Multiple Gluon Exchange Webs

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  Three loops MGEWs connecting four lines
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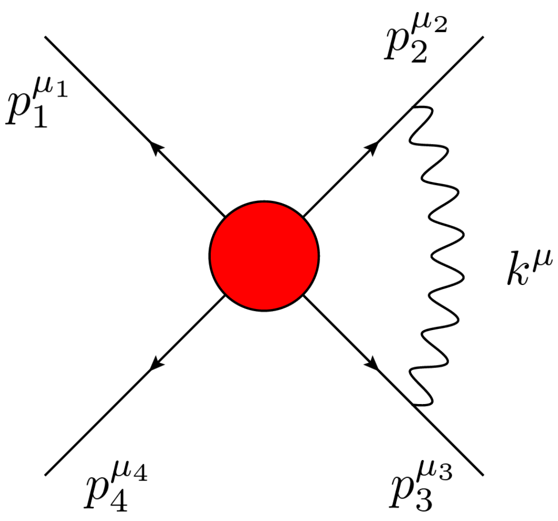
- Conclusion and outlook

Infrared divergences

- Gauge theory scattering amplitudes are affected by long distance singularities, which arise when propagators go on the mass shell



$$p_i \cdot p_j = O(Q^2) \gg \Lambda^2_{QCD}$$

if $p_i^2 = 0$

- Soft region

$$k^\mu = \lambda Q \qquad \lambda \ll 1$$

- Collinear region

$$k^+ \sim Q$$
$$k^- = \lambda^2 Q$$
$$k^\perp = \lambda Q$$

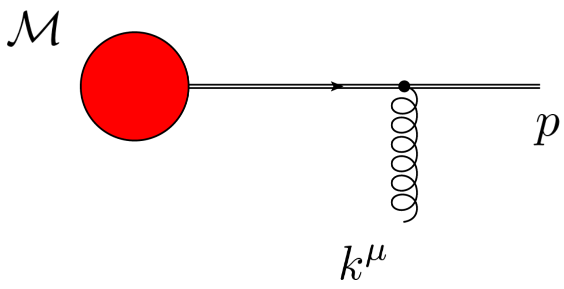
The pattern of infrared divergences is important both for phenomenology and theory:

- analytic subtraction of the singularities in infrared safe observables
- resummation of the large logarthmic enhancements

- insight on high orders in perturbation theory

Eikonal expansion and Wilson lines

The eikonal approximation is a key tool for the analysis of soft divergences:



$$\mathcal{M}^\mu = i\, g_s\, T^a \overline{u}(p) \gamma^\mu \frac{\not p + \not k}{(p-k)^2} \mathcal{M} \simeq i g_s T^a \overline{u}(p) \frac{[-\not p \gamma^\mu + 2p^\mu]}{-2p \cdot k} \mathcal{M}$$

$$= -i g_s T^a \frac{p^\mu}{p \cdot k} \times \overline{u}(p)\mathcal{M} = -i g_s T^a \frac{\beta^\mu}{\beta \cdot k} \times \overline{u}(p)\mathcal{M}$$

$$p^\mu = Q\, \beta^\mu$$

We get the Feynman rule for soft gluon emission:

- it does not depend on the energy of the hard parton, but only on its velocity
- it does not depend on the spin
- it depends on the colour charge through the representation of T

