New Frontiers in Theoretical Physics	Cortona, 30.05.2014

Heavy dense QCD and nuclear matter on the lattice

HIC for FAIR
Helmholtz International Center

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Bundesministerium für Bildung und Forschung

- Introduction: the QCD phase diagram: what we do and do not know
- 3d effective lattice theory derived by strong coupling methods JHEP 1102 (2011) 057
- Cold and dense QCD: transition to nuclear matter PRL 110 (2013), arXiv:1403.4162

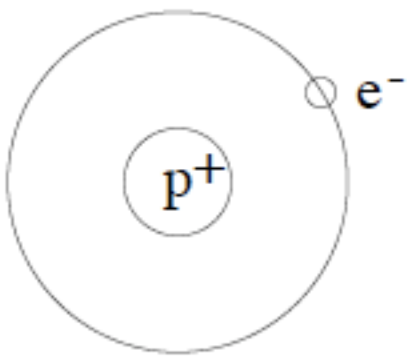
Quantum Chromodynamics, theory of strong interactions

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2}\,\mathrm{Tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{3} \bar{\psi}_i[\gamma_\mu D_\mu + m_i]\psi_i$$

$$m_u \sim 3\text{MeV}, \quad m_d \sim 6\text{MeV}, \quad m_s \sim 120\text{MeV} \;\Rightarrow\; N_f \approx 2+1$$

weak vs. strong coupling:

QED



$M_e = 0.5$ MeV

$M_p = 938$ MeV

$E_{bind} = 13.6$ eV

hydrogen (e.m. force)

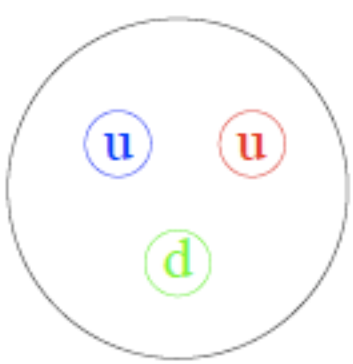
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

photons → gluons

e,p → quarks

gauge group U(1) → gauge group SU(3)

QCD



$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

proton (strong force)

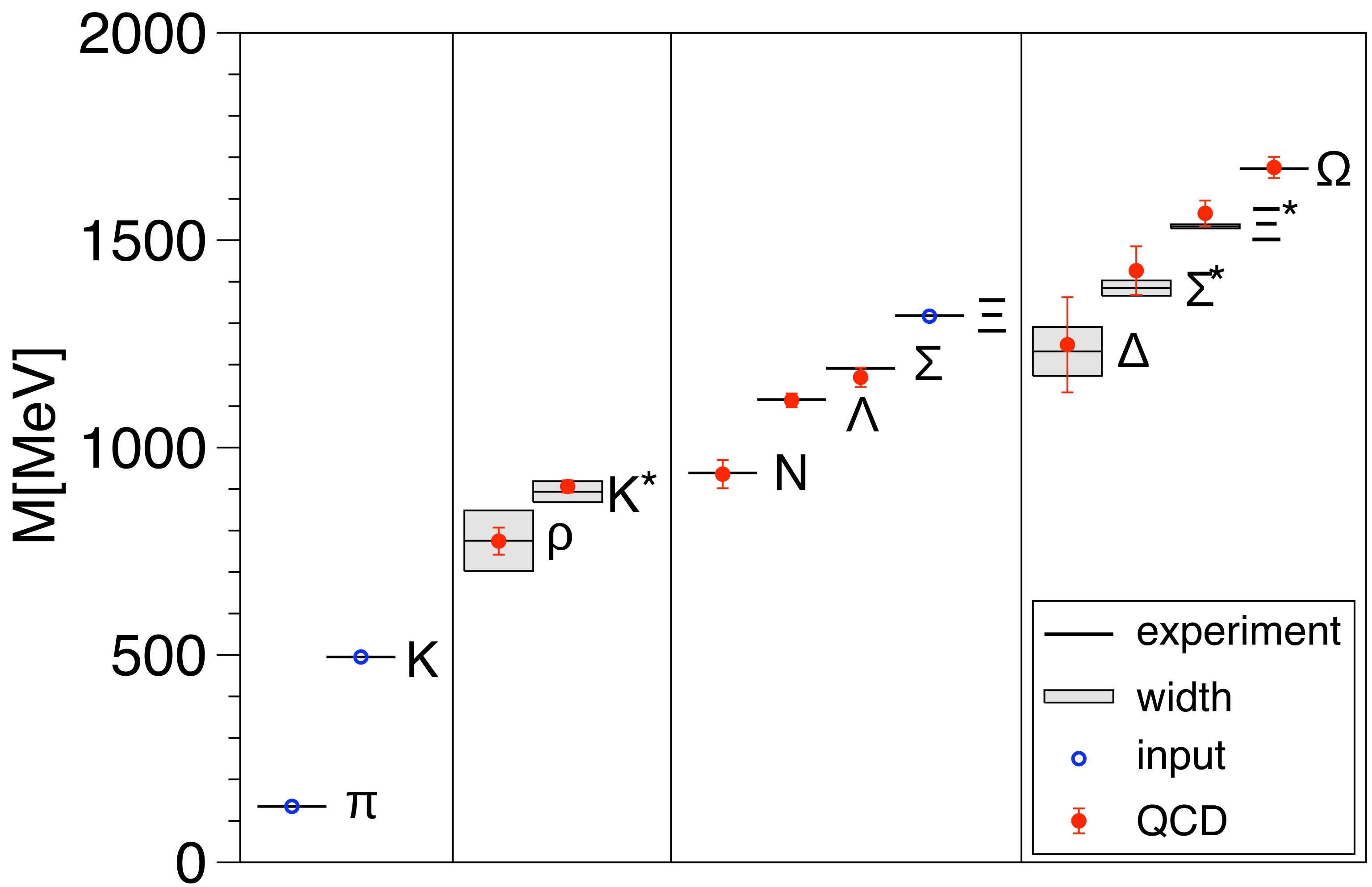
$$\alpha_s = \frac{g^2}{4\pi} \approx 1$$

⇒**Confinement, non-perturbative**

gluon self-interaction!

Light hadron spectrum from the lattice

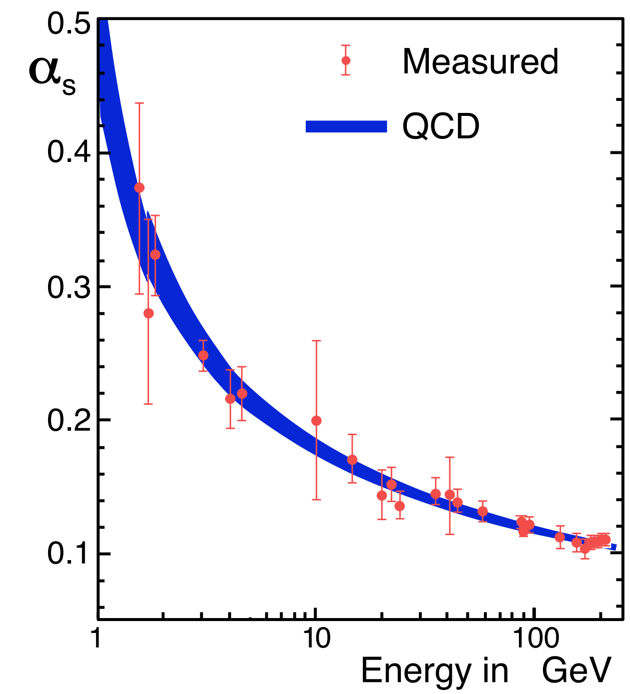
BMW collaboration (Budapest, Marseille, Wuppertal) 2010



mesons=
quark anti-quark states

QCD is the correct theory for strong interactions also at low energy!

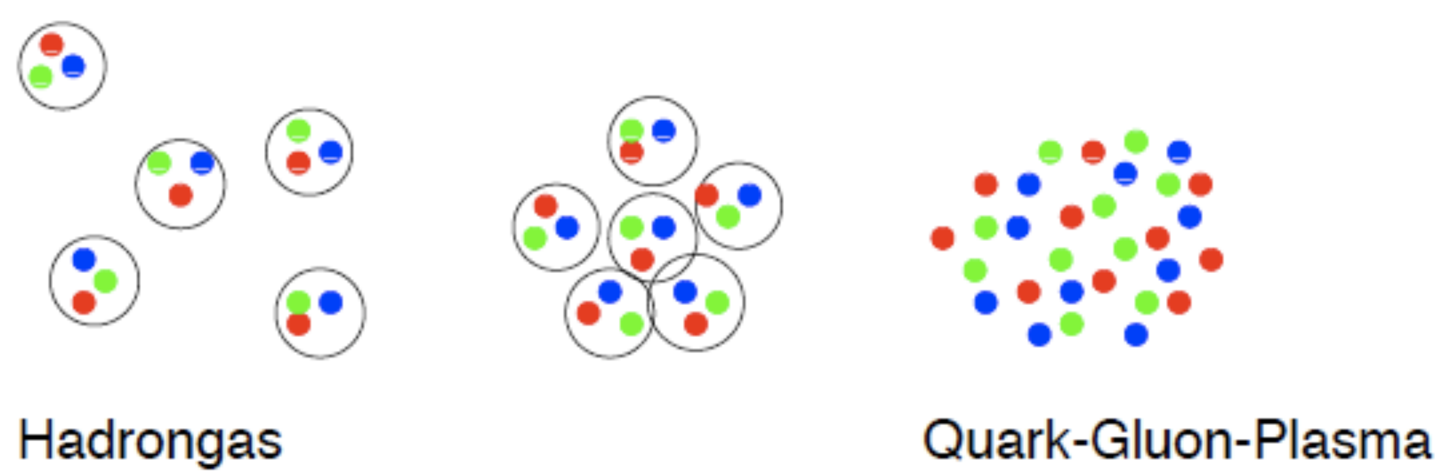
QCD at high temperature/density: change of dynamics



asymptotic freedom $\alpha_s(p \to \infty) \to 0$

T, μ_B

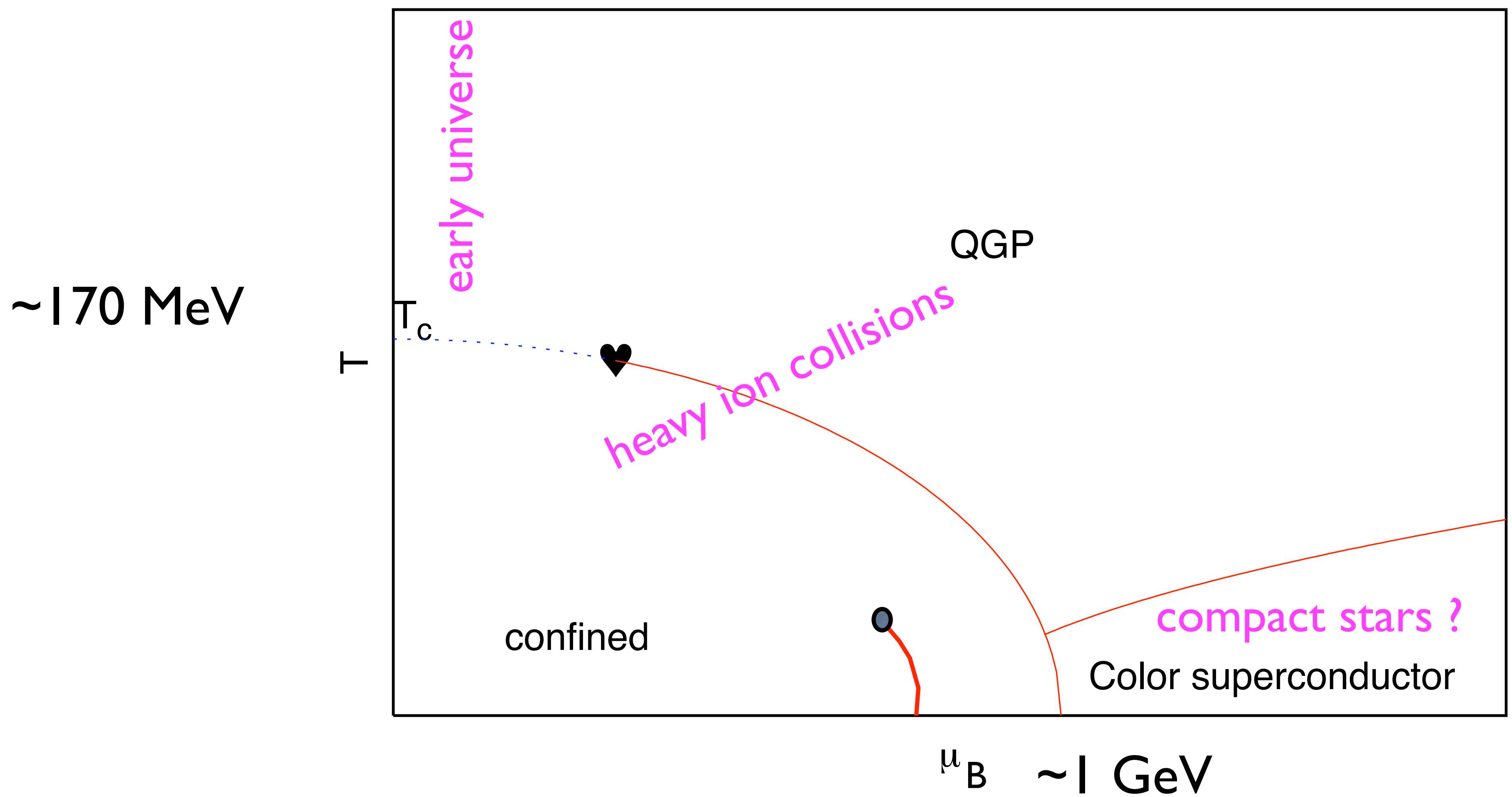
Phase transitions?



Order parameters: $\langle \bar{\psi}\psi \rangle, \langle \psi\psi \rangle$

chiral condensate , Cooper pairs

QCD phase diagram: theorist's science fiction

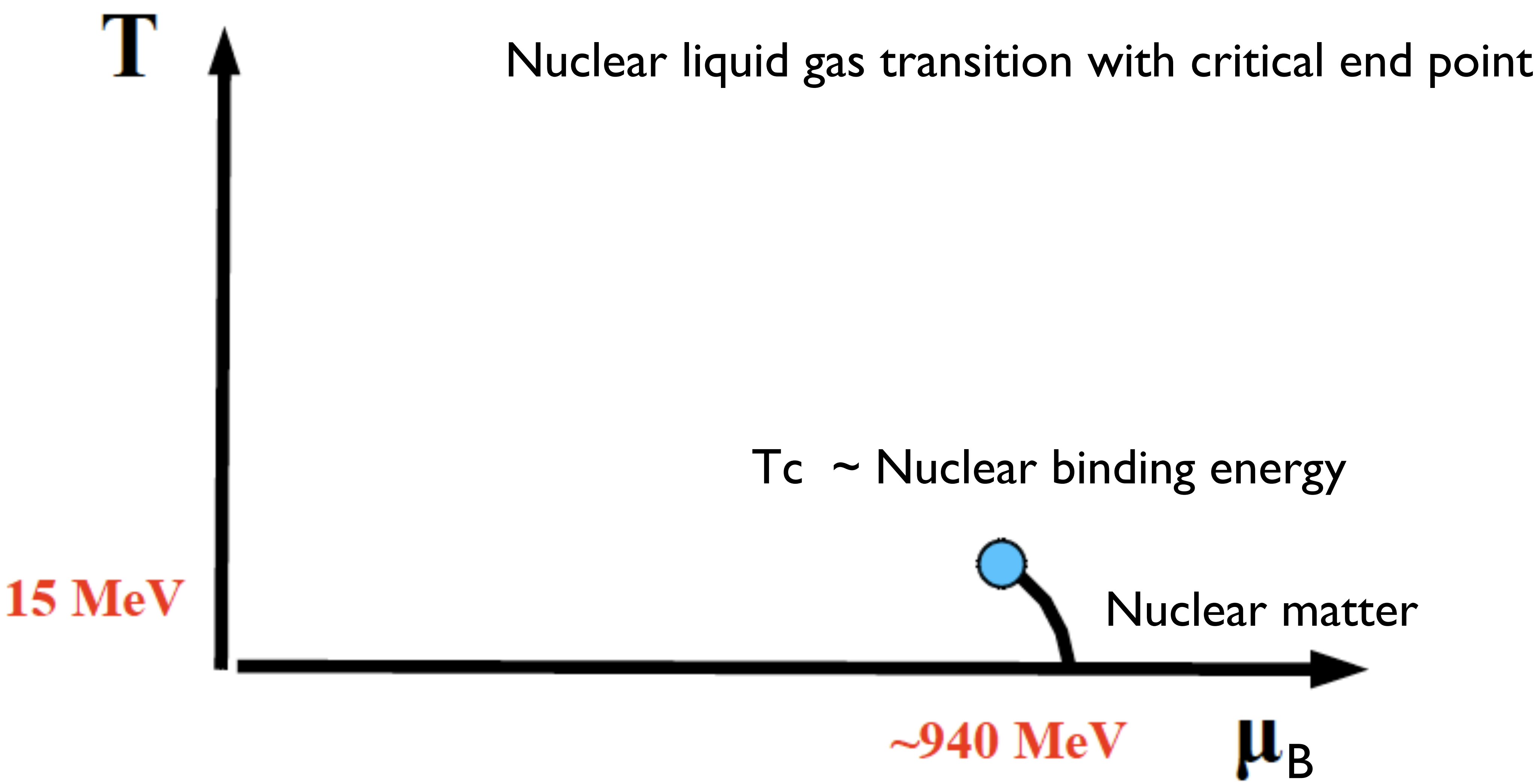


Until 2001: no finite density lattice calculations, sign problem!

Expectation based on simplifying models (NJL, linear sigma model, random matrix models, ...) and symmetry arguments

Check this from first principles QCD!

Experimentally established phase diagram:

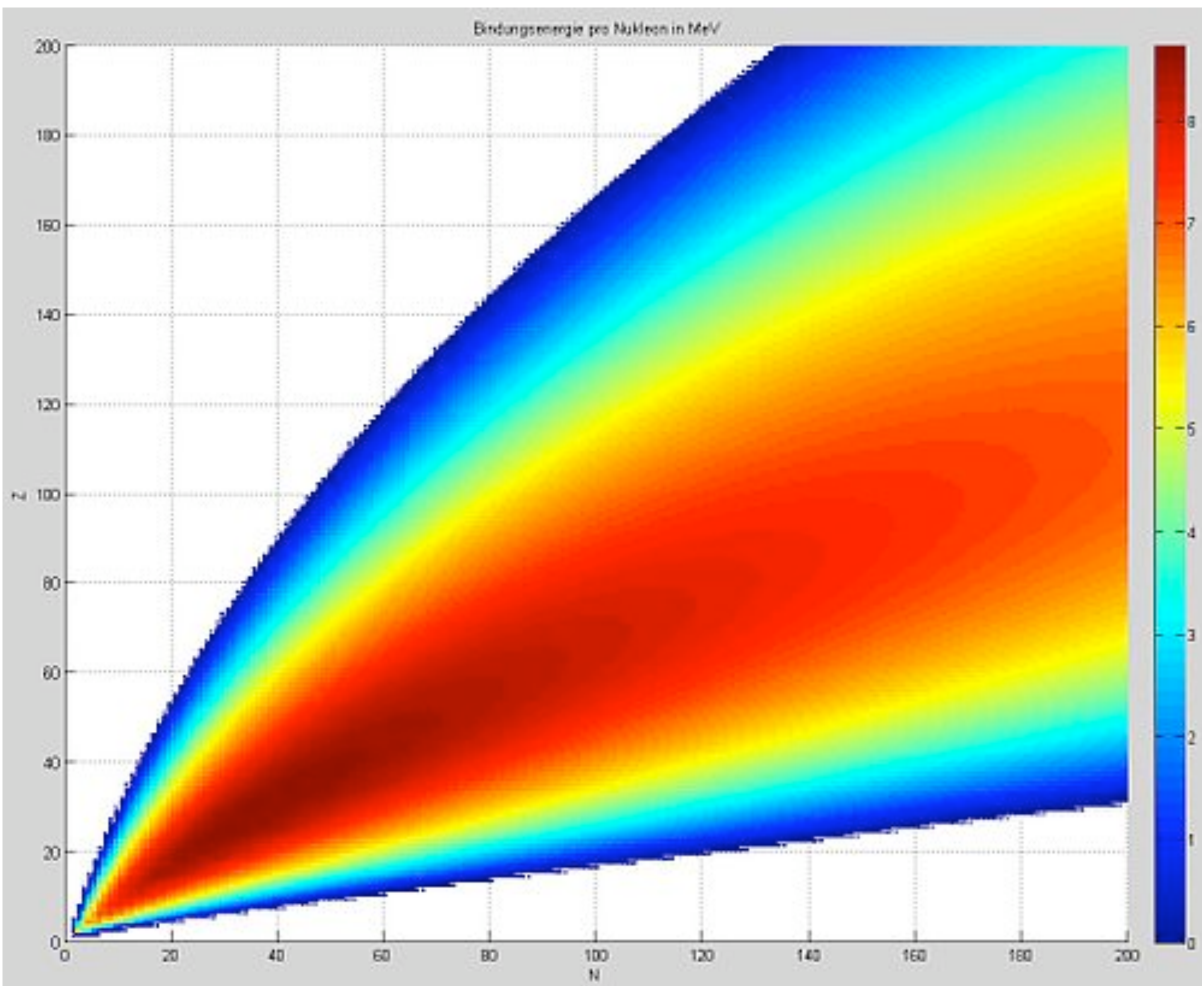


Nuclear physics

~100 years old, still no fundamental description!

