over a vast literature but is mainly based on these papers
  - I.P., "Strings in an arbitrary constant magnetic field with arbitrary constant metric and stringy form factors," JHEP **1106** (2011) 138 [arXiv:1101.5898 [hep-th]].
  - I.P., "Green functions and twist correlators for N branes at angles," Nucl. Phys. B **866** (2013) 87 [arXiv:1206.1431 [hep-th]].
  - I.P, "Correlators of arbitrary untwisted operators and excited twist operators for N branes at angles," arXiv:1401.6797 [hep-th].
  - I.P., "Canonical quantization of a string describing N branes at angle," to appear

(Some) Credits

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T. T. Burwick, R. K. Kaiser and H. F. Muller, 1991
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and

J. J. Atick, L. J. Dixon, P. A. Griffin, D. Nemeschansky, 1988
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E. Corrigan, D. B. Fairlie, 1975
J. H. Schwarz, C. C. Wu, 1974
P. Hermansson, B. E. W. Nilsson, A. K. Tollsten, A. Watterstam, 1990
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E. Gava, K. S. Narain and M. H. Sarmadi, 1997
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M. Bertolini, M. Billo, A. Lerda, J. F. Morales and R. Russo, 2006
A. Lawrence and A. Sever, 2007
D. Duo, R. Russo, S. Sciuto, 2007
J. P. Conlon and L. T. Witkowski, 2011

Plan of the talk

1. Introduction and motivation
   - The setup
   - Local expansion: string with $N = 2$ twists and excited twist fields
2. The main result
   - Quick examples of the main result
3. Conclusions

Introduction and motivation

The big picture: why?

- We would like to do "phenomenology" from string in a humble way start from the observed gauge group and matter:
  - consider D-brane worlds $\longrightarrow$ but $G_{GUT} \leq SU(5)$!
  - add instantons in order to get some needed/wanted features (Majorana masses, Yukawa couplings)

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  - stringy instantonic calculus
  - Melvin background and its T-dual versions
  - type II and heterotic compactifications on orbifolds

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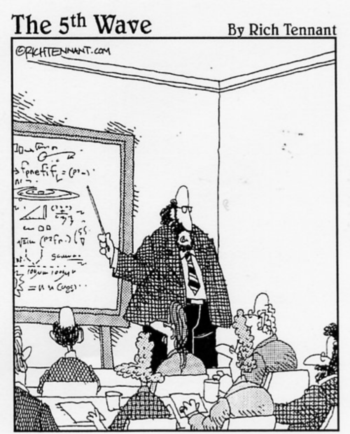
Therefore it is worth having a complete control over the correlators involving all kinds of twist fields.

My true personal motivation



"After the discovery of 'antimatter' and 'dark matter', we have just confirmed the existence of 'doesn't matter', which does not have any influence on the Universe whatsoever."

Figure : I was bothered by not been able to deal with twist fields as one does with spin fields

The setup

The setup

The Euclidean action for a string configuration is given by

$$S_E = \frac{1}{4\pi\alpha'} \int d\tau_E \int_0^\pi d\sigma \, (\partial_\alpha X^I)^2 = \frac{1}{4\pi\alpha'} \int_H d^2u \, (\partial_u X \bar{\partial}_{\bar{u}} \bar{X} + \bar{\partial}_{\bar{u}} X \partial_u \bar{X})$$

Pictorially

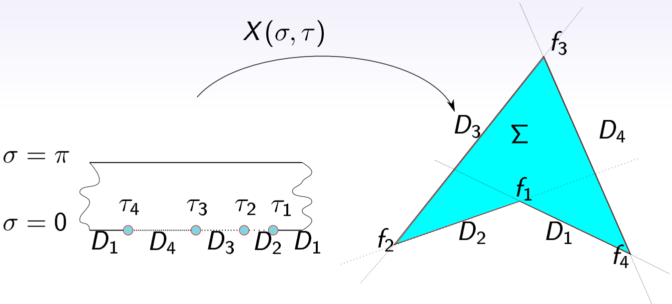
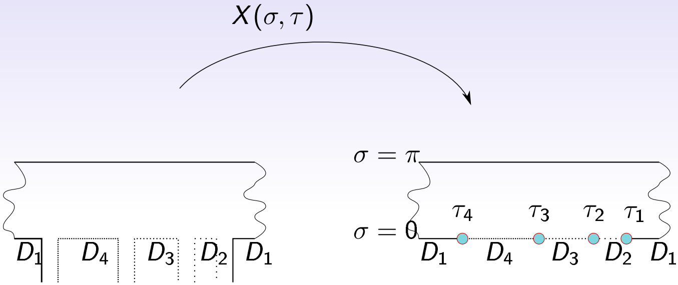


Figure : Map from the worldsheet to the target polygon Σ with a plain in and out string. The map $X(\sigma, \tau)$ folds the $\sigma = 0$ starting from $\tau = -\infty$ in a counterclockwise direction.

Go to details on b.c.

The setup: from interactions to boundary conditions

Pictorially



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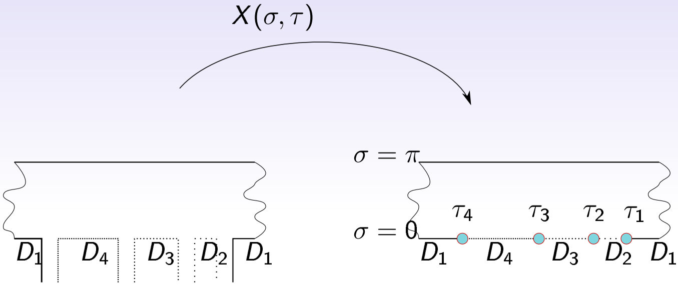

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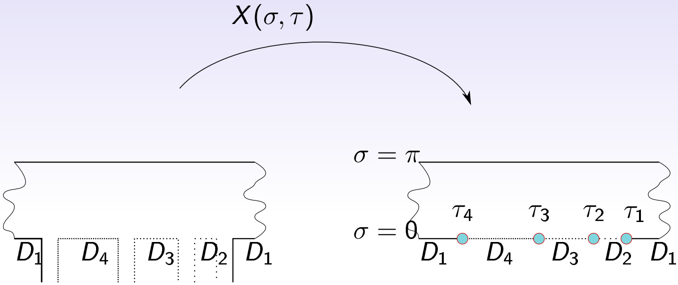

- We are mapping interactions to boundary conditions.
- This is also what done in path integral approach.
- Surely it works for ground states which are "pointlike".

