# Higher Spin Theories and Holography

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# Massless higher spins

- Consistent interactions of *massless* higher spin fields (gauge fields!) are highly constrained
- In flat space, no consistent theory of interacting massless higher spin fields of spin $s > 2$ (Coleman-Mandula, Weinberg…)
- However, with non-zero cosmological constant, Vasiliev explicitly constructed ('89-'92) consistent fully non-linear theories of interacting massless higher spin fields (in arbitrary dimensions). No smooth flat space limit.
- These theories involve infinite towers of higher spin fields, including in particular the *graviton (s=2).* Hence, they are in particular theories of gravity.

# Higher spins in AdS

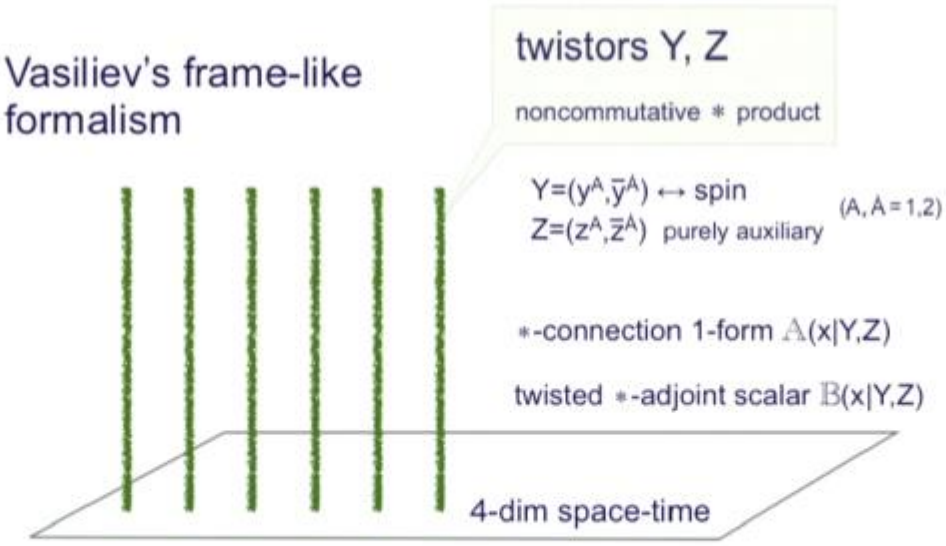
- Vasiliev wrote down a set of consistent gauge invariant equations of motion. They admit a vacuum solution which is AdS space (or dS if the cosmological constant is positive. Will focus on AdS case in this talk).
- In the simplest bosonic 4d theory, the linearized spectrum around the AdS vacuum consists of an infinite tower of higher spin fields plus a scalar

$$\begin{array}{lll} \text{Spectrum :} & s = 1, 2, 3, \ldots, \infty & \text{gauge fields} \\ & s = 0, \quad m^2 = -2/\ell_{AdS}^2 & \text{scalar} \end{array}$$

# Vasiliev equations

- The Vasiliev equations in 4d

$$
\begin{aligned}
&d_x\hat{A} + \hat{A} * \hat{A} = f_*(B * K)dz^2 + \overline{f}_*(B * \overline{K})d\bar{z}^2, \\
&d_x B + \hat{A} * B - B * \pi(\hat{A}) = 0.
\end{aligned}
$$



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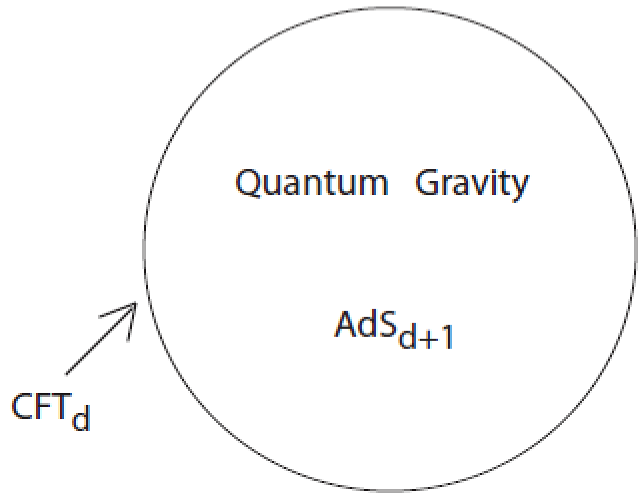
\end{aligned}
$$

- Essentially, *A* contains the metric and all other higher spin fields, and *B* contains the scalar field and the curvatures (weyl tensors) of the HS fields.
- The linearized equations are the standard equations for a scalar, linearized graviton and free massless HS fields (Fronsdal).