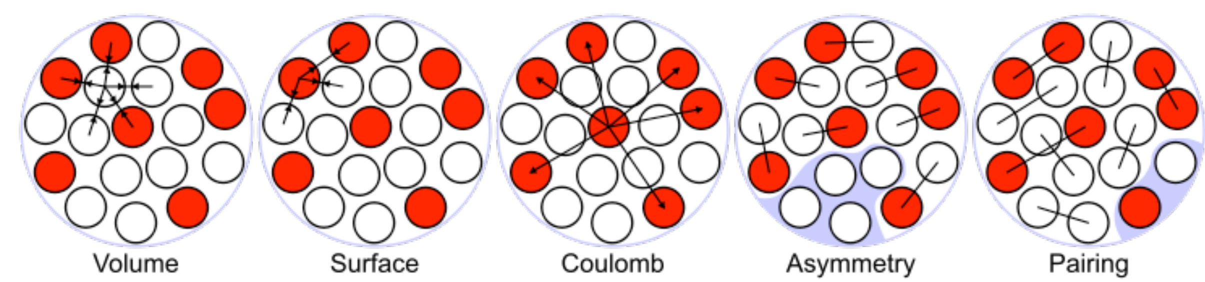
Bethe-Weizsäcker droplet model:

Z



N

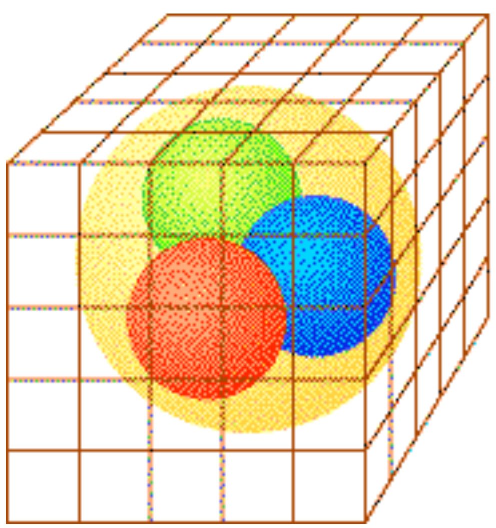
Binding energy per nucleon



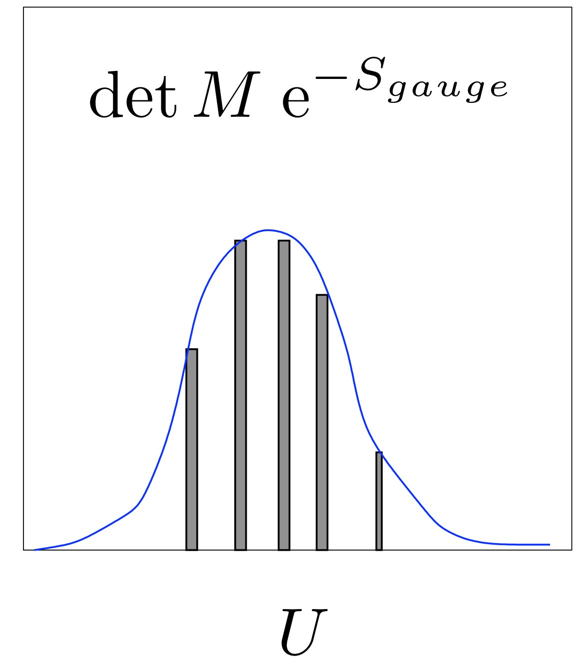
QFT descriptions: -Fetter-Walecka model (nucleons + mesons)
-Skyrme model (skyrmions)

Lattice QCD + Monte Carlo method

QCD partition fcn: $Z = \int DU \prod_f \det M(\mu_f, m_f; U)\, e^{-S_{gauge}(\beta;U)}$ $\beta = \frac{2N_c}{g^2}$



links=gauge fields



Monte Carlo by importance sampling

>10^8 dimensional integral

$N_t = 4, 6$

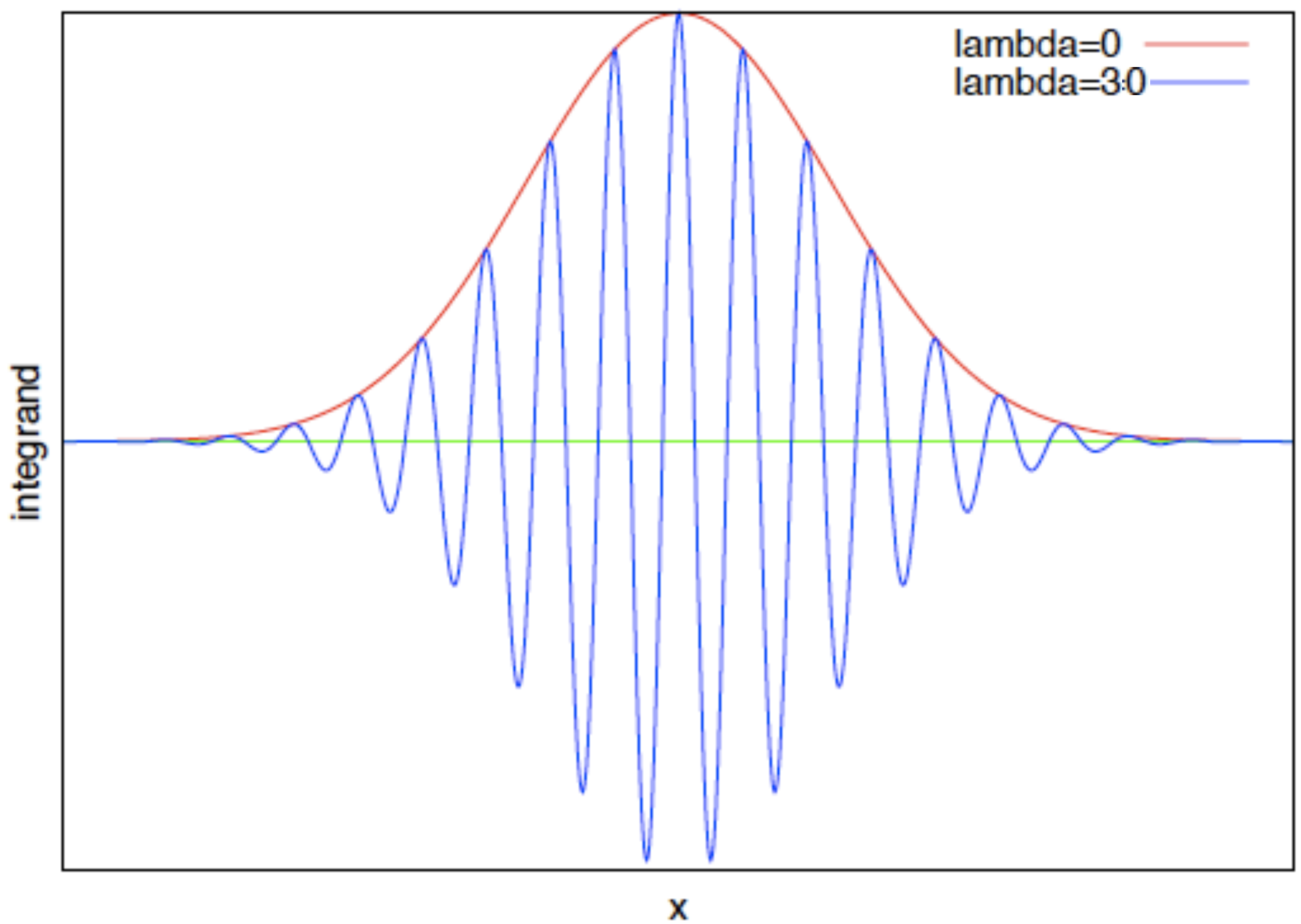
$T = \frac{1}{aN_t}$ Continuum limit: $N_t \to \infty, a \to 0$ $a \sim 0.3, 0.2$ fm

Sign problem: $\det(M)$ complex for SU(3), $\mu \neq 0$

importance sampling requires positive weights

Sign problem: 1-dim. illustration

- Example: $Z(\lambda) = \int dx \exp(-x^2 + \mathbf{i}\lambda\mathbf{x})$



- $Z(\lambda)/Z(0) = \exp(-\lambda^2/4)$: exponential cancellations

QCD: ~ exp. prop. to volume, chemical potential

Theory: how to calculate p.t., critical temperature

deconfinement/chiral phase transition $\longrightarrow$ quark gluon plasma

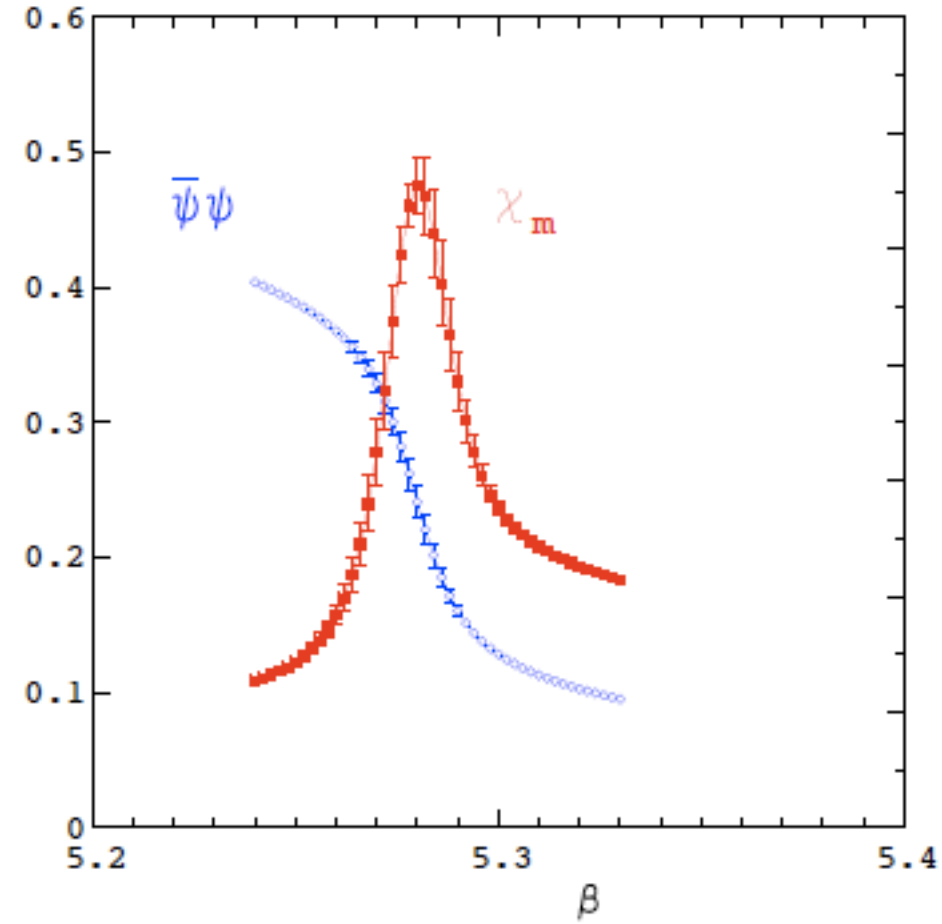
"order parameter":

chiral condensate $\langle\bar{\psi}\psi\rangle$

generalized susceptibilities:

$$\chi = V(\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2)$$

$$\Rightarrow \chi_{max} = \chi(\beta_c) \quad \Rightarrow T_c$$



lattice coupling β, viz. T

only pseudo-critical on finite V!

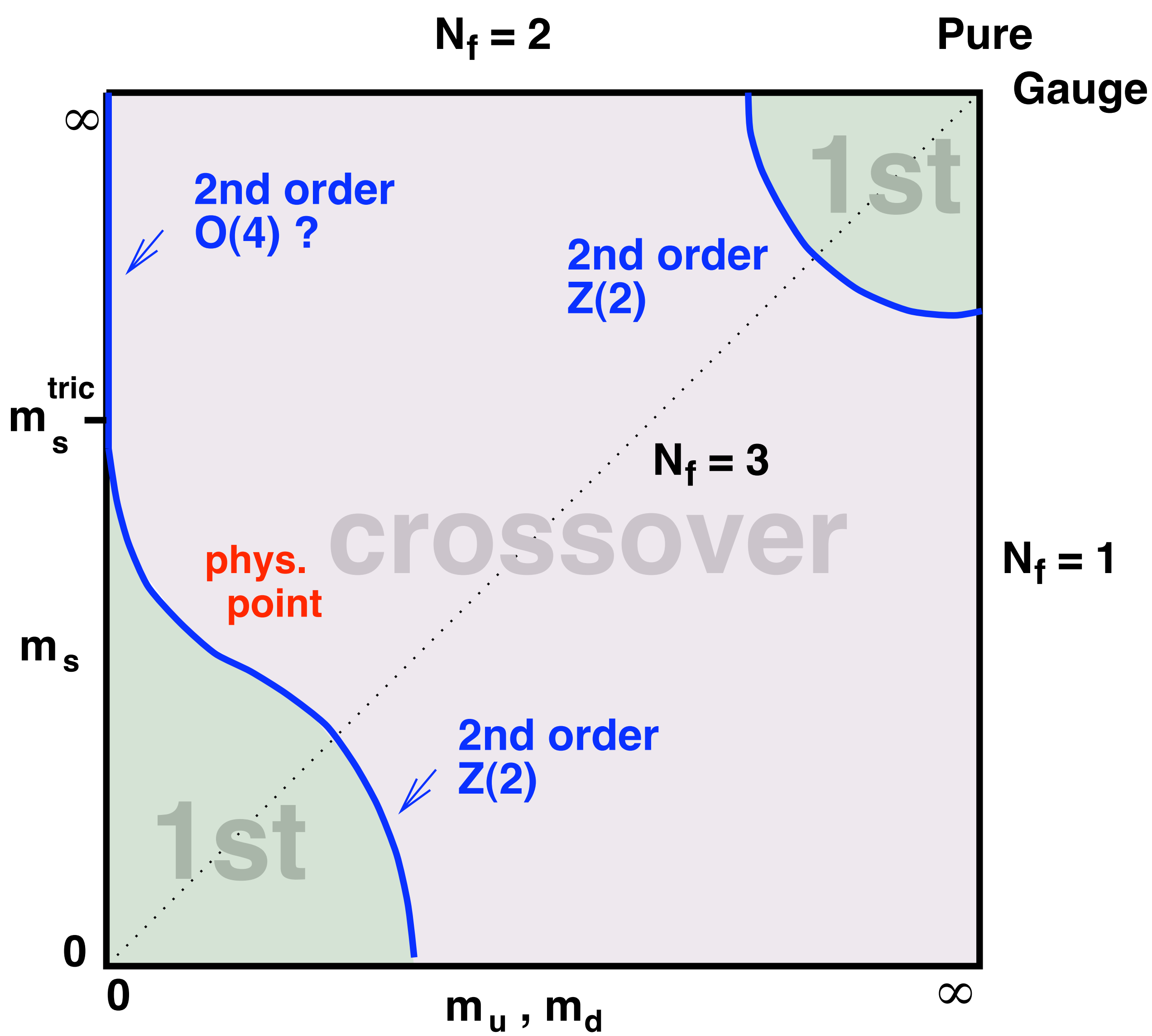
Order of transition:

finite volume scaling

$$\chi_{max} \sim V^{\sigma}$$

$\sigma = 1$	1st order
σ = crit. exponent	2nd order
$\sigma = 0$	crossover

The order of the p.t., arbitrary quark masses $\mu = 0$



deconfinement p.t.:
breaking of global $Z(3)$

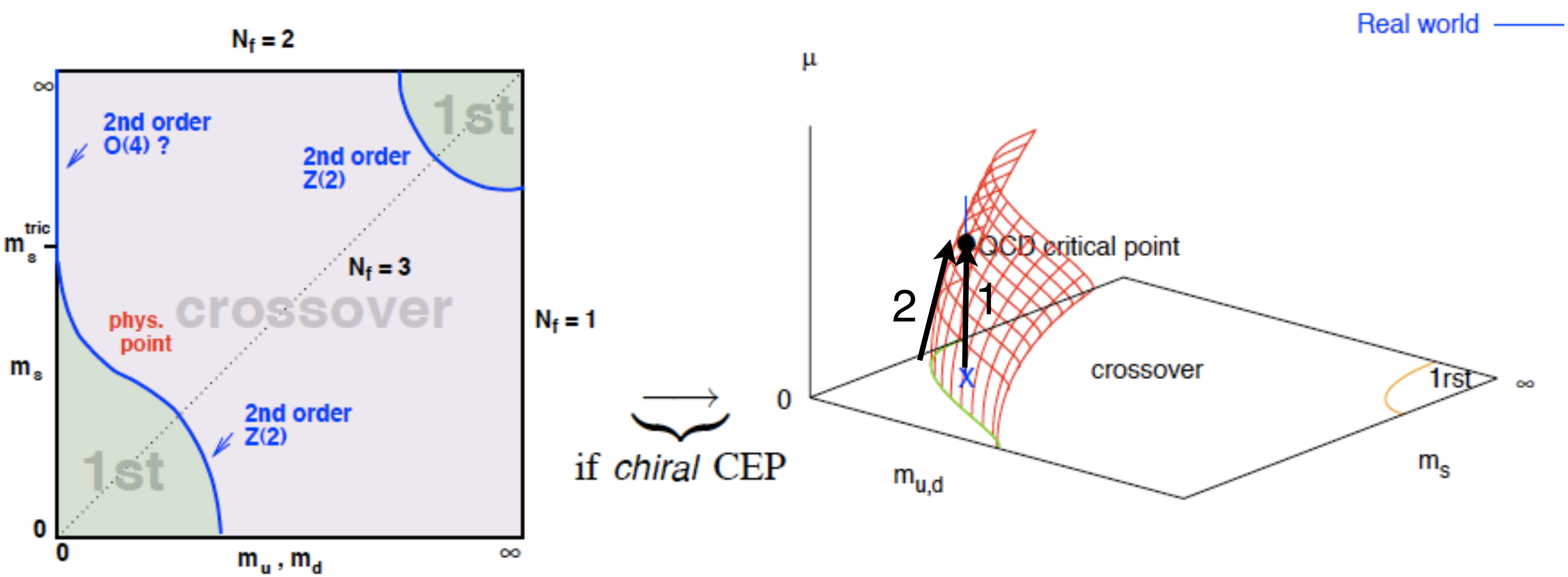
chiral p.t.
restoration of global

$SU_L(N_f) \times SU_R(N_f) \times U_V(1) \times U_A(1)$

↑
anomalous

- physical point: crossover in the continuum
  Aoki et al. 06
- chiral critical line on coarse lattice
  de Forcrand, O.P. 07

Much harder: is there a QCD critical point?



Two strategies:

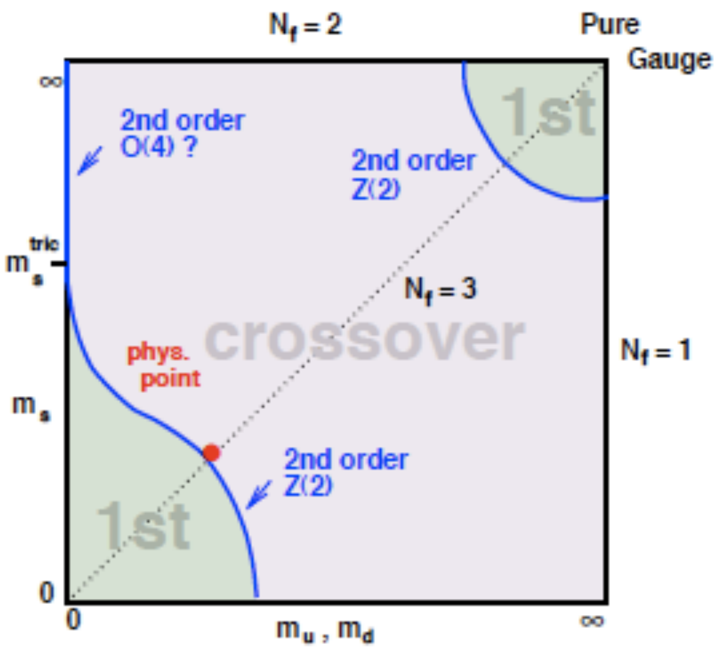
1 follow vertical line: $m = m_{\text{phys}}$, turn on μ

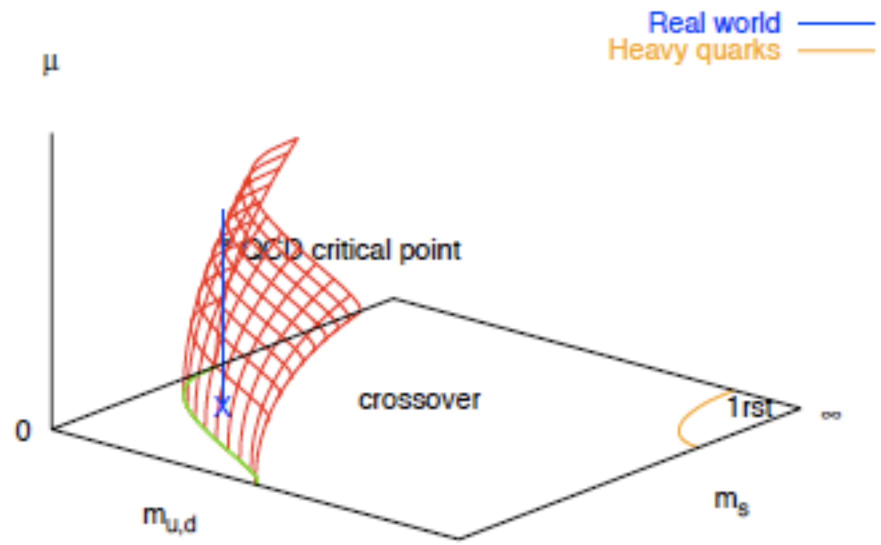
2 follow critical surface: $m = m_{\text{crit}}(\mu)$

Approach 1: some signals (reweighting, extrapolated Taylor series, canonical) systematics not yet controlled !