The setup: different sectors

At given no. of branes there are different inequivalent sectors
Labeled by M no. of convex angles minus 2.

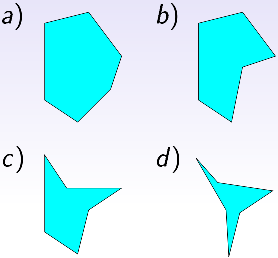

Figure : The four different cases with $N = 6$. a) $M = 4$. b) $M = 3$. c) $M = 2$. d) $M = 1$.

The intuitive reason: we need go through the straight line, i.e. no twist, if we want to go from a reflex angles to a more usual convex one.
One sector is more equal than the others: $M = 1$! It has holomorphic classical solution.

Local expansion: string with $N = 2$ twists and excited twist fields

Zooming and usual twisted string

The local picture

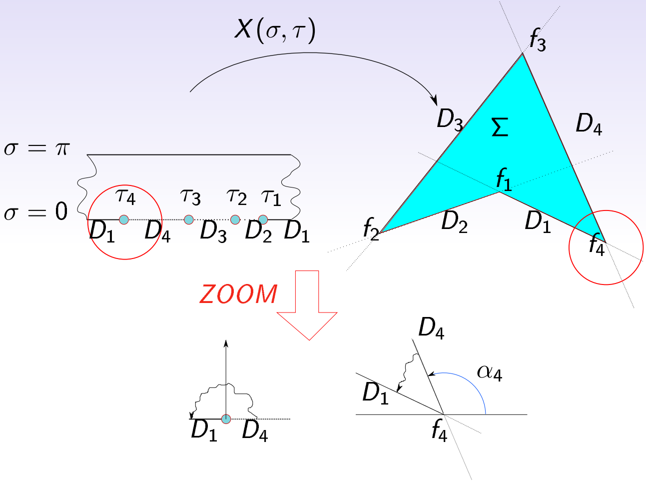

Figure : Zoom locally and get the usual twisted string.

f_t is the interaction point in space.

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A quantum part

$$X_q(u, \bar{u}; \{x_t, \alpha_t\}) = +i\frac{1}{2}\sqrt{2\alpha'}e^{i\pi\alpha_1}\sum_{n=0}^{\infty}\left[\frac{\bar{\alpha}_{n+\bar{\epsilon}}}{n+\bar{\epsilon}}u^{-(n+\bar{\epsilon})} - \frac{\alpha^{\dagger}_{n+\epsilon}}{n+\epsilon}u^{n+\epsilon}\right]$$
$$+i\frac{1}{2}\sqrt{2\alpha'}e^{i\pi\alpha_1}\sum_{n=0}^{\infty}\left[-\frac{\bar{\alpha}^{\dagger}_{n+\bar{\epsilon}}}{n+\bar{\epsilon}}\bar{u}^{n+\bar{\epsilon}} + \frac{\alpha_{n+\epsilon}}{n+\epsilon}\bar{u}^{-(n+\epsilon)}\right]$$

$(\epsilon = \alpha_{t+1} - \alpha_t + \theta(\alpha_t - \alpha_{t+1})$ the is the angle between the two branes; $\bar{\epsilon} = 1 - \epsilon)$

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The splitting into a classical and quantum part is needed for the existence of a conserved product between modes.

Go to details

Abstract excited twists and states in twisted Hilbert space (1)

In twisted Hilbert space there are the non normalized! states

$$\prod_{n=0}^{\infty} \left(n! \alpha^{\dagger}_{n+\epsilon} \right)^{N_n} \left(n! \bar{\alpha}^{\dagger}_{n+\bar{\epsilon}} \right)^{\bar{N}_n} | T \rangle$$

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All other states correspond the (generically **non primary**) abstract operators

$$\left[\prod_{n=0}^{\infty} \left(\partial_u^{n+1} X\right)^{N_n} \left(\partial_u^{n+1} \bar X\right)^{\bar N_n} \sigma_{\epsilon,f}\right](x)$$

the excited twists.
Notice that f.x. all $\bar N_n = 0$ are primary.

Abstract excited twists and states in twisted Hilbert space(2)

The notation

$$\left[\prod_{n=0}^{\infty} \left(\partial_u^{n+1} X\right)^{N_n} \left(\partial_u^{n+1} \bar{X}\right)^{\bar{N}_n} \sigma_{\epsilon,f}\right](x)$$

is non standard but better than the usual one since it does not use a symbol for each field

$$[\partial_u X \sigma_{\epsilon,f}](x) \leftrightarrow \tau_\epsilon(x), \qquad [\partial_u \bar{X} \sigma_{\epsilon,f}](x) \leftrightarrow \bar{\tau}_\epsilon(x),$$
$$[(\partial_u X)^2 \sigma_{\epsilon,f}](x) \leftrightarrow \omega_\epsilon(x), \qquad [(\partial_u \bar{X})^2 \sigma_{\epsilon,f}](x) \leftrightarrow \bar{\omega}_\epsilon(x),$$

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However this notation can be partially misleading since it is *not* true that

$$\partial_u^2 X(u, \bar{u})\ \sigma_{\epsilon,f}(x) \sim \frac{1}{(u-x)^{\#}} (\partial_u^2 X \sigma_{\epsilon,f})(x) + \dots$$

but

$$\partial_u^2 X(u, \bar{u}) \sigma_{\epsilon,f}(x) = (u-x)^{\epsilon-2}\ (\epsilon-1)\ (\partial_u X \sigma_{\epsilon,f})(x) + (u-x)^{\epsilon-1}\ \epsilon\ (\partial_u^2 X \sigma_{\epsilon,f})(x) + \dots$$

The main result

The main result in few words

For branes at angle on R^2 (T^2) the generic correlator

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- x_t $(t = 1, \ldots N)$ positions on ws of twists
- f_t intersections in space of two consecutive branes
- $\pi\epsilon_t$ angles between two consecutive branes

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- classical solution $X^I_{cl}(u, \bar{u}; \{x_t, f_t, \epsilon_t\})$

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- a lot of patience

Quick examples of the main result (1)

- On $\mathbb{C} = \mathbb{R}^2$ with open string fields $X(u, \bar{u}) = X^z(u, \bar{u}) \in \mathbb{C}$ and $\bar{X}(u, \bar{u}) = X^{\bar{z}}(u, \bar{u}) = X^*(u, \bar{u}) \in \mathbb{C}$ with $u = x + iy \in H$ (the upper half plane)

