Flux tubes in the SU(3) vacuum: London penetration depth and coherence length

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The color confinement problem

Figure: $q\bar{q}$ pair at distance R in the QCD vacuum

Deconfined phase

$$E_0(R) \overset{R \to \infty}{\longrightarrow} 2m$$

Confined phase

$$E(R) \longrightarrow \sigma R, \ \sqrt{\sigma} = 420 \text{ MeV}$$

At the scale of color confinement non perturbative methods are needed

Dual superconductivity

Dual superconductor picture of confinement in QCD was proposed by Mandelstam and 't Hooft.

[G. 't Hooft, in High Energy Physics, EPS International Conference, (1975)]

[S. Mandelstam, Phys. Rep. 23, (1976)]

QCD vacuum as a dual superconductor

- Color confinement due to the dual Meissner effect produced by the condensation of chromomagnetic monopoles
- Chromoelectric field connecting a $q\bar{q}$ static pair squeezed inside a tube structure: Abrikosov vortex

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SUPERCONDUCTIVITY	DUAL SUPERCONDUCTIVITY
$F^{\mu\nu}$	$\tilde{F}^{\mu\nu}$
Electric charges condensate (Cooper pairs)	Magnetic monopoles condensate
Magnetic Abrikosov flux tubes	Chromoelectric dual Abrikosov flux tubes

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The dual superconductor model

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Coherence length and London penetration depth
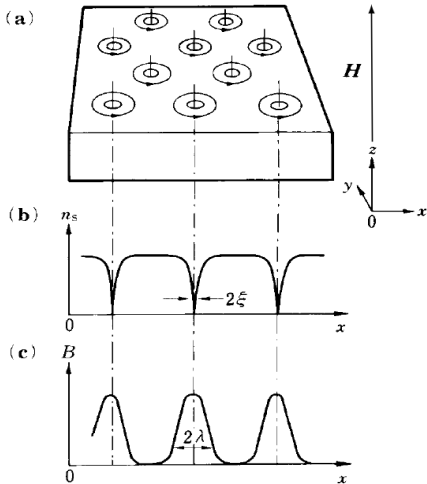
[A. C. Rose-Innes and E. H. Rhoderick, Introduction to Superconductivity (Pergamon Press, Second edition, 1978)]

- λ London penetration depth: characteristic length of the exponential decrease of $\vec{B}$ in a superconductor
- ξ Coherence length: length scale on which the density of Cooper pairs can change appreciably (distance between the two electrons in a pair)

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The dual superconductor model

Fitting functions for $E_l(x_t)$ shape

Here enters the dual superconductor model

- Ordinary superconductivity: magnetic field as function of the distance from a vortex line in the mixed state
- Two different expressions coming from the London model or, equivalently, the Ginzburg-Landau theory
- Dual analogy

1 Vortex as a line singularities

$$E_l(x_t) = \frac{\Phi}{2\pi} \mu^2 K_0(\mu x_t), \quad x_t > 0, \quad \lambda \gg \xi \leftrightarrow \kappa \ll 1$$

[P. Cea and L. Cosmai, Nucl. Phys. B, Proc. Suppl. 30, (1993)]

2 Cylindrical vortex

$$E_l(x_t) = \frac{\phi}{2\pi} \frac{1}{\lambda \xi_v} \frac{K_0(R/\lambda)}{K_1(\xi_v/\lambda)},$$

[J. R. Clem, J. Low Temp. Phys. 18, 427 (1975)]

[P. Cea, L. Cosmai, and A. Papa, Phys. Rev. D 86, (2012)]

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Fitting function in our work

$$E_l(x_t) = \frac{\phi}{2\pi} \frac{\mu^2}{\alpha} \frac{K_0[(\mu^2 x_t^2 + \alpha^2)^{1/2}]}{K_1[\alpha]} \qquad x_t \geq 0,$$

$$R = \sqrt{x_t^2 + \xi_v^2}, \qquad \mu = \frac{1}{\lambda}, \qquad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}, \qquad \kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha} \left[1 - K_0^2(\alpha)/K_1^2(\alpha)\right]^{1/2}.$$

1. ϕ external flux
2. $\mu = 1/\lambda$ London penetration depth inverse
3. $1/\alpha = \lambda/\xi_v$ with ξ_v variational core-radius parameter
4. $\kappa = \lambda/\xi$ Ginzburg-Landau parameter

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Lattice Gauge Theories

Lattice formulation

Quantization

- Feynman path integral formulation in Euclidean space
- For pure gauge theory $\langle \hat{\mathcal{O}}_1...\hat{\mathcal{O}}_n \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \hat{\mathcal{O}}_1...\hat{\mathcal{O}}_n e^{-S_E}$,
  $S_E = \frac{1}{4} \int d^4x F_{\mu\nu}^a(x) F^{a\mu\nu}(x)$ and $Z = \int \mathcal{D}A_\mu e^{-S_E}$
- LGT techniques based on the interpretation of e^{-S_E} in the partition function Z as a Boltzmann probability distribution for A_μ variables.

