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Gauge Invariant Gluon Field Strength correlators in the presence of a Magnetic Background

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Outline

- Motivations of this work
- Description of the new parametrization of the correlators for $B \neq 0$
- Numerical Results : Lattice measurements and determination of G_2 and λ
- Conclusions

There are situations where $|eB|$ is comparable with the energy scales of QCD:

- Heavy Ion Collisions ($B \sim 10^{15}\,T$) (e.g. [1103.4239])
- EW cosmological phase transition ($B \sim 10^{16}\,T$) (e.g. [Grasso, Rubinstein 2001])
- Magnetars (a type of neutron stars)($B \sim 10^{10}\,T$)[Duncan, Thompson 92]

Note: $|eB| = 0.06 GeV^2 \rightarrow B = 10^{15}\,T$

Uses of the correlators

- High energy phenomenology (hadron scattering)
- Stochastic models of QCD (heavy quarkonium systems)

Non Perturbative quantities can be extracted from it, e.g.

- the gluon condensate G_2 [D'Elia, Di Giacomo, Meggiolaro 97]
- the correlation length(s).
- estimates of the string tension (through stochastic vacuum model)

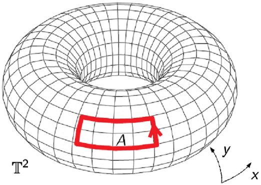
Effects of the B field can propagate to the gluon sector thanks to the quark fields.

- How it is introduced:

$$U_\mu(n) \Rightarrow U_\mu(n)e^{iaA_\mu^{EM}(n)}$$

- B quantization on a torus (due to the topology):

$$qB = \frac{2\pi}{a^2}\frac{b}{L_x L_y}$$



- Due to the ultraviolet cutoff, we will have physically significant results only when

$$qB << \frac{2\pi}{a^2}$$

$$\Delta_{\mu_1\nu_1,\mu_2\nu_2}(z_1 - z_2) = \frac{1}{N_c} \langle \mathrm{Tr}\ F_{\mu_1\nu_1}(z_1)\Phi(z_1, z_2)F_{\mu_2\nu_2}(z_2)\Phi(z_2, z_1)\rangle$$

$\Phi(z_1, z_2)$ parallel transport operator along a *straight path* from z_1 to z_2.

- Translationally invariant
- Gauge invariant
- It is a tensor
- Inherits index symmetries from $F_{\mu\nu} = i\,[\mathcal{D}_\mu, \mathcal{D}_\nu]$

On the lattice:

$$\hat{\Delta}_{\mu_1\nu_1,\mu_2\nu_2} = \mathrm{Re}\langle \mathrm{Tr}\left[\Omega_{\mu_1\nu_1}(n+\hat{\rho}d)S(n+\hat{\rho}d,n)\Omega_{\mu_2\nu_2}(n)S^\dagger(n+\hat{\rho}d,n)\right]\rangle$$

where $\Omega_{\mu\nu}(n)$ is the traceless antihermitean part of the parallel transport around the plaquette n.

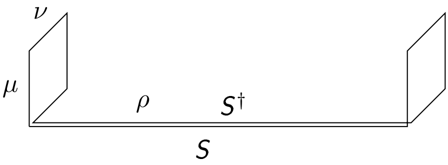


Figure : Depiction of a *perpendicular* correlator on the lattice

Symmetries of Δ entail this is the most general form:

$$\Delta_{\mu_1\nu_1,\mu_2\nu_2}(z) = (\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2} - \delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})\, D(z) +$$
$$\frac{1}{2}\left(\frac{\partial}{\partial z_{\mu_1}}(z_{\mu_2}\delta_{\nu_1\nu_2} - z_{\nu_2}\delta_{\nu_1\mu_2}) + \frac{\partial}{\partial z_{\nu_1}}(z_{\nu_2}\delta_{\mu_1\mu_2} - z_{\mu_2}\delta_{\mu_1\nu_2})\right) D_1(z)$$

Scalar function parametrization on the lattice, suitable for "medium range":

$$D(z) = \frac{a_0}{z^4} + A_0 \exp\left(-\frac{z}{\lambda_A}\right)$$
$$D_1(z) = \frac{a_1}{z^4} + A_1 \exp\left(-\frac{z}{\lambda_A}\right)$$

D and D_1 have a *perturbative* and *nonperturbative* part.