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- the following boundary correlator on a single brane (i.e. untwisted sector)

$$\langle \partial_x \bar{X}(x_1, x_1)\ \partial_x X(x_2, x_2)\ (\partial_x^2 X \partial_x \bar{X})(x_3, x_3)\ \rangle$$

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- it is given by

$$\begin{aligned} = & \partial_{x_1} \partial_{x_3}^2 G^{\bar{z}z}_{U,bou}(x_1, x_3)\ \partial_{x_2} \partial_{x_3} G^{z\bar{z}}_{U,bou}(x_2, x_3) \\ & + \partial_{x_1} \partial_{x_3} G^{\bar{z}\bar{z}}_{U,bou}(x_1, x_3)\ \partial_{x_2} \partial_{x_3}^2 G^{zz}_{U,bou}(x_2, x_3) \end{aligned}$$

where $G^{IJ}_{U,bou}(x_1, x_2)$ is the boundary Green function for Untwisted boundary conditions between two points $x_1, x_2 \in R$ on the boundary of the upper plane boundary ($G^{zz} \neq 0$ since brane breaks rotations)
- other possible terms like

$$\partial_{x_1} \partial_{x_2} G^{\bar{z}z}_{U,bou}(x_1, x_2)\ \partial_{x_3}^2 \partial_{x_3} [G^{z\bar{z}}_{U,bou}(x_2, x_3)]_{regularized}$$

are absent because of normal ordering

Quick examples of the main result (2)

- Twisted case: the boundary correlator on N branes at angles

$$\langle \partial_x \bar{X}(x_1, x_1)\ \partial_x X(x_2, x_2)\ (\partial_x^2 X \partial_x \bar{X})(x_3, x_3)\ \prod_{t=1}^{N} \sigma_{\epsilon_t}(x_t) \rangle$$

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$$= \langle \prod_{t=1}^{N} \sigma_{\epsilon_t}(x_t) \rangle \Big\{ \partial_{x_1} \partial_{x_3}^2 G_{bou}^{\bar{z}z}(x_1, x_3)\, \partial_{x_2} \partial_{x_3} G_{bou}^{z\bar{z}}(x_2, x_3)$$

$$+ \partial_{x_1} \partial_{x_3} G_{bou}^{\bar{z}\bar{z}}(x_1, x_3)\, \partial_{x_2} \partial_{x_3}^2 G_{bou}^{zz}(x_2, x_3) \text{ as before}$$

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$$+ \partial_{x_1} \partial_{x_2} G^{\bar z z}_{bou}(x_1,x_2)\, \partial^2_{x_3} \partial_{y_3}|_{y_3=x_3} \Delta^{z \bar z}_{bou}(x_3,y_3) \text{ left over from norm. ord.}$$

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$$+ \partial_{x_1} \partial_{x_2} G_{bou}^{\bar{z}z}(x_1,x_2)\, \partial_{x_3}^2 \partial_{y_3}|_{y_3=x_3} \Delta_{bou}^{z\bar{z}}(x_3,y_3) \quad \text{left over from norm. ord.}$$

$$+ \partial_{x_1} \partial_{x_2}^2 G_{bou}^{\bar{z}z}(x_1,x_2)\, \partial_{x_3}^2 X_{cl}(x_2)\, \partial_{x_3} \bar{X}_{cl}(x_3) + \partial_{x_1} \bar{X}_{cl}(x_1)\, \partial_{x_2} X_{cl}(x_2)\, \partial_{x_3}^2 \partial_{y_3}|_{y_3=x_3} \Delta_{bou}^{z\bar{z}}(x_3,y_3)$$
$$+ \partial_{x_1} \partial_{x_3}^2 G_{bou}^{\bar{z}z}(x_1,x_3)\, \partial_{x_2} X_{cl}(x_2)\, \partial_{x_3} \bar{X}_{cl}(x_3) + \partial_{x_1} \bar{X}_{cl}(x_1)\, \partial_{x_3}^2 X_{cl}(x_3)\, \partial_{x_2} \partial_{x_3} G_{bou}^{z\bar{z}}(x_2,x_3)$$
$$+ \partial_{x_1} \partial_{x_3} G_{bou}^{\bar{z}\bar{z}}(x_1,x_3)\, \partial_{x_2} X_{cl}(x_2)\, \partial_{x_3}^2 X_{cl}(x_3) + \partial_{x_1} \bar{X}_{cl}(x_1)\, \partial_{x_3} \bar{X}_{cl}(x_3)\, \partial_{x_2} \partial_{x_3}^2 G_{bou}^{zz}(x_2,x_3)$$
$$+ \partial_x \bar{X}_{cl}(x_1,x_1)\, \partial_x X_{cl}(x_2,x_2)\, \partial_x^2 X_{cl}(x_3,x_3)\, \partial_x \bar{X}_{cl}(x_3,x_3) \Big\} \quad \text{from classical solution } X_{cl}$$

where $G_{bou}^{IJ}(x,y)$ is the boundary Green function for twisted b.c.
and $\Delta_{bou}^{IJ}(x,y)$ its regularized version.
A NUMBER OF DETAILS HAVE BEEN OMITTED!

Quick examples of the main result (3)

- Twisted case: the boundary correlator and excited twists on N branes at angles

$$\langle \partial_x \bar{X}(\hat{x}_1, \hat{x}_1)\ (\partial_x X \sigma_{\epsilon_1})(x_1, x_1)\ (\partial_x^2 X \partial_x \bar{X} \sigma_{\epsilon_2})(x_2, x_2)\ \prod_{t=3}^{N} \sigma_{\epsilon_t}(x_t)\rangle$$

where $(\partial_x^2 X \partial_x \bar{X} \sigma_{\epsilon_2})$ is the excited twist defined very roughly as $\lim_{u \to x_2} (\partial_x^2 X \partial_x \bar{X})(u, \bar{u}) \sigma_{\epsilon_2}(x_2)$

