# Superfluid transport of information in turning flocks of starlings

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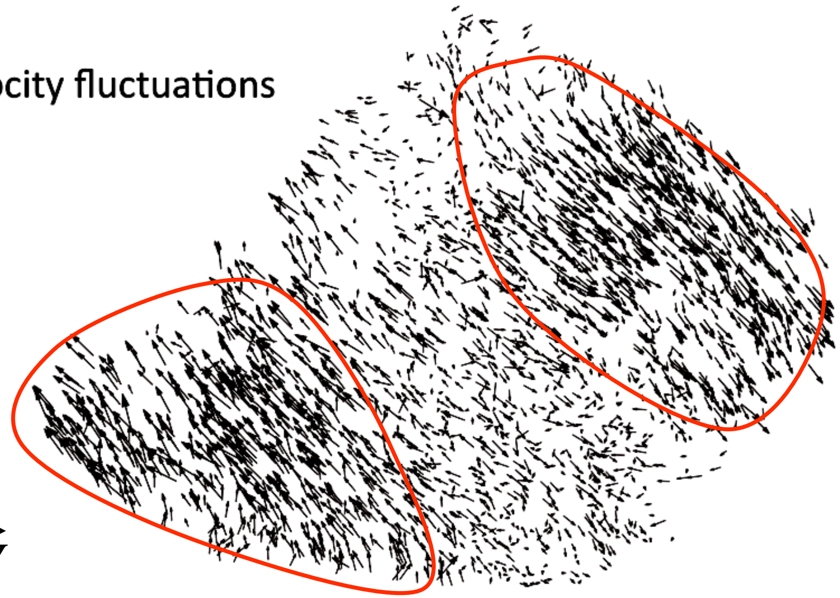
why should theoretical physics bother with this?