# Higher spins and AdS/CFT

- From the point of view of AdS/CFT correspondence, it is not too surprising that such theories exist.




- The AdS/CFT correspondence is an *exact equivalence*, or *duality*, between quantum gravity in AdS and a conformal quantum field theory that can be thought of as living at the boundary of AdS

# Higher spins and AdS/CFT

- If Vasiliev theory defines a consistent quantum gravity theory in AdS, what is its CFT dual?
- Consider a free theory of N free complex scalar fields in 3d

$$S = \frac{1}{2} \int d^3x \partial_\mu \phi_i^* \partial^\mu \phi^i \,, \qquad i = 1, \ldots, N$$

- It has a U(N) global symmetry under which the scalar transforms as a *vector*. (This is different from familiar examples of AdS/CFT, where the CFT side is usually a gauge theory with matrix (adjoint) type fields).

# Higher spins and AdS/CFT

- By virtue of being free, it is easy to see that this theory has an infinite tower of conserved HS currents

$$J_{\mu_1 \cdots \mu_s} = \phi_i^* \partial_{(\mu_1} \cdots \partial_{\mu_s)} \phi^i + \ldots$$
$$\partial^\mu J_{\mu\mu_2 \cdots \mu_s} = 0\,, \qquad \Delta(J_s) = s+1$$

- If we consider U(N) invariant operators (*singlet sector*), these currents, together with the scalar operator

$$J_0 = \phi_i^* \phi^i\,, \qquad \Delta = 1$$

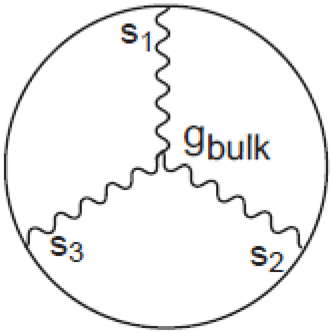
are all the “single trace” primaries of the CFT.

# Higher spins and AdS/CFT

- By the usual AdS/CFT dictionary, single trace primary operators in the CFT are dual to single particle states in AdS.
- *Conserved* currents are dual to *gauge* fields (massless HS fields)
- A scalar operator is dual to a bulk scalar with $\Delta(\Delta - d) = m^2$
- This precisely matches the spectrum of Vasiliev's bosonic theory in $AdS_4$.

# Higher spins and AdS/CFT

- The dual higher spin fields are necessarily interacting in order to reproduce the non-vanishing correlation functions of HS currents in the CFT




- The coupling constant scales as $g_{\rm bulk} \sim \dfrac{1}{\sqrt{N}}$ (or in terms of Newton's constant $G_N \sim 1/N$).

- The large N limit corresponds as usual to weak interactions in the bulk.

# Higher spins and AdS/CFT

- We could also consider a theory of N free real scalars, and look at the O(N) singlet sector
- The spectrum of single trace primaries now includes a scalar plus all the *even* spin HS currents
- On Vasiliev's theory side, this corresponds to a consistent truncation of the equations which retains only the even spin gauge fields ("Minimal bosonic HS theory").

$$
\begin{array}{lll}
\text{Spectrum :} & s = 2, 4, 6, \ldots, \infty & \text{gauge fields} \\
 & s = 0, \quad m^2 = -2/\ell^2_{AdS} & \text{scalar}
\end{array}
$$

# The interacting O(N) model

- The conjecture that the (singlet sector of) the O(N)/U(N) vector model is dual to the bosonic Vasiliev theory was made by Klebanov and Polyakov (2002). A crucial observation is that one can also consider the Wilson-Fisher IR fixed point reached by a relevant "double trace" deformation of the free theory.

$$S = \int d^3x \left[ \frac{1}{2} \left( \partial_\mu \phi^i \right)^2 + \frac{\lambda}{4} (\phi^i \phi^i)^2 \right]$$

# The interacting O(N) model

- The IR fixed point ("critical vector model") is an *interacting* CFT whose single trace spectrum has a scalar operator of dimension $\Delta = 2 + O(1/N)$ and slightly broken HS current of dimension $\Delta = s + 1 + O(1/N)$
- It is dual to the *same* Vasiliev theory dual to the free vector model, with the only difference that the $m^2=-2$ bulk scalar is assigned the alternate $\Delta=2$ boundary condition.
- HS symmetry broken by 1/N corrections. Loop effect from AdS point of view.