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$$= \langle \prod_{t=1}^{N} \sigma_{\epsilon_t}(x_t) \rangle \Big\{ \partial_{v_2} [(v_2 - x_2)^{\bar{\epsilon}_2} \partial_{\hat{x}_1} \partial_{v_2} G^{\bar{z}z}(\hat{x}_1, \hat{x}_1; v_2, \bar{v}_2)] \Big|_{v_2 = x_2}$$
$$\times [(v_1 - x_1)^{\bar{\epsilon}_1} (v_2 - x_2)^{\epsilon_2} \partial_{v_1} \partial_{v_2} G^{z\bar{z}}(v_1, \bar{v}_1; v_2, \bar{v}_2)]|_{v_t = x_t}$$
$$+ [(v_2 - x_2)^{\epsilon_2} \partial_{\hat{x}_1} \partial_{v_2} G^{\bar{z}\bar{z}}(\hat{x}_1, \hat{x}_1; v_2, \bar{v}_2)]|_{v_2 = x_2}$$
$$\times \partial_{v_2} [(v_1 - x_1)^{\bar{\epsilon}_1} (v_2 - x_2)^{\bar{\epsilon}_2} \partial_{v_1} \partial_{v_2} G^{zz}(v_1, \bar{v}_1; v_2, \bar{v}_2)]|_{v_t = x_t} \quad \text{"as before"}$$

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$$\times [(v_1 - x_1)^{\bar\epsilon_1} (v_2 - x_2)^{\epsilon_2} \partial_{v_1} \partial_{v_2} G^{z \bar z}(v_1, \bar v_1; v_2, \bar v_2)]|_{v_t = x_t}$$

$$+ [(v_2 - x_2)^{\epsilon_2} \partial_{\hat x_1} \partial_{v_2} G^{\bar z \bar z}(\hat x_1, \hat x_1; v_2, \bar v_2)]|_{v_2 = x_2}$$

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$$+ [(v_1 - x_1)^{\bar\epsilon_1} \partial_{\hat x_1} \partial_{v_1} G^{\bar z z}(\hat x_1, \hat x_1; v_1, \bar v_1)]|_{v_1 = x_1}$$

$$\times \partial_{v_2} [(u_2 - x_2)^{\bar\epsilon_2} (v_2 - x_2)^{\epsilon_2} \partial_{u_2} \partial_{v_2} \Delta^{z \bar z}(u_2, \bar u_2; v_2, \bar v_2)]|_{u_2 = v_2 = x_2}$$ left over from norm. ord.

$+$ terms with classical contributions $\Big\}$

The Reggeon vertex (1)

Is it possible to generate the previous correlators in a “mechanical” way?

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$$\langle \partial_x \bar{X}(x_1, x_1)\ \partial_x X(x_2, x_2)\ (\partial_x^2 X \partial_x \bar{X})(x_3, x_3)\ \rangle$$
$$= \frac{\partial}{\partial c_{(1)1}} \frac{\partial}{\partial \bar{c}_{(2)1}} \frac{\partial^2}{\partial \bar{c}_{(3)2} \partial c_{(3)1}} V(\{c_{(i)n}, \bar{c}_{(i)n}\})\Big|_{c=0}$$

where

- $V(\{c_{(i)n}, \bar{c}_{(i)n}\})$ is the Reggeon vertex
- $c_{(i)n}$ with i associated with x_i
- $c_{(i)n}$ with n associated with the numeber of derivatives $\partial_{x_i}^n$

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Easy to derive for the untwisted correaltors.
More complicated with the twisted ones

The untwisted Reggeon vertex (2)

- Map untwisted abstract operator to a realization in an untwisted Hilbert space.
  E.g. in untwisted Hilbert space

$$(\partial_x^2 X \partial_x \bar{X})(x_3, x_3) = \frac{\partial^2}{\partial \bar{c}_{(3)2} \partial c_{(3)1}} \mathcal{S}(c_{(3)}, \bar{c}_{(3)})$$

$$+ \mathcal{S}(c_{(3)}, \bar{c}_{(3)}) = : e^{\sum_{n=0}^{\infty} \left[\bar{c}_{(3)n} \partial_x^n X_{op}(x_3, x_3) c_{(3)n} \partial_x^n \bar{X}_{op}(x_3, x_3) \right]} : = : e^{\sum_{n=0}^{\infty} c_{(3)nI} \partial_x^n \bar{X}_{op}^I(x_3, x_3)} :$$

The Sciuto-Della Selva-Saito vertex $\mathcal{S}$ is the generating function of this map.

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- Compute the generating function of all correlators with L untwisted vertices in untwisted Hilbert space

$$V_L(\{c_{(i)n}, \bar{c}_{(i)n}\}) = \langle 0 | \mathcal{S}(c_{(1)}, \bar{c}_{(1)}) \dots \mathcal{S}(c_{(L)}, \bar{c}_{(L)}) | 0 \rangle$$

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-

$$= \prod_{1 \le i < j \le L} e^{\sum_{n,m=0}^{\infty} c_{(i)nI} \; c_{(j)mJ} \; \partial_{x_i}^n \partial_{x_j}^m G_U^{IJ}(x_i, x_j)}$$

  with $c_{(i)n} = c_{(i)n\bar{z}} = c^z_{(i)n}$ and $\bar{c}_{(i)n} = c_{(i)nz} = c^{\bar{z}}_{(i)n}$.

The Reggeon vertex (3)

The idea is to generalize the untwisted computation

$$V_L(\{c_{(i)n}, c_{(i)n}\}) = \langle 0|\mathcal{S}(c_{(1)}, \bar{c}_{(1)}) \ldots \mathcal{S}(c_{(L)}, \bar{c}_{(L)})|0\rangle$$

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to the twisted case

$$V_{N+L}(\{c_{(i)n}, d_{(t)n}\}) = \langle 0_{out}|\mathcal{S}_{T(1)}(c_{(1)}, \bar{c}_{(1)}) \dots \mathcal{S}_{T(L)}(c_{(L)}, \bar{c}_{(L)}) \times \\ \times \mathcal{T}_{(1)}(d_{(1)}, \bar{d}_{(1)}) \dots \mathcal{T}_{(N)}(d_{(N)}, \bar{d}_{(N)})|0_{in}\rangle$$

We need understanding

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We need understanding

- the in vacuum $|0_{in}\rangle$
- the out vacuum $\langle 0_{out}|$ Details

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- the Sciuto-Della Selva-Saito $\mathcal{S}_T(c_{(i)}, \bar{c}_{(i)})$ for the untwisted matter in the twisted sectors Details

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$$\begin{aligned} V_{N+L}(\{c_{(i)n}, d_{(t)n}\}) = & \langle 0_{out}| \mathcal{S}_{T(1)}(c_{(1)}, \bar{c}_{(1)}) \dots \mathcal{S}_{T(L)}(c_{(L)}, \bar{c}_{(L)}) \times \\ & \times \mathcal{T}_{(1)}(d_{(1)}, \bar{d}_{(1)}) \dots \mathcal{T}_{(N)}(d_{(N)}, \bar{d}_{(N)})|0_{in}\rangle \end{aligned}$$

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Details on computation

The Reggeon vertex (3)

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to the twisted case

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Details on computation

- It is also possible and more normal to perform the previous computation in path integral formalism

Sketch of computation

The Reggeon vertex (4)

The final result for L untwisted vertices and N twisted ones.