# velocity fluctuations in a flock

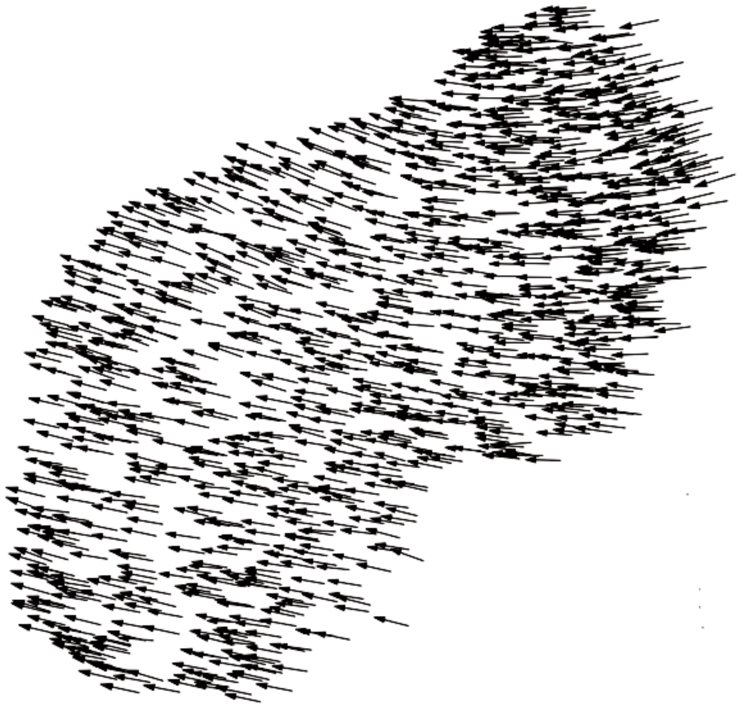
A: full velocities

velocity fluctuations

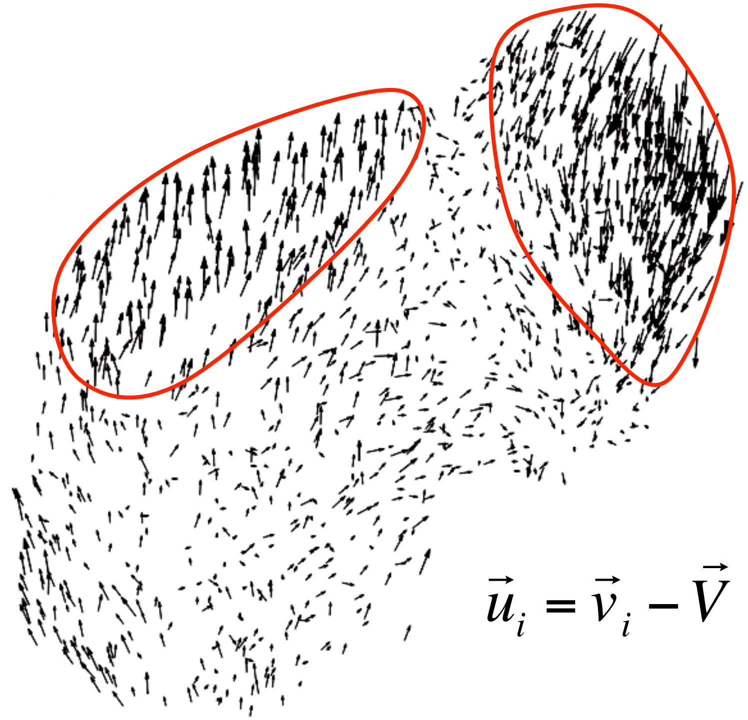


$$\vec{u}_i = \vec{v}_i - \vec{V}$$

A: full velocities

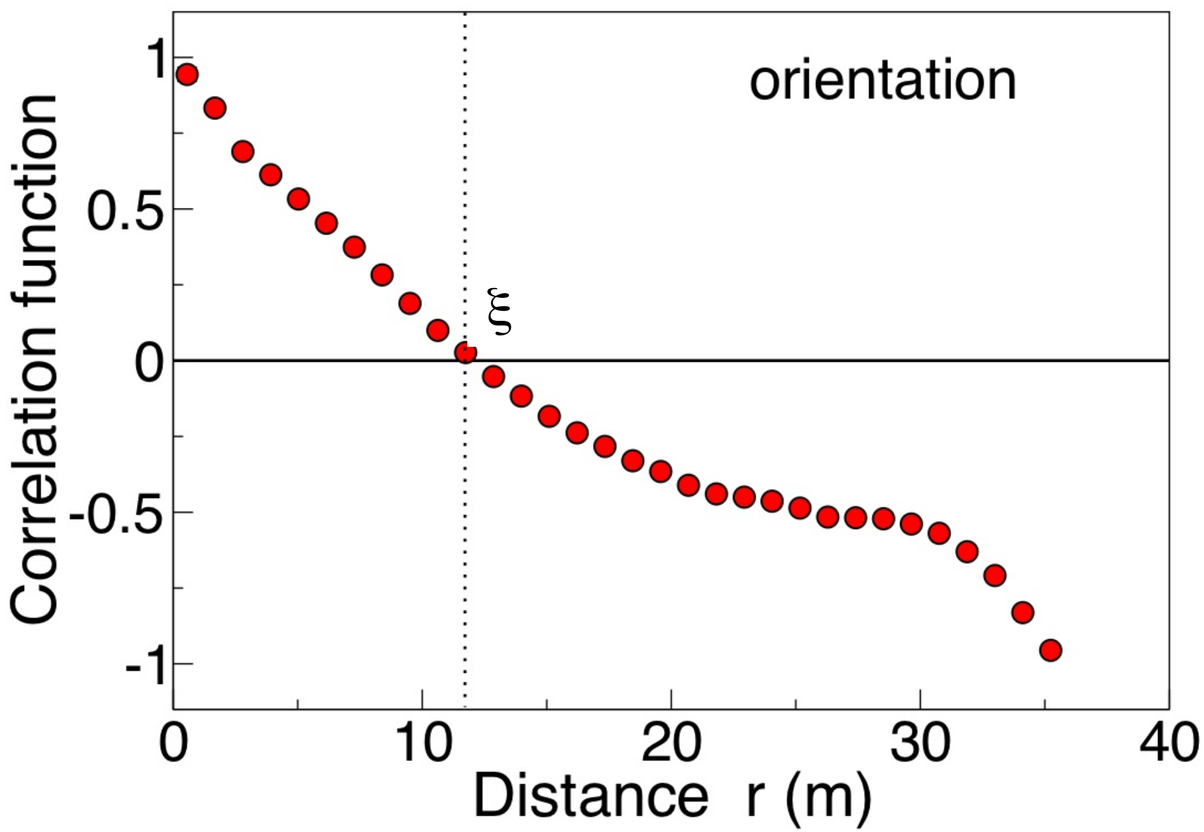


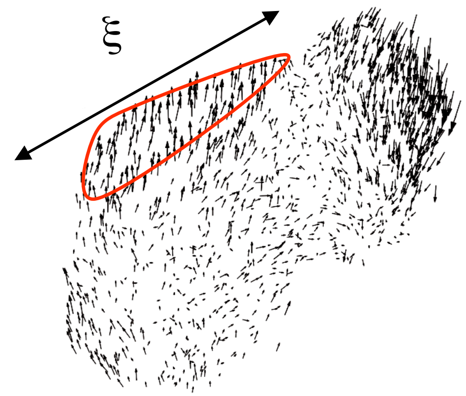
B: velocity fluctuations



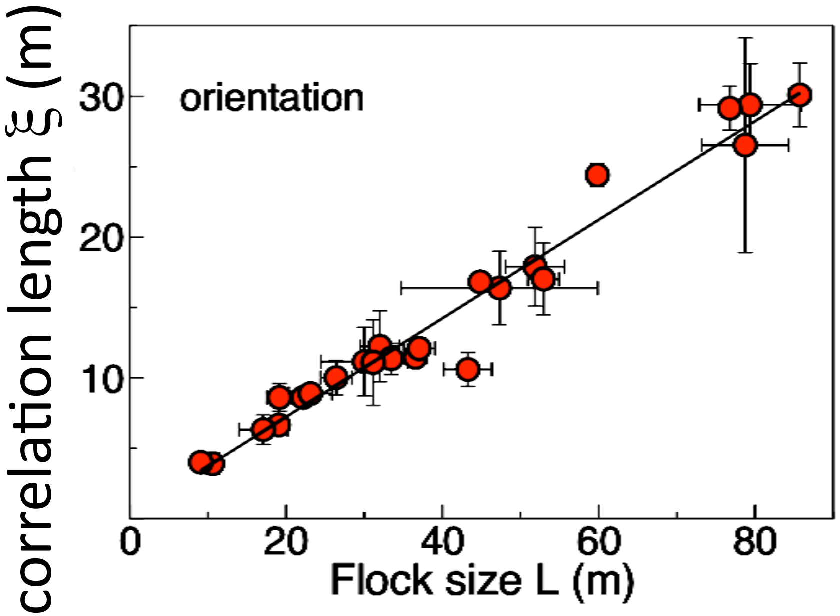
$$\vec{u}_i = \vec{v}_i - \vec{V}$$

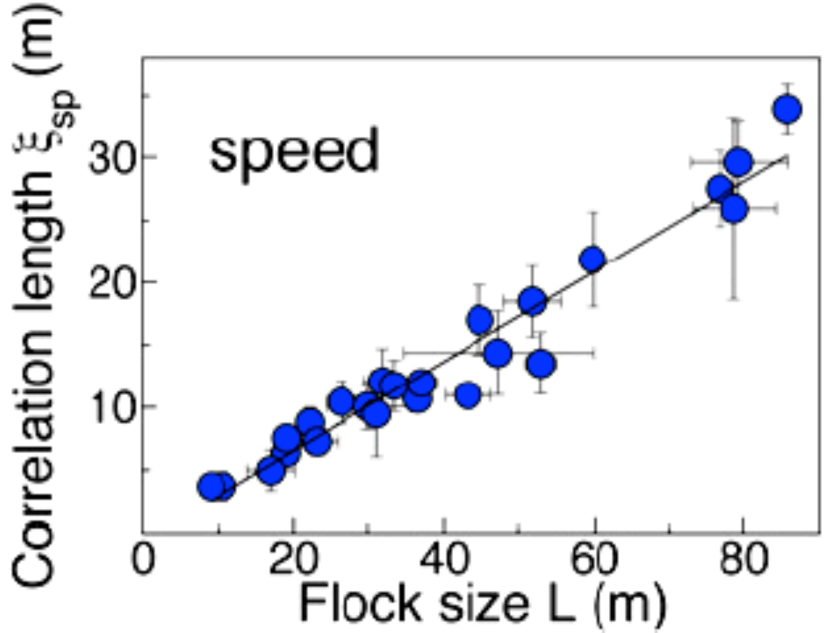
correlation function



## scale-free correlations

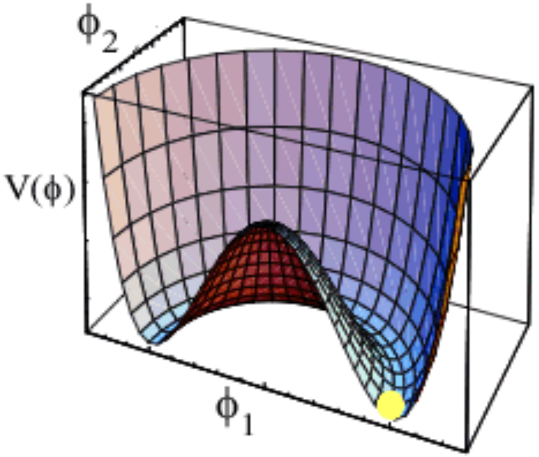



are these two phenomena equally surprising/unsurprising?

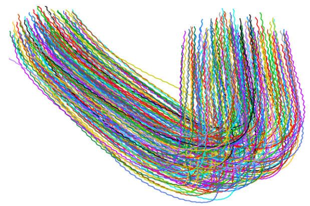
## starlings vs Goldstone's theorem

all birds are flying in the same direction!