# Higher spins and AdS/CFT

- One can also consider a fermionic version of the vector model

$$S = \frac{1}{2} \int d^3x \bar{\psi}_i \gamma^\mu \partial_\mu \psi^i$$

- The single trace spectrum is similar to the scalar case, there is an infinite tower of conserved HS current, plus a parity odd scalar $\bar{\psi}_i \psi^i$ of dimension 2.
- It turns out that there is indeed a Vasiliev’s theory with such a single particle spectrum: at linearized level, the only difference is that the bulk scalar is now a pseudo-scalar. This is called “type B”, while the theory dual to scalars “type A”.
- At non-linear level “type A” and “type B” theories have different interactions (for instance, the graviton cubic coupling is different: fermion and boson CFT’s have different <TTT>).

# Higher spins and AdS/CFT

- One can also consider an interacting (UV) fixed point corresponding to a $(\bar{\psi}_i \psi^i)^2$ deformation of the free theory. It corresponds again to alternate boundary conditions in the bulk.
- Summary of higher spin/vector model dualities

| | HS A-type | HS B-type |
|---|---|---|
| $\Delta$=1 scalar b.c. | Free U(N)/O(N) scalar | Critical U(N)/O(N) fermion |
| $\Delta$=2 scalar b.c. | Critical U(N)/O(N) scalar | Free U(N)/O(N) fermion |

# Some comments

- Pure HS gauge theories have exactly the right spectrum to be dual to *vector models* (adjoint theories have many more single trace operators)
- The restriction to singlet sector can be implemented by *gauging* the U(N)/O(N) symmetry, and taking the limit of zero gauge coupling. In 3d, we can do this with Chern-Simons gauge theory.
- In the large N limit with $\lambda = N/k$ fixed (k is the CS level), the singlet sector of the free (critical) theories correspond to the limit $\lambda \to 0$.

# HS/CS vector model dualities

- This point of view has suggested a generalization of the higher spin AdS/CFT duality to the vector models coupled to Chern-Simons theory (SG et al, Aharony, Gur-Ari, Yacoby)
- They were conjectured to be dual to parity breaking versions of Vasiliev's theory in $AdS_4$.

CS+vector model $\leftarrow\rightarrow$ parity breaking HS theory

- These HS theories involve extra parameters which can be mapped to $\lambda$ and allow an interpolation between "type A" and "type B" theories.
- On the CFT side, this has suggested a novel bose-fermi duality relating theories of bosons coupled to CS to theories of fermions coupled to CS, with N $<\!-\!>$ k (generalizes level-rank duality).

# Free energy on $S^3$

- These HS/vector model dualities have been explicitly tested so far at the level of 3-point correlation functions (SG-Yin, Maldacena-Zhiboedov, Didenko-Skorvstov…)
- We would like to make a new type of test based on a different observable of the CFT: the partition function of the free energy on a round sphere $S^3$, $F=-\log Z$.
- It is an interesting quantity that for any RG flow satisfies $F_{UV} > F_{IR}$.
- For a CFT, it is also related to the entanglement entropy across a circle.

# Free energy on $S^3$

- In the CFT, it is simply defined as the log of the partition function of the theory on $S^3$ (generalization to $S^d$ is straightforward)

$$F = -\log Z \qquad\qquad Z = \int D\phi e^{-S}$$

$$S = \int d^3x \sqrt{g} \left( \partial_\mu \phi^i \partial^\mu \phi^i + \frac{R}{8} \phi^i \phi^i \right)$$

- This is straightforward to compute in the free theory: need to evaluate the determinant of the kinetic operator

$$F = \frac{1}{2} \log \det \left( -\nabla^2 + 3/4 \right)$$

# Free energy on $S^3$

- The explicit computation gives (Klebanov, Pufu, Safdi)

$$F = \frac{N}{2}\sum_{n=0}^{\infty}(n+1)^2 \log[(n+1/2)(n+3/2)] = N\left(\frac{\log 2}{8} - \frac{3\zeta_3}{16\pi^2}\right)$$

for N real scalars, and twice this result for N complex scalars. Trivial N dependence.

