Anisotropy in the $Q\overline{Q}$-potential in a magnetic field

Francesco Negro

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UniPi

Istituto Nazionale
di Fisica Nucleare

INFN

Collaborators: Claudio Bonati, Massimo D'Elia, Marco Mariti, Michele Mesiti and Francesco Sanfilippo.

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Outline

- Phenomenological Motivation
- Static $Q\overline{Q}$ potential
- Lattice QCD & Magnetic Fields (eB)
- Static $Q\overline{Q}$ potential in the presence of (eB)
- Conclusions

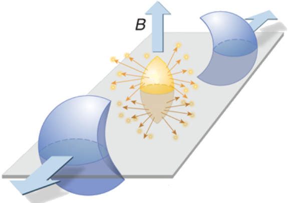
Phenomenological Motivation

QCD is a strongly interacting theory.
In the IR regime the coupling constant is large, hence it displays many non-perturbative properties (confinement, chiral simmetry breaking, ...).

ElectroWeak corrections are often small if compared to the Strong int.
But: what happens if we consider the presence of an external magnetic field, eB, large enough to be comparable with the scale Λ_{QCD}?

- Astrophysics - in a class of neutron stars, called **magnetars**: $eB \sim 10^{10}$ T
  [Duncan and Thompson, '92]
- Cosmology - during the **ElectroWeak phase transition**: $eB \sim 10^{16}$ T
  [Vachaspati, '91]
- Heavy ion collisions - at LHC in
  non-central HIC: $eB \sim 10^{15}$ T $\sim 15 m_\pi^2$
  [Skokov, Illarionov and Toneev, '09]

$1 \text{ GeV}^2 \sim 5 \cdot 10^{15}$ T



The static $Q\overline{Q}$-Potential

$V_{Q\overline{Q}}$ is a non-perturbative feature of QCD.

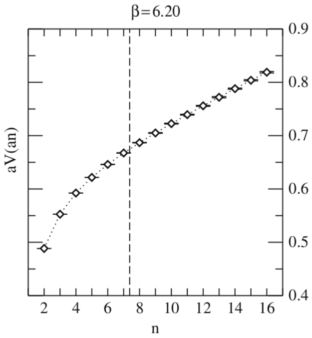
A property of gauge fields only.

Coulomb term + linear term

$\rightarrow$ **Cornell Potential**

$$V_{Q\overline{Q}}(\vec{r}) = C + \sigma|\vec{r}| + \frac{\alpha}{|\vec{r}|}$$

- $\sigma \equiv$ String Tension
- $\alpha \equiv$ Coulomb Parameter



- It gives a description of the phenomenon of confinement
- Spectrum for heavy mesons: e.g. charmonia and bottomonia

$\rightarrow$ NR bound states of heavy quarks ($c\overline{c}$,$b\overline{b}$)

POSSIBLE DEPENDENCE ON eB?

PRESENCE OF ANISOTROPIES?

The Lattice QCD approach

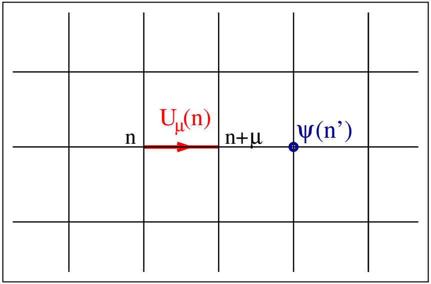
NP properties of QCD? $\Longrightarrow$ **Lattice QCD**

Lattice QCD in 3 steps:

1 − Feynman path integral formulation of the Euclidean theory

2 − We regularize the integral by introducing a finite lattice:
UV cut-off $\rightarrow a$ IR cut-off $\rightarrow V$

3 − We get a well defined multidimensional integral
$\rightarrow$ Monte Carlo approach



$$\langle \hat{O} \rangle = \frac{\int \mathcal{D}U O[U] e^{-S_E[U]}}{\int \mathcal{D}U e^{-S_E[U]}}, \quad \text{where } U_\mu(x) = \exp(i g_s a A_\mu(x)).$$

Integration variables $\longrightarrow$ the links (elementary parallel transporters);
3×3 matrices belonging to the Gauge Group $SU(3)$.

Lattice QCD & Magnetic fields

The degrees of freedom for QED are abelian phases $u_\mu(n)$.
They enter the fermionic action:

$$S_E = S_{YM}^{SU(3)} + S_{Ferm}, \quad \text{where}$$

$$S_{Ferm} = \sum_n \Big[am\overline{\chi}(n)\chi(n) + \frac{1}{2}\sum_\mu \eta_\mu(n)\left(\overline{\chi}(n)U_\mu(n)u_\mu(n)\chi(n+\hat{\mu}) + \right.$$
$$\left. - \overline{\chi}(n)U_\mu^\dagger(n-\hat{\mu})u_\mu^\dagger(n-\hat{\mu})\chi(n-\hat{\mu})\right)\Big] .$$

We fix them in order to produce the desired magnetic field.
We will consider uniform magnetic field along the Z direction.