- Associate: space index $\leftrightarrow I$ with $I = z, \bar{z}$
  untwisted $\leftrightarrow c_{(i)nI}$ with $i = 1, \ldots L$,
  twisted $\leftrightarrow d_{(t)nI}$ with $t = 1, \ldots N$

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- The generating function is

$$V_{N+L}(c,d) = \lim_{\{u_t\}\to\{x_t\}} \langle \sigma_{\epsilon_1,f_1}(x_1) \dots \sigma_{\epsilon_N,f_N}(x_N) \rangle \times V_{class} \times V_{self\ \ int} \times V_{int}$$

with

$$V_{class} = \prod_{t=1}^{N} e^{\sum_{n=1}^{\infty} d_{(t)nI} \partial_{u_t}^{n-1} [(u_t - x_t)^{\epsilon_{tI}} \partial_u X_{cl}^I(u_t, \bar{u}_t)]} \times \prod_{i=1}^{L} e^{\sum_{n=0}^{\infty} c_{(i)nI} \partial_{x_i}^{n} X_{cl}^I(x_i, x_i)}$$

where $X_{cl}^I(u, \bar{u})$ is the classical solution.

The Reggeon vertex (4)

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with

$$V_{self\ \ interaction} =$$

$$\prod_{t=1}^{N} e^{\frac{1}{2}\sum_{n,m=1}^{\infty} d_{(t)nI}\, d_{(t)mJ}\, \partial_{u_t}^{n-1} \partial_{v_t}^{m-1} [(u_t - x_t)^{\epsilon_{tI}} (v_t - x_t)^{\epsilon_{tJ}} \partial_u \partial_v \Delta_{(t)}^{IJ}(u_t, \bar{u}_t; v_t, \bar{v}_t; \{x_{\bar{t}}, \epsilon_{\bar{t}}\})]|_{v_t = u_t}}$$

$$\times \prod_{i=1}^{L} e^{\frac{1}{2}\sum_{n=0}^{\infty} c_{(i)nI} \sum_{m=0}^{\infty} c_{(i)mJ}\, \partial_{x_i}^{n} \partial_{\hat{x}_i}^{m} \Delta_{(i)}^{IJ}(x_i, x_i; \hat{x}_i, \hat{x}_i; \{x_t, \epsilon_t\})|_{\hat{x}_i = x_i}}$$

where $\Delta_{(t)}^{IJ}$ is the Green function regularized at point x_t

The Reggeon vertex (4)

The final result for L untwisted vertices and N twisted ones.

- Associate: space index $\leftrightarrow I$ with $I = z, \bar{z}$
  untwisted $\leftrightarrow c_{(i)nI}$ with $i = 1, \ldots L$,
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- The generating function is

$$V_{N+L}(c,d) = \lim_{\{u_t\}\to\{x_t\}} \langle \sigma_{\epsilon_1,f_1}(x_1)\ldots\sigma_{\epsilon_N,f_N}(x_N)\rangle \times V_{classical} \times V_{self\ \ interaction} \times V_{interactions}$$

with

$$V_{interactions} =$$
$$\prod_{1\le t<\hat{t}\le N} e^{\sum_{n,m=1}^{\infty} d_{(t)nI} d_{(\hat{t})mJ} \partial_{u_t}^{n-1} \partial_{v_{\hat{t}}}^{m-1} [(u_t - x_t)^{\epsilon_t I} (v_{\hat{t}} - x_{\hat{t}})^{\epsilon_{\hat{t}} J} \partial_u \partial_v G^{IJ}(u_t,\bar{u}_t; v_{\hat{t}}, \bar{v}_{\hat{t}}; \{x_{\bar{t}}, \epsilon_{\bar{t}}\})]}$$
$$\times \prod_{1\le i<j\le L} e^{\sum_{n=0}^{\infty} c_{(i)nI} \sum_{m=0}^{\infty} c_{(j)mJ} \partial_{x_i}^{n} \partial_{x_j}^{m} G^{IJ}(x_i, x_i; x_j, x_j; \{x_t, \epsilon_t\})}$$
$$\times \prod_{1\le t\le N} \prod_{1\le j\le L} e^{\sum_{n=1}^{\infty} d_{(t)nI} c_{(j)mJ} \partial_{u_t}^{n-1} \partial_{x_j}^{m} [(u_t - x_t)^{\epsilon_t I} \partial_u G^{IJ}(u_t, \bar{u}_t; x_j, x_j; \{x_{\bar{t}}, \epsilon_{\bar{t}}\})]}$$

The Reggeon vertex (4)

Our case L untwisted vertices and N twisted ones.
Putting all together the generating function is

$$V_{N+L}(c,d) = \lim_{\{u_t\}\to\{x_t\}} \langle \sigma_{\epsilon_1,f_1}(x_1)\dots\sigma_{\epsilon_N,f_N}(x_N)\rangle$$

$$\times \prod_{t=1}^{N} \Bigg\{ e^{\sum_{n=1}^{\infty} d_{(t)nI}\partial_{u_t}^{n-1}[(u_t-x_t)^{\epsilon_{tI}}\partial_u X_{cl}^I(u_t,\bar u_t)]}$$

$$\times e^{\frac{1}{2}\sum_{n,m=1}^{\infty} d_{(t)nI}d_{(t)mJ}\partial_{u_t}^{n-1}\partial_{v_t}^{m-1}[(u_t-x_t)^{\epsilon_{tI}}(v_t-x_t)^{\epsilon_{tJ}}\partial_u\partial_v\Delta^{IJ}_{(N,M)(t)}(u_t,\bar u_t;v_t,\bar v_t;\{x_{\bar t},\epsilon_{\bar t}\})]|_{v_t=u_t}}\Bigg\}$$

$$\times \prod_{i=1}^{L} \Bigg\{ e^{\sum_{n=0}^{\infty} c_{(i)nI}\partial_{x_i}^{n} X_{cl}^I(x_i,x_i)}$$