- One can also perform the calculation in the critical theory (in a large N expansion), with the result

$$F^{\text{critical}} = F^{\text{free}} - \frac{\zeta_3}{8\pi^2} + O(1/N)$$

# Free energy on $S^3$ from the bulk

- The challenge is: can we reproduce these results from the bulk? In particular, can we see the vanishing of the subleading corrections in the large N expansion of the free energy from the HS dual to the free theory?
- How do we compute F from the bulk?

$$Z_{\rm CFT} = Z_{\rm bulk}$$

- We "simply" have to compute the partition function of the Vasiliev's theory on the Euclidean $AdS_4$ vacuum

$$ds^2 = d\rho^2 + \sinh^2 \rho \, d\Omega_3$$

# Free energy on $S^3$ from the bulk

- In practice, we should compute the path integral of the bulk theory, where we expand the metric around $AdS_4$ and integrate over all quantum fluctuations

$$
\begin{aligned}
Z_{\mathrm{bulk}} &= \int D\varphi_{(0)} D g_{\mu\nu} D\varphi_{(s)} e^{-S[g=g_0+h,\varphi_{(0)},\varphi_{(s)}]} \\
&= e^{-\frac{1}{G_N}F^{(0)}-F^{(1)}-G_N F^{(2)}+\ldots} = e^{-F_{\mathrm{bulk}}}
\end{aligned}
$$

- Here $G_N$ is Newton's constant, which scales as $1/G_N \sim N$.

# Free energy on $S^3$ from the bulk

- The explicit bulk action is not well understood (proposals by *Douroud, Smolin*; *Boulanger, Sundell*), but in terms of physical fields is expected to take a form

$$S \sim \frac{1}{G_N}\int d^4x\sqrt{g}\,\Big(R+\Lambda+R^3+R^4+\dots \\ +\varphi_{(s)}\Delta_s\varphi_{(s)}+\sum C_{s_1s_2s_3}\partial^{k_1}\varphi_{s_1}\partial^{k_2}\varphi_{s_2}\partial^{k_3}\varphi_{s_3}+\dots\Big)$$

- The leading term $\frac{1}{G_N}F^{(0)}$ in the bulk free energy corresponds to evaluating this action on the $AdS_4$ background metric, with all other fields set to zero.
- This is already very non-trivial, as it requires to know the form of all the higher derivative corrections in the metric sector (we know they are non-trivial from knowledge of correlation functions).

# Free energy on $S^3$ from the bulk

- One would like to show that

$$S_{\mathrm{classical}}[g_{\mu\nu} = AdS_4, \varphi_{(s)} = 0] = \frac{1}{G_N} F^{(0)} = N\left(\frac{\log 2}{4} - \frac{3\zeta_3}{8\pi^2}\right)$$

- This is one of the outstanding open problems in testing HS/vector model dualities.
- While we cannot show this (yet), we can start by something simpler, namely assume that this tree level piece works, and compute the one-loop contribution $F^{(1)}$ to the bulk free energy.

# The one-loop piece

- Let us now concentrate on the calculation of the one-loop contribution $F^{(1)}$ to the bulk free energy

$$e^{-\frac{1}{G_N}F^{(0)}-F^{(1)}-G_N F^{(2)}+\ldots} = e^{-F_{\text{bulk}}}$$

- Even if we don't know the full action, we know that the linearized equations correspond to standard kinetic terms for all the higher spin fields, so we assume a canonical quadratic action

$$S_{(2)} = \int d^4x\sqrt{g}\left(\varphi_{(0)}(-\nabla^2-2)\varphi_{(0)} + \sum_{s=1,2,\ldots}\varphi_{(s)}\Delta_s\varphi_{(s)}\right)$$

# The one-loop piece

- One can introduce spin s-1 ghosts, then after decomposing physical and ghost fields into their irreducible parts, the contribution to the one-loop free energy of each HS field is the ratio of determinants on symmetric traceless transverse fields

$$\frac{\left[\det^{STT}_{s-1}\left(-\nabla^2+s^2-1\right)\right)\right]^{\frac{1}{2}}}{\left[\det^{STT}_{s}\left(-\nabla^2+s(s-2)-2\right)\right]^{\frac{1}{2}}}$$