Discretization

- Lattice regularization
- Discretized action in terms of $SU(3)$ links $U_\mu(n) = e^{igaA_\mu(n)}$

$$S = \beta \sum_{n, \mu > \nu} [1 - \frac{1}{3}\mathrm{ReTr}\, U_{\mu\nu}(n)],$$
$$U_{\mu\nu}(n) = U_\mu(n)\, U_\nu(n + \hat{\mu})\, U_\mu^\dagger(n + \hat{\nu})\, U_\nu^\dagger(n)$$

Connected correlator from previous studies

$$\rho_W^{\mathrm{conn}} = \frac{\langle \mathrm{tr}\left(WLU_P L^\dagger\right)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(U_P)\mathrm{tr}(W)\rangle}{\langle \mathrm{tr}(W)\rangle}$$
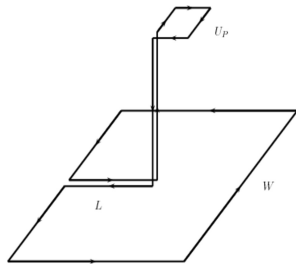
[A. Di Giacomo, M. Maggiore, S. Olejnik, Nucl.Phys. B347 (1990)]

[P. Cea, L. Cosmai, Phys.Rev. D52 (1995)]

- Continuum limit

$$\rho_W^{\mathrm{conn}} \xrightarrow{a\to 0} a^2 g\left[\langle F_{\mu\nu}\rangle_{q\bar{q}} - \langle F_{\mu\nu}\rangle_0\right]$$

- Color field strength tensor

$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\,\rho_W^{\mathrm{conn}}(x)$$

- W Wilson loop
- L Schwinger line
- U_p Plaquette

- $E_i(x)$, $B_i(x)$ by changing $U_P = U_{\mu\nu}(x)$ orientation.
- $E_l(x_t)$ component dominates at T=0.

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Another color field probe

Connected correlator with Polyakov loops

$$\rho_P^{\mathrm{conn}} = \frac{\left\langle \mathrm{tr}\left(P\left(x\right) L U_P L^\dagger\right) \mathrm{tr} P\left(y\right)\right\rangle}{\left\langle \mathrm{tr}\left(P\left(x\right)\right) \mathrm{tr}\left(P\left(y\right)\right)\right\rangle}$$

$$- \frac{1}{3} \frac{\left\langle \mathrm{tr}\left(P\left(x\right)\right) \mathrm{tr}\left(P\left(y\right)\right) \mathrm{tr}\left(U_P\right)\right\rangle}{\left\langle \mathrm{tr}\left(P\left(x\right)\right) \mathrm{tr}\left(P\left(y\right)\right)\right\rangle}$$

- Color field strength tensor

$$F_{\mu\nu}\left(x\right) = \sqrt{\frac{\beta}{6}} \rho_P^{\mathrm{conn}}\left(x\right).$$

- ρ_P^{conn} suited for the $T \neq 0$ case

[A. Di Giacomo, M. Maggiore, S. Olejnik, Nucl.Phys. B347 (1990)]
[P. Skala, M. Faber, and M. Zach, Nucl. Phys. B494 (1997)]
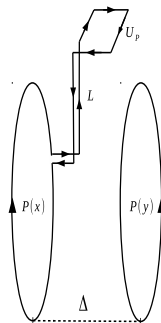
- $P(x)$, $P(y)$ Polyakov lines separated by a distance Δ
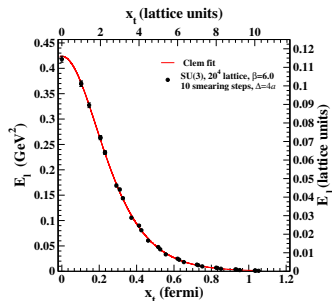
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The measuring process at a glance