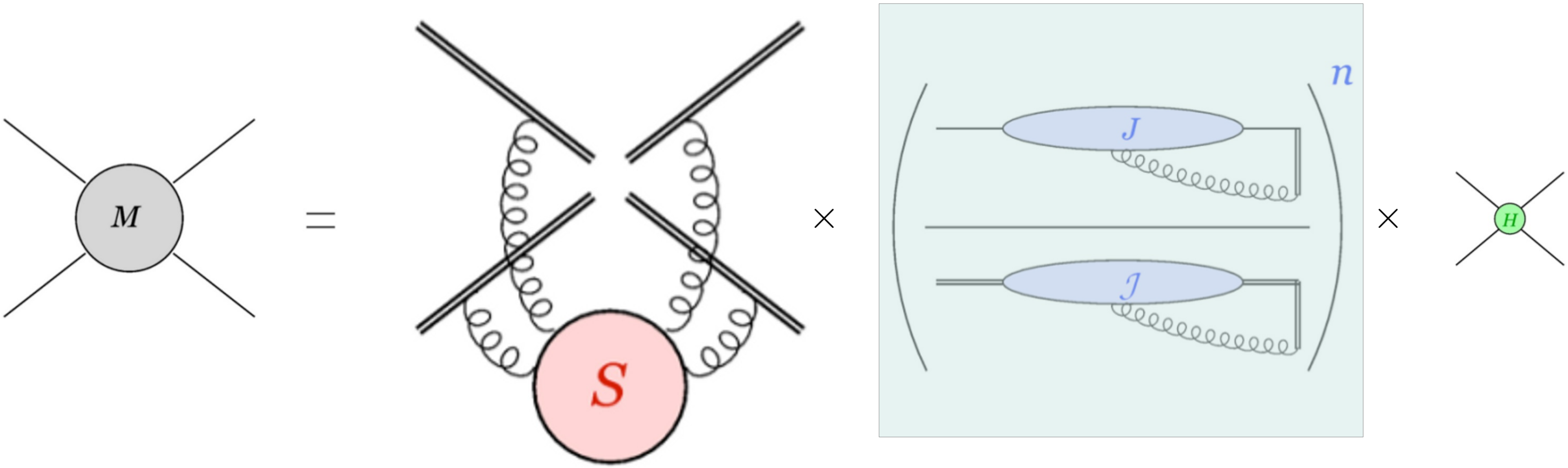
The eikonal Feynman rules are obtained by replacing the hard parton with a Wilson line in the direction of the classical trajectory

$$\Phi_\beta(\infty, 0) = \mathcal{P} \exp\left[\int_0^\infty d\sigma\, \beta_\mu\, A^\mu(\sigma \beta) \right]$$

Soft gluon corrections are taken into account by computing correlators of Wilson lines

Factorisation theorem

Both soft and collinear singularities of scattering amplitudes can be factorised



(Dixon, Magnea, Sterman '08)

$$\mathrm{M}\left(\frac{p_i \cdot p_j}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \mathcal{Z}\left(\frac{p_i \cdot p_j}{\mu_F^2}, \alpha_s(\mu_F^2), \epsilon\right) \cdot \mathcal{H}\left(\frac{p_i \cdot p_j}{\mu^2}, \frac{\mu^2}{\mu_F^2}, \alpha_s(\mu^2)\right)$$

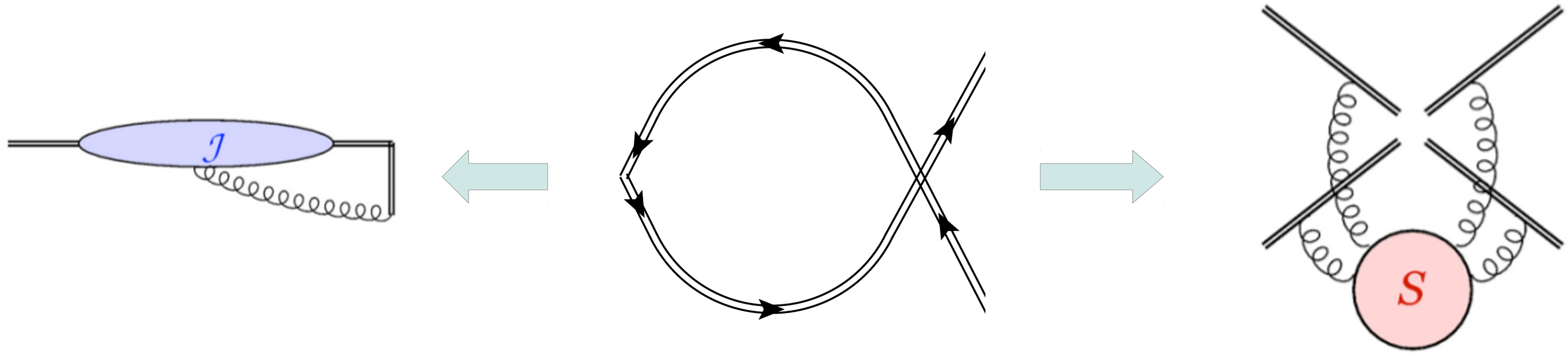
$S = \langle 0 | \prod_{i=1}^{n} \Phi_{\beta_i}(\infty, 0) | 0 \rangle$ Generates all the soft divergences