# One-loop free energy

- We have to compute

$$F_{1-\mathrm{loop}} = \frac{1}{2}\log\det\left(-\nabla^2 - 2\right) + \frac{1}{2}\sum_{s=1}^{\infty}\left[\log\det\nolimits_s\left(-\nabla^2 - 2 + s(s-2)\right) - \log\det\nolimits_{s-1}\left(-\nabla^2 + s^2 - 1\right)\right]$$

- Luckily, a large part of the calculation was already done in a series of papers by Camporesi and Higuchi in the ’90’s.
- They computed the spectral zeta function (Laplace transform of the heat kernel) for $-\nabla^2 + \kappa^2$ operators acting on STT fields of arbitrary spin.

# AdS Spectral zeta function

- The explicit spectral zeta function in AdS is

$$\zeta_{(\Delta,s)}(z) = \left( \frac{\int \mathrm{vol}_{AdS_{d+1}}}{\int \mathrm{vol}_{S^d}} \right) \frac{2^{d-1}}{\pi} g(s) \int_0^\infty du \, \frac{\mu_s(u)}{\left[u^2 + \left(\Delta - \frac{d}{2}\right)^2\right]^z}$$

with $\Delta(\Delta - d) - s = \kappa^2$

- In the present case of d=3

$$\mathrm{vol}_{AdS_4} = \frac{4}{3}\pi^2 \,, \qquad \mathrm{vol}_{S^3} = 2\pi^2$$

$$\mu_s(u) = \frac{\pi u}{16}\left[u^2 + \left(s + \frac{1}{2}\right)^2\right] \tanh \pi u \,, \qquad g(s) = 2s + 1$$

# AdS Spectral zeta function

- In terms of the spectral zeta function, the contribution to the one-loop free energy is then obtained as

$$F^{(1)}_{(\Delta,s)} = -\frac{1}{2}\zeta'_{(\Delta,s)}(0) - \frac{1}{2}\zeta_{(\Delta,s)}(0)\log\left(\ell^2\Lambda^2\right)$$

- Importantly, in every even dimensional bulk spacetime, there is a logarithmic divergence proportional to the value of the spectral zeta function at z=0. (It is related to the bulk conformal anomaly).

# UV finiteness

- For the duality to be exact and Vasiliev theory to be “UV complete”, this divergence should not be present in the full HS theory: the bulk theory should be finite.
- While each spin contributes a log divergence, can the divergence cancel in the sum over the infinite tower of fields?

$$
\begin{aligned}
F^{(1)}\Big|_{\log-\mathrm{div}} &= -\frac{1}{2}\left(\zeta_{(1,0)}(0)+\sum_{s=1}^{\infty}\left(\zeta_{(s+1,s)}(0)-\zeta_{(s+2,s-1)}(0)\right)\right)\log\left(\ell^2\Lambda^2\right) \\
&= \left(\frac{1}{360}+\sum_{s=1}^{\infty}\left(\frac{1}{180}-\frac{s^2}{24}+\frac{5s^4}{24}\right)\right)\log\left(\ell^2\Lambda^2\right)
\end{aligned}
$$

# UV finiteness

- It appears natural to regulate this sum with the usual Riemann zeta-function regularization (we will come back to the question of regularization later). Recall that ζ(0)=-1/2, and ζ(-2)=ζ(-4)=0. So

$$\frac{1}{360} + \sum_{s=1}^{\infty} \left( \frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24} \right) = 0$$

- So Vasiliev's theory is one-loop finite!
- Regularization can be understood as natural analytic continuation of spectral zeta function.
- The same result holds for the theory with even spins only, and regardless of boundary conditions on the scalar.
- This is similar to the cancellation of UV divergences in N>4 SUGRA in $AdS_4$ , but here we have a purely bosonic theory (with an *infinite* number of fields).