Our investigation in few steps

- Realize the smearing over a thermalized field configuration

- Measure $E_l(x_t)$ through ρ_P^{conn} and changing plaquette position

- Fit the shape of $E_l(x_t)$

- Extract from the fit the values of the parameters ϕ, μ, λ/ξ_v, κ

- Analyze the behavior of parameters with smearing looking for a plateau

- Deduce London penetration depth and coherence length values from a scaling analysis
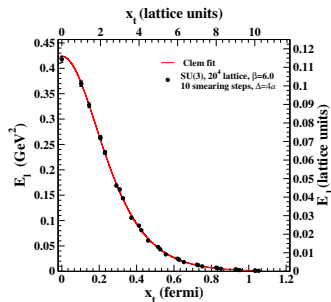
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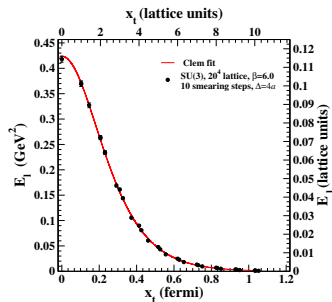- Deduce London penetration depth and coherence length values from a scaling analysis

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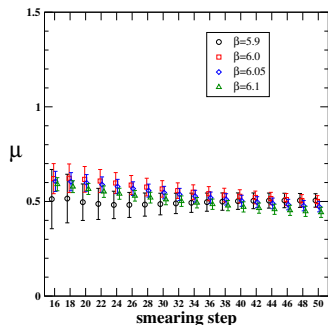- Deduce London penetration depth and coherence length values from a scaling analysis

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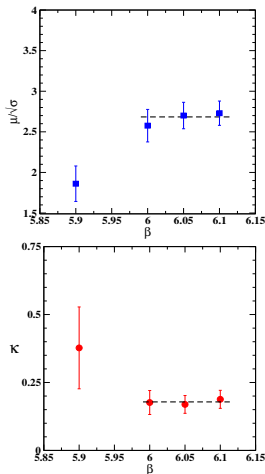- Deduce London penetration depth and coherence length values from a scaling analysis

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Details about simulations

Technicalities

Lattice and correlator features

- Size 20^4 and periodic boundary conditions
- Distance between Polyakov loops $\Delta = 4a, 6a, 8a$

LGT and action

- $SU(3)$ pure gauge LGT
- Wilson action $S = \beta \sum_{x, \mu > \nu} [1 - \frac{1}{3} \mathrm{ReTr} U_{\mu\nu}(x)]$, with $5.9 < \beta < 6.1$

Algorithms

- Cabibbo-Marinari algorithm combined with overrelaxation
- APE smearing procedure to increase signal-to-noise ratio

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Details about simulations

Smearing procedure: motivations and method

In view of the $T \neq 0$ case

- need to replace the previously used cooling mechanism
- Smearing guarantees:
  - local suppression of quantum fluctuations
  - maintenance of thermal fluctuations

Through the APE smearing procedure
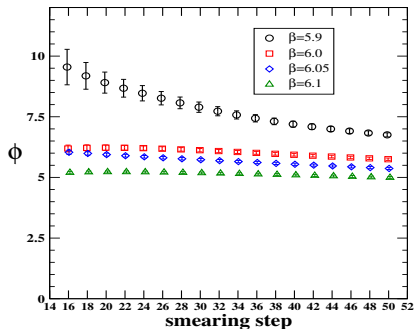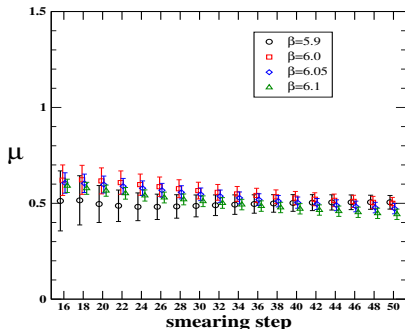
[M. Albanese et al., Phys. Lett. B 192 (1987)]

$$
\begin{aligned}
C_{\mu\nu}(x) &= U_\nu(x)U_\mu(x+\hat\nu)U_\nu^\dagger(x+\hat\mu) \\
&+ U_\nu^\dagger(x-\hat\nu)U_\mu(x-\hat\nu)U_\nu(x-\hat\nu+\hat\mu)
\end{aligned}
$$

$$
\tilde{U}_\mu(x) = \mathcal{P}_{SU(3)}[(1-\alpha)U_\mu(x) + \frac{\alpha}{6}\sum_{\mu\neq\nu} C_{\mu\nu}(x)], \qquad \alpha = 0.5
$$

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Results from the fit: looking for a plateau

Parameters vs. smearing

Figure: ϕ vs smearing ($\Delta = 6a$)

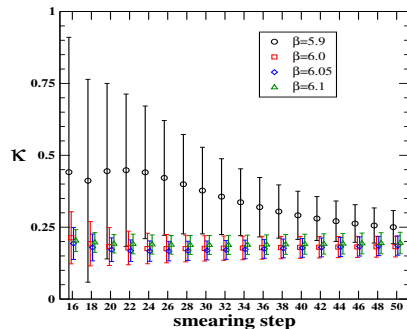Figure: μ vs smearing ($\Delta = 6a$)