On the lattice we are interested only in $\Delta_{\mu\nu,\mu\nu}$.
Thus we have 24 nonzero correlation functions, which can be grouped into 2 equivalence classes:

- $D_{||}(z)$; if $\rho = \mu$ or $\rho = \nu$;
- $D_{\perp}(z)$; if $\rho \neq \mu$ and $\rho \neq \nu$;

$$F^{em}_{\mu\nu} \neq 0 \Rightarrow \text{ a lot of possible new terms!}$$

Euclidean $SO(4)$ symmetry is explicitly broken to $SO(2) \otimes SO(2)$.
On the lattice, we have the equivalences $x \sim y$, $z \sim t$
New equivalence classes of interest:

Class	**Elements** $(\mu\nu, \rho)$	**"Parent"**	**Link dir** ρ
$A_{xy}(d)$	(12,1) , (12,2)	$\parallel$	(12)
$A_{zt}(d)$	(12,3) , (12,4)	$\perp$	(34)
$B_{xy}(d)$	(13,1) , (14,1) , (23,2) , (24,2)	$\parallel$	(12)
$B_{zt}(d)$	(13,3) , (14,4) , (23,3) , (24,4)	$\parallel$	(34)
$C_{xy}(d)$	(13,2) , (14,2) , (23,1) , (24,1)	$\perp$	(12)
$C_{zt}(d)$	(13,4) , (14,3) , (23,4) , (24,3)	$\perp$	(34)
$D_{xy}(d)$	(34,1) , (34,2)	$\perp$	(12)
$D_{zt}(d)$	(34,3) , (34,4)	$\parallel$	(34)

We choose to parametrize each class independently, in a form similar to the traditional one. One possible choice is

$$D^{(class)}(z) = \frac{a_0^{(class)}}{z^4} + A_0^{(class)} \exp\left(-\frac{z}{\lambda_A^{(class)}}\right)$$

Note: There are some constraints on the parameters

The Gluon Condensate is used in the SVZ sum rules [Shifman,Vainshtein,Zakharov 1979]

$$G_2 = \frac{1}{4\pi^2}\langle F^a_{\mu\nu} F^{a\ \mu\nu}\rangle$$

On the lattice, it is linked to the correlator through an OPE:

$$\frac{1}{2\pi^2}\sum_{\mu<\nu}\Delta_{\mu\nu,\mu\nu}(z) \underset{z\to 0}{\sim} C_1(z)\langle 1\rangle + C_g(z)G_2 + \sum_{f=1}^{N_f} C_f(z) m_f \langle : \bar{q}_f q_f :\rangle + \dots$$

Its empiric value is $0.024 \pm 0.011\, GeV^4$ [Narison, 96]
For small quark masses

$$\frac{dG_2}{dm_f} = -\frac{24}{b}\langle \bar{q}_f q_f\rangle$$

$\langle \bar{q}_f q_f\rangle \simeq -0.01\, GeV^3$, $b = 11 - \frac{2}{3}N_f$

Cooling

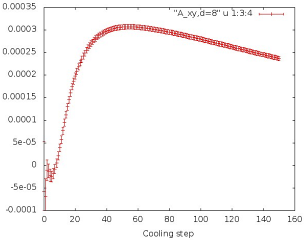
Cooling was used as the noise reduction technique.

- A step of the cooling algorithm consists in substituting each gauge link with one that reduces the local action contribution.
- For $SU(2)$

$$U'_\mu(n) = \frac{1}{D} \sum_{\nu \neq \mu} U^\dagger_\nu(n+\hat{\mu}) U_\mu(n+\hat{\nu}) U_\nu(n)$$

is the one that locally minimizes the action.

- We can't measure very short distance correlators reliably
- $c_{max} \propto d^2$
- We take the real value of the correlation function at the maximum
- Systematic error (in addition the the statistical one)



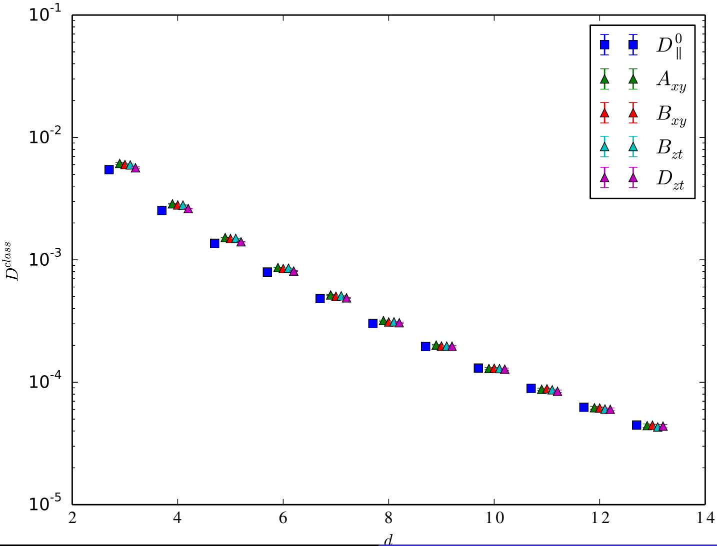
$$\delta_{sys} = D(c_{max}) - \frac{1}{2}\left(D(c_{max}+1) + D(c_{max}-1)\right)$$