# The finite part

- Having shown that the log divergence cancels, we can move on to the computation of the finite contribution. This is considerably more involved. Computing the derivative of the spectral zeta-function, the result is expressed as

$$F^{(1)} = -\frac{1}{2}\mathcal{I}(-1/2, 0) - \frac{1}{2}\sum_{s=1}^{\infty}\left[\mathcal{I}(s-1/2, s) - \mathcal{I}(s+1/2, s-1)\right]$$

with:

$$\mathcal{I}(\nu, s) = \frac{1}{3}(2s+1)\int_0^{\nu} dx \left[\left(s+\frac{1}{2}\right)^2 x - x^3\right]\psi(x+\frac{1}{2})$$

# The finite part

- After a somewhat lengthy calculation we find

$$\mathcal{I}\left(-\frac{1}{2},0\right) = -\frac{1}{3}\int_{-1/2}^{0} dx\left(\frac{x}{4}-x^3\right)\psi(x+\frac{1}{2}) = \frac{11}{1152}-\frac{11\log 2}{2880}-\frac{\log A}{8}-\frac{\zeta(3)}{8\pi^2}+\frac{5\zeta'(-3)}{8}$$

$$\sum_{s=1}^{\infty}\left[\mathcal{I}\left(s-\frac{1}{2},s\right)-\mathcal{I}\left(s+\frac{1}{2},s-1\right)\right]$$
$$=-\frac{11}{1152}+\frac{11\log 2}{2880}+\frac{\log A}{8}-\frac{5\zeta'(-3)}{8}-\frac{\zeta'(-2)}{2}$$

- Recalling that $\zeta'(-2) = -\frac{\zeta(3)}{4\pi^2}$, the higher spin tower precisely cancels the scalar!

# The finite part

- So we conclude that the one-loop bulk free energy in Vasiliev's type A theory with $\Delta$=1 boundary condition for the scalar is exactly zero

$$F^{(1)} = 0$$

- This is precisely consistent with the fact that in the dual free CFT the large N expansion should be trivial.

# Δ=2 and the critical vector model

- We can also easily do the calculation with Δ=2 boundary condition on the scalar. Only the scalar contribution is affected, and one finds

$$-\frac{1}{2}\mathcal{I}(\Delta=2,0) = -\frac{1}{2}\mathcal{I}(\Delta=1,0) - \frac{\zeta_3}{8\pi^2}$$

- So the final result is

$$F^{(1)} = -\frac{\zeta_3}{8\pi^2}$$

exactly consistent with the non-trivial large N expansion in the critical scalar theory.

# The minimal HS theory

- We can repeat the same calculation in the minimal theory, with even spins only, which should be dual to the O(N) vector model.
- Here we find a surprise. The total one loop free energy is *not* zero, but it is equal to

$$F_{\rm min}^{(1)} = \frac{\log 2}{8} - \frac{3\zeta(3)}{16\pi^2}$$

- This is precisely equal to the value of the $S^3$ free energy of a single real conformal scalar field…Why?

# The minimal HS theory

- So far we have always assumed that Newton's constant is given by $G_N^{-1} = cN$ . But there can in principle be subleading corrections in the map between $G_N$ and N.
- Because the one-loop piece is precisely proportional to the expected classical piece, this suggests that the result can be consistent with the duality if we assume a shift N->N-1 so that the classical piece is

$$\frac{1}{G_N} F^{(0)}_{\min} = (N-1)\left(\frac{\log 2}{8} - \frac{3\zeta(3)}{16\pi^2}\right)$$

which combined with the one-loop piece would give the expected result for F.

# One-loop shift

- This effect may perhaps be thought as a finite “one-loop renormalization” of the bare coupling constant in Vasiliev’s theory, somewhat similar to the one-loop shift of the level in Chern-Simons gauge theory
- The fact that the shift is simply an integer is consistent with the idea that the coupling constant in Vasiliev’s theory should be quantized (*Maldacena-Zhiboedov*).

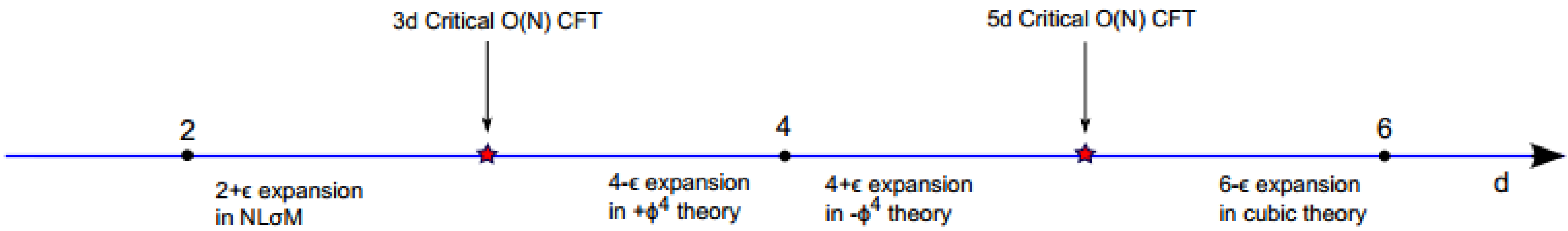
# General dimensions

- There is a formulation of Vasiliev's theory in arbitrary dimensions. The equations of motion have a $AdS_{d+1}$ vacuum solution, and the linearized spectrum around this background is