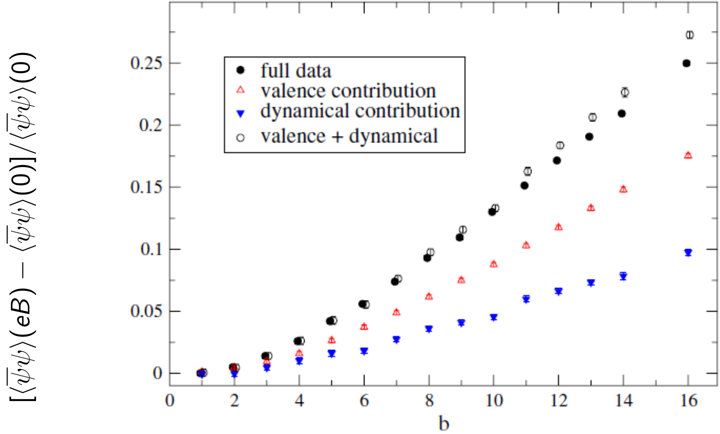
The magnetic field can influence the gluon field through quark loops.

HOW MUCH?

Chiral Condensate at non-zero (eB)

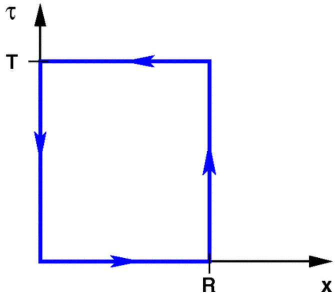
Deviation: dependence of the chiral condensate $\langle\overline{\psi}\psi\rangle$ on the field (eB).

[D'Elia and N, '11]



We disentangled the contribution due to the modification of the gluon fields: 40% of the total signal.

The $Q\overline{Q}$ potential from Lattice QCD



It is possible to extract the static inter-quark potential by computing the **Wilson Loop** observables

$$\mathbf{W}(\vec{\mathbf{R}}, \mathbf{T})$$

They are rectangular $R \times T$ loops built up of link variables $U_\mu(n)$.

$$aV_{Q\overline{Q}}(a\vec{n}) = \lim_{n_t \to \infty} \log\left(\frac{\mathbf{W}(a\vec{n}, an_t)}{\mathbf{W}(a\vec{n}, a(n_t+1))}\right)$$

They correspond to the following process:

- creation of a quark-antiquark pair at distance $\vec{\mathbf{R}}$
- imaginary time propagation for an interval of time $\mathbf{T}$
- annihilation of the pair.

$$\langle \mathbf{W}(\vec{\mathbf{R}}, \mathbf{T}) \rangle \simeq C \exp\left(-\mathbf{T}\, V_{Q\overline{Q}}(\vec{\mathbf{R}})\right)$$

The $Q\overline{Q}$ potential at $eB \neq 0$

Rotation simmetry is broken by the magnetic field $e\vec{B} = eB\hat{Z}$.

We cannot average over Wilson Loops with different spatial orientations!

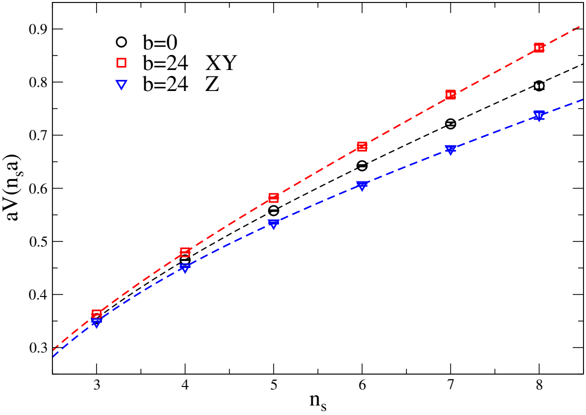
We get different results for parallel Z and perpendicular $X - Y$ directions.

Simulation Details:

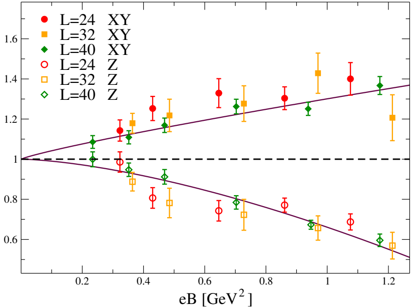
- QCD with $N_f = 2 + 1$
- Physical Quark masses
- State of art discretizations

For this plot:

- $a = 0.1249$ fm
- Volume $\rightarrow$ 40^4
- $eB = 0.7$ GeV^2



String Tension and Coulomb Parameter



Lattices:

$(24^4, 0.2173 \text{ fm})$
$(32^4, 0.1535 \text{ fm})$
$(40^4, 0.1249 \text{ fm})$