$J_i = \langle 0 | \Phi_n(\infty, 0) \Psi(0) | p \rangle$ Generates all the collinear divergences

$\mathcal{J}_i = \langle 0 | \Phi_n(\infty, 0) \Phi_{\beta_i}(0, -\infty) | 0 \rangle$ Subtracts the double counting of soft and collinear

The soft anomalous dimension

Singularities of soft and jet functions are cusp and cross divergences of Wilson loops



The evolution equations for the cusp and the cross are

$$\mu \frac{d}{d\mu} \mathcal{J}\left(\frac{(\beta_i \cdot n)^2}{n^2}, \alpha_s(\mu^2)\right) = -\gamma_{\mathcal{J}\,i}\, \mathcal{J}\left(\frac{(\beta_i \cdot n)^2}{n^2}, \alpha_s(\mu^2)\right)$$

$\gamma_{\mathcal{J}_i}$ Collinear anomalous dimension

$$\mu \frac{d}{d\mu} S\left(\beta_i \cdot \beta, \alpha_s(\mu^2)\right) = -S\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2)\right) \cdot \Gamma_S$$

Γ_S Soft anomalous dimension matrix

The information about soft singularities is encoded in the anomalous dimension

$$\mu \frac{d}{d\mu} \mathcal{Z}\left(\frac{p_i \cdot p_j}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = -\mathcal{Z}\left(\frac{p_i \cdot p_j}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) \cdot \Gamma\left(\frac{p_i \cdot p_j}{\mu^2}, \alpha_s(\mu^2)\right)$$

The dipole formula

In recent years an ansatz for the soft anomalous dimension at all orders has been proposed

$$\Gamma = -\frac{1}{4}\widehat{\gamma}_K(\alpha_s)\sum_{(i,j)}\log\left(\frac{2|p_i\cdot p_j|e^{-i\pi\lambda_{ij}}}{\mu^2}\right)\boxed{T_i\cdot T_j}+\sum_{i=1}^{n}\gamma_i(\alpha_s)\,\mathbf{1}$$

(Gardi, Magnea '09; Becher, Neubert '09)

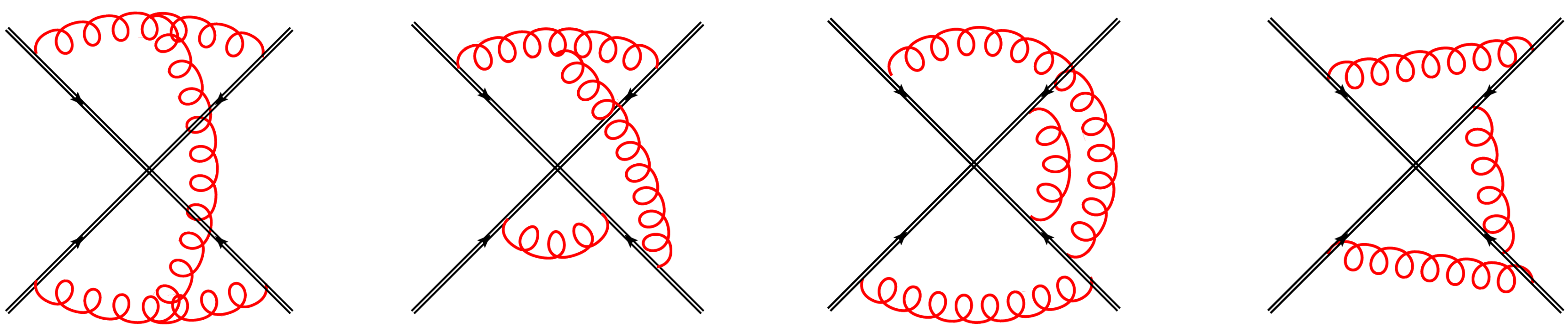
Only dipole correlations!

$$\widehat{\gamma}_K = \frac{\gamma_K}{\mathcal{C}} = 2\frac{\alpha_s}{\pi} + \left[N_c\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{5}{9}n_f\right]\left(\frac{\alpha_s}{\pi}\right)^2 + O\left(\alpha_s^3\right)$$

Cusp anomalous dimension
(Korchemsky, Radyushkin, '87)