- scale-free correlation of the *orientation* fluctuations are quite natural
- scale-free correlation of the *speed* fluctuations are not so natural

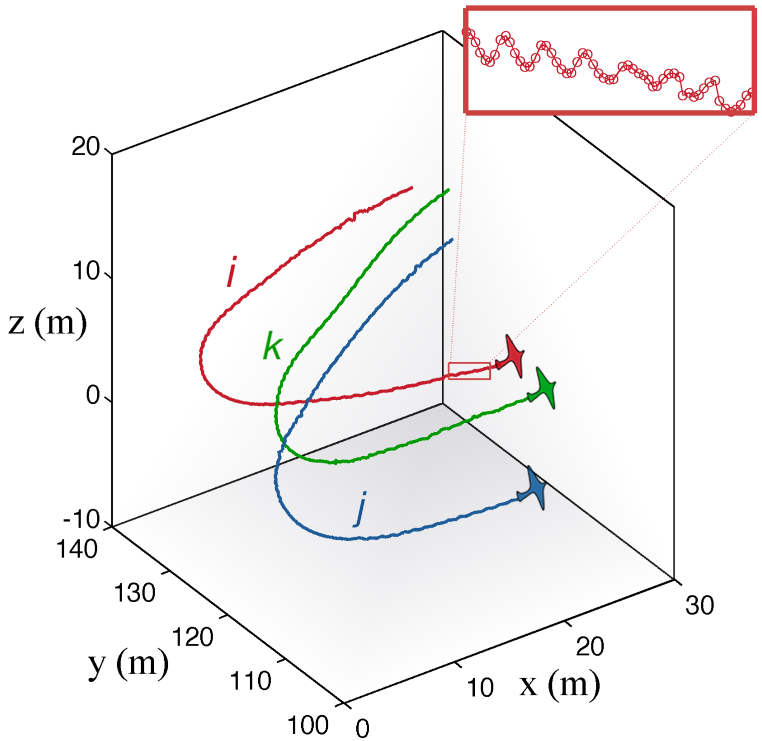
# turning as collective decision-making

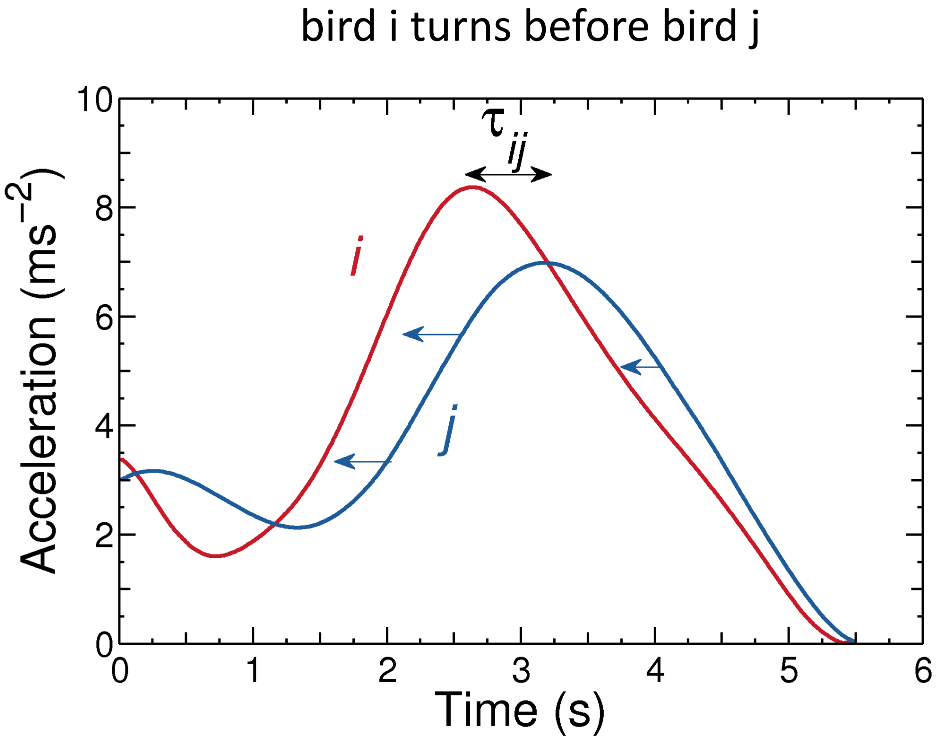


- does the decision to turn originate locally?

- how does the information to turn propagate across the flock?

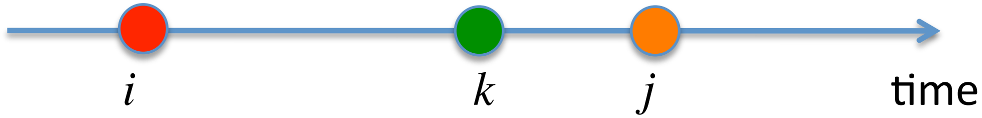
## mutual delay $\tau_{ij}$



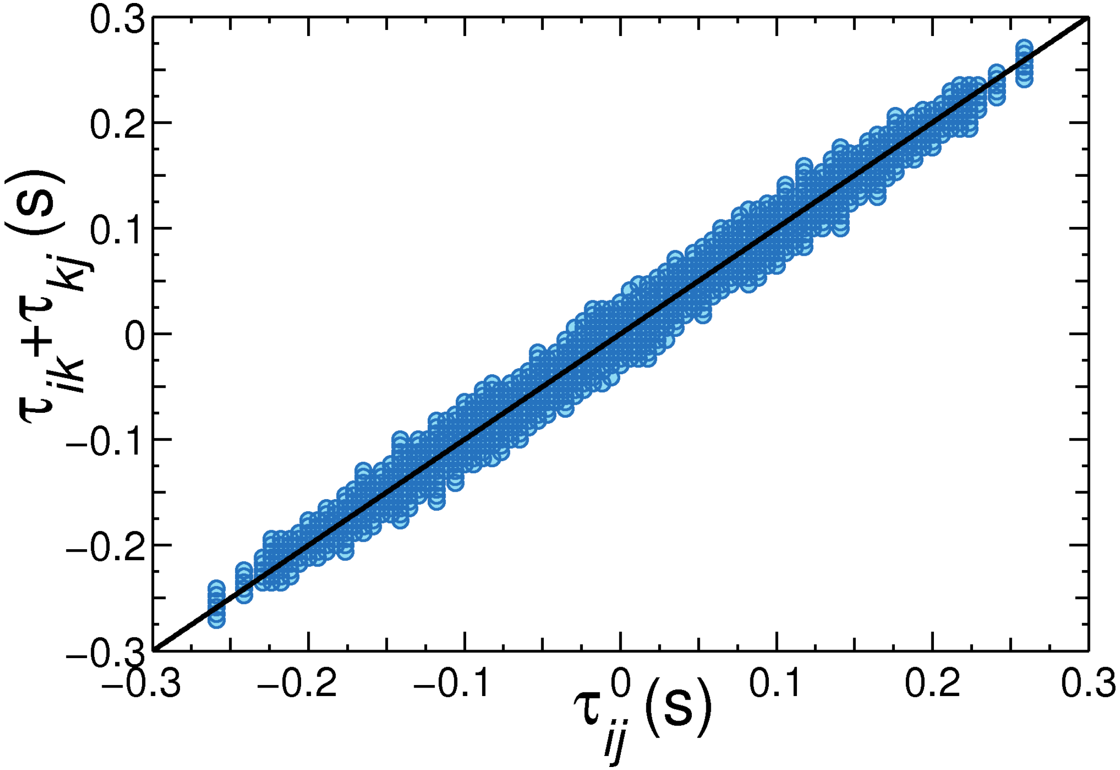




find the delay $\tau_{ij}$ that maximizes the overlap between the two the accelerations

## time ordering check




$$\tau_{ij} = \tau_{ik} + \tau_{kj}$$



# birds ranking

rank birds according to their mutual delays $\tau_{ij}$

| rank | turning time delay |
|---|---|
| 1 | 0 ms – *first bird to turn* |
| 2 | 18 ms |
| 3 | 27 ms |
| 4 | 33 ms |
| 5 | 35 ms |
| 6 | 38 ms |
| 7 | 43 ms |
| 8 | 41 ms |
| 9 | 45 ms |
| ... | ... |