$$\times e^{\frac{1}{2}\sum_{n=0}^{\infty} c_{(i)nI}\sum_{m=0}^{\infty} c_{(i)mJ}\partial_{x_i}^{n}\partial_{\hat x_i}^{m}\Delta^{IJ}_{(N,M),bou(i)}(x_i,\hat x_i;\{x_t,\epsilon_t\})|_{\hat x_i=x_i}}\Bigg\}$$

$$\times \prod_{1\le t<\hat t\le N} e^{\sum_{n,m=1}^{\infty} d_{(t)nI}d_{(\hat t)mJ}\partial_{u_t}^{n-1}\partial_{v_{\hat t}}^{m-1}[(u_t-x_t)^{\epsilon_{tI}}(v_{\hat t}-x_{\hat t})^{\epsilon_{\hat t J}}\partial_u\partial_v G^{IJ}_{(N,M)}(u_t,\bar u_t;v_{\hat t},\bar v_{\hat t};\{x_{\bar t},\epsilon_{\bar t}\})]}$$

$$\times \prod_{1\le i<j\le L} e^{\sum_{n=0}^{\infty} c_{(i)nI}\sum_{m=0}^{\infty} c_{(j)mJ}\partial_{x_i}^{n}\partial_{x_j}^{m} G^{IJ}_{(N,M),bou}(x_i,x_j;\{x_t,\epsilon_t\})}$$

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Conclusions

We have shown that to compute any correlator involving excited twisted fields and untwisted vertices are needed three ingredients

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to get the amplitudes is almost impossible since the Green function is a product of generalization of hypergeometric functions.
Branes at angles Green function in NOT the same Green function for magnetized branes!

Details

The setup: boundary conditions

We put the following boundary conditions

$$e^{-i\pi\alpha_t}\partial_y X^z(u,\bar u)|_{u=x+i0^+} + e^{i\pi\alpha_t}\partial_y X^{\bar z}(u,\bar u)|_{u=x+i0^+} = 0 \qquad x_t < x < x_{t-1}$$

$$e^{-i\pi\alpha_t} X^z(u,\bar u)|_{u=x+i0^+} - e^{i\pi\alpha_t} X^{\bar z}(u,\bar u)|_{u=x+i0^+} = 2ig_t \qquad x_t < x < x_{t-1}$$

They mean

- brane D_t is on the segment $x_t < x < x_{t-1}$
- brane D_t has Dirichlet boundary condition in the orthogonal direction 2_t

$$\sqrt{2} i X^{2_t} = e^{-i\pi\alpha_t} X^z - e^{i\pi\alpha_t} X^{\bar z} = 2ig_t \qquad (1)$$

  hence $\sqrt{2} g_t \in R$ is the distance of the brane from the origin
- brane D_t has Neumann boundary condition in the parallel direction 1_t

$$\sqrt{2} X^{1_t} = e^{-i\pi\alpha_t} X^z + e^{i\pi\alpha_t} X^{\bar z} \qquad (2)$$

Back to Setup

The Hermitian product for modes (1)

- Have a time dependent world-sheet since the boundary conditions vary with time.

Back to Twisted string

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- Start from K-G current for any two 2-vectors $F_{1,2} = (f^z_{1,2}, f^{\bar{z}}_{1,2})$

$$j_\alpha(F_1, F_2) = i[(f_1^I)^* \partial_\alpha f_2^I - (\partial_\alpha f_1^I)^* f_2^I]$$

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- It is conserved on solutions.

Back to Twisted string

The Hermitian product for modes (2)

- Consider on half an annulus $S(r_0, r_1)$ in the upper half plane

$$0 = \int_{S(r_0,r_1)} d * j = \int_{|u|=r_1} *j - \int_{|u|=r_0} *j + \int_{[r_0,r_1]} *j + \int_{[-r_1,-r_0]} *j$$

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- Good? Only if $G(r) - G(-r)$ does not depend on past bck values.
  This requires F to have quantum boundary conditions

$$e^{-i\pi\alpha_t}\partial_y f^z(u,\bar{u})|_{u=x+i0^+} + e^{i\pi\alpha_t}\partial_y f^{\bar{z}}(u,\bar{u})|_{u=x+i0^+} = 0 \qquad x_t < x < x_{t-1}$$
$$e^{-i\pi\alpha_t} f^z(u,\bar{u})|_{u=x+i0^+} - e^{i\pi\alpha_t} f^{\bar{z}}(u,\bar{u})|_{u=x+i0^+} = 0 \qquad x_t < x < x_{t-1}$$

  Same for having a self-adjoint $\partial_u \bar{\partial}_{\bar{u}}$!

The Hermitian product for modes (3)

- Quantum boundary condition implies split

$$X^I(u,\bar u) = X^I_{cl}(u,\bar u;\{x_t, g_t, \alpha_t\}) + X^I_q(u,\bar u;\{x_t,\epsilon_t\})$$

with X_{cl} classical solution, X_q quantum fluctuation to be quantized

Back to twisted string

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- For the usual magnetic branes get the well known "weird" Hermitian form

$$(F_1,F_2) = \int_0^\pi i\ F_1^\dagger \overset{\leftrightarrow}{\partial}_\tau F_2 d\sigma \ + iF_1^\dagger \mathcal{F}_0 F_2|_{\sigma=0} - iF_1^\dagger \mathcal{F}_\pi F_2|_{\sigma=\pi}$$

  where $\mathcal{F}_{IJ}s$ are the magnetic fields.

Back to twisted string

Refined overlap condition

In principle it is possible to study the quantum modes of the $\partial_u \bar{\partial}_{\bar{u}}$ with quantum boundary conditions.

Back to twisted string

Back to reggeon

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$$X_q(u,\bar{u};\{D_N, D_t\})$$
  - at transition "time" like $|u| = x_t$ require match of the two quantum expansions as
$$X_q(u,\bar{u};\{D_N, D_{t+1}\})\big|_{|u|=x_t^-} = X_q(u,\bar{u};\{D_N, D_t\})\big|_{|u|=x_t^+}$$

Back to twisted string
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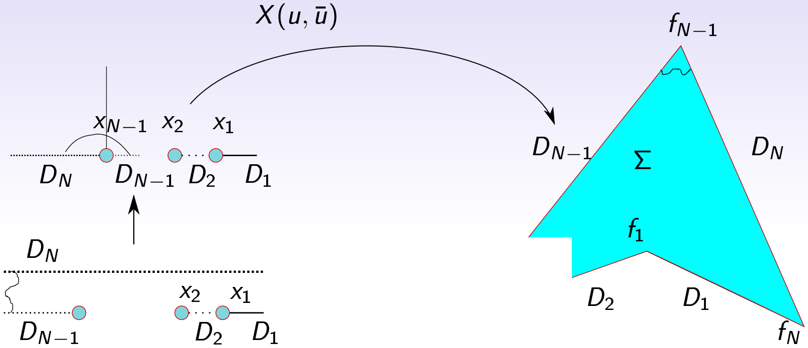
In and out vacua in presence of N twist fields (1)

We consider the configuration



We use the improved overlap

Hence we take the in vacuum to be the twisted vacuum corresponding to the usual $N = 2$ twisted string

$$|0_{in}\rangle = |\, T_{D_{N-1} D_N}\rangle$$

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In and out vacua in presence of N twist fields (2)

What about $\langle 0_{out}|$?