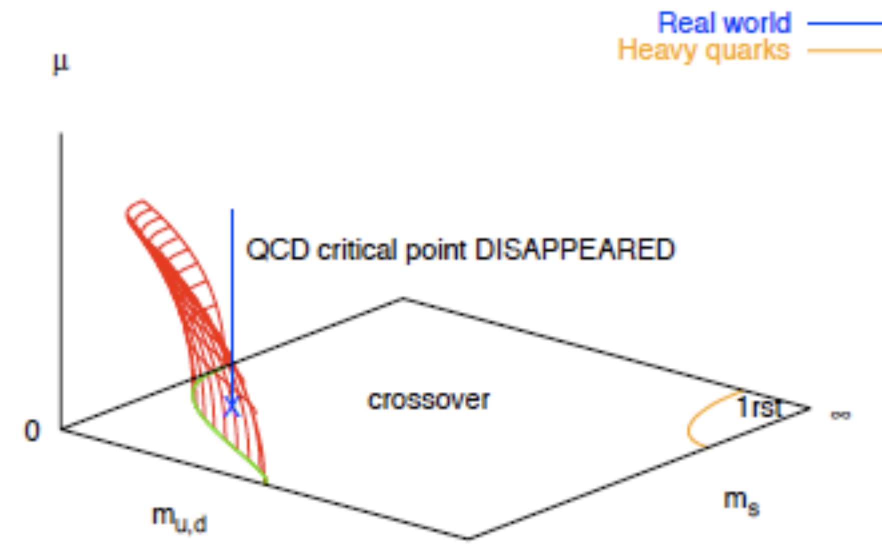
Fodor, Katz; Gavai, Gupta; Alexandru, Liu

Approach 2: curvature of chiral critical surface





$c_1 > 0$



$c_1 < 0$

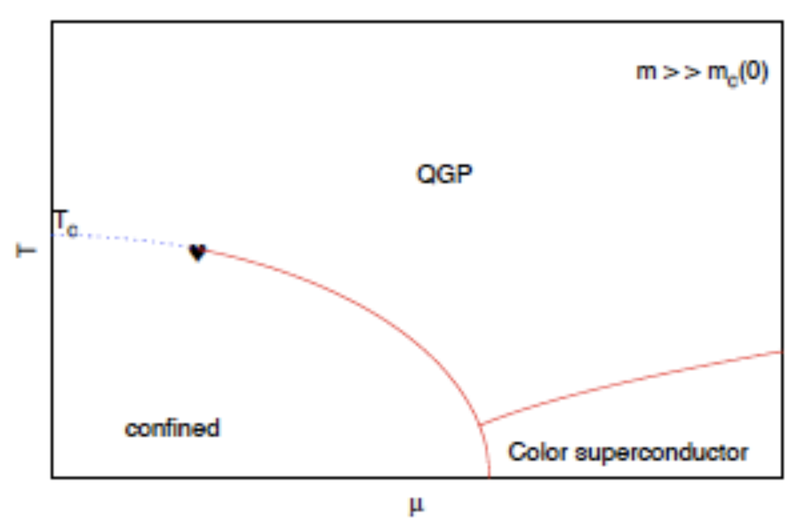
● : critical quark mass m_c for $N_f = 3$

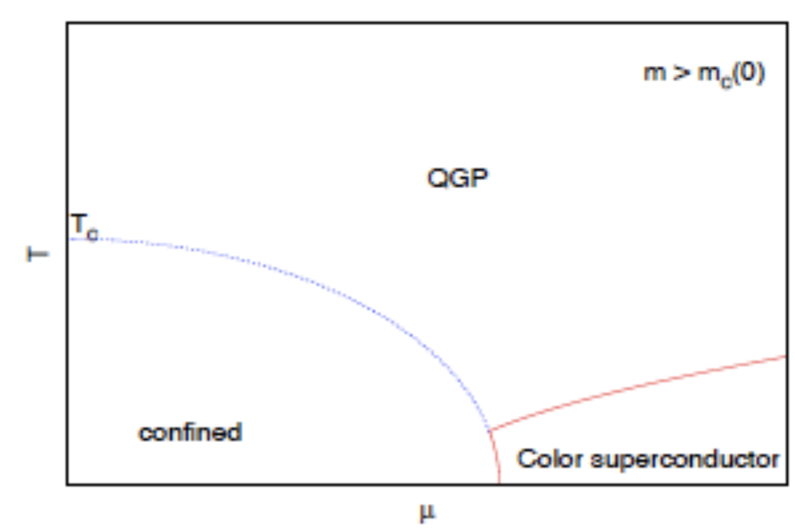
Possible scenarios:

existence and location of chiral critical end-point determined by

- distance phys. point – crit. surface
- curvature crit. surface, c_1

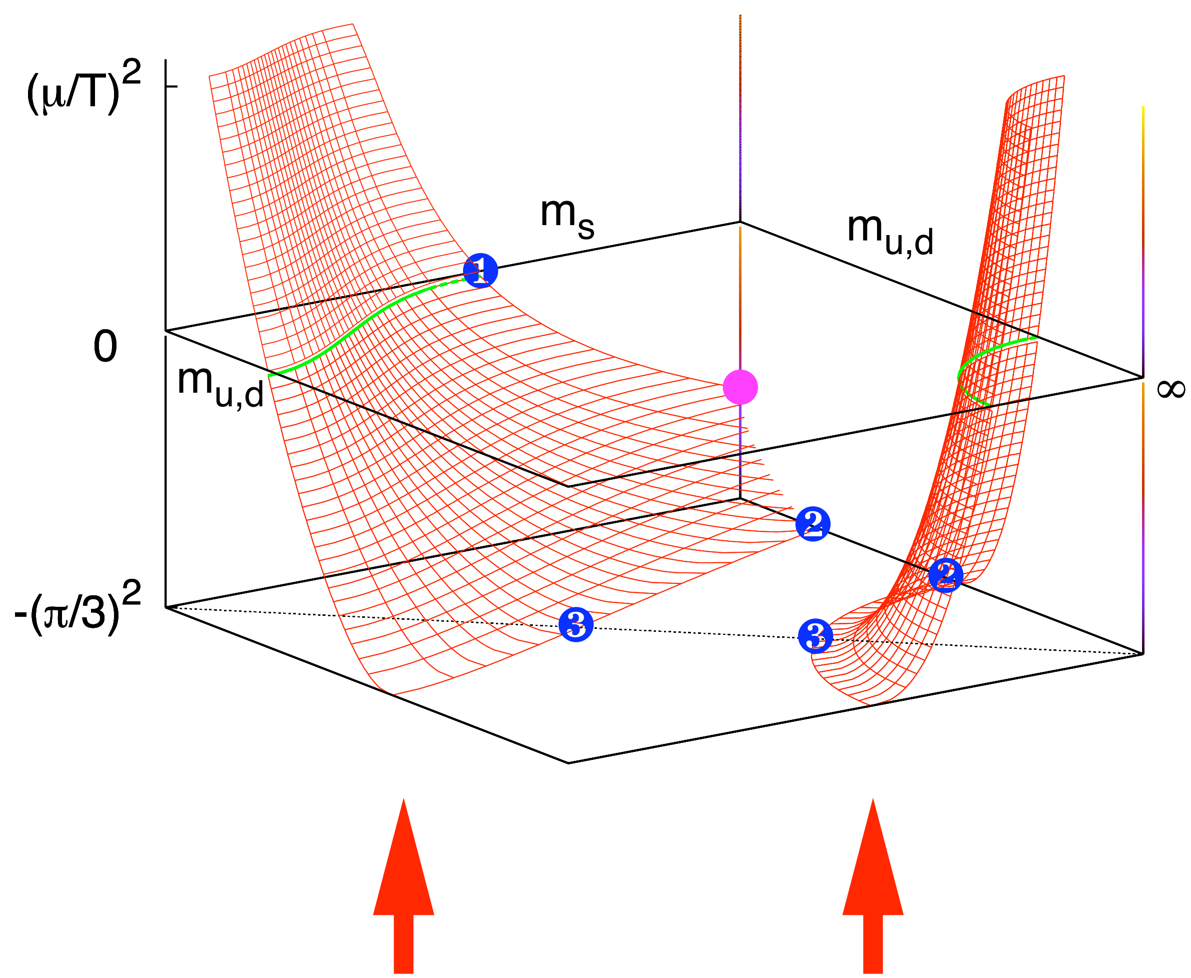
$$\frac{m_c(\mu)}{m_c(0)} = 1 + c_1 \left(\frac{\mu}{\pi T}\right)^2 + c_2 \left(\frac{\mu}{\pi T}\right)^4 + \ldots$$





Chiral and deconfinement critical surfaces

Real chemical potential:
sign problem

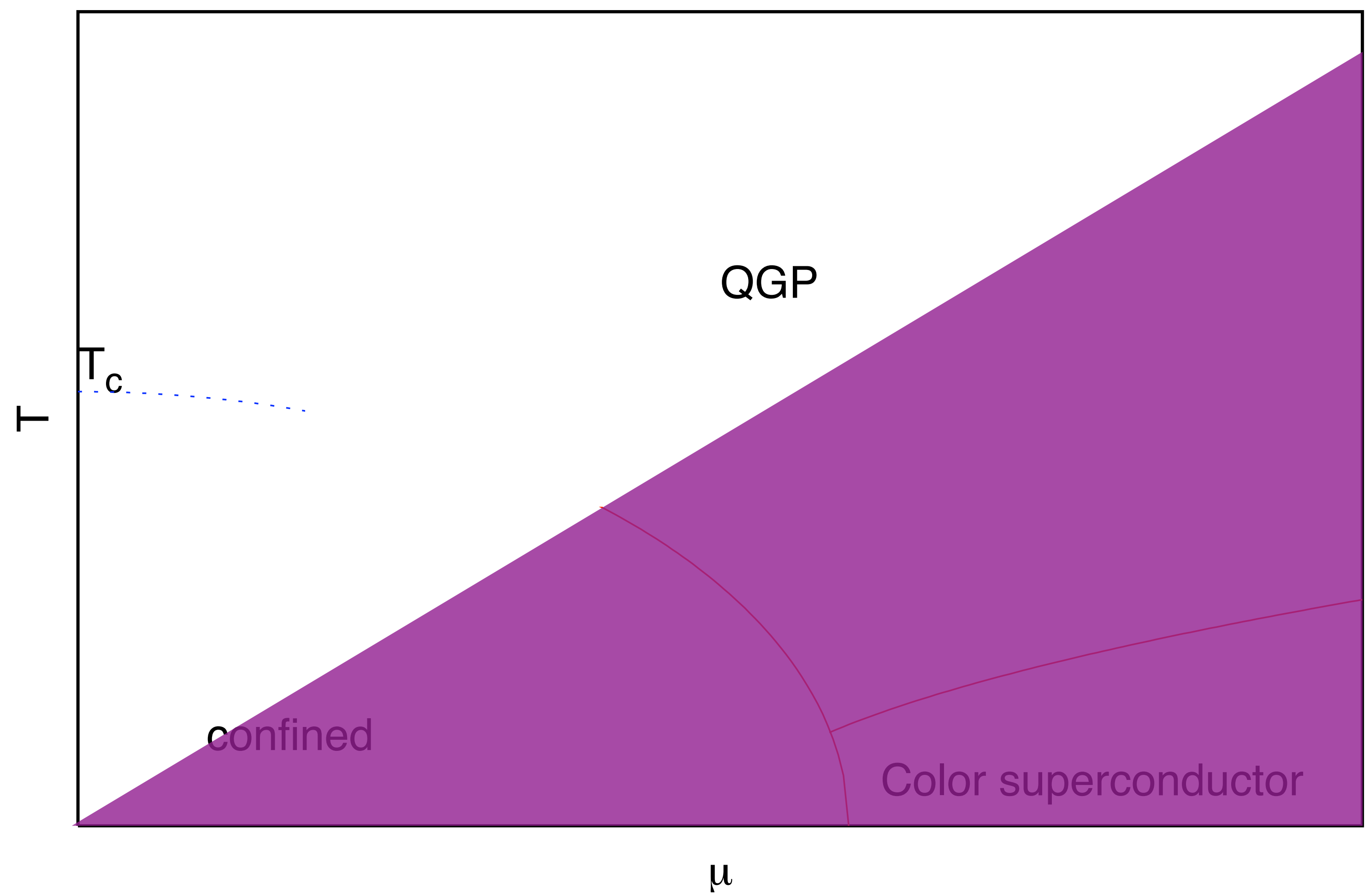


Imaginary chemical potential:
no sign problem!

de Forcrand, O.P.
D'Elia, Lombardo
Bonati, D'Elia, Sanfillippo
...

shape, sign of curvatures determined by tricritical scaling!

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond

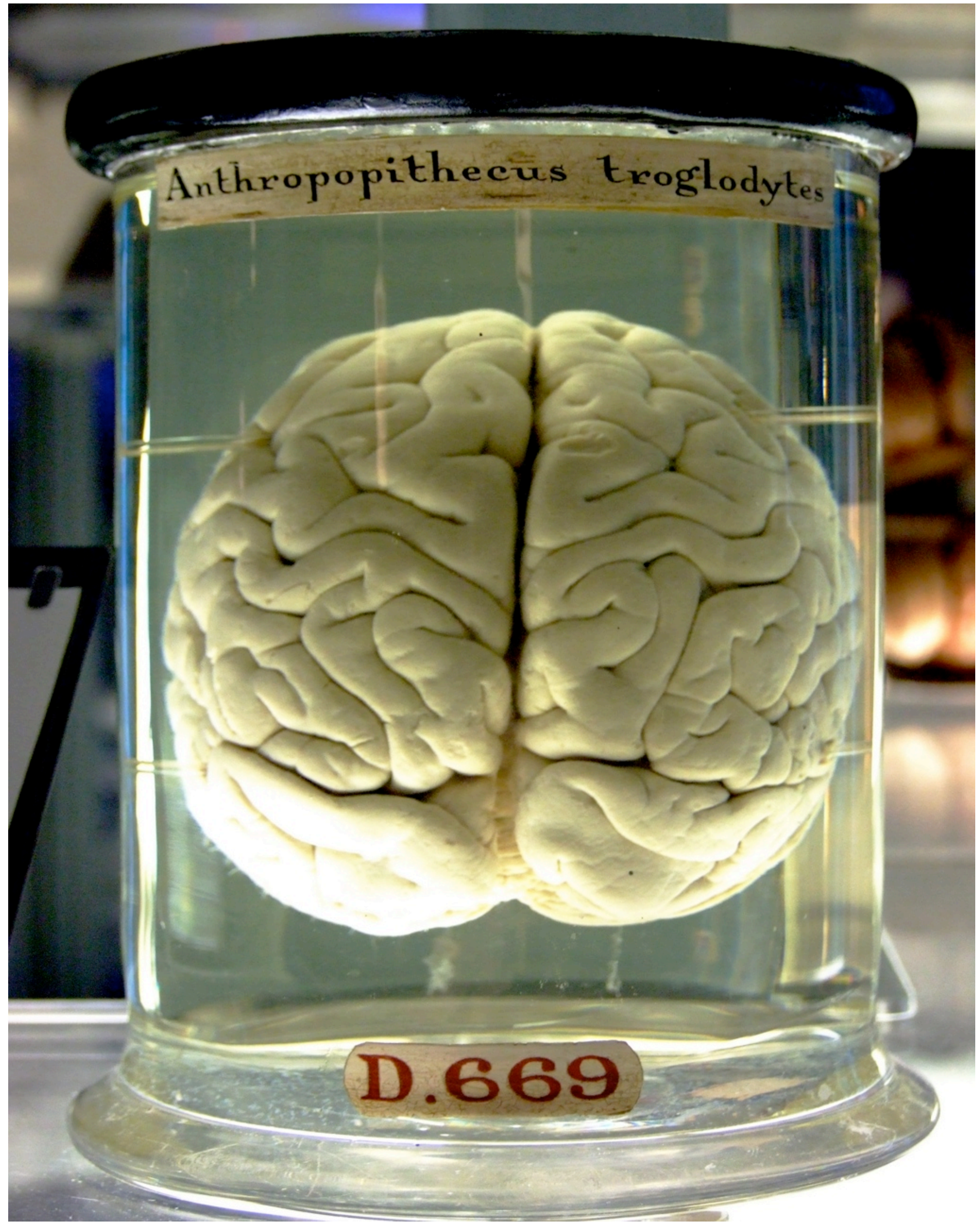
New computational avenues in Lattice QCD:

"(Wall)Time is Money (CPU hrs)"

CPU ⟶ GPU

Here, very old-fashioned approach: BPU!

Biological Processing Unit!



The effective lattice theory approach

with M. Fromm, J. Langelage, S. Lottini, M. Neuman

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \; \det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion

- Result: 3d spin model of QCD
- Step II: sign problem milder: Monte Carlo, complex Langevin
- Numerical simulations in 3d without fermion matrix inversion, very cheap!