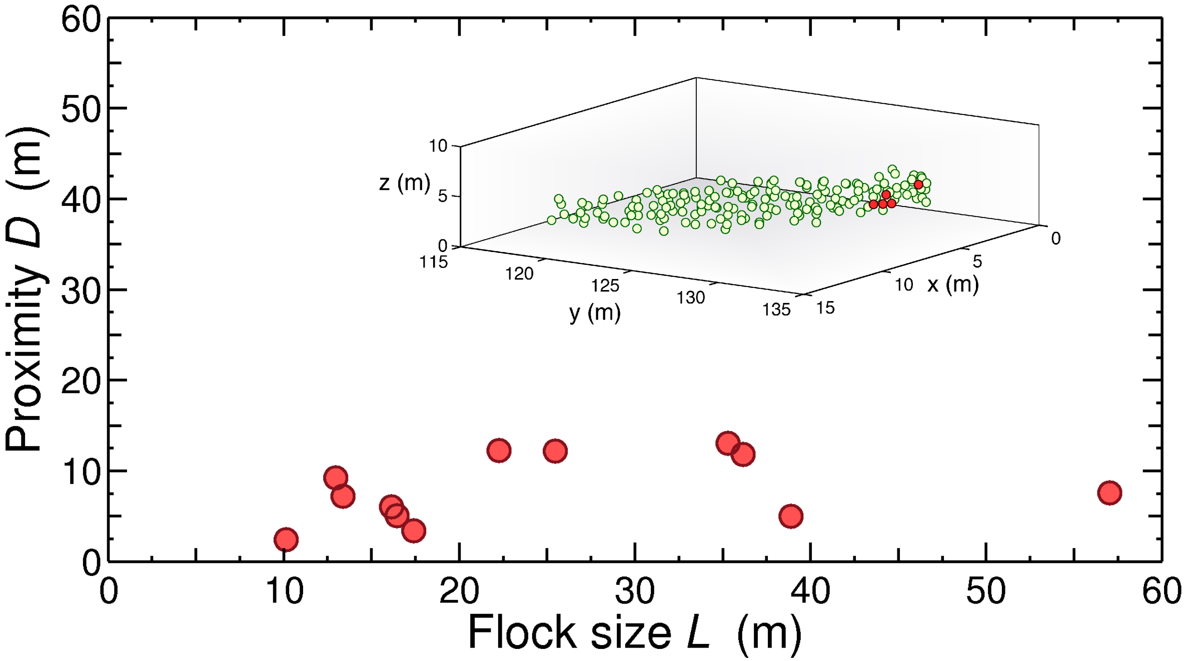
flock_20111214_ACQ4_N154

# localized origin of the turn

`nucleus' = first 5 birds in the rank

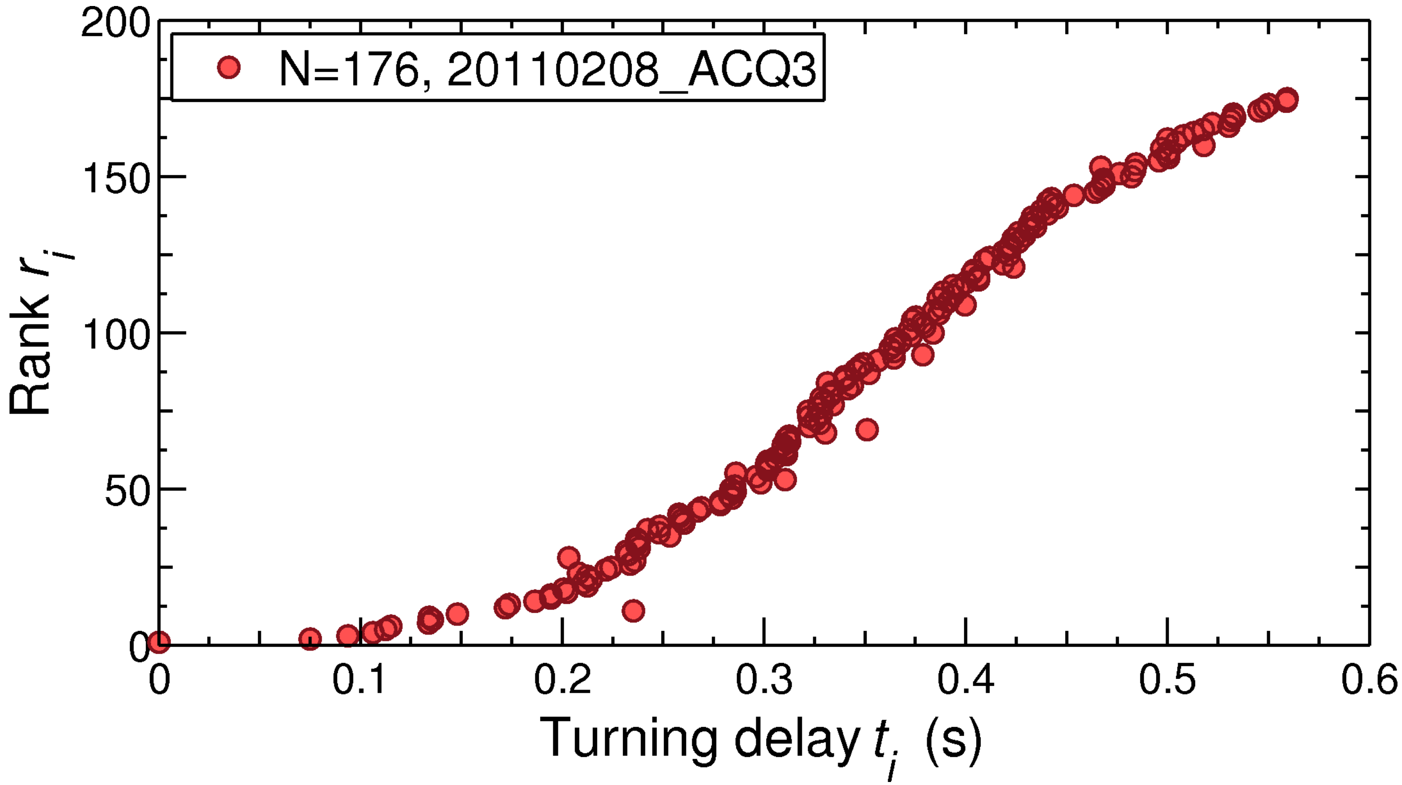
| rank | delay |
|---|---|
| 1 | 0 ms |
| 2 | 18 ms |
| 3 | 27 ms |
| 4 | 33 ms |
| 5 | 35 ms |
| 6 | 41 ms |
| 7 | 43 ms |
| 8 | 45 ms |
| 9 | 48 ms |
| 10 | 49 ms |
| 11 | 51 ms |
| ... | ... |

spatial size of the nucleus does not scale with L




the turn starts localized and then it propagates across the flock what is the dispersion relation?

