Higher Spins on (A)dS in the worldline formalism

Roberto Bonezzi

Department of Physics and Astronomy, University of Bologna
and INFN, Sezione di Bologna

New Frontiers in Theoretical Physics, Cortona, May 30 2014

Motivations

- The worldline formalism proved to be useful to compute various one-loop quantities of QFT's, such as n-point functions, propagators, effective actions and so on

Motivations

- The worldline formalism proved to be useful to compute various one-loop quantities of QFT's, such as n-point functions, propagators, effective actions and so on
- O(N) spinning particles in flat space describe quite efficiently higher spin fields (spin $\frac{N}{2}$ in $D = 4$)

Motivations

- The worldline formalism proved to be useful to compute various one-loop quantities of QFT's, such as n-point functions, propagators, effective actions and so on
- O(N) spinning particles in flat space describe quite efficiently higher spin fields (spin $\frac{N}{2}$ in $D = 4$)
- Coupling to curved space can provide useful information about their quantum properties on (A)dS backgrounds

Motivations

- The worldline formalism proved to be useful to compute various one-loop quantities of QFT's, such as n-point functions, propagators, effective actions and so on
- O(N) spinning particles in flat space describe quite efficiently higher spin fields (spin $\frac{N}{2}$ in $D = 4$)
- Coupling to curved space can provide useful information about their quantum properties on (A)dS backgrounds
- Non-commutative products can be represented in the worldline phase space $\longrightarrow$ Maybe useful for Vasiliev theories

Motivations

- The worldline formalism proved to be useful to compute various one-loop quantities of QFT's, such as n-point functions, propagators, effective actions and so on
- O(N) spinning particles in flat space describe quite efficiently higher spin fields (spin $\frac{N}{2}$ in $D = 4$)
- Coupling to curved space can provide useful information about their quantum properties on (A)dS backgrounds
- Non-commutative products can be represented in the worldline phase space $\longrightarrow$ Maybe useful for Vasiliev theories
- Gauge symmetries of the field theory are taken into account by worldline symmetries

Motivations

- The worldline formalism proved to be useful to compute various one-loop quantities of QFT's, such as n-point functions, propagators, effective actions and so on
- O(N) spinning particles in flat space describe quite efficiently higher spin fields (spin $\frac{N}{2}$ in $D = 4$)
- Coupling to curved space can provide useful information about their quantum properties on (A)dS backgrounds
- Non-commutative products can be represented in the worldline phase space $\longrightarrow$ Maybe useful for Vasiliev theories
- Gauge symmetries of the field theory are taken into account by worldline symmetries
- No need of a field theory action

Outline

- O(N) spinning particles and higher spin fields in flat space

Outline

- O(N) spinning particles and higher spin fields in flat space
- Coupling to (A)dS and effective action with HS loop

Outline

- O(N) spinning particles and higher spin fields in flat space
- Coupling to (A)dS and effective action with HS loop
- Dimensional reduction and HS in odd dimensions

Outline

- O(N) spinning particles and higher spin fields in flat space
- Coupling to (A)dS and effective action with HS loop
- Dimensional reduction and HS in odd dimensions
- Outlook and future directions

O(N) spinning particle in flat space

Consider the worldline action ($i = 1, ..., N$)

$$S = \int dt \Big[p_\mu \dot{x}^\mu + \tfrac{i}{2} \psi_i^\mu \dot{\psi}_{\mu i} - \tfrac{1}{2} p^2 \Big]$$

O(N) spinning particle in flat space

Consider the worldline action ($i = 1, ..., N$)

$$S = \int dt \Big[p_\mu \dot{x}^\mu + \tfrac{i}{2} \psi_i^\mu \dot{\psi}_{\mu i} - \tfrac{1}{2} p^2 \Big]$$

It enjoys O(N)-extended WL supersymmetry, generated by

O(N) spinning particle in flat space

Consider the worldline action ($i = 1, ..., N$)

$$S = \int dt \Big[p_\mu \dot{x}^\mu + \tfrac{i}{2} \psi_i^\mu \dot{\psi}_{\mu i} - \tfrac{1}{2} p^2 \Big]$$

It enjoys O(N)-extended WL supersymmetry, generated by

$$H = \frac{p^2}{2} \,, \quad Q_i = \psi_i^\mu \, p_\mu \,, \quad J_{ij} = \tfrac{i}{2} [\psi_i^\mu, \psi_{\mu j}]$$

O(N) spinning particle in flat space

Consider the worldline action ($i = 1, ..., N$)

$$S = \int dt \left[p_\mu \dot{x}^\mu + \tfrac{i}{2} \psi_i^\mu \dot{\psi}_{\mu i} - \tfrac{1}{2} p^2 \right]$$

It enjoys O(N)-extended WL supersymmetry, generated by

$$H = \frac{p^2}{2} \;, \quad Q_i = \psi_i^\mu \, p_\mu \;, \quad J_{ij} = \tfrac{i}{2} [\psi_i^\mu , \psi_{\mu j}]$$

The generators obey the following supersymmetry algebra

$$\begin{aligned} \{Q_i, Q_j\} &= 2\, \delta_{ij}\, H \\ [J_{ij}, Q_k] &= i\, \delta_{jk}\, Q_i - i\, \delta_{ik}\, Q_j \\ [J_{ij}, J_{kl}] &= i\, \delta_{jk}\, J_{il} - i\, \delta_{ik}\, J_{jl} - i\, \delta_{jl}\, J_{ik} + i\, \delta_{il}\, J_{jk} \end{aligned}$$