Effective one-coupling theory for SU(3) YM

- Leading order graph in case of $N_\tau = 4$:

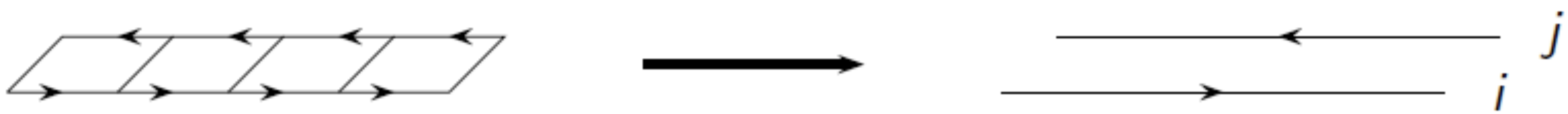


Figure: 4 plaquettes in fundamental representation lead to a 2 Polyakov loop interaction term

- Integration of spatial link variables leads to

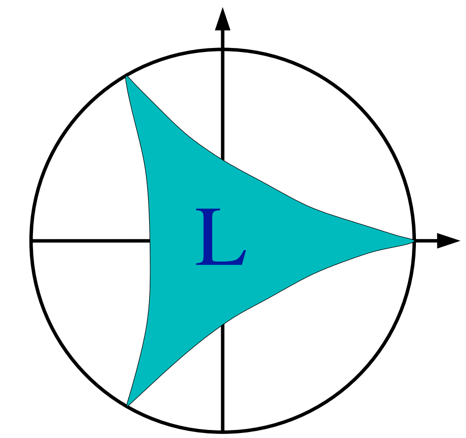
$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \,\mathrm{tr}\, W_j$$

- Possible generalizations: larger distance, higher dimensional representations, larger number of loops involved, ...
- *Here*: Decorate LO graph with additional spatial and temporal plaquettes

Effective one-coupling theory for SU(3) YM

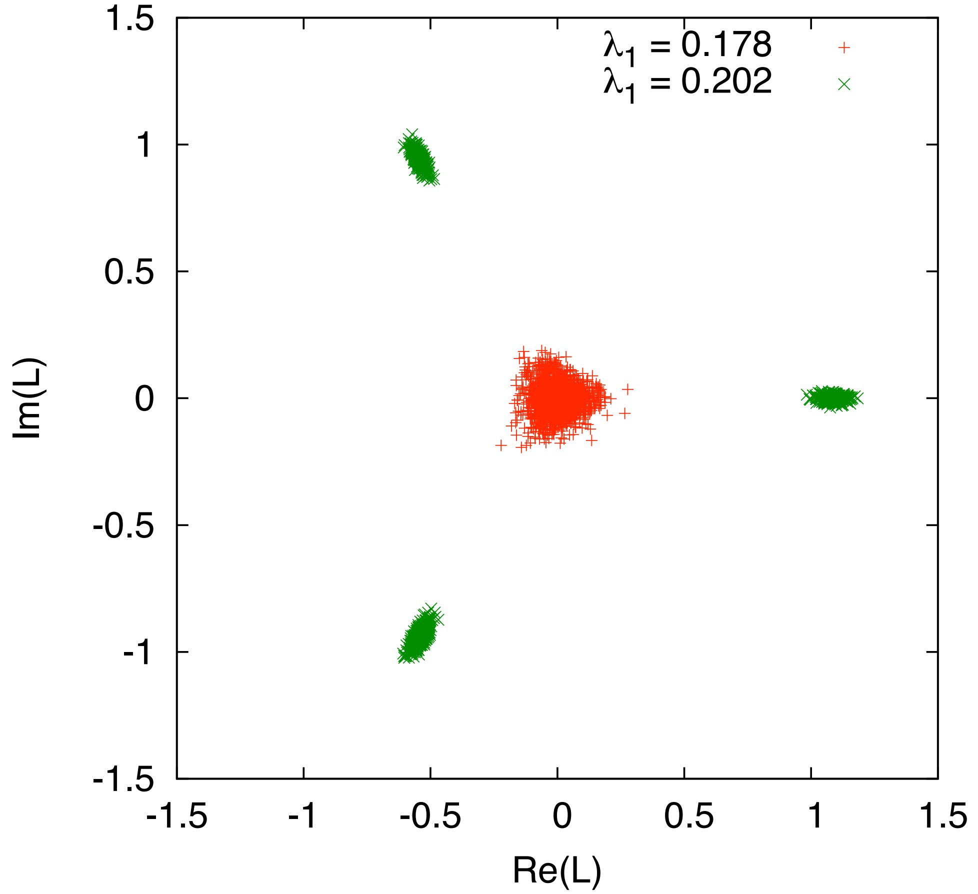
(L= Tr W)

$$Z = \int [dL] \exp\left[-S_1 + V_{SU(3)}\right]$$
$$= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] *$$
$$* \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re} L_i^3 - |L_i|^4}$$

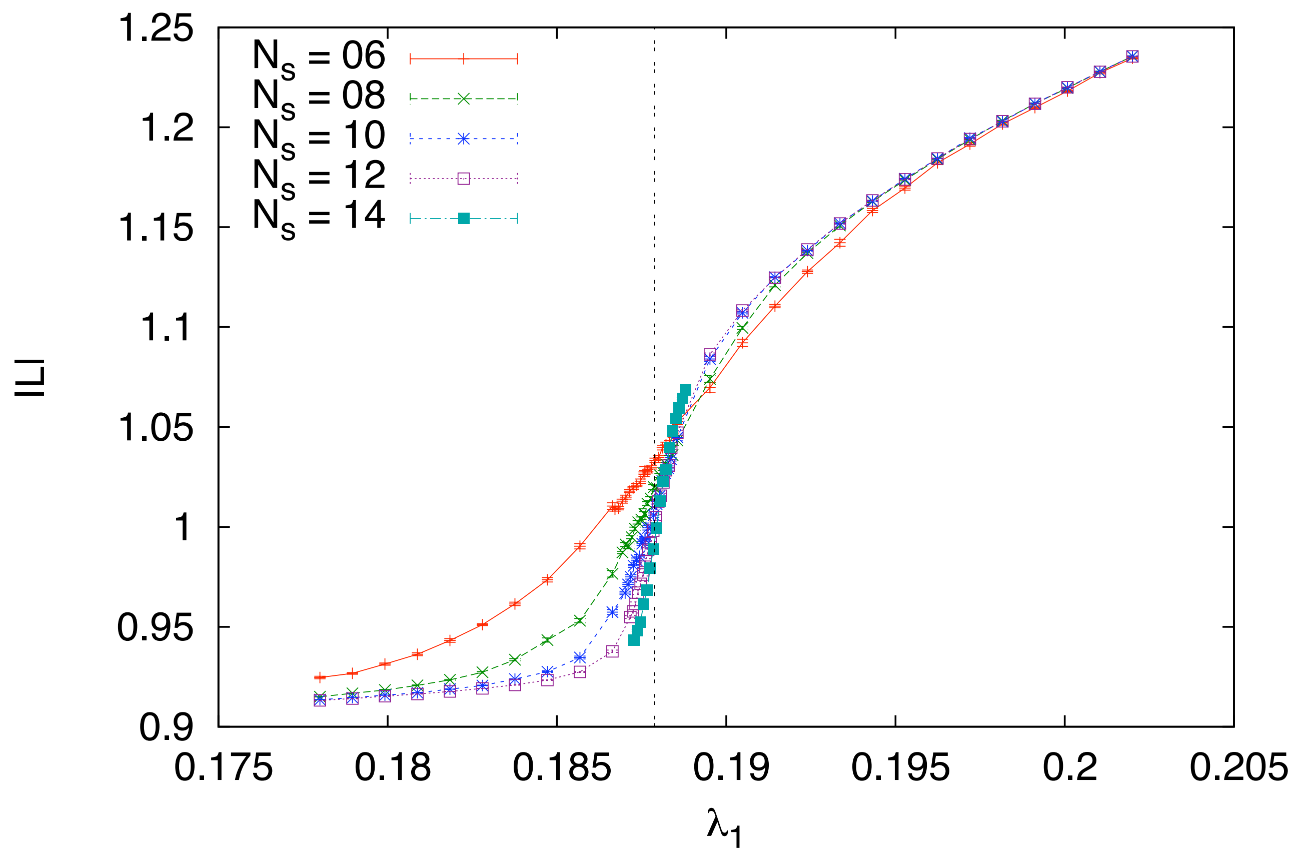


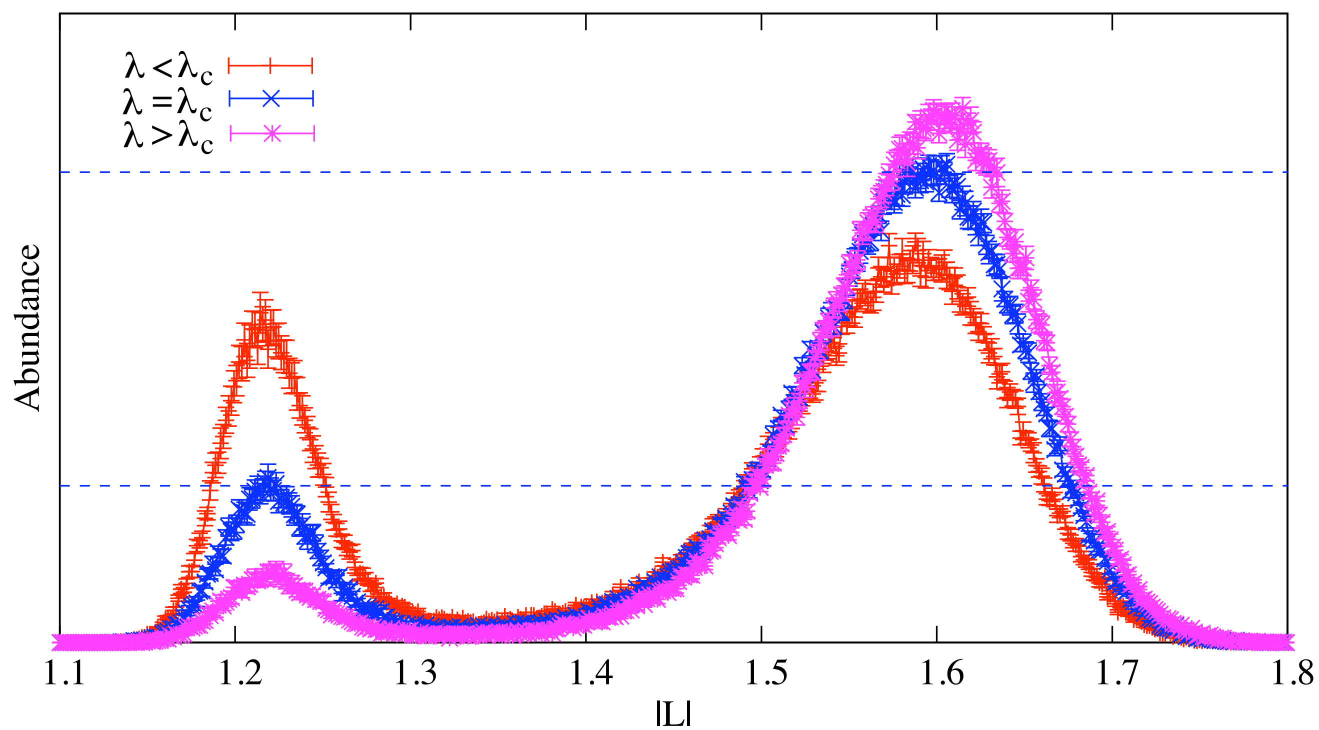
$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

Numerical results for SU(3)



Order-disorder transition





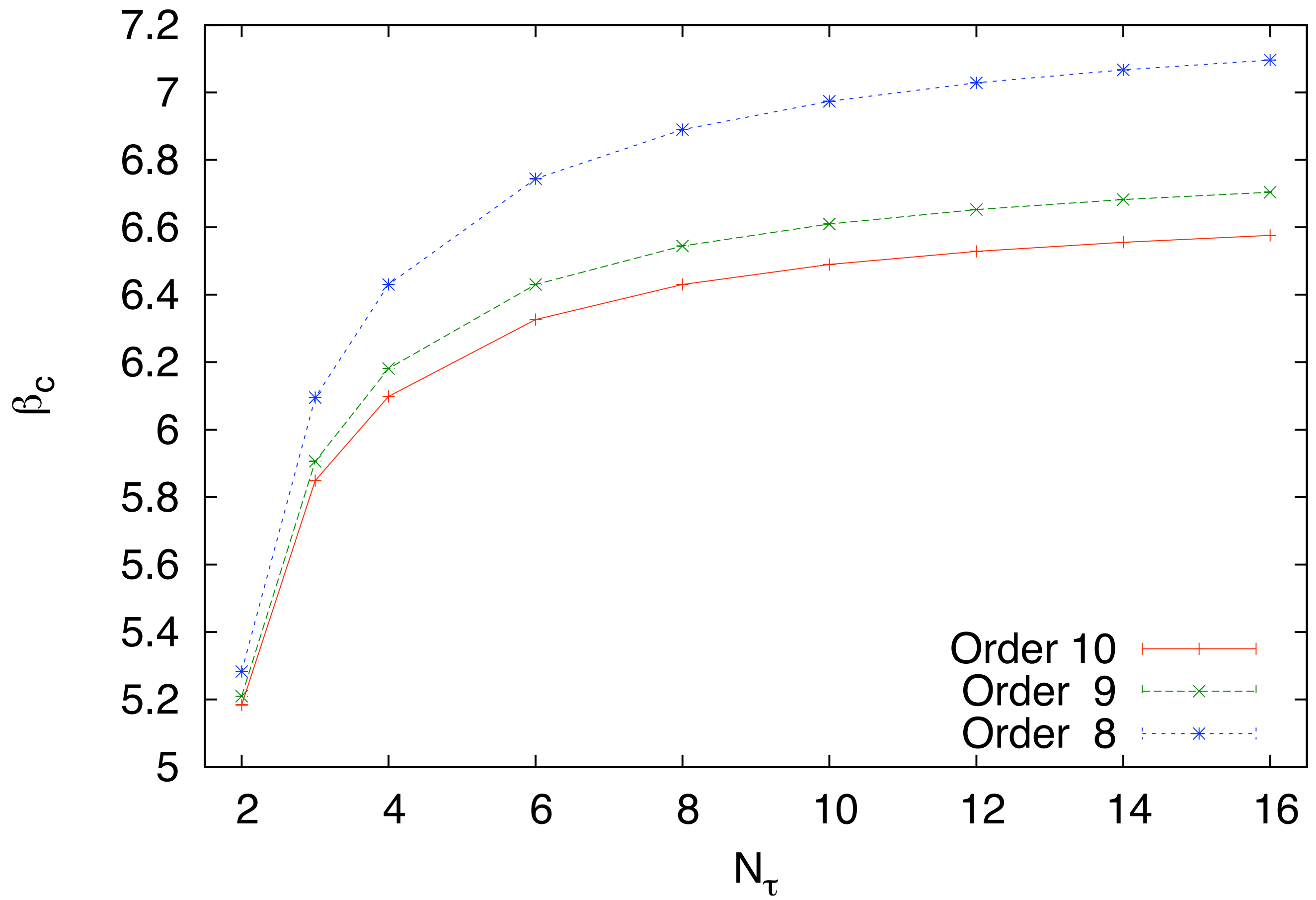
First order phase transition for SU(3) in the thermodynamic limit!

Mapping back to 4d finite T Yang-Mills

Inverting

$$\lambda_1(N_\tau, \beta) \rightarrow \beta_c(\lambda_{1,c}, N_\tau)$$

...points at reasonable convergence

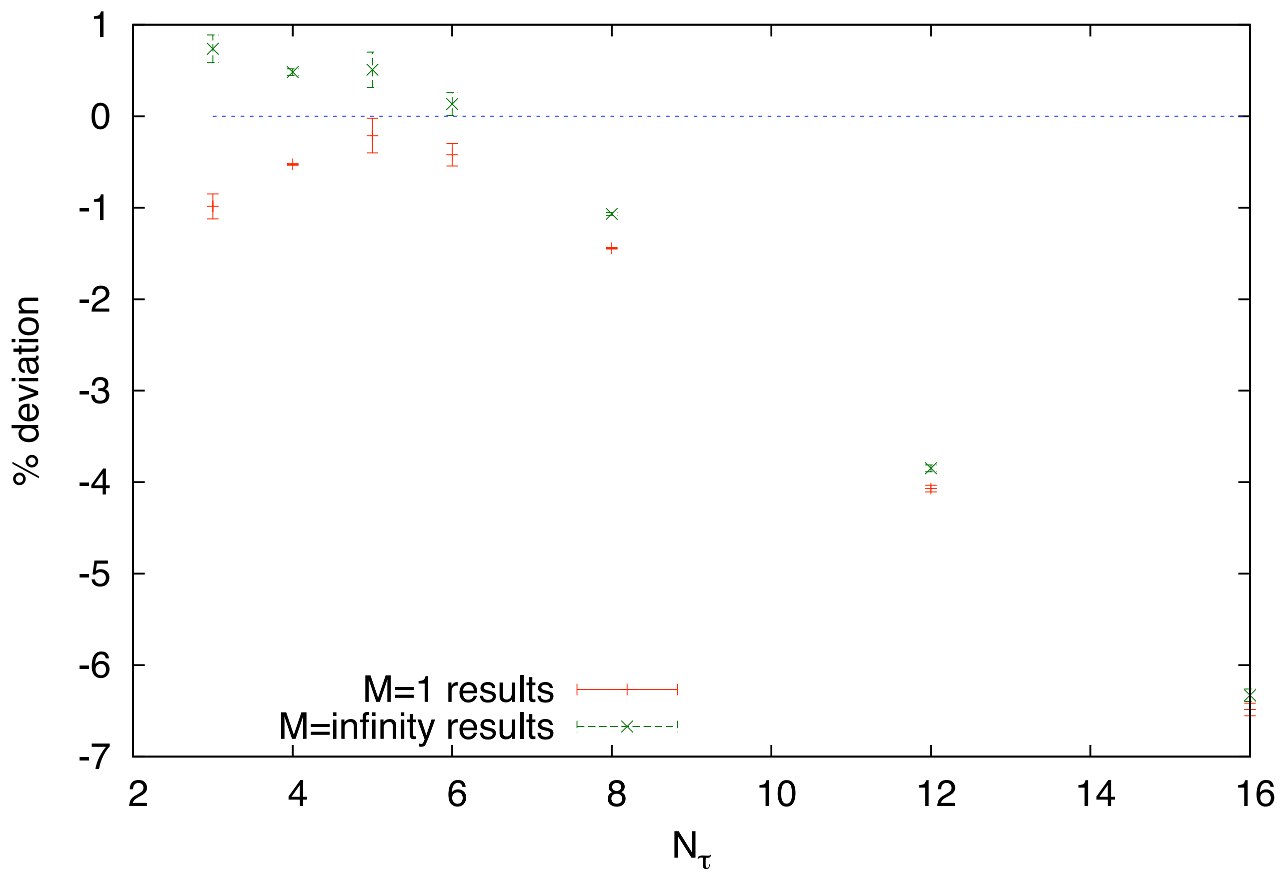


SU(3)

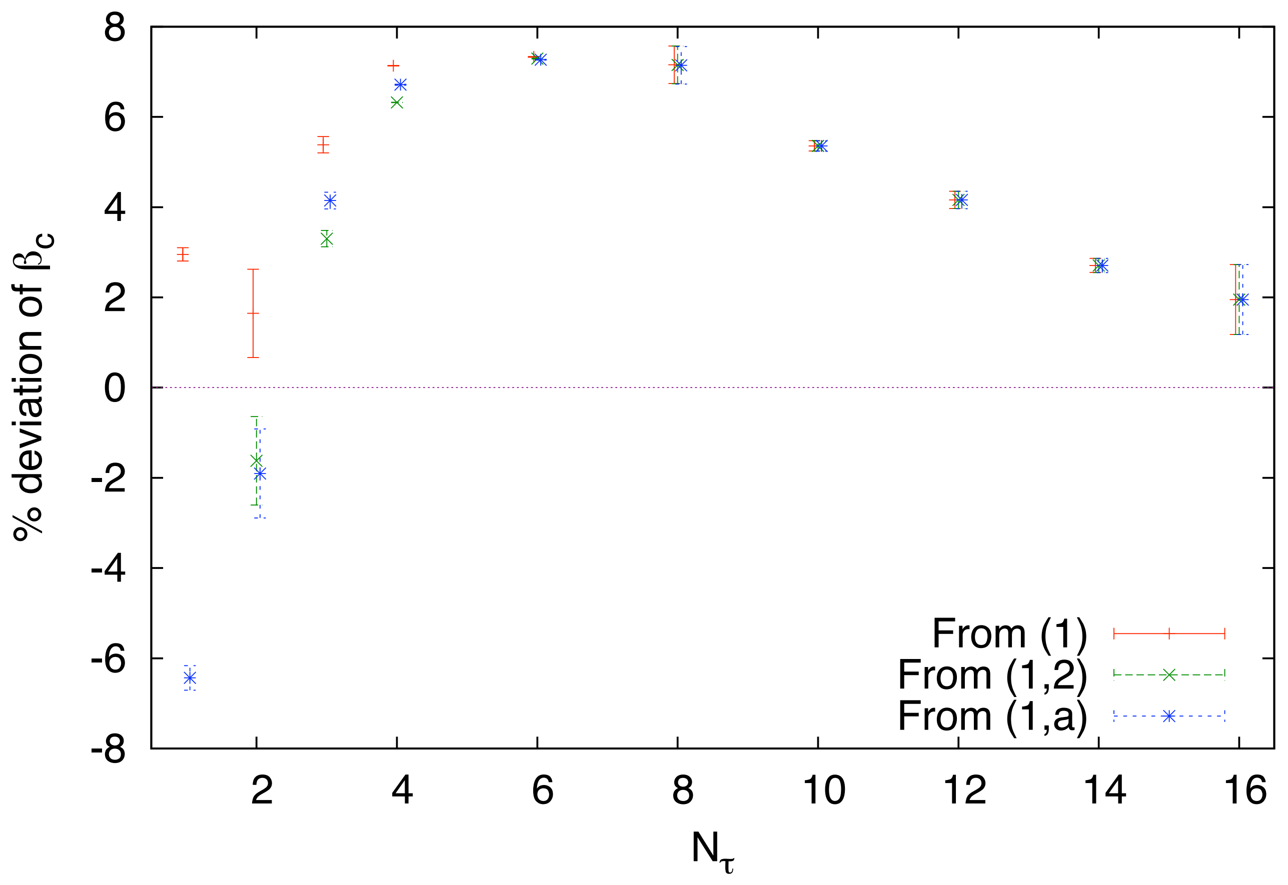
Comparison with 4d Monte Carlo

Relative accuracy for β_c compared to the full theory

SU(2)

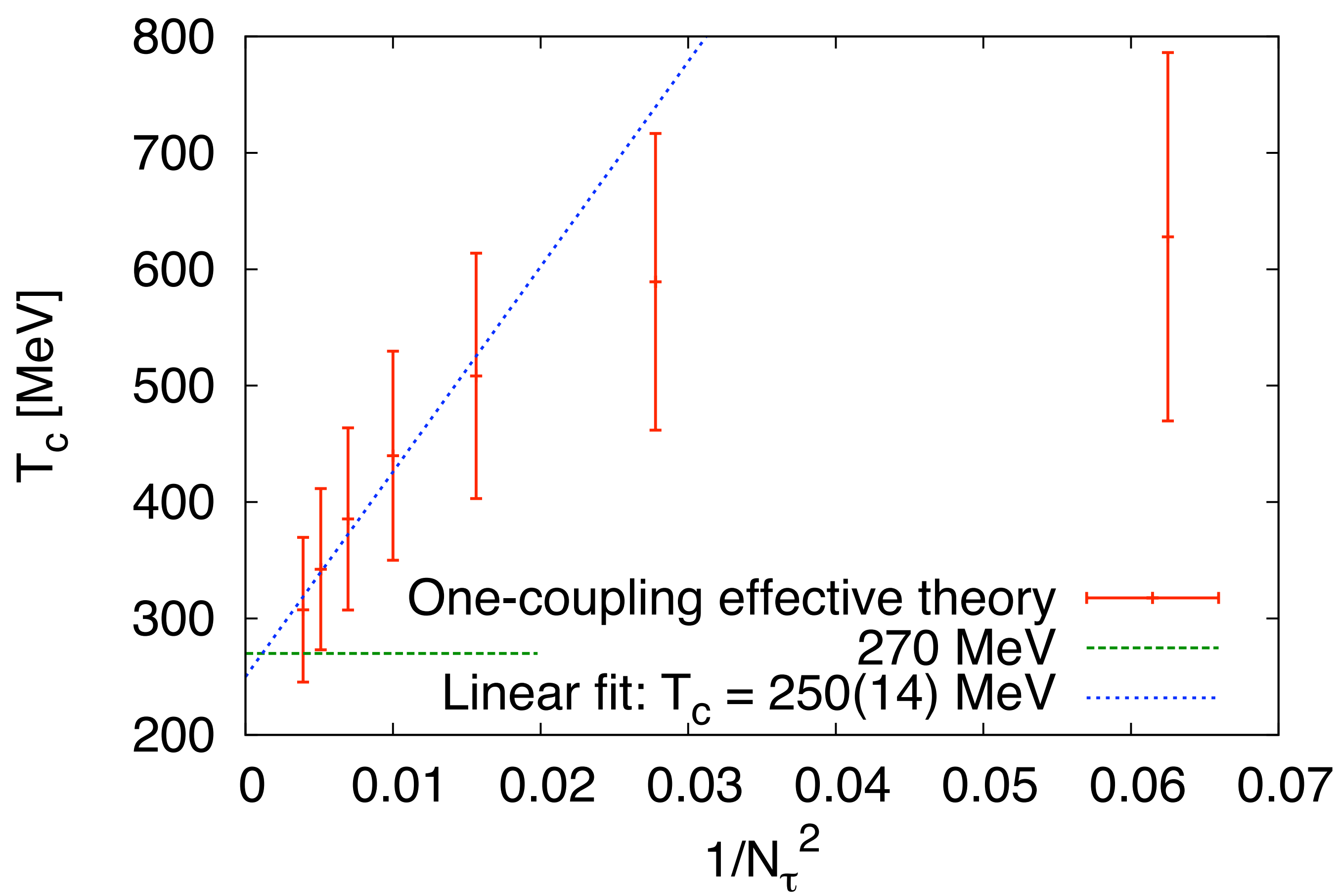


SU(3)



Note: influence of additional couplings checked explicitly!