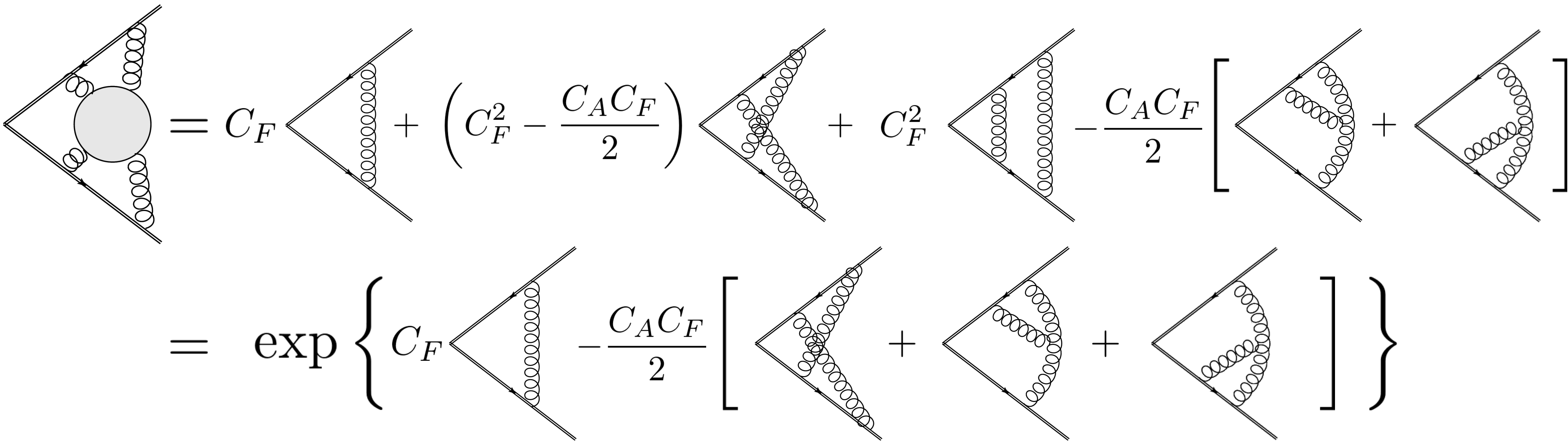
$\gamma_i(\alpha_s)$ Jet anomalous dimension

This formula is exact at two loops, the first possible corrections are four partons correlations arising at the next order. We need to compute explicitly the correlators of four Wilson lines at three loops, including



Webs and non abelian exponentiation

Correlators of Wilson lines are obtained by exponentiating a subset of Feynman diagrams with modified colour factors, called webs: the simplest example is with two lines



(Sterman '81, Gatheral, Frenkel, Taylor '83)

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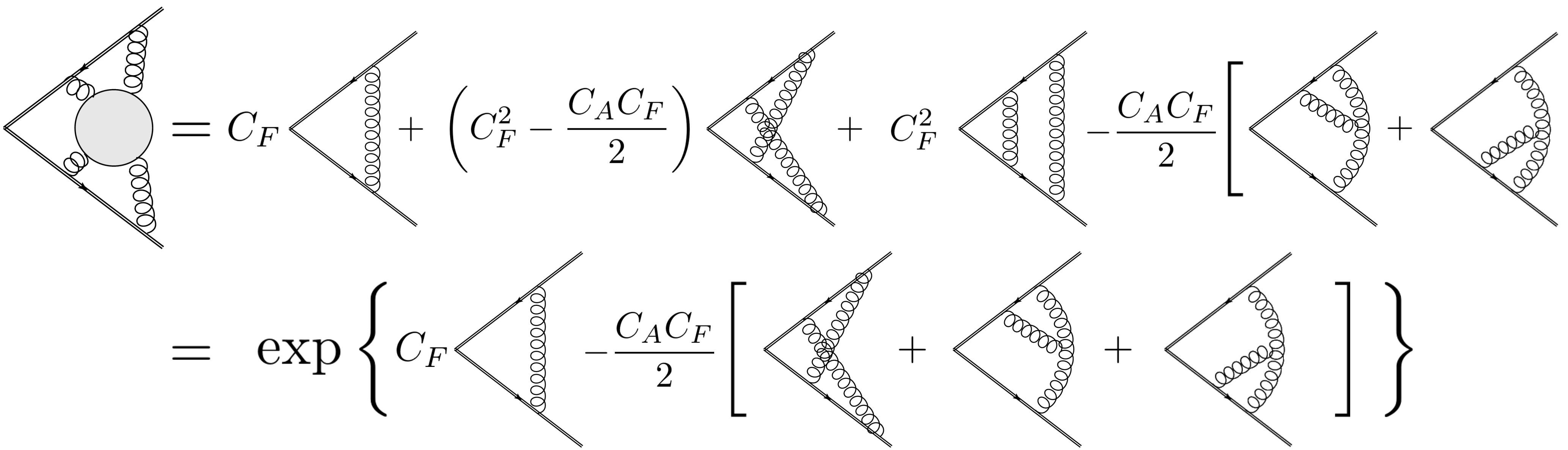