Ratios:

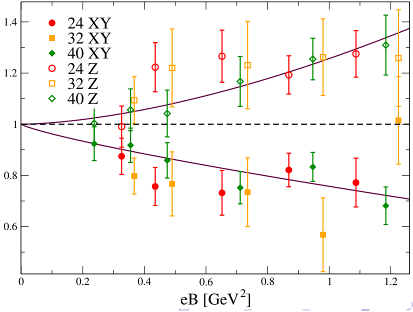
$R^{\mathcal{O}}_{DIR} = \mathcal{O}_{DIR}(eB)/\mathcal{O}(0)$

$\longleftarrow$ **String Tension ratio** $(R^{\sigma}_{XY}, R^{\sigma}_{Z})$

$$\frac{\sigma_{XY}(eB)}{\sigma(0)} > \frac{\sigma_Z(eB)}{\sigma(0)}$$

Coulomb Param. ratio $(R^{\alpha}_{XY}, R^{\alpha}_{Z}) \longrightarrow$

$$\frac{\alpha_{XY}(eB)}{\alpha(0)} < \frac{\alpha_Z(eB)}{\alpha(0)}$$



The solid lines corresponds to fit on 40^4 data according to the power law

$$R = 1 + A \cdot (eB)^C$$

Possible Phenomenology

Modification of the heavy meson spectrum [Alford and Strickland, '13]

NR treatment of the heavy quark-antiquark pair in the presence of eB.
Hamiltonian with the $eB \equiv 0$ Cornell Potential & spin-spin interaction

At $eB \simeq 0.3$ GeV^2:
$\eta_c, \; J/\Psi \qquad \Delta m/m \sim 10\%$
$\eta_b, \; \Upsilon \qquad \Delta m/m \sim 1\%$

Mixings of the singlet and triplet states.

Computed with an isotropic potential $\longrightarrow$ Further effect due to the anisotropy at $eB \neq 0$.

Work in progress with A. Rucci

Possible effect in Heavy Ion Collisions

An influence on cross-sections and decay rates has been pointed out.
[Strickland, Nohorona et al. '13]

In Heavy Ion Collisions the dynamic which determines the final particle multiplicity is very complicated.

Even small corrections may have large effects on the final multiplicities.

Conclusions and Perspectives

Conclusions:

- Determiantion of $V_{Q\overline{Q}}$ at non-zero eB.
- Gauge fields gets modified by $eB \longrightarrow$ anisotropy
- Determination of σ and α
- Possible phenomenological relevance

Open questions:

- Complete angular dependence of $V_{Q\overline{Q}}$: still missing.
- Vanishing string tension along Z for large enough eB?
- What happens at finite temperature T? Influence on HIC?
- Need for a direct lattice computation of the spectrum!

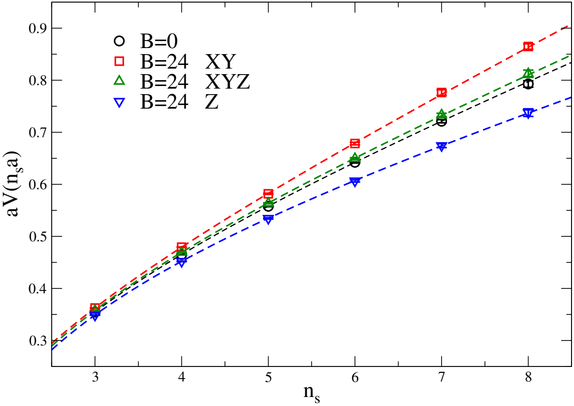
Thank you!

Backup - XYZ average

We cannot average over Wilson Loops with different spatial orientations!
But: what happens if we try to do it?

The dependence on eB apparently disappears!

That is why previous studies on this subject didn't reported any significant dependence of the potential on the magnetic field.



Backup - Timescales in HIC

- *Formation time of the plasma* $\rightarrow$ $\tau_f \sim 0.2$ fm

- *Temperature* [Zhao and Rapp, '11]

Assumption for the deconfinement temperature: $T_c \simeq 170$ MeV

T	RHIC at 200 GeV	LHC at 2.76 TeV
$T > 2T_c$	-	$\tau_f < \tau < 1\,fm/c$
$T_c < T < 2T_c$	$\tau_f < \tau < 3\,fm/c$	$1\,fm/c < \tau < 6\,fm/c$
$T = T_c$	$3\,fm/c < \tau < 5\,fm/c$	$6\,fm/c < \tau < 9\,fm/c$

- *Magnetic Field*

Estimate of *eB* time evolution @ RHIC for $Au - Au$ collisions for two values of $\sqrt{s_{NN}}$.

As the collision energy increases the magnetic field increases, but it gets more shrinked in time.

[Skokov, Illarionov and Toneev, '09]