Continuum limit feasible!



-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

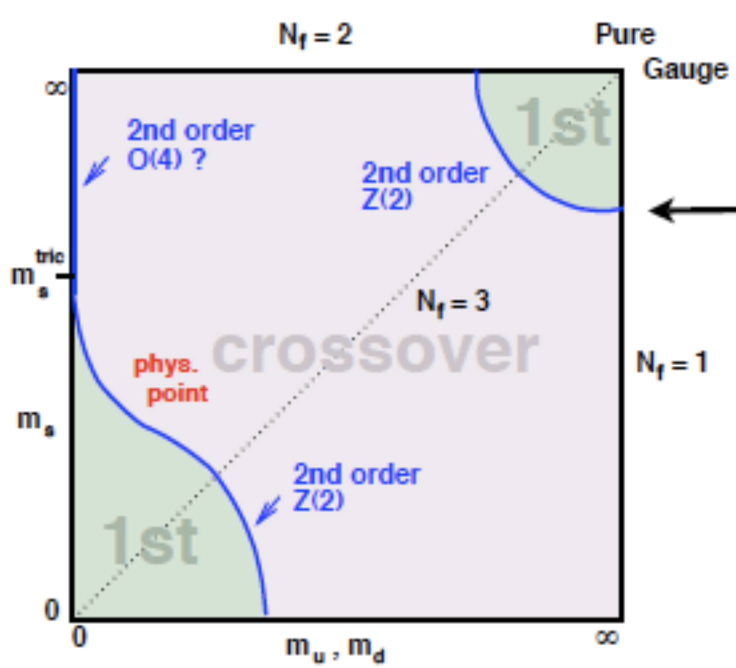
Including heavy, dynamical Wilson fermions

Expand in the *hopping parameter* $\kappa = 1/(2aM + 8)$:

$$-S_{\text{eff}} = \sum_i \lambda_i(u, \kappa, N_\tau) S_i^{\text{S}} - 2N_f \sum_i \left[h_i(u, \kappa, \mu, N_\tau) S_i^{\text{A}} + \overline{h}_i(u, \kappa, \mu, N_\tau) S_i^{\dagger\text{A}} \right]$$

Now, keep only $\lambda_1 S_1^{\text{S}}$ and $h_1 S_1^{\text{A}} + \overline{h}_1 S_1^{\dagger\text{A}}$

NLO: $\sim \kappa^2$

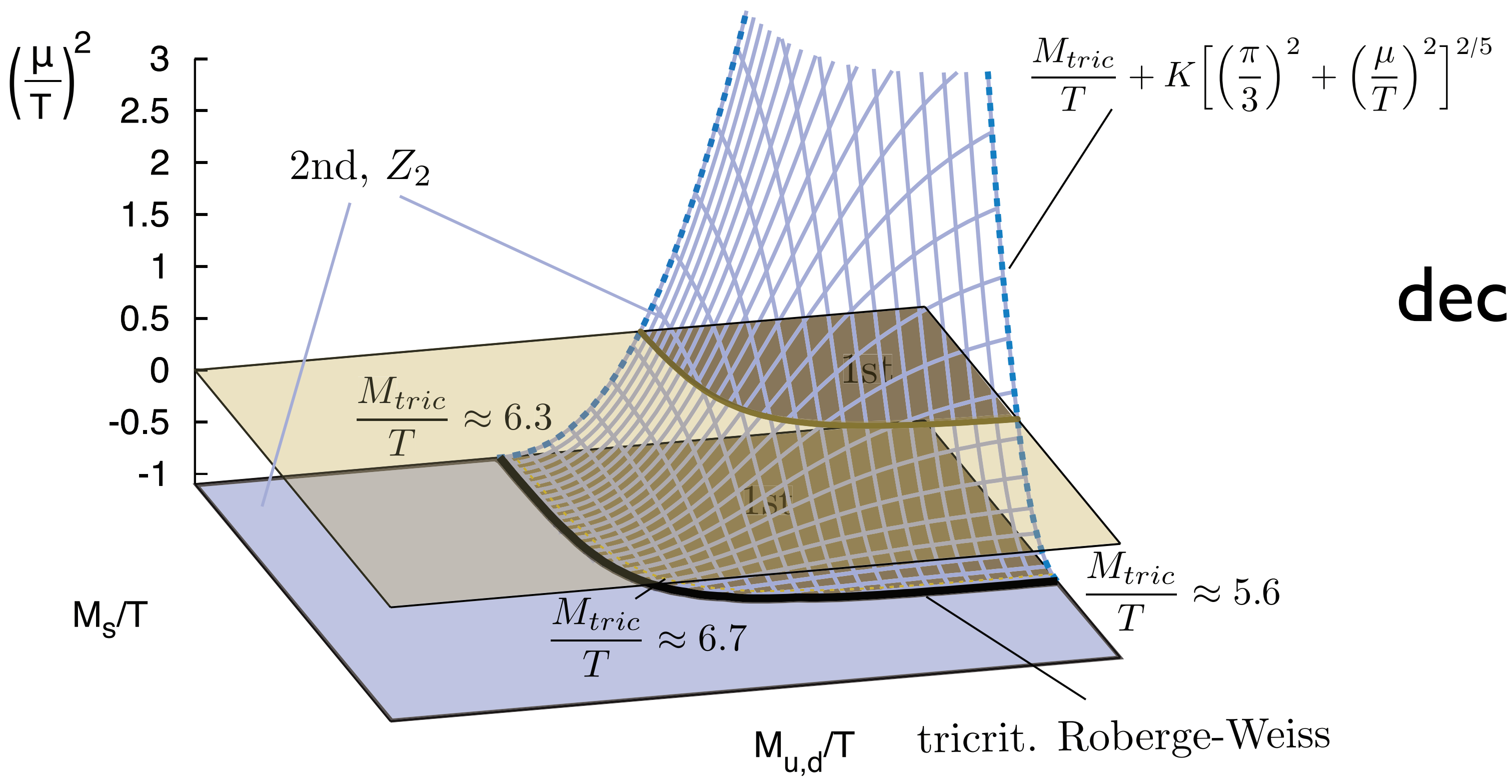


	eff. theory		4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!

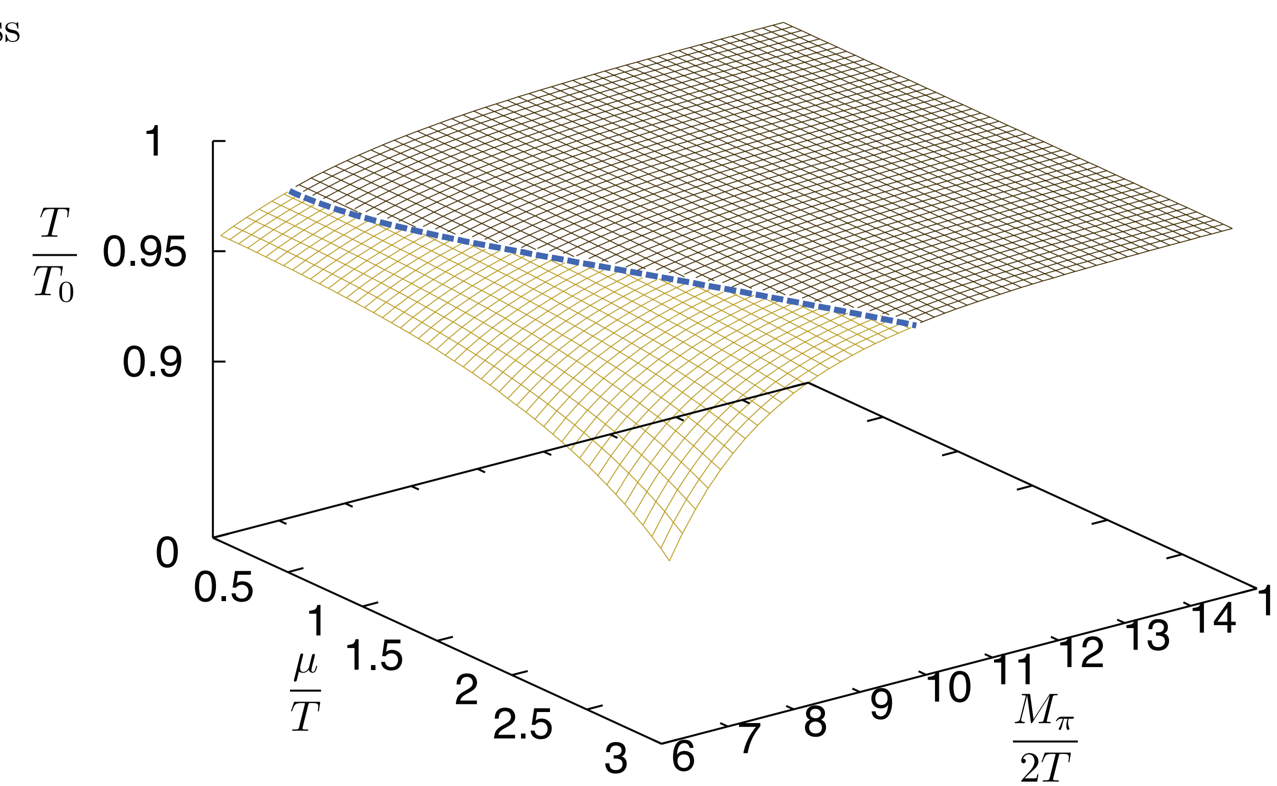
The deconfinement transition at all densities

Metropolis algorithm,
Complex Langevin



phase diagram for Nf=2, Nt=6

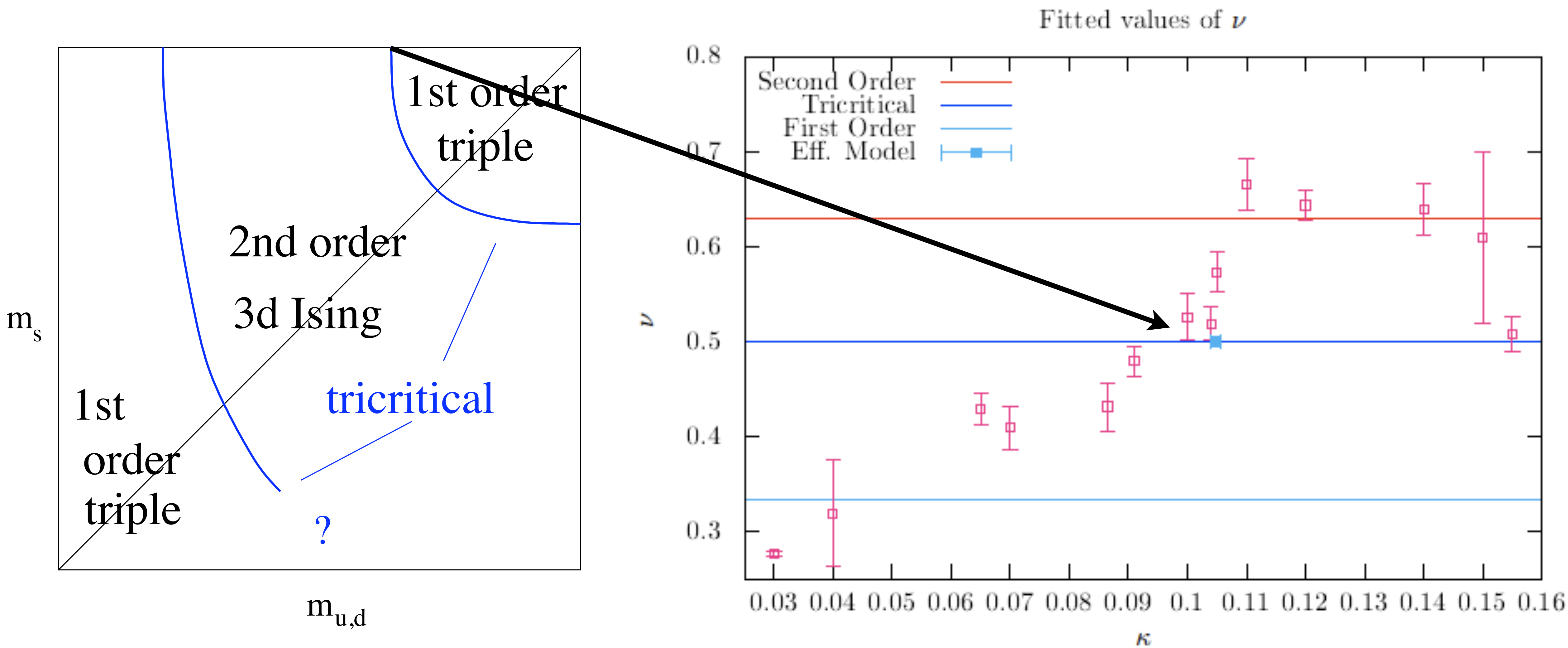
deconfinement critical surface



Roberge-Weiss transition, eff. th. against full 4d

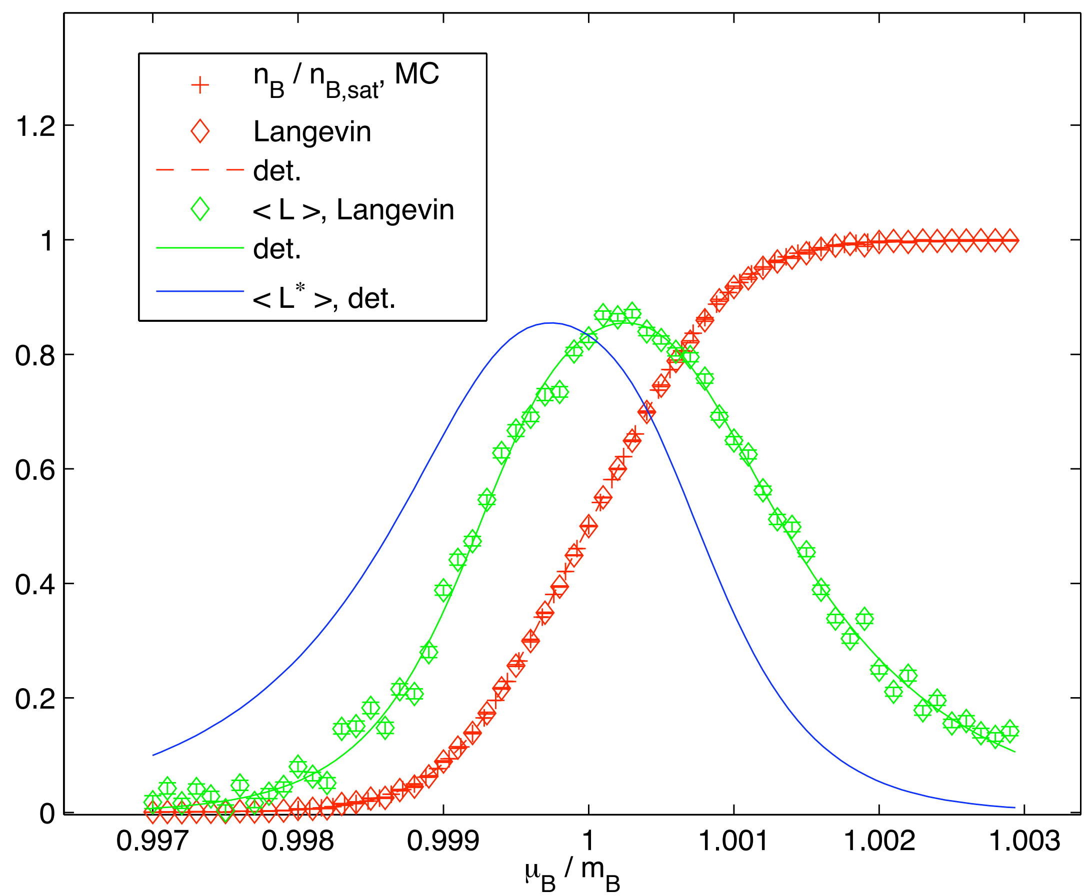
Pinke, O.P. 14

critical exponent distinguishes order of p.t.



$$\mu = i\frac{\pi T}{3}$$

Cold and dense heavy QCD



$m_\pi = 20\ \mathrm{GeV}, T = 10\ \mathrm{MeV}, a = 0.17\ \mathrm{fm}$

$\beta = 5.7, \kappa = 0.0000887, N_\tau = 116$

"Silver blaze" property
+ lattice saturation (Pauli principle)

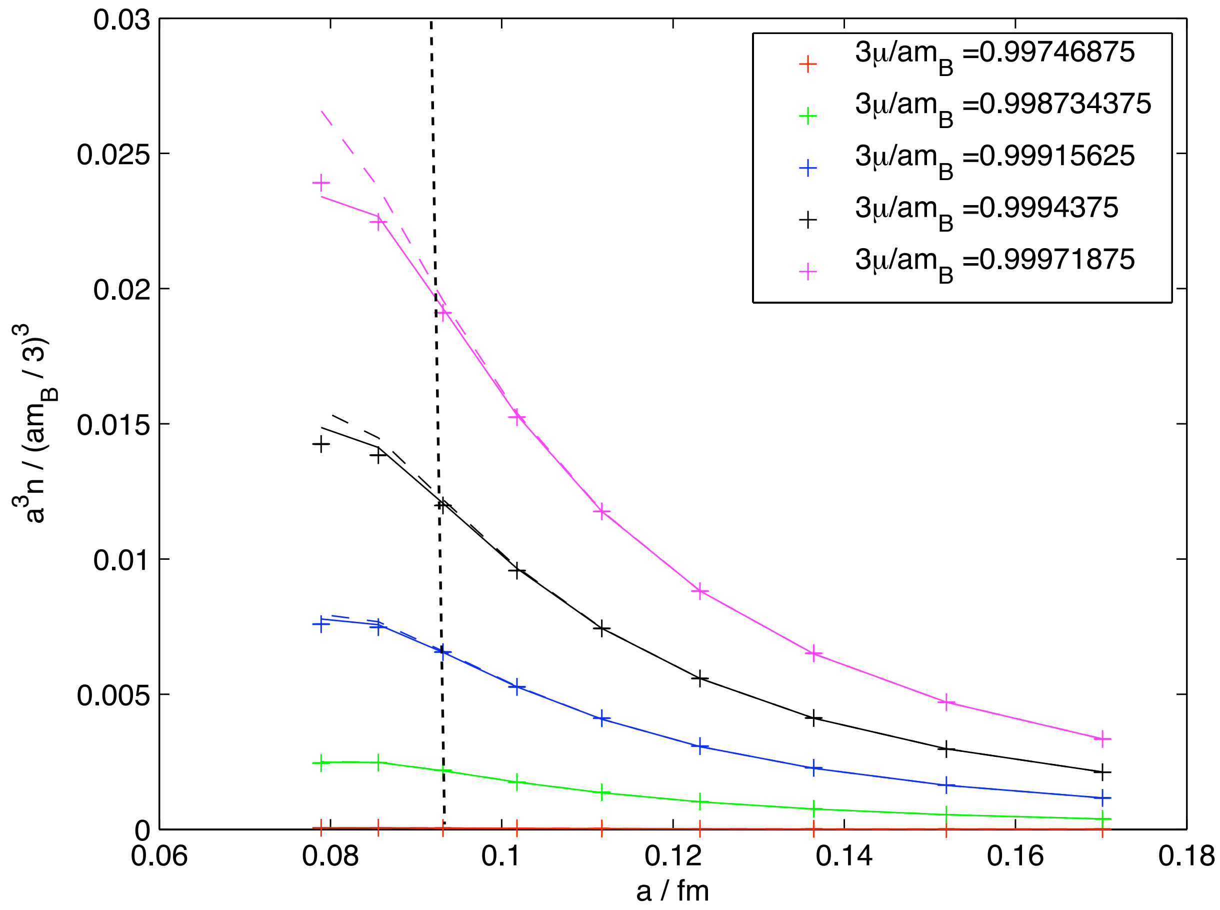
Analytic strong coupling soln. valid!