(Sterman '81, Gatheral, Frenkel, Taylor '83)

The proof of the exponentiation in the multi line case is more recent (Gardi, Laenen, Stavenga, White '10)

$$S_{bare}(\beta_i \cdot \beta_j, \alpha_s(\mu^2)) = \langle 0 | \prod_{i=1}^{n} \Phi_{\beta_i}(\infty, 0) | 0 \rangle \equiv \exp\left[w\right] = \exp\left[\sum_{D,D'} \mathcal{F}(D) \, \mathcal{R}_{DD'} \, \mathcal{C}(D') \right]$$

Kinematic factor of diagram D

Mixing matrix of combinatoric origin

Colour factor of diagram D'

Renormalisation of webs

Eikonal integrals are scaleless, so they vanish in dimensional regularization because infrared divergences cancel against ultraviolet divergences. After renormalisation they are defined by the UV counterterm

$$S_R\left(\gamma_{ij}, \alpha_s(\mu_R^2), \mu^2, \epsilon_{IR}\right) = Z\left(\gamma_{ij}, \alpha_s(\mu_R^2), \epsilon_{UV}, \mu^2\right) \cdot S_{bare} \equiv Z\left(\gamma_{ij}, \alpha_s(\mu_R^2), \epsilon_{UV}, \mu^2\right)$$

$$\gamma_{ij} \equiv 2\frac{\beta_i \cdot \beta_j}{\sqrt{\beta_i^2 \beta_j^2}}$$

We get the anomalous dimension from the ultraviolet counterterm of the soft function

$$\mu\frac{d}{d\mu} Z\left(\gamma_{ij}, \alpha_s(\mu_R^2), \epsilon_{UV}\right) = -Z\left(\gamma_{ij}, \alpha_s(\mu_R^2), \epsilon_{UV}\right) \cdot \Gamma$$

Anomalous dimension in practice: we isolate the UV pole introducing an infrared regulator and get a non-trivial regularised soft function, which can be expanded in ε

$$S_{bare}^{reg} = \exp\left[\sum_n w^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n\right], \qquad \text{with } w^{(n)} = \sum_{j=-n} \epsilon^j \, w^{(n,j)}$$

The anomalous dimension at each order is a combination of the single pole of the webs and commutators of the decompositions in lower order webs

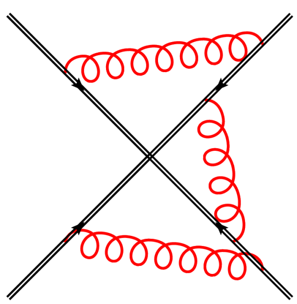
$$\Gamma^{(1)} = -2w^{(1,-1)}$$

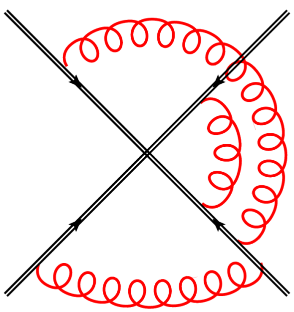
$$\Gamma^{(2)} = -4w^{(2,-1)} - 2\left[w^{(1,-1)}, w^{(1,0)}\right]$$

(Gardi, Smillie, White '11)

Multiple Gluon Exchange Webs

We focus on the webs that we get by taking only the quadratic part of the QCD lagrangian and don't contain gluons self interactions (MGEW). At three loops, MGEWs can introduce four Wilson lines correlations:





(Gardi '13)

However, these correlations disappear in the combination of the ultraviolet pole and the subtraction terms. The results are in the form

$$\Gamma^{(3)} = \left(\frac{\alpha_s}{\pi}\right)^3 \int_0^1 dx_1 dx_2 dx_3\, p_0(x_1, \alpha_{ij})\, p_0(x_2, \alpha_{jk})\, p_0(x_3, \alpha_{kl}) \times \phi(\{x_i\})$$

$$p_0(x, \alpha) = \frac{1}{x^2 + (1-x)^2 + \left(\alpha + \frac{1}{\alpha}\right) x(1-x)}$$

$$\gamma \equiv -\left(\alpha + \frac{1}{\alpha}\right)$$

product of logarithms!