$$\text{Spectrum}: \qquad s = 1, 2, 3, \ldots, \infty \qquad \text{gauge fields}$$
$$s = 0, \quad m^2 = -2(d-2) \quad \text{scalar}$$

- This spectrum is in one-to-one correspondence with the single trace primaries of a free scalar vector model in dimension d (the scalar bilinear has dimension $\Delta$=d-2). Above d=3, there are no interacting IR fixed points dual to alternate boundary conditions. But there is a UV fixed point in d=5 of the scalar theory with $\phi^4$ interaction: dual to Vasiliev theory in $AdS_6$ with alternate b.c. on scalar.

# The critical O(N) theory in d=5




- The 5d fixed point may be understood as either the UV fixed point of the $\phi^4$ theory, or (*Fei, SG, Klebanov*) as the *IR fixed point* of the cubic theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g_1}{2}\sigma\phi^i\phi^i + \frac{g_2}{6}\sigma^3$$

- The latter theory has a perturbative, IR stable and unitary fixed point for d=6-$\varepsilon$ and sufficiently large N. This fixed point can be shown to be indentical to the dimensional continuation of the fixed point of the $\phi^4$ theory at large N $\phi^4$ (*Vasiliev et al, Petkou, Lang-Ruhl…*)

# General dimensions

- Given the spectrum of Vasiliev theory in general dimensions, it is natural to conjecture that the singlet sector of the free scalar vector model in dimension d is dual to the Vasiliev's theory in $AdS_{d+1}$.
- The spectral zeta functions of the totally symmetric HS fields in general dimension are known (*Camporesi-Higuchi*). It is then natural to repeat the one loop calculations in general dimensions (*SG, Klebanov, Safdi).*
- In all odd d, there are UV logarithmic divergences spin by spin. Summing over all spins, the UV divergence always vanishes. Vasiliev theory is one-loop UV finite in *any* dimension.
- Finite part of $F^{(1)}$ is consistent with AdS/CFT in all dimensions (for minimal theories, this requires the shift N->N-1 as found earlier).

# Conclusion and summary

- Consistent interacting theories of massless higher spins can be constructed if the cosmological constant is non-zero. They involve infinite towers of fields of all spins.
- The Vasiliev theory in AdS was conjectured to be exactly dual to simple vector model CFT's.
- Vasiliev theories provide exact AdS dual not only to free theories, but also to interesting interacting theories such as the critical (Wilson-Fisher) O(N) model, the Gross-Neveu model, $CP^N$ model, theories involving Chern-Simons gauge fields...(Also suggested the existence of new interacting CFT's with O(N) symmetry in 5 dimensions *(Fei, SG, Klebanov)* , dual to Vasiliev theory in $AdS_6$).

# Conclusion and summary

- We have recently obtained new simple tests of higher spin/vector model dualities, by comparing partition functions on both sides of the duality.
- The classical bulk contribution is still out of reach (lacking understanding of the Lagrangian), but the one-loop calculation is well defined and can be done explicitly in general dimensions.
- In all dimensions, we find that one-loop UV divergences in the Vasiliev theory vanish due to the contribution of the infinite tower of spins. Is higher spin gravity a "UV complete" model of quantum gravity? Connection to string theory?

# Conclusion and summary

- More to be done: loop corrections to correlation functions, understand action for Vasiliev equations, study other non-trivial solutions of the theory (e.g. mass deformations of CFT? Black holes?)…
- Vasiliev theory is an interesting model of quantum gravity, and higher spin/vector model dualities are in some sense simplest example of AdS/CFT. Prove the duality directly? (*Jevicki et al*, *Douglas et al*, *Pando Zayas-Peng*, *Leigh et al…*).