- The algebra is first class $\rightarrow$ It can be gauged to make the (super)symmetries local

- The algebra is first class $\rightarrow$ It can be gauged to make the (super)symmetries local
- One introduces WL gauge fields $e(\tau)$, $\chi_i(\tau)$ and $a_{ij}(\tau)$ to be coupled to the symmetry generators

- The algebra is first class $\rightarrow$ It can be gauged to make the (super)symmetries local
- One introduces WL gauge fields $e(\tau)$, $\chi_i(\tau)$ and $a_{ij}(\tau)$ to be coupled to the symmetry generators
- By doing so one ends up with the O(N) spinning particle action:

$$S = \int_0^1 dt \left[p_\mu \dot{x}^\mu + \frac{i}{2}\psi_i^\mu \dot{\psi}_{\mu i} - eH - i\chi_i Q_i - \frac{1}{2} a_{ij} J_{ij} \right]$$

- The algebra is first class $\rightarrow$ It can be gauged to make the (super)symmetries local
- One introduces WL gauge fields $e(\tau)$, $\chi_i(\tau)$ and $a_{ij}(\tau)$ to be coupled to the symmetry generators
- By doing so one ends up with the O(N) spinning particle action:

$$S = \int_0^1 dt \Big[p_\mu \dot{x}^\mu + \frac{i}{2}\psi_i^\mu \dot{\psi}_{\mu i} - eH - i\chi_i Q_i - \frac{1}{2} a_{ij} J_{ij} \Big]$$

In configuration space it reads

$$S = \int_0^1 dt \Big[\frac{1}{2e} \left(\dot{x}^\mu - i\chi_i \psi_i^\mu \right)^2 + \frac{i}{2}\psi_i^\mu \left(\delta_{ij}\partial_t - a_{ij} \right) \psi_{\mu j} \Big]$$

- The algebra is first class $\rightarrow$ It can be gauged to make the (super)symmetries local
- One introduces WL gauge fields $e(\tau)$, $\chi_i(\tau)$ and $a_{ij}(\tau)$ to be coupled to the symmetry generators
- By doing so one ends up with the O(N) spinning particle action:

$$S = \int_0^1 dt \Big[p_\mu \dot{x}^\mu + \tfrac{i}{2}\psi_i^\mu \dot{\psi}_{\mu i} - eH - i\chi_i Q_i - \tfrac{1}{2} a_{ij} J_{ij} \Big]$$

In configuration space it reads

$$S = \int_0^1 dt \Big[\frac{1}{2e} \left(\dot{x}^\mu - i\chi_i \psi_i^\mu \right)^2 + \tfrac{i}{2}\psi_i^\mu \left(\delta_{ij}\partial_t - a_{ij} \right) \psi_{\mu j} \Big]$$

At the quantum level Dirac constraints

$$T_A |R\rangle = 0 \ , \quad \text{with} \quad T_A := \left(J_{ij}, Q_i, H \right)$$

Canonical quantization: Fronsdal equations

Consider the simple case of $D = 4$ and even $N = 2s$

Canonical quantization: Fronsdal equations

Consider the simple case of $D = 4$ and even $N = 2s$

- Complexify fermions to get s pairs of oscillators:

$$\left\{ \psi_I^\mu, \bar{\psi}^{\nu J} \right\} = \delta_I^J \, \eta^{\mu\nu} \quad I, J = 1, ..., s$$

Canonical quantization: Fronsdal equations

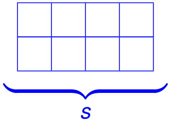
Consider the simple case of $D = 4$ and even $N = 2s$

- Complexify fermions to get s pairs of oscillators:

$$\left\{ \psi_I^\mu, \bar{\psi}^{\nu J} \right\} = \delta_I^J \, \eta^{\mu\nu} \quad I, J = 1, ..., s$$

- J_{ij} constraints $\rightarrow$ physical states are tensors characterized by a $2 \times s$ Young tableau

$$|R\rangle \sim R_{\mu_1 \nu_1 \ldots \mu_s \nu_s}(x) \quad \rightarrow$$



Canonical quantization: Fronsdal equations

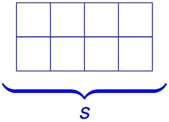
Consider the simple case of $D = 4$ and even $N = 2s$

- Complexify fermions to get s pairs of oscillators:

$$\left\{\psi_I^\mu, \bar{\psi}^{\nu J}\right\} = \delta_I^J \, \eta^{\mu\nu} \qquad I, J = 1, \ldots, s$$

- J_{ij} constraints $\rightarrow$ physical states are tensors characterized by a $2 \times s$ Young tableau

$$|R\rangle \sim R_{\mu_1\nu_1\ldots\mu_s\nu_s}(x) \quad \rightarrow$$



and traceless: $\mathrm{Tr}^{IJ}|R\rangle = 0 \;\rightarrow\; R^{\mu}{}_{\nu_1\ldots\mu\nu_s} = 0$

Canonical quantization: Fronsdal equations

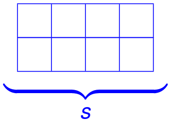
Consider the simple case of $D = 4$ and even $N = 2s$

- Complexify fermions to get s pairs of oscillators:

$$\left\{\psi_I^\mu, \bar{\psi}^{\nu J}\right\} = \delta_I^J\, \eta^{\mu\nu} \qquad I, J = 1, ..., s$$

- J_{ij} constraints $\rightarrow$ physical states are tensors characterized by a $2 \times s$ Young tableau

$$|R\rangle \sim R_{\mu_1\nu_1\ldots\mu_s\nu_s}(x) \