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Results from the fit: looking for a plateau

Parameters vs. smearing

Figure: λ/ξ_v vs smearing ($\Delta = 6a$)

Figure: κ vs smearing ($\Delta = 6a$)

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From lattice to physical units

Setting the scale

Scaling of the plateau values of $a\mu$ with the string tension through the parametrization.

$$
\begin{aligned}
\sqrt{\sigma}(g) &= f_{\mathrm{SU}(3)}(g^2)[1 + 0.2731\,\hat{a}^2(g) \\
&- 0.01545\,\hat{a}^4(g) + 0.01975\,\hat{a}^6(g)]/0.01364
\end{aligned}
$$

$$
\hat{a}(g) = \frac{f_{\mathrm{SU}(3)}(g^2)}{f_{\mathrm{SU}(3)}(g^2(\beta=6))}, \quad \beta = \frac{6}{g^2}, \quad 5.6 \leq \beta \leq 6.5
$$

$$
f_{\mathrm{SU}(3)}(g^2) = \left(b_0 g^2\right)^{\frac{-b_1}{2b_0^2}} \exp\left(\frac{-1}{2b_0 g^2}\right), \quad b_0 = \frac{11}{(4\pi)^2}, \quad b_1 = \frac{102}{(4\pi)^4}
$$

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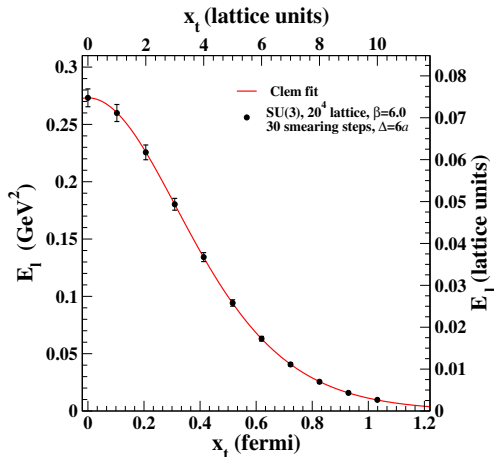From lattice to physical units
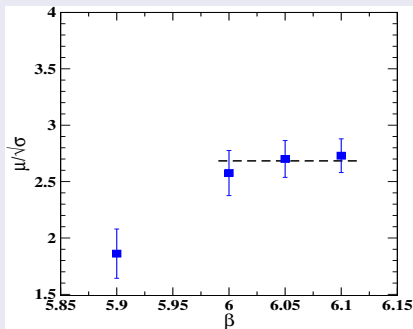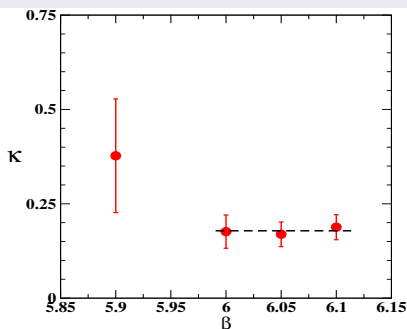
Field in lattice and physical units

Figure: (color online). Longitudinal chromoelectric field E_l versus x_t, in lattice units and in physical units, for $\Delta = 6a$ and after 30 smearing steps

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Scaling

Parameters scaling behavior

$\mu/\sqrt{\sigma} = 2.684(97)$

$\kappa = 0.178(21)$

$\lambda = {}^1\!/_\mu = 0.1750(63)$ fm

$\xi = 0.983(121)$ fm

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   - Lattice Gauge Theories
   - Chromoelectric field on the lattice

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3. Numerical data
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4. Penetration depth and coherece lengths
   - From lattice to physical units
   - Scaling

5. **Conclusions**

Summary and conclusions

Towards $T \neq 0$

- Connected correlator built with Polyakov, rather than Wilson, loops
- Smearing, rather than cooling, procedure

Dual superconductor scenario

- SU(3) vacuum as a type-I dual superconductor in agreement with [A. Shibata, K.-I. Kondo, S. Kato, and T. Shinohara, Phys. Rev. D 87, (2013)]
- λ in agreement with [P. Cea, L. Cosmai, and A. Papa, Phys. Rev. D 86, (2012)][P. Bicudo, M. Cardoso, and N. Cardoso, PoS LATTICE2013 (2014) 495]
- Relation to the "intrinsic width" of the flux tube [M. Caselle and P. Grinza, J. High Energy Phys. 11 (2012) 174.] to be investigated