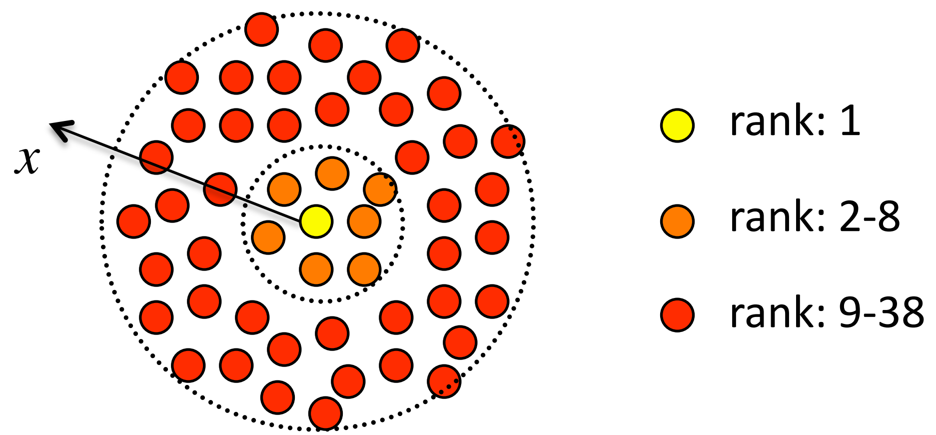
ranking curve



# ranking and propagation

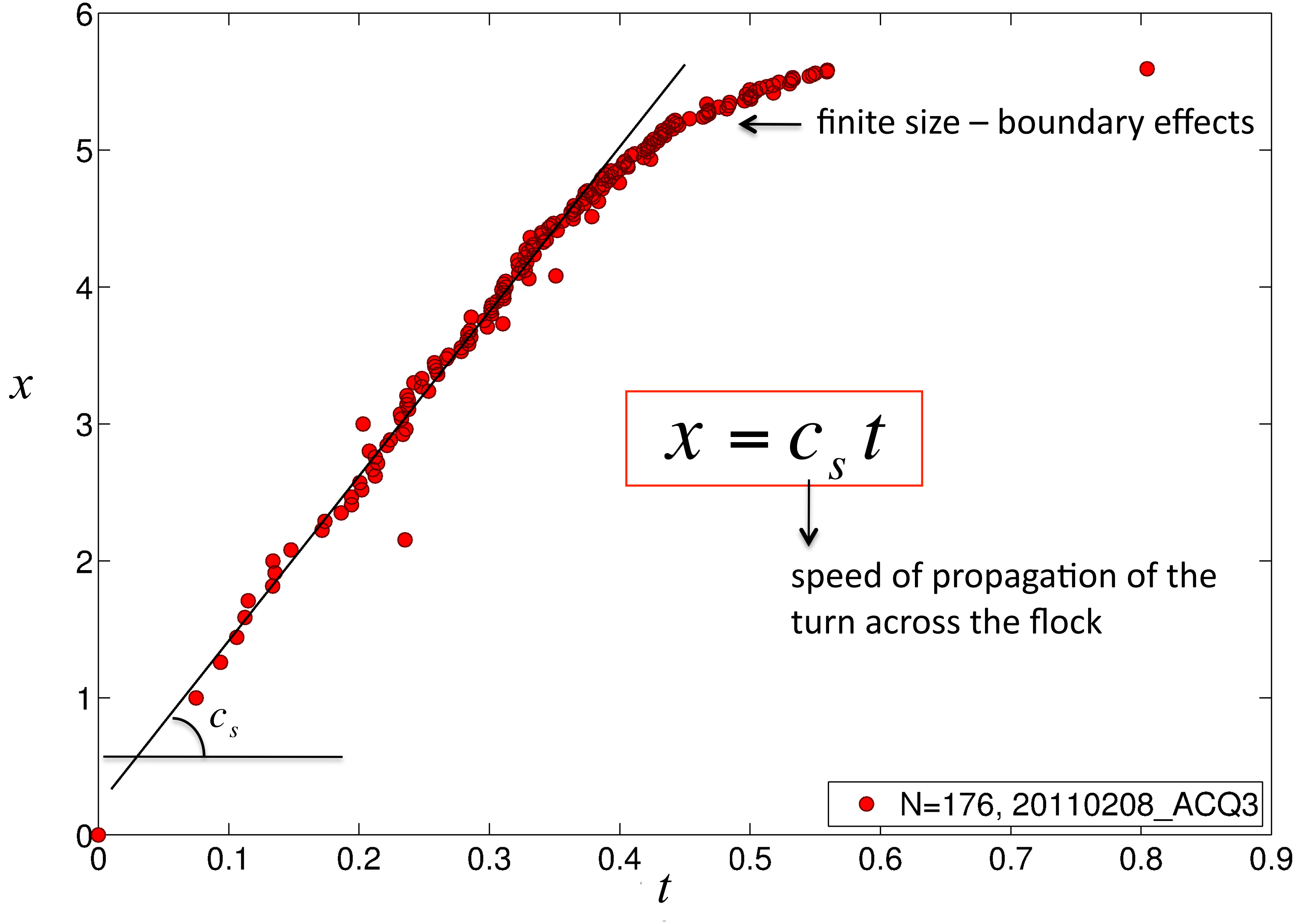
if the turn starts localized then:

*rank = (density $\rho$) x (distance traveled by the turn $x$)$^3$*

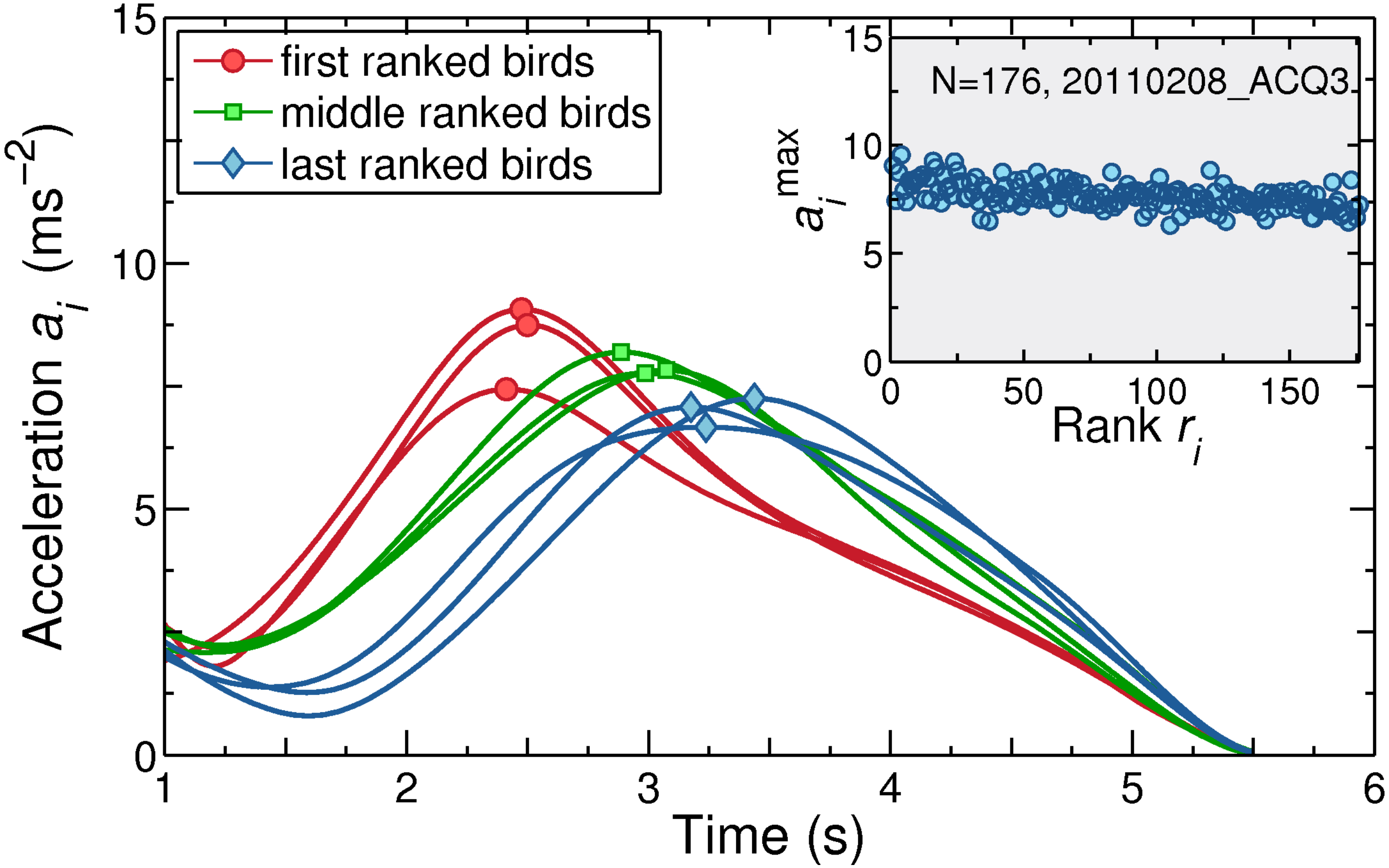



$$x(t) = \left[\frac{rank(t)}{\rho}\right]^{1/3}$$

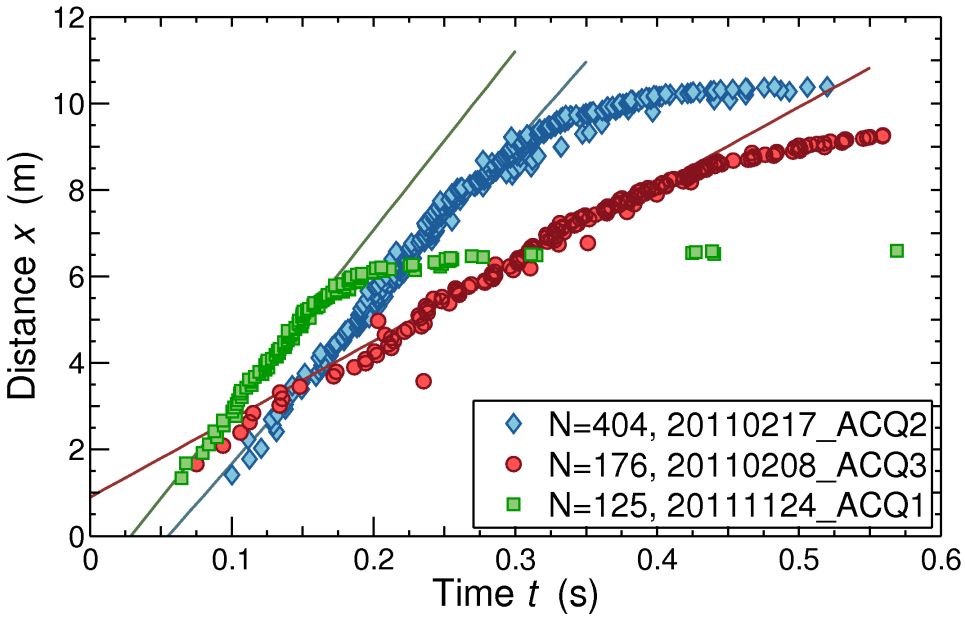
# linear dispersion law

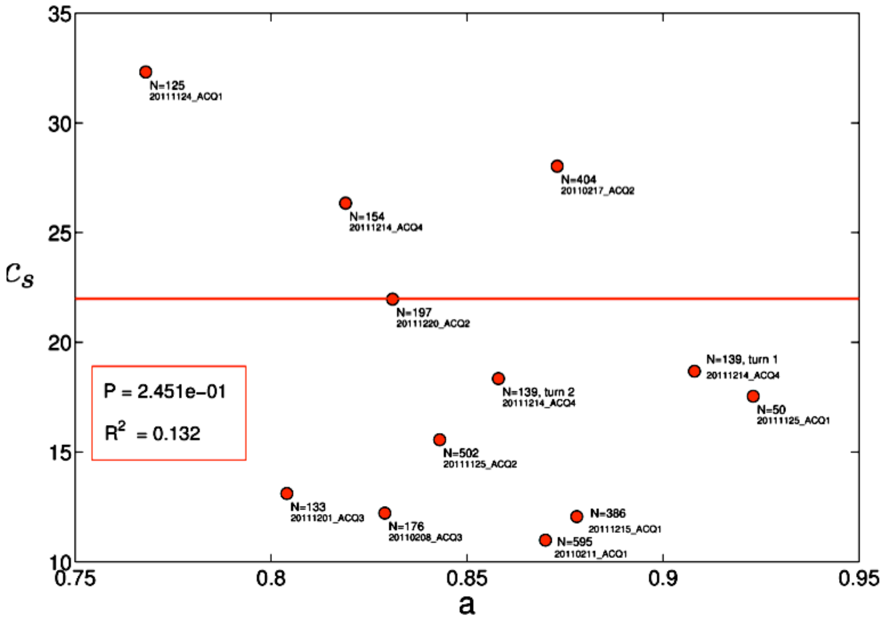


## very weak attenuation



# great variability of the speed of propagation $c_s$




nearest neighbours distance

what does $c_s$ depend on?

## questions

- why a linear propagation law?

  *spin waves (orientation), not density waves*

- why a very weak attenuation?

- how to make sense of the variability of $c_s$ ?

theoretical physics description

# old theory of flocking

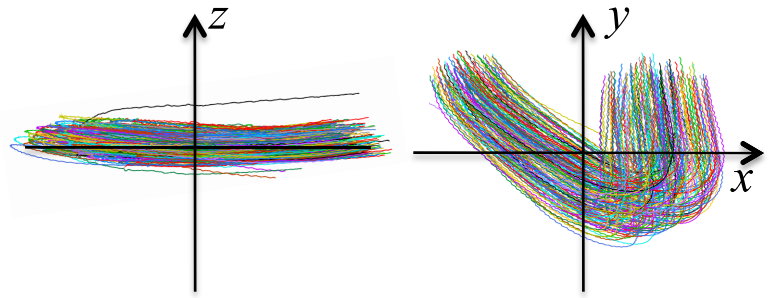
$$\boxed{\vec{v}_i(t+1) = \vec{v}_i(t) + J\sum_{j\in i}\vec{v}_j(t) + \vec{\xi}_i}$$

Vicsek flocking model

$$\frac{d\vec{v}_i}{dt} = -\frac{\partial H}{\partial \vec{v}_i} + \vec{\xi}_i \qquad H = -J\sum_{<ij>}\vec{v}_i\cdot\vec{v}_j$$

fixed velocity modulus

- planar order parameter - introduce the phase φ: $v_i^x + iv_i^y = v\,e^{i\varphi_i}$