$\lambda(\beta = 5.7, N_\tau = 115) \sim 10^{-27}$

Continuum extrapolation

Scaling with lattice spacing:

$$\frac{n_{\rm lat}(\mu)}{m_B^3} = \frac{n_{\rm cont}(\mu)}{m_B^3} + A(\mu)a + B(\mu)a^2 + \ldots$$



Solid/dashed lines: analytic strong coupling limit with/without $\mathcal{O}(\kappa^2)$:

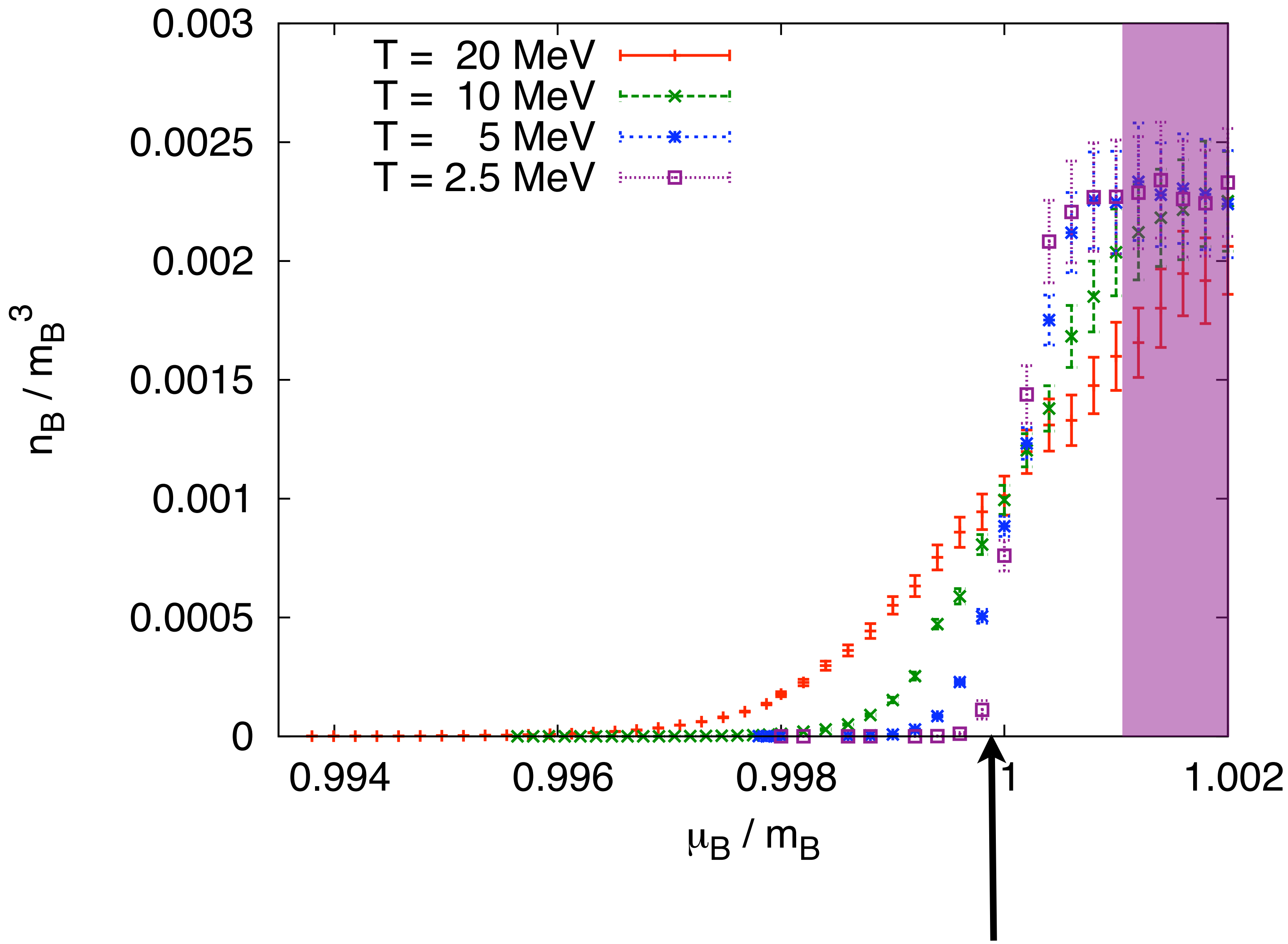
Breakdown of hopping series!

Onset transition to cold nuclear matter

PRL 13

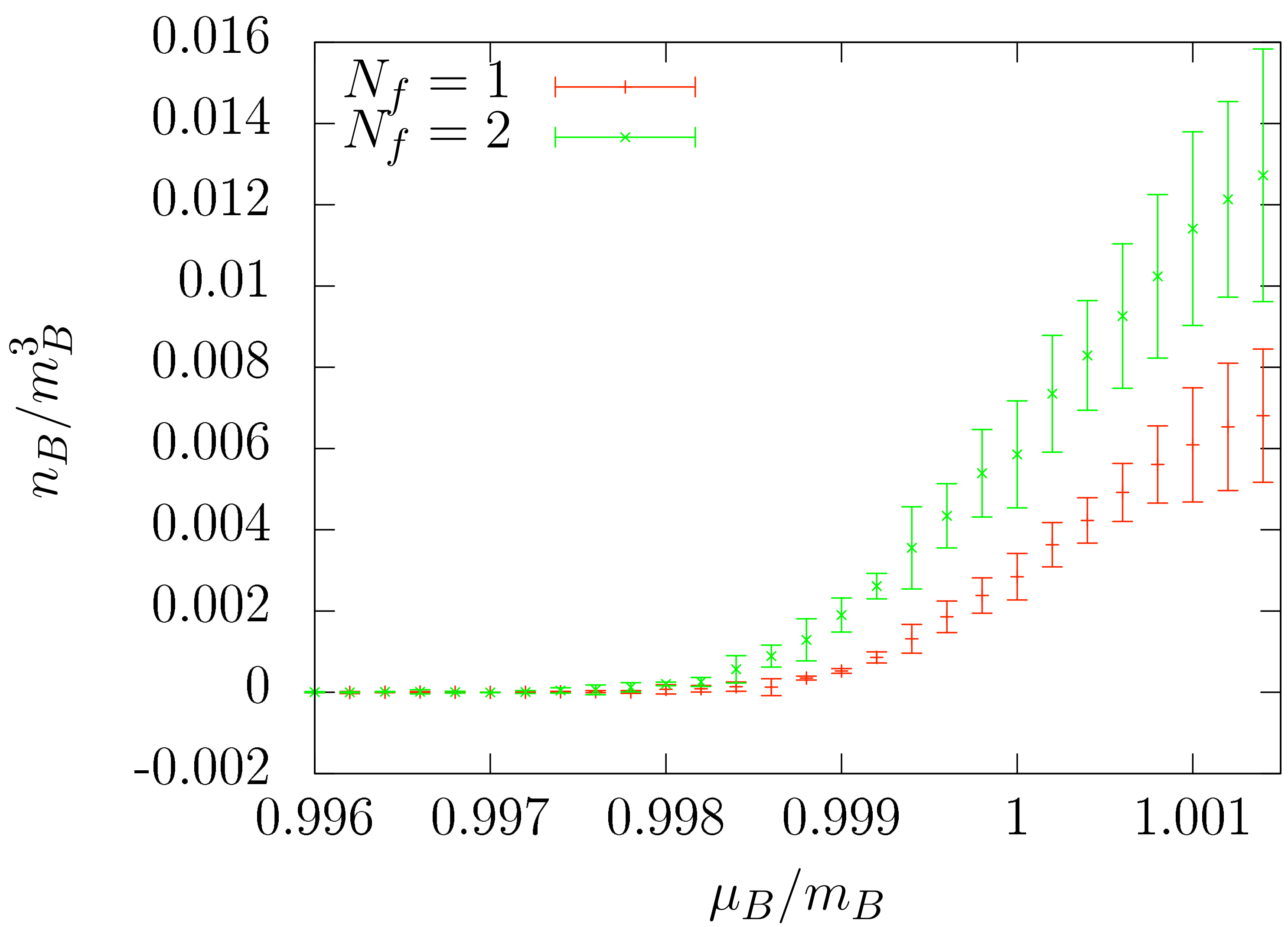
... with very heavy quarks $m_\pi = 20 \text{ GeV}$

continuum limit with 5-7 lattice spacings per point



$$\frac{\mu}{T} \sim 4000$$

The equation of state for nuclear matter



$$S_{eff} \sim \kappa^n u^m, \quad n+m=4$$

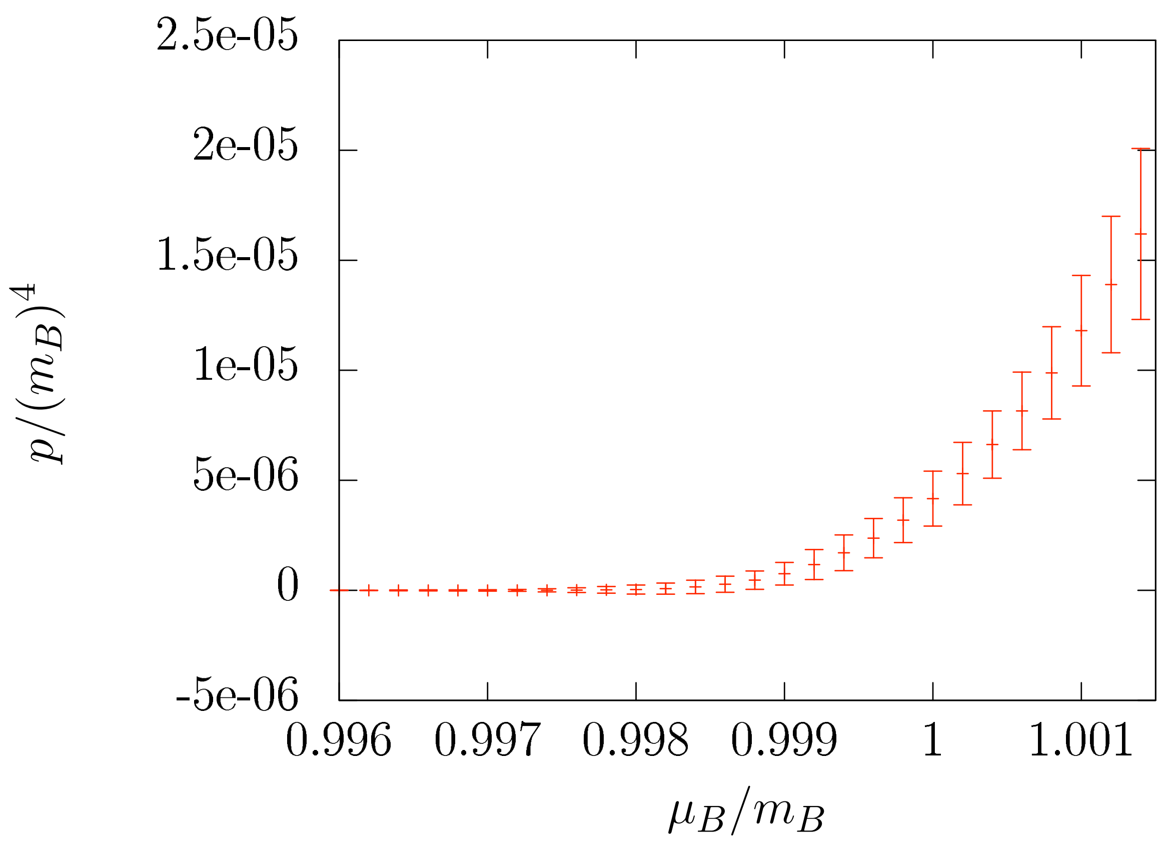
$$m_\pi = 20\ \mathrm{GeV}, T = 10\ \mathrm{MeV}$$

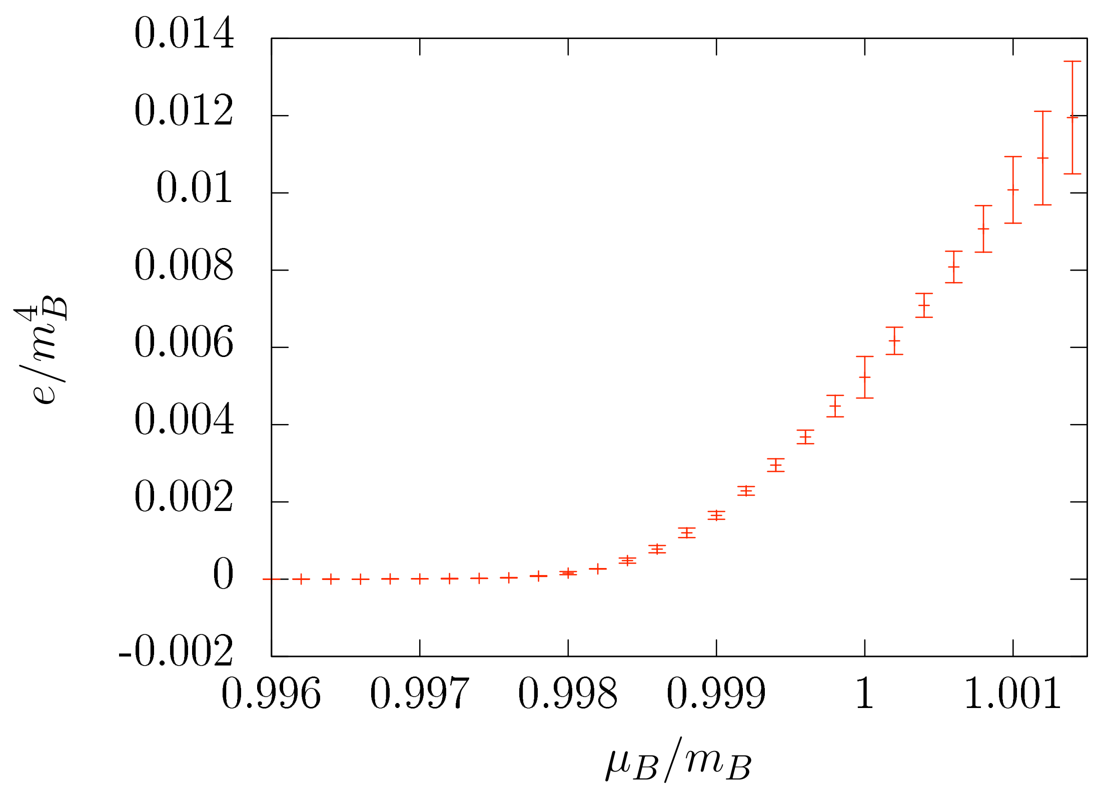
Effect of binding between baryons: $\mu_c < m_B$

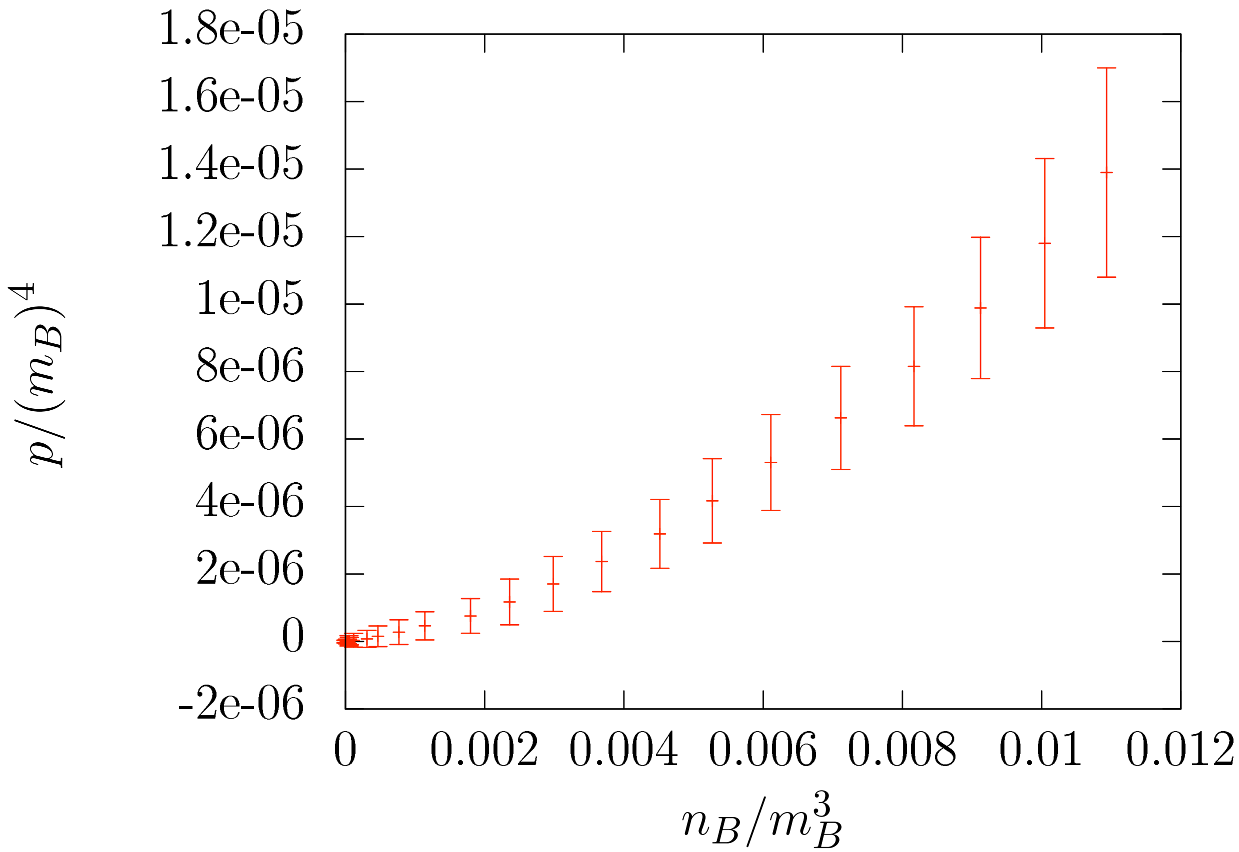
Binding energy per nucleon: $\epsilon = \dfrac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