The contribution to the anomalus dimension of MGEW connecting four lines are factorised functions of different cusp angles.

Exploring MGEWs

MGEWs have a surprisingly simple structure, summarised by

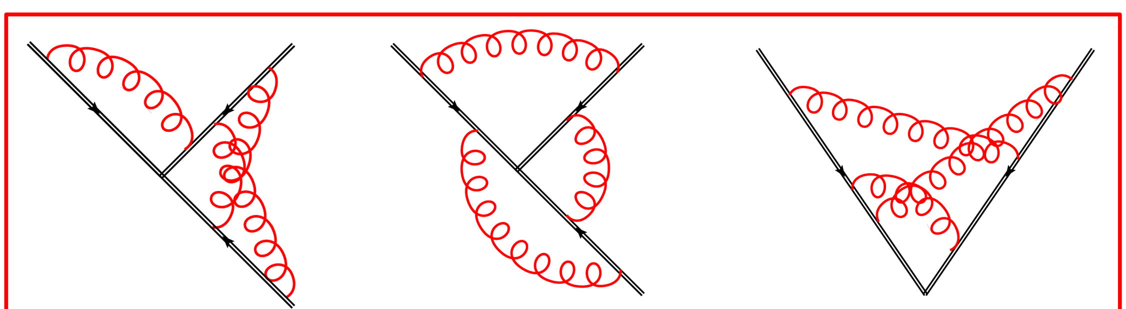
- the kinematic dependence is factorised in the different cusp angles. The symbols of the functions appearing in the anomalous dimension have an alphabet composed of only two letters $\left\{\alpha_{ij}, \eta_{ij} \equiv \frac{\alpha_{ij}}{1-\alpha_{ij}^2}\right\}$ where α_{ij} is defined by $\gamma_{ij} \equiv -\left(\alpha_{ij} + \frac{1}{\alpha_{ij}}\right)$

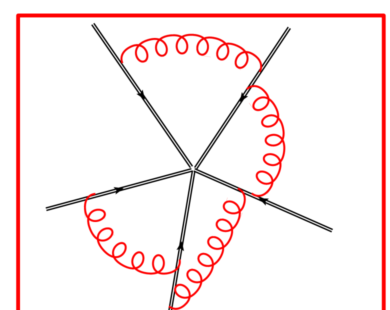
- we find a basis of functions with these features

$$M_{k,l,n}(\alpha) = \frac{1}{r(\alpha)} \int_0^1 dx\, p_0(x,\alpha)\, \log^k\left(\frac{q(x,\alpha)}{x^2}\right) \log^l\left(\frac{x}{1-x}\right) \log^n\left(\tilde{q}(x,\alpha)\right)$$

$$r(\alpha) = \frac{1+\alpha^2}{1-\alpha^2}\,,\; q(x,\alpha) = \frac{1}{p_0(x,\alpha)}\,,\; \tilde{q}(x,\alpha) = \log\frac{1-(1-\alpha)x}{1+\frac{1-\alpha}{\alpha}x}$$

We check that more entangled configurations at 3 and 4 loops are written in terms of the basis:





(Gardi, Harley, Magnea, White, GF, to appear)

Conclusion

Summarising the results

- the structure of infrared divergences of scattering amplitudes is determined by a soft anomalous dimension, which at two loops level introduces only dipole correlations
- possible corrections from the webs at three loops without gluon self interactions (MGEWs) are still factorised, at the level of the kinematic dependence. It has been conjectured that all the MGEW share this feature
- we directly compute all the remaining MGEWs at three loops and one at four loops. Factorisation holds in all these cases and we find a basis of functions which describes all the webs we have already computed.

The next steps will investigate

- the factorisation (and alphabet) in MGEWs at higher, possibly all orders
- check the basis of functions
- Non-MGEW diagrams at three loops and possible corrections to the dipole formula (Almelid, Duhr, Gardi).