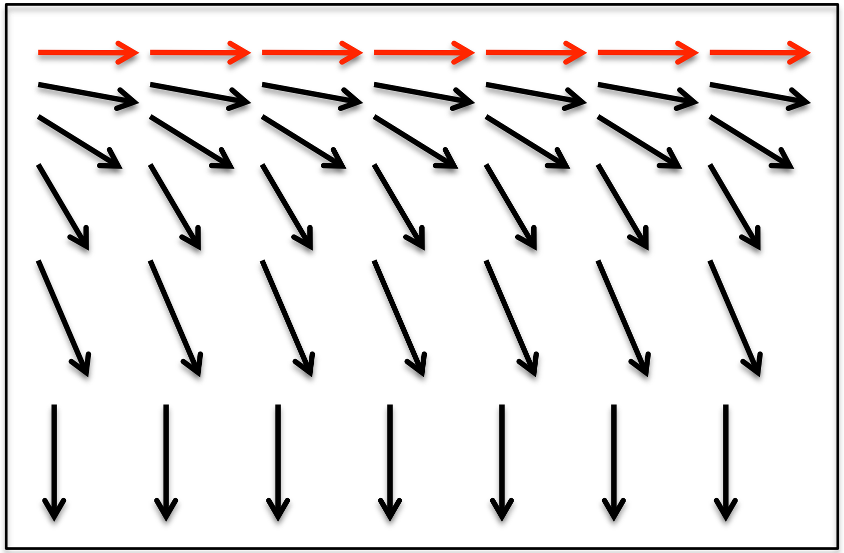

- high polarization (low T) - spin wave expansion:

$$\varphi \sim 0 \quad \Rightarrow \quad H = \frac{1}{2}J\sum_{<ij>}(\varphi_i - \varphi_j)^2 = \frac{1}{2a}J\int d^3x\left[\vec{\nabla}\varphi(x,t)\right]^2$$

$$\frac{\partial\varphi}{\partial t} = -\frac{\delta H}{\delta\varphi} = \nabla^2\varphi \qquad \omega = ik^2 \qquad x\sim\sqrt{t}$$

- damping ✗
- diffusive propagation ✗

bottom line: classic spin waves are diffusive



$$\omega = ik^2$$

## two problems with the old theory

The theory is rotationally invariant - all flight directions are the same
however, there is **no conservation law**.

$$\varphi_i \longrightarrow \varphi_i + \delta\varphi$$ *the phase is the generator of the symmetry*

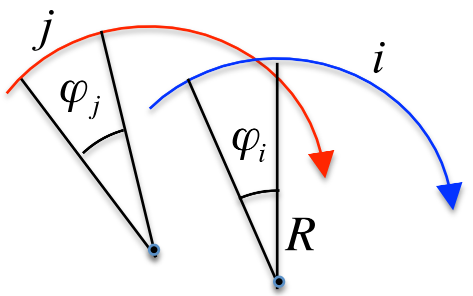
*`An excitation of a conserved field cannot be relaxed locally. It must be transported away.'*
[HH69]

The theory completely **neglects behavioural inertia.**

$$\dot{\varphi} = F_s = \nabla^2\varphi \qquad \textit{rather than:} \qquad \ddot{\varphi} = F_s$$

this is biologically implausible: a bird cannot perform a U-turn in 1 time-step

## superfluid theory of flocking




we introduce the birds' spin $s_z$

$s_z$ is the generator of the rotation parametrized by $\varphi$

$$\{\vec{v}, s_z\} = \frac{\partial \vec{v}}{\partial \varphi} = i\vec{v} \qquad v_i^x + i v_i^y = v\, e^{i\varphi_i}$$

reinstate the `kinetic' term: $H = \int d^3x \left[\frac{1}{2} J \left(\vec{\nabla}\varphi\right)^2 + \frac{s_z^2}{2\chi}\right]$ $\chi$ is a generalized inertia

canonical equations for the conjugated pair $(\varphi, s_z)$:

$$\begin{cases} \dfrac{\partial \varphi}{\partial t} = \dfrac{\delta H}{\delta s_z} = \dfrac{s_z}{\chi} \\[2ex] \dfrac{\partial s_z}{\partial t} = -\dfrac{\delta H}{\delta \varphi} = J\, \nabla^2 \varphi \end{cases}$$

conservation law – continuity equation: $\dfrac{\partial s_z}{\partial t} - \vec{\nabla} \cdot \vec{j}_z = 0$ with: $\vec{j}_z = J\, \vec{\nabla}\varphi$

this is Model F of Hohenberg-Halperin ['69], mathematically equivalent to the quantum lattice gas model of Matsubara-Matsuda ['56] for superfluid liquid helium

## new spin-wave equation

equations of motion

$$\frac{\partial \varphi}{\partial t} = \frac{1}{\chi} s_z \qquad \frac{\partial s_z}{\partial t} = J\, \nabla^2 \varphi$$

$$\Rightarrow \quad \frac{\partial^2 \varphi}{\partial t^2} - \frac{J}{\chi} \nabla^2 \varphi = 0$$

D'Alembert equation $\quad c_s = \sqrt{\frac{J}{\chi}}$

linear dispersion law: $\quad \omega = c_s k \qquad x = c_s t$ ✔

this is the law of second-sound propagation in superfluid Helium

# predictions of the new theory

speed of propagation of the turn across the flock: $c_s = \sqrt{\frac{J}{\chi}}$

the coupling $J$ can be measured through the order parameter $\Phi$ :

$$\Phi = \left| \frac{1}{N} \sum_i \vec{v}_i \right| = 1 - \frac{1}{2} \frac{1}{N} \sum_i \varphi_i^2 = 1 - \frac{1}{2} \langle \varphi^2 \rangle$$

$$P(\varphi) \sim \exp\left( -\frac{1}{2} \beta \int d^3x \, J(\vec{\nabla}\varphi)^2 \right) = \exp\left( -\frac{1}{2} \beta \int d^3k \, J \, k^2 \varphi_k \varphi_{-k} \right)$$

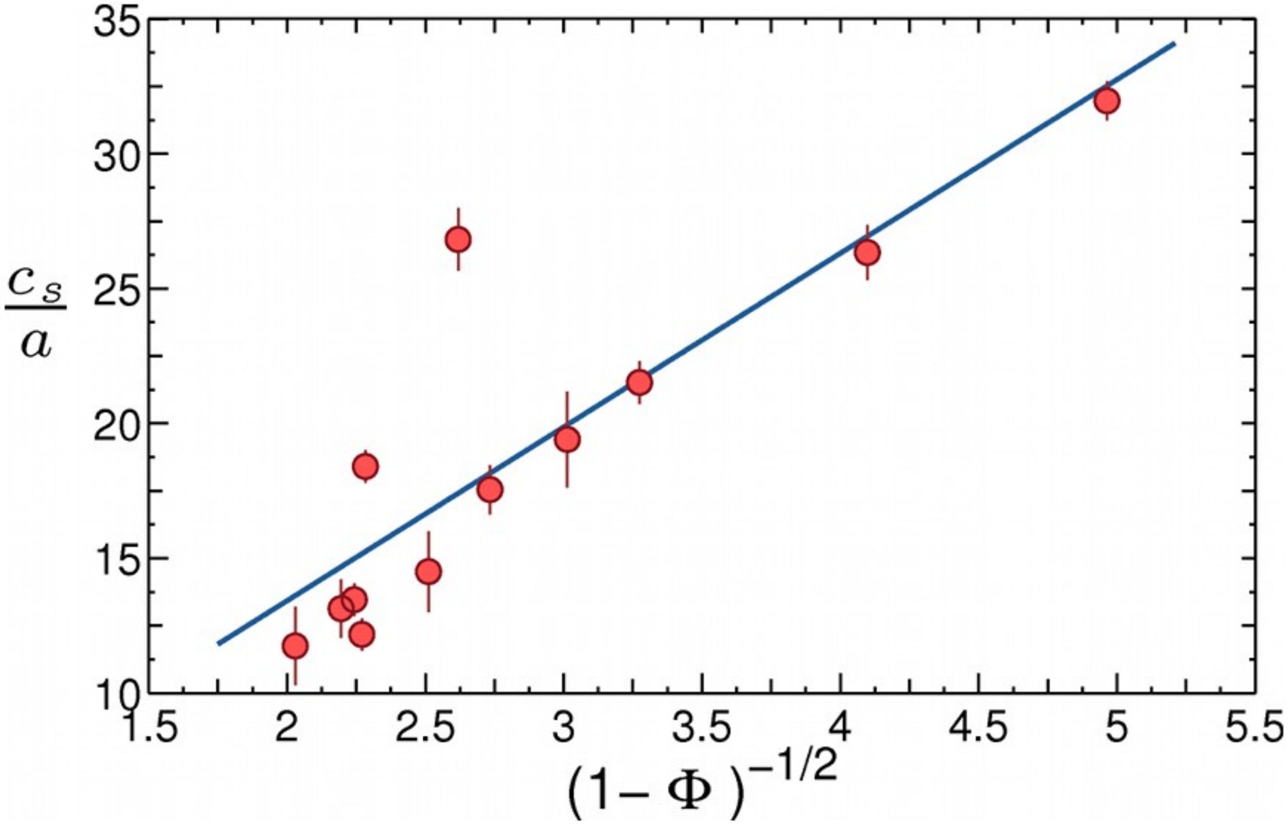
$$\Phi = 1 - \frac{1}{2} \langle \varphi^2 \rangle = 1 - \int d^3k \, \frac{1}{\beta J k^2} \sim 1 - \frac{1}{\beta J} \quad \Rightarrow \quad J = \frac{1/\beta}{1 - \Phi}$$