The equation of state for nuclear matter, Nf=2

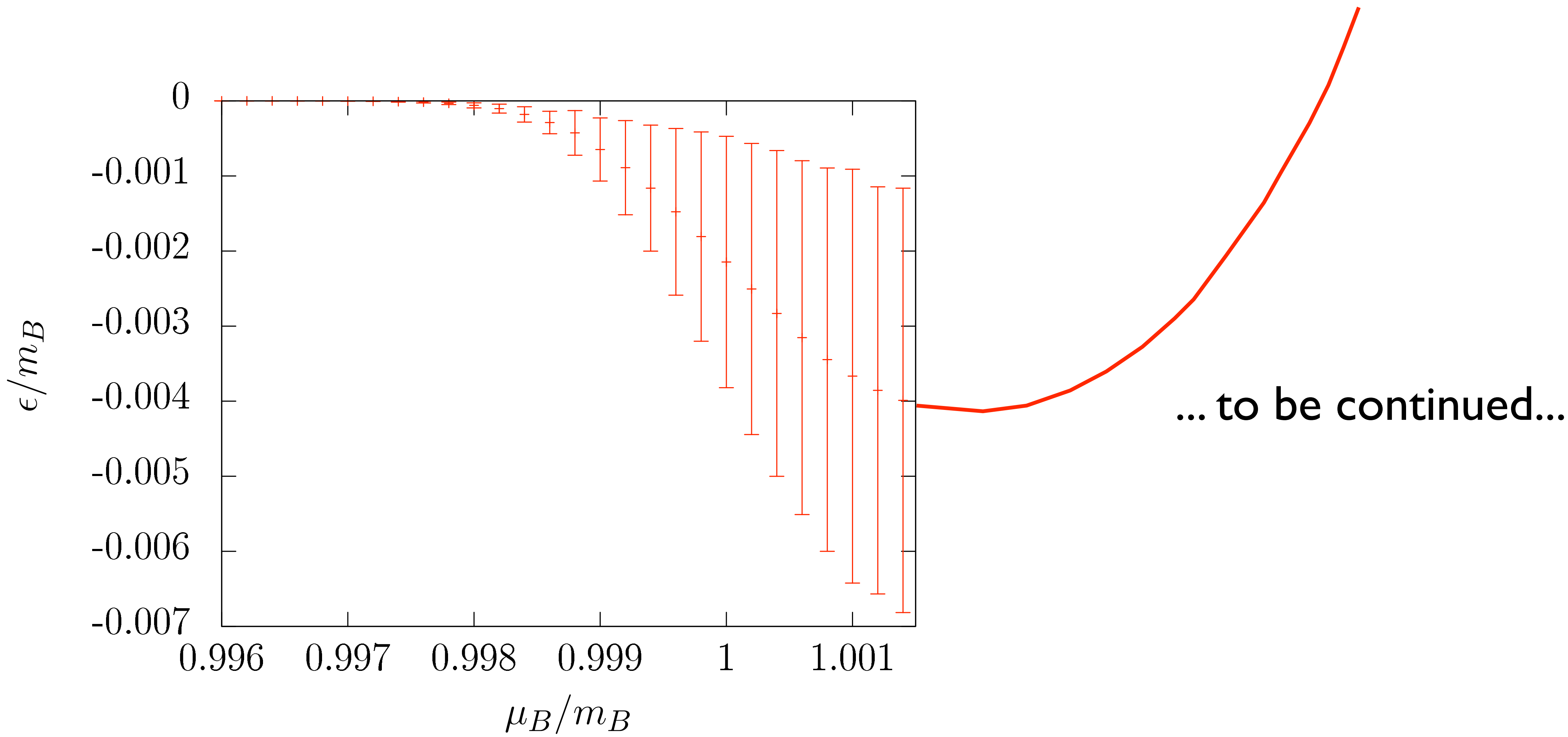






Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



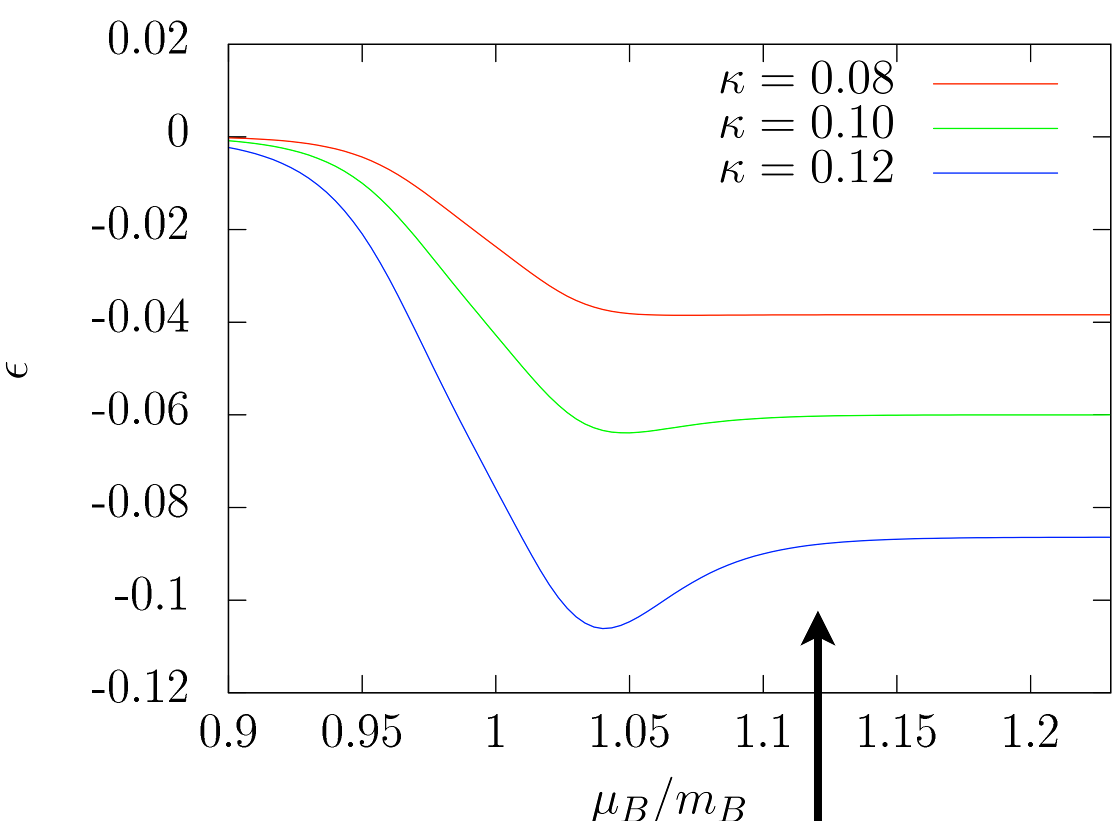
Minimum: access to nucl. binding energy, nucl. saturation density!

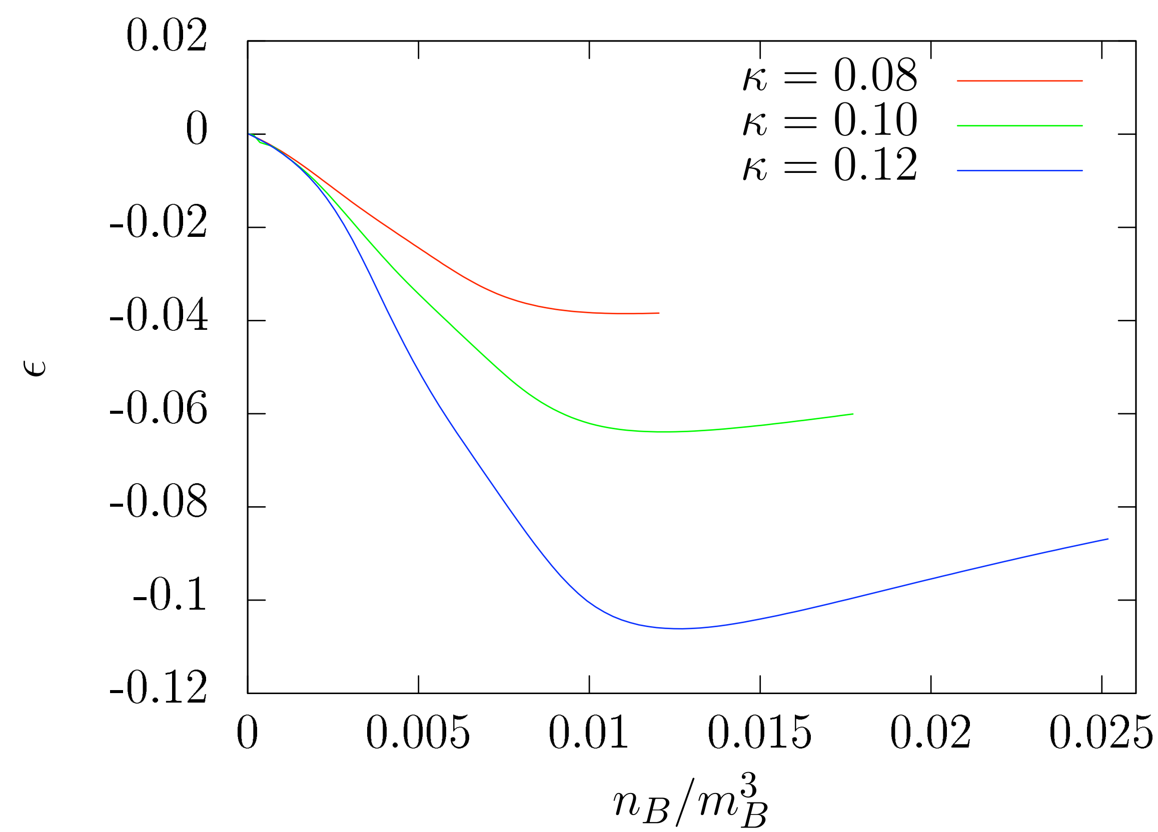
$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

Quark mass dependence of the binding energy:

Expect short range nucl. potential for heavy pions, $V \sim \frac{e^{-m_\pi r}}{r}$

Analytic solution, finite lattice spacing:

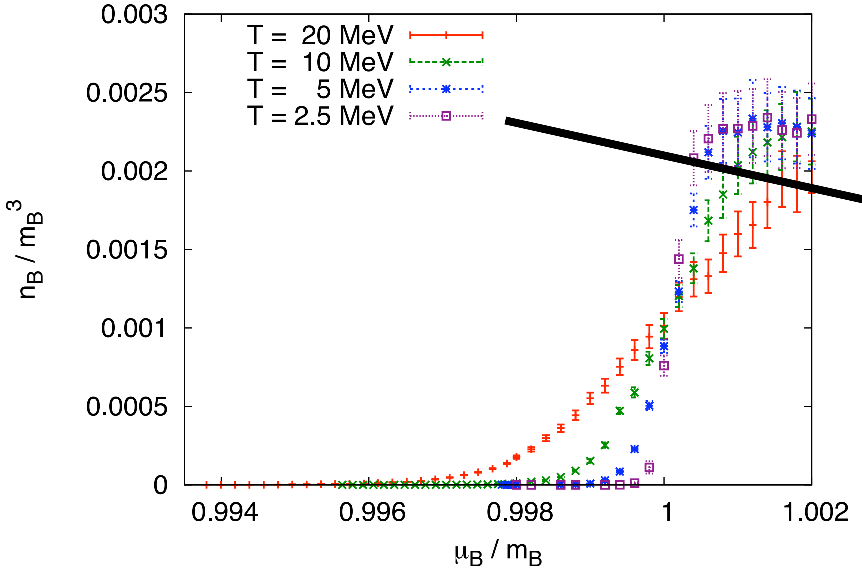




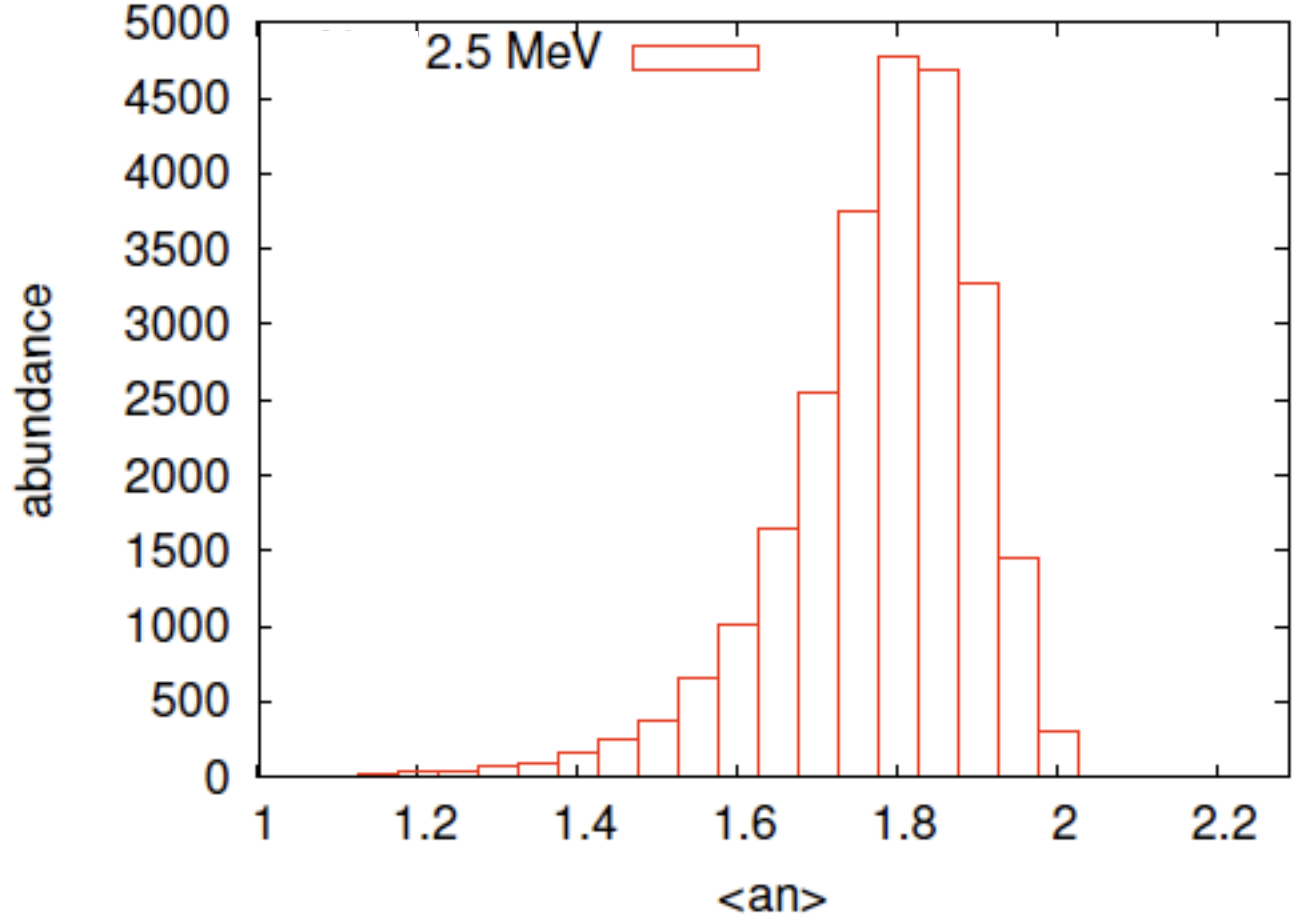
quark mass

lattice saturation

Order of the onset transition?

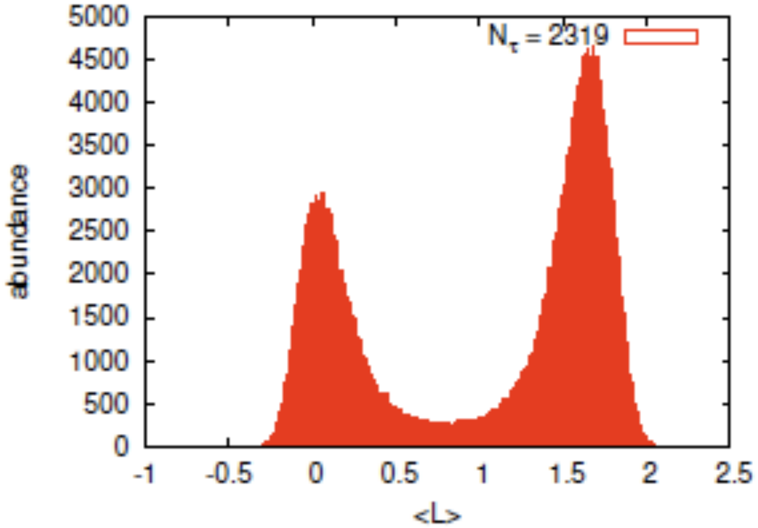


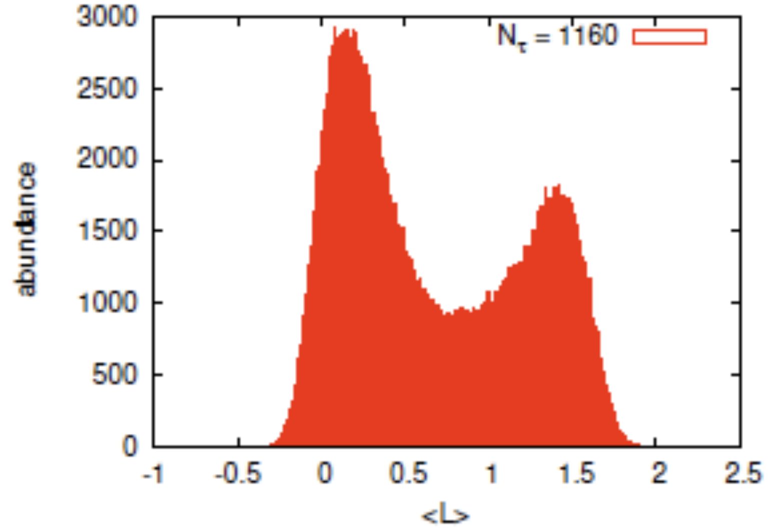
Distribution of fermion density: crossover!?

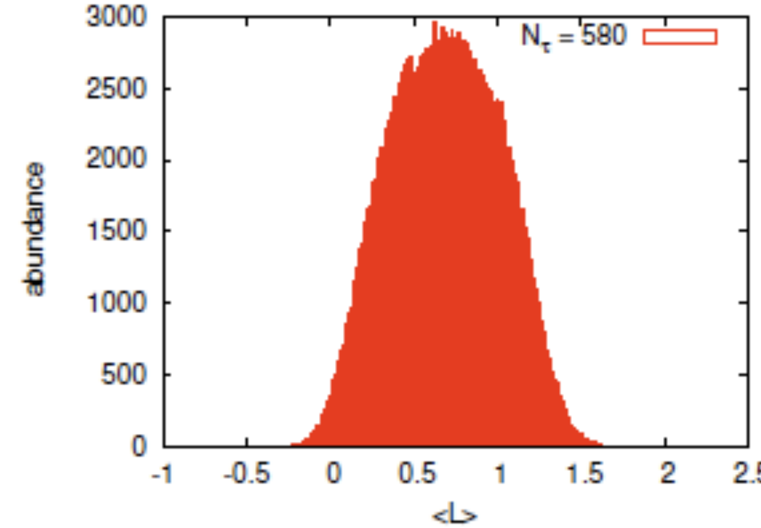


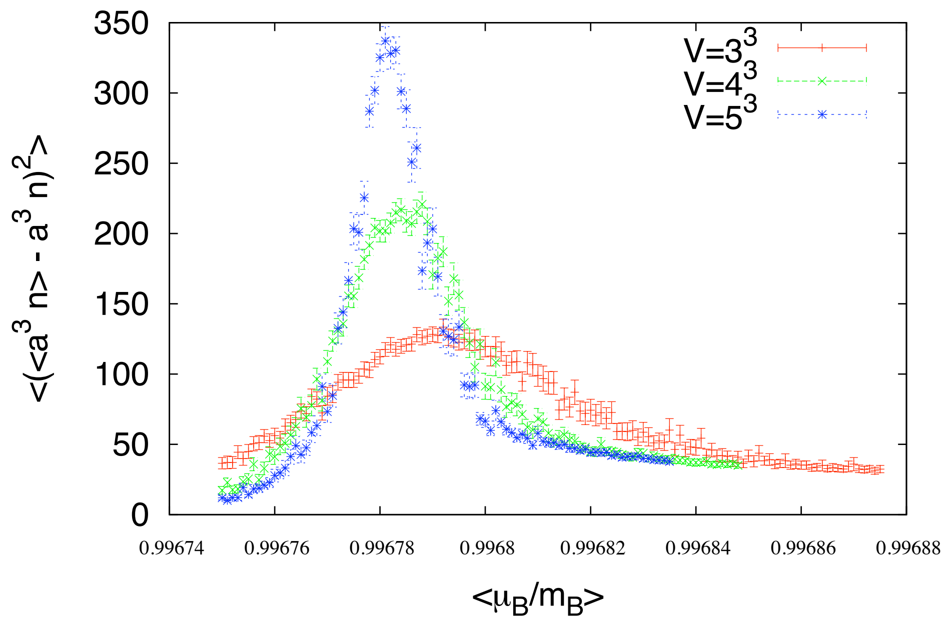
Reason: expect short range nucl. potential for heavy pions, $V \sim \frac{e^{-m_\pi r}}{r}$

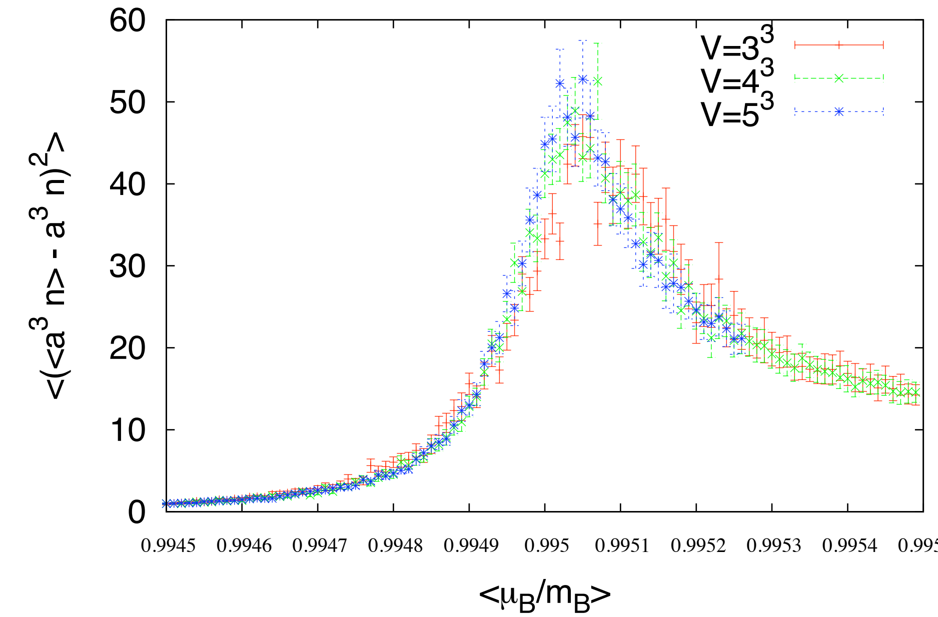
Lighter quarks: First order signal + endpoint!