Simulation Details

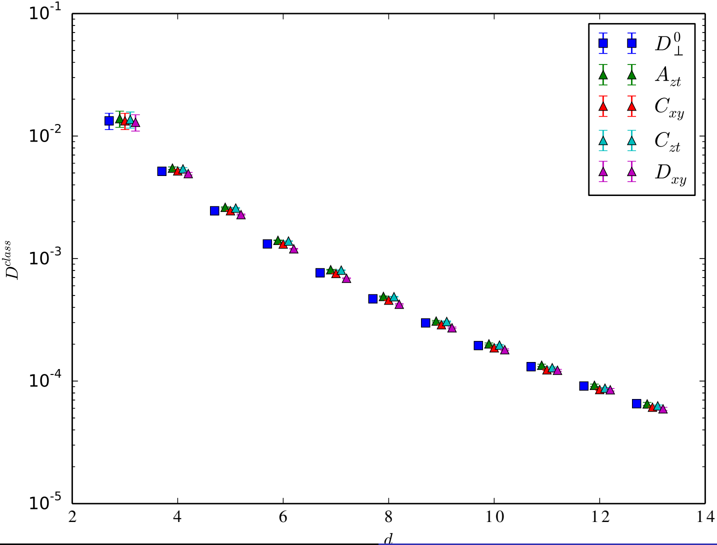
- Lattice 24^4, lattice spacing $a = 0.125 fm$, $m_\pi = 480 MeV$
- $N_f = 2$ rooted staggered
- Values of eB ranging from 0 to $1.5 GeV^2$

Jobs ran on the GPU farm of the INFN - Sezione di Genova

Correlators, Parallel classes
$eB = 1.47 GeV^2$



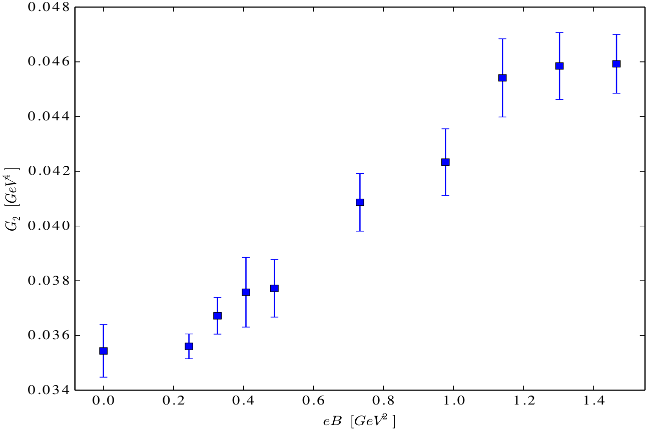
Correlators, Perpendicular classes
$eB = 1.47 GeV^2$



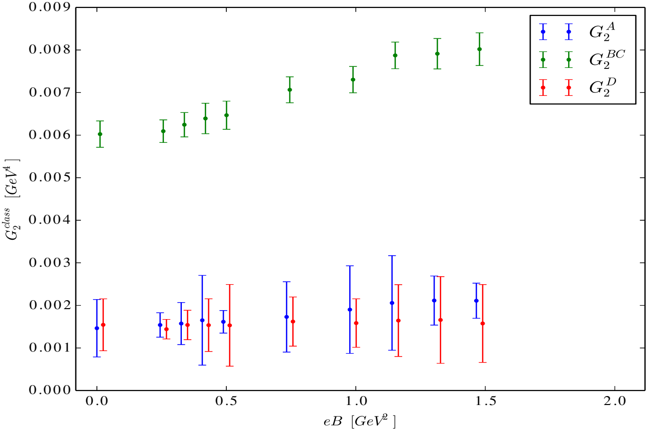
Fit strategy

- We tried a number of different parametrizations
- We decreased the number of independent parameters
  - Based on evidence, we assumed the perturbative part was independent of B and independent on the class
  - We let all correlation lengths free
  - We let the NP parameters free

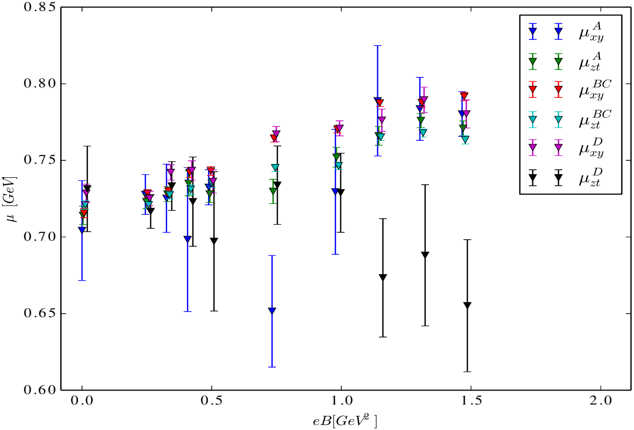
Increment of the Gluon Condensate: gluon catalysis.



Contributions to G_2 from different plaquettes



Effects on the correlation lengths



Conclusions

We evidenced effects of B on the gluon fields

- Significant increase in the gluon condensate
- Diverse effects on the correlation length(s)

Possible developments

- Measurement at the physical point
- Study of the $B \neq 0$ and $T \neq 0$ case

Conclusions

Thanks for your attention!

Dependence on B of the perturbative parameters