$$\Rightarrow \quad \boxed{c_s = \sqrt{\frac{1}{\beta\chi}} \frac{a}{\sqrt{1 - \Phi}}}$$

*the speed of propagation of the turn must be larger in more ordered flocks*

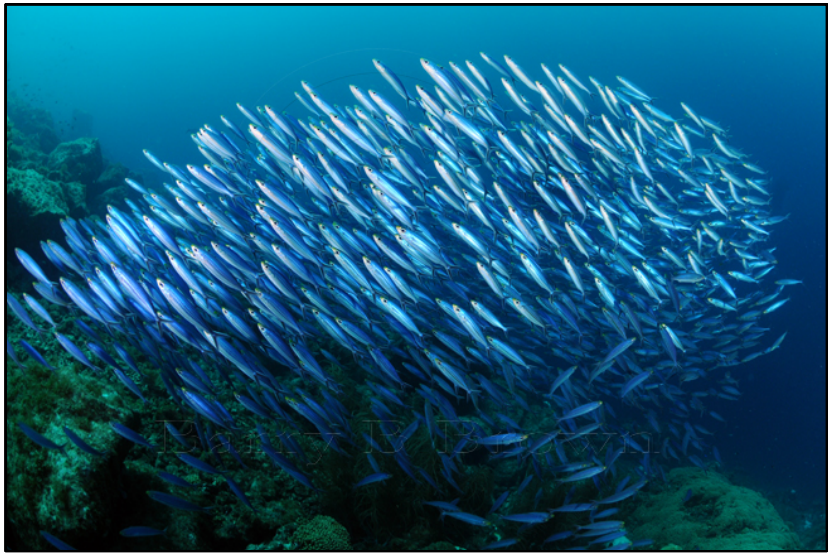
both $\Phi$ and $c_s$ are experimentally accessible quantities

## experimental test of the new theory



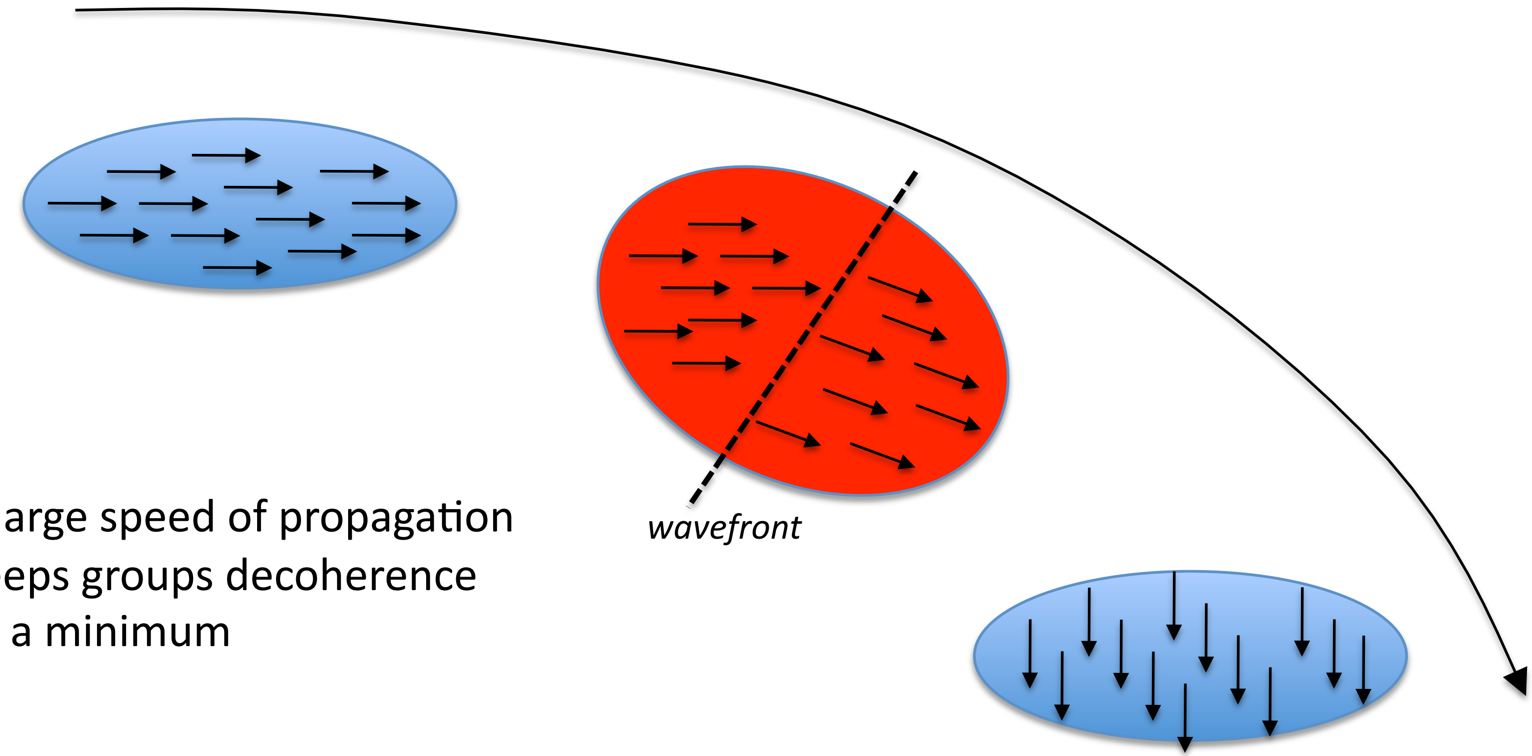

$$c_s \propto \frac{a}{\sqrt{1-\Phi}}$$

## why natural groups are so polarized ?



*starling flocks have packing fraction of the order of 0.01*
*and polarization around 0.98*

# the group is fragile during the decision




a large speed of propagation
keeps groups decoherence
to a minimum

$$c_s \sim \frac{1}{\sqrt{1-\Phi}}$$

to achieve a large speed of propagation of the information, a large polarization is necessary

# conclusions

the link between swift decision-making and large polarization may be the evolutionary drive behind the strong ordering observed in many living groups

the mathematical equivalence between flocking and superfluidity shows that symmetries and conservation laws work in biology too

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