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In and out vacua in presence of N twist fields (2)

What about $\langle 0_{out}|$?

- Compute Green function

$$G^{IJ}(u, \bar{u}; v, \bar{v}; \{x_t, \epsilon_t\})$$

in the usual way

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- consider the operatorial definition of the (derivative of) Green function

$$\partial_u \partial_v G^{IJ}(u,\bar{u};v,\bar{v};\{x_t,\epsilon_t\}) = \frac{\langle 0_{out}|\partial_u X^I_q(u,\bar{u})\partial_v X^J_q(v,\bar{v})|0_{in}\rangle}{\langle 0_{out}|0_{in}\rangle}$$

- take $|u|, |v| < x_{N-1}$ so we can write

$$\partial_u \partial_v G^{IJ} = \frac{\langle 0_{out}|\partial_u X^I_{\{D_{N-1},D_N\},q}(u,\bar{u})\partial_v X^J_{\{D_{N-1},D_N\},q}(v,\bar{v})|T_{D_{N-1}D_N}\rangle}{\langle 0_{out}|T_{D_{N-1}D_N}\rangle}$$

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In and out vacua in presence of N twist fields (3)

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In and out vacua in presence of N twist fields (3)

What about $\langle 0_{out}|$?

- Use mode expansion and normal order the result

$$\partial_u \partial_v \Delta^{IJ}_{(N,M)(N-1)} = \frac{\langle 0_{out}| : \partial_u X^{I(-)}_{\{D_{N-1},D_N\},q}(u) \partial_v X^{J(-)}_{\{D_{N-1},D_N\},q}(v) : | T_{D_{N-1} D_N}\rangle}{\langle 0_{out}| T_{D_{N-1} D_N}\rangle}$$

with

$$\Delta^{IJ}_{(N,M)(N-1)}(u,\bar{u};v,\bar{v};\{x_t,\epsilon_t\}) = G^{IJ}(u,\bar{u};v,\bar{v};\{x_t,\epsilon_t\}) - G^{IJ}_{N=2,\{D_{N-1},D_N\}}(u,\bar{u};v,\bar{v})$$

the regularized Green function.

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the regularized Green function.

- derive

$$\langle 0_{out}| \sim \langle T_{D_1 D_N}| e^{B_{\bar{z}\bar{z}}\alpha\alpha + B_{zz}\bar{\alpha}\bar{\alpha} + B_{\bar{z}z}\alpha\bar{\alpha}}$$

with $B \sim \Delta^{IJ}$

Back to Reggeon

SDS vertex for untwisted vertices (1)

The SDS vertex maps an abstract operator to an operatorial realization.
The map for an untwisted abstract operator to its operatorial realization in twisted Hilbert space is

$$\mathcal{S}_T(c,\bar{c}) = :e^{\sum_{n=0}^{\infty}\left[\bar{c}_n \partial_x^n X_{op\ T}(x+i0^+,x-i0^+)+c_n \partial_x^n \bar{X}_{op\ T}(x+i0^+,x-i0^+)\right]}:$$

$$\exp\left\{\frac{1}{2}\sum_{n,m=0}^{\infty} \bar{c}_{nI}\ c_{mJ}\ \partial_{x_1}^n\ \partial_{x_2}^m \Delta_{bou\ T}^{IJ}(x_1;x_2)|_{x_1=x_2=x}\right\}$$

There is a new piece

$$\Delta_{bou\ T}^{IJ}(x_1;x_2) = G_{N=2\ T}^{IJ}(x_1+i0^+,x_1-i0^+;x_2+i0^+,x_2-i0^+) - G_U^{IJ}(x_1+i0^+,x_1-i0^+;x_2+i0^+,x_2-i0^+)$$

the left over of the "minimal subtraction".

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SDS vertex for untwisted vertices (2)

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- Consider "simplest" untwisted vertex in untwisted Hilbert space

$$: e^{ik^I X^I_{op\ Untwisted}(x)} :$$

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- Consider "simplest" untwisted vertex in untwisted Hilbert space

$$: e^{ik^I X^I_{op\ Untwisted}(x)} :$$

- can be derived from non normal ordered vertex by a point splitting procedure

$$: e^{ik^I X^I_{op\ Untwisted}(x)} := \lim_{\eta \to 0} \mathcal{N}(\eta) e^{ik^I [X^{I(-)}_{op\ Untwisted}(x e^{-\eta}) + X^{I(+)}_{op\ Untwisted}(x)]}$$

with $\mathcal{N}(\eta)$ a regularization factor

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- OK since realizations in twisted Hilbert reproduce the usual OPEs!

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SDS vertex for untwisted vertices (3)

Two examples

- to the boundary tachyonic vertex $e^{i\bar{k}X(x,x)+ik\bar{X}(x,x)}$ corresponds the operatorial realization

$$x^{-\alpha' k_{\parallel D}^2} e^{-\frac{1}{2}R^2(\epsilon)\alpha' k_{\parallel D}^2} \; : e^{i(\bar{k}X(x,x)+k\bar{X}(x,x))} :$$

with $R^2(\epsilon) = 2\psi(1) - \psi(\epsilon) - \psi(\bar{\epsilon}) > 0$ and $\psi(z) = \frac{d \ln \Gamma(z)}{dz}$ the digamma function and $k_{\parallel D}$ is the part of the momentum, parallel to the brane

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  with $R^2(\epsilon) = 2\psi(1) - \psi(\epsilon) - \psi(\bar{\epsilon}) > 0$ and $\psi(z) = \frac{d \ln \Gamma(z)}{dz}$ the digamma function and $k_{\| D}$ is the part of the momentum, parallel to the brane