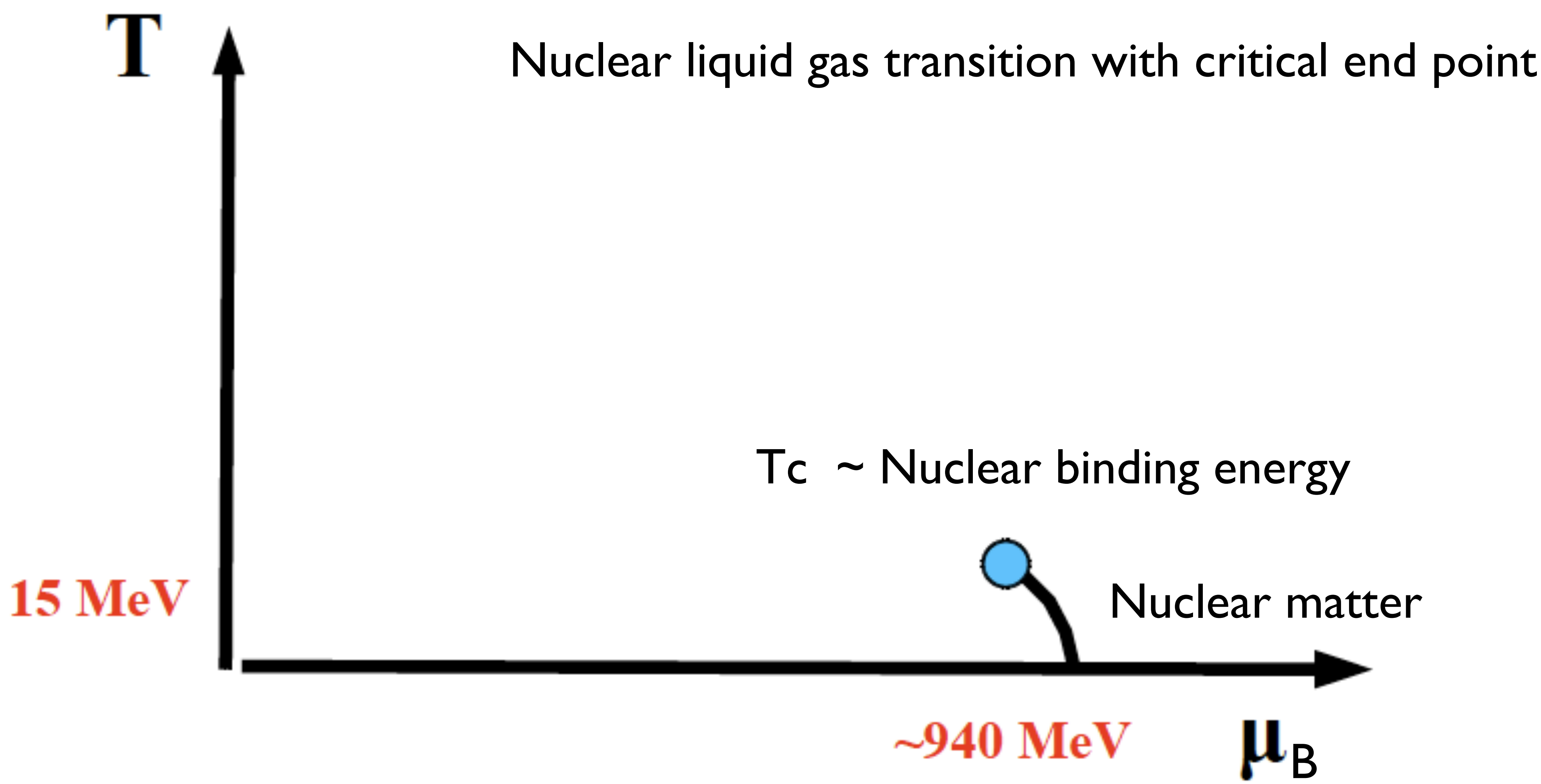


- $O(k^4)$:Stretching the hopping series, $\kappa = 0.12, \beta = 5.6$
- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

attn: no convergence yet!

all features of liquid gas transition!!!

Within reach of effective lattice QCD!



Can we get high enough orders for light quarks?!?

Conclusions

- No chiral critical point for $\mu/T \lesssim 1$
- New effective lattice theory allows to simulate heavy quarks at all densities
- Onset transition to (heavy) nuclear matter seen from lattice QCD!
- Higher orders, smaller quark masses?

Backup slides

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \; = \sqrt{2}$$

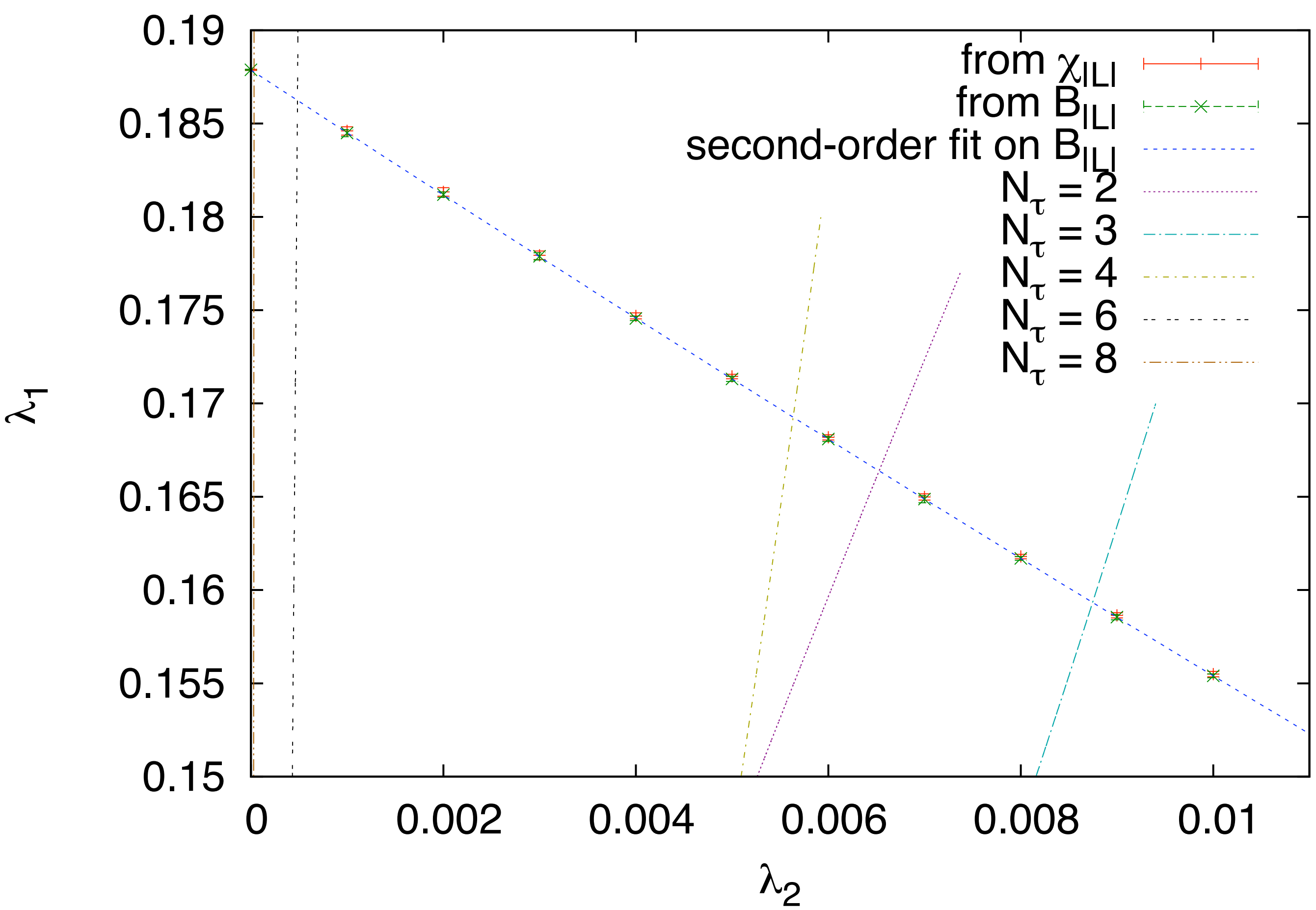
$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \; = 2$$

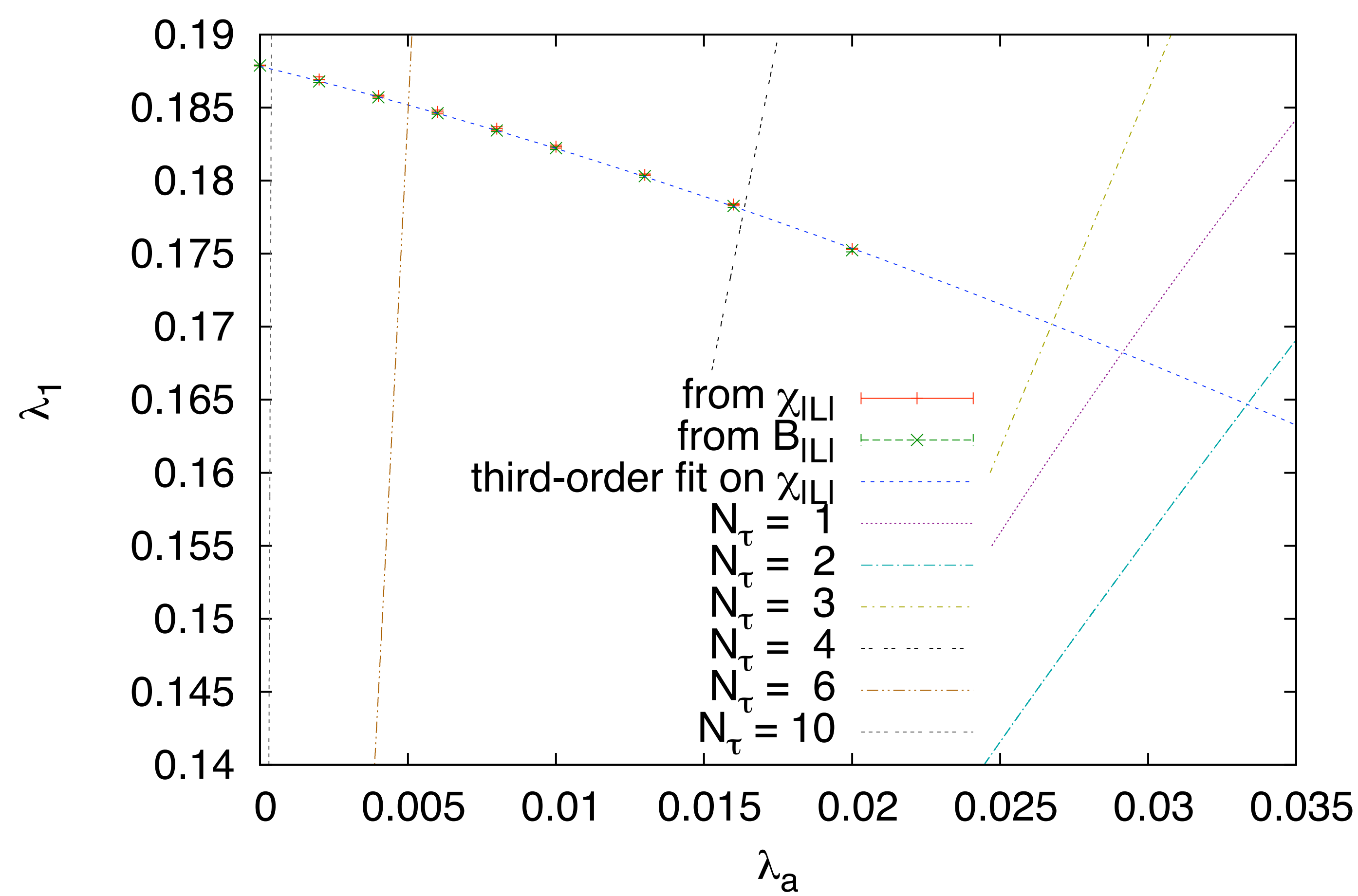
as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

The influence of a second coupling

NLO-couplings: next-to-nearest neighbour, adjoint rep. loops





...gets very small for large N_τ !

Cold and dense QCD I: static, strong coupling limit

For T=0 (at finite density) anti-fermions decouple $N_f = 1, h_1 = C, h_2 = 0$

$$C_f \equiv (2\kappa_f e^{a\mu_f})^{N_\tau} = e^{(\mu_f - m_f)/T}, \; \bar{C}_f(\mu_f) = C_f(-\mu_f)$$

$$Z(\beta = 0) = \left[\prod_f \int \mathrm{d}W \left(1 + C_f L + C_f^2 L^* + C_f^3\right)^2 \right]^{N_s^3}$$

$$\xrightarrow{T \to 0} \left[1 + 4C^{N_c} + C^{2N_c}\right]^{N_s^3}$$

Free gas of baryons!

Quarkyonic?

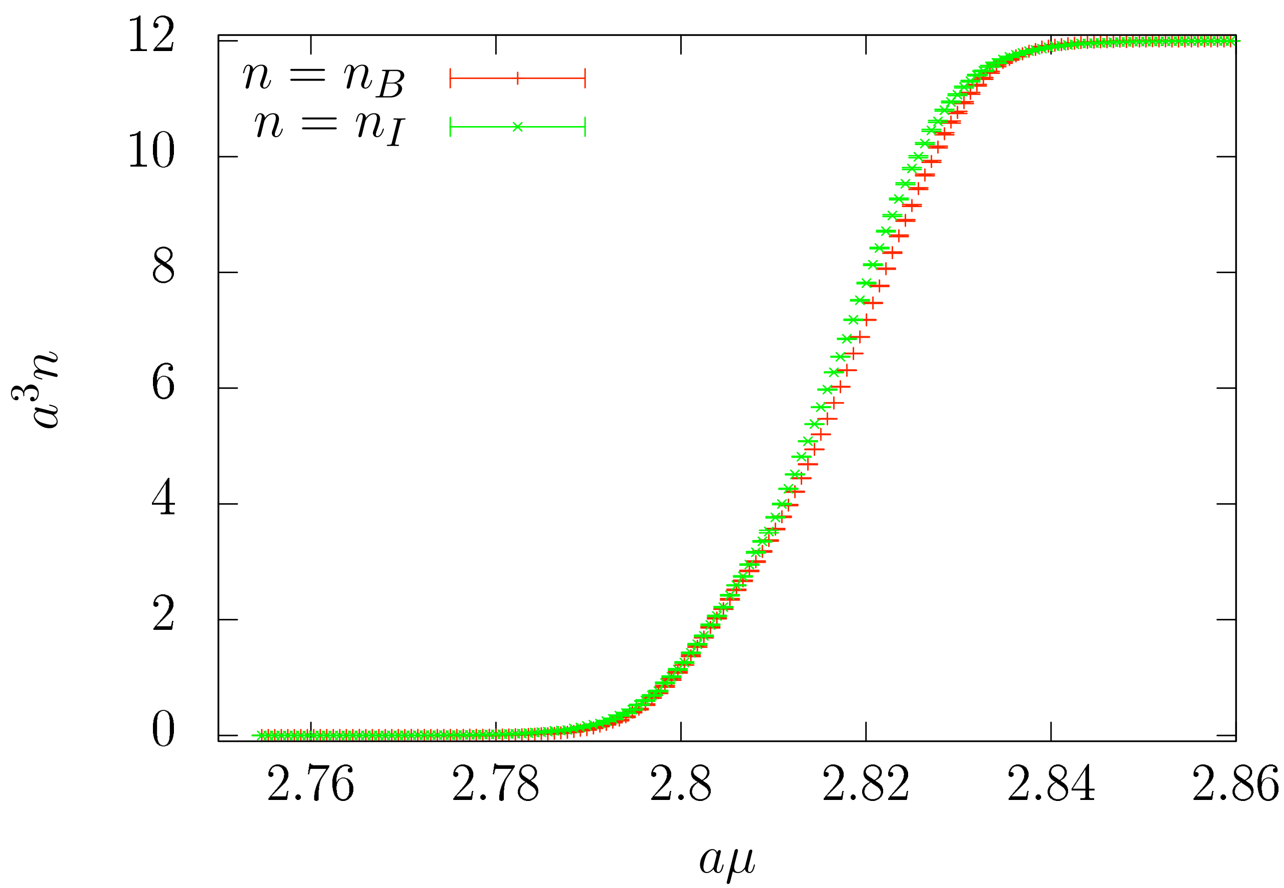
$$n = \frac{T}{V}\frac{\partial}{\partial\mu}\ln Z = \frac{1}{a^3}\frac{4N_c C^{N_c} + 2N_c C^{2N_c}}{1 + 4C^{N_c} + C^{2N_c}}$$

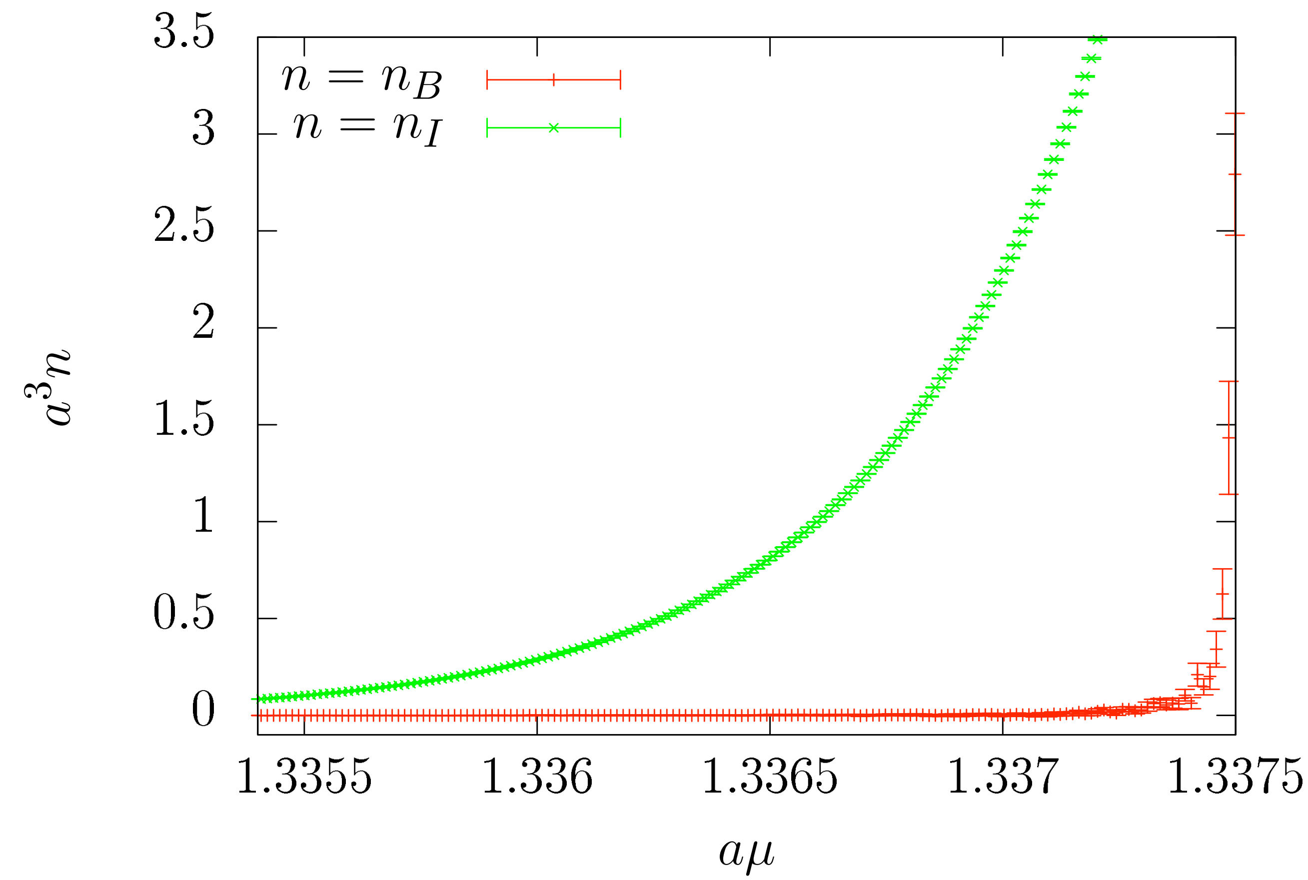
$$\lim_{\mu \to \infty}(a^3 n) = 2N_c$$

Sivler blaze property + saturation!

$$\lim_{T \to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

Finite isospin vs baryon chemical potential





heavy quarks

$$\frac{m_\pi}{2} \approx \frac{m_B}{3}$$

light quarks

$$\frac{m_\pi}{2} < \frac{m_B}{3}$$

onset at smaller chemical potential

Observable to identify order of p.t.:

$$\delta B_Q = B_4(\delta Q) = \frac{\langle(\delta Q)^4\rangle}{\langle(\delta Q)^2\rangle^2}$$

$$B_4(x) = 1.604 + bL^{1/\nu}(x - x_c) + \ldots$$