- we can also compute the SDS for chiral operators: to the chiral operator $(\partial_u^2 X\, \partial_u X\, \partial_u \bar{X})(u)$ corresponds

$$\begin{aligned} &: (\partial_u^2 X \partial_u X \partial_u \bar{X})(u) : + \partial_u^2 \partial_v \Delta_c^{z\bar{z}}|_{v=u} \partial_u X + \partial_u \partial_v \Delta_c^{z\bar{z}}|_{v=u} \partial_u^2 X \\ &=: (\partial_u^2 X \partial_u X \partial_u \bar{X})(u) : - \frac{k_\epsilon k_{\bar{\epsilon}} \epsilon(1-\epsilon)(2-\epsilon)}{2u^3} \partial_u X + \frac{k_\epsilon k_{\bar{\epsilon}} \epsilon(1-\epsilon)}{2u^2} \partial_u^2 X \end{aligned}$$

  with $k_\epsilon = -i\frac{1}{2}\sqrt{2\alpha'} e^{i\pi\alpha_t}$ and $k_{\bar{\epsilon}} = -i\frac{1}{2}\sqrt{2\alpha'} e^{-i\pi\alpha_t}$

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SDS for excited twists (1)

- The main observation

$$\partial_u^{n-1}\left[u^{\bar\epsilon}\partial_u X_{op}(u,\bar u)\right] = (n-1)!\; k_\epsilon \alpha^\dagger_{n-1+\epsilon} + O(u)$$

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- therefore a normal ordered products of these operators gives directly an excited twist state, e.g.

$$\lim_{u\to 0} : \partial_u^{n-1}\left[u^{\bar\epsilon}\partial_u X_{op}(u,\bar u)\right]\partial_u^{m-1}\left[u^{\epsilon}\partial_u \bar X_{op}(u,\bar u)\right] : |T\rangle$$
$$= k_\epsilon k_{\bar\epsilon}(n-1)!(m-1)!\alpha^\dagger_{n-1+\epsilon}\bar\alpha^\dagger_{m-1+\epsilon}|T\rangle = \left(\partial^n X \partial^m \bar X \sigma_{\epsilon,f}\right)(0)|0\rangle_{SL(2)}$$

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- then the SDS vertex is

$$\mathcal{T}_T(d,\bar d) =$$
$$\lim_{u\to 0} :\exp\left\{\sum_{n=1}^{\infty}\left[\bar d_n \partial_u^{n-1}\left[u^{\bar\epsilon}\partial_u X_{op\,T}(u,\bar u)\right] + d_n\partial_u^{n-1}\left[u^{\epsilon}\partial_u \bar X_{op\,T}(u,\bar u)\right]\right]\right\}:$$

since

$$\left[\prod_{n=1}^{\infty}\left(\partial_u^n X\right)^{N_n}\left(\partial_u^n \bar X\right)^{\bar N_n}\sigma_{\epsilon,f}\right](0)|0\rangle_{SL(2)} \leftrightarrow \lim_{u\to 0}\prod_{n=1}^{\infty}\frac{\partial^{N_n}}{\partial \bar d_n^{N_n}}\frac{\partial^{\bar N_n}}{\partial d_n^{\bar N_n}}\mathcal{T}(d,\bar d)\Big|_{d=0}|T\rangle$$

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SDS for excited twists (2)

What if the twist field is not located at $x = 0$?

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$$\mathcal{T}(d, \bar{d}) =$$

$$\lim_{u \to x} : \exp \left\{ \sum_{n=1}^{\infty} \left[\bar{d}_n \partial_u^{n-1} \left[(u - x)^{\bar{\epsilon}} \partial_u X_{op}(u, \bar{u}) \right] + d_n \partial_u^{n-1} \left[(u - x)^{\epsilon} \partial_u \bar{X}_{op}(u, \bar{u}) \right] \right] \right\} :$$

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This is what needed for exciting the other twist fields hidden in the boundary conditions discontinuities.

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Reggeon vertex for N excited twist fields and L untwisted states

We have now all ingredients to compute

$$V_{N+L}(\{c_{(i)n}, d_{(t)n}\}) = \langle 0_{out}| \mathcal{S}_{T(1)}(c_{(1)}, \bar{c}_{(1)}) \dots \mathcal{S}_{T(L)}(c_{(L)}, \bar{c}_{(L)}) \times \\ \times \mathcal{T}_{(1)}(d_{(1)}, \bar{d}_{(1)}) \dots \mathcal{T}_{(N)}(d_{(N)}, \bar{d}_{(N)}) |0_{in}\rangle$$

and get the stated result.

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Notice that for the interactions not in the in Hilbert state we need to use the overlap condition to analytically continue them into the in Hilbert state.
In particular the relations are fundamental

$$[\mathcal{S}(c_{(i)}, \bar{c}_{(i)})|_{Hilbert(D_t D_N)}]_{analytically\ \ cont.} \sim e^{c^I c^J [G_{U(D_{N-1})} - G_{U(D_t)}]} \\ \mathcal{S}(c_{(i)}, \bar{c}_{(i)})|_{Hilbert(D_{N-1} D_N)}$$

and

$$[\mathcal{T}(d_{(t)}, \bar{d}_{(t)})|_{Hilbert(D_t D_N)}]_{analytically\ \ cont.} \sim e^{d^I d^J [G_{N=2,(D_{N-1} D_N)} - G_{N=2,(D_t D_N)}]} \\ \mathcal{T}(d_{(t)}, \bar{d}_{(t)})|_{Hilbert(D_{N-1} D_N)}$$

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The path integral approach

The path integral amounts to computing

$$V_{N+L}(\{c_{(i)}, d_{(t)}\}) = \int_{\mathcal{M}(\{x_t, \epsilon_t, f_t\})} \mathcal{D}X \; e^{-S_E} \prod_{i=1}^{L} \mathcal{S}_{abs}(c_{(i)}, \bar{c}_{(i)}) \; \prod_{t=1}^{N} \mathcal{T}_{abs}(d_{(t)}, \bar{d}_{(t)})$$

where

- $\mathcal{M}(\{x_t, \epsilon_t, f_t\})$ is the space of string configurations satisfying the desired boundary conditions
- $\mathcal{S}_{abs}(c_{(i)}, \bar{c}_{(i)})$ is the abstract operator version of the SDS vertex
- $\mathcal{T}_{abs}(d_{(t)}, \bar{d}_{(t)})$ is the abstract operator version of the SDS vertex

Since the integral is quadratic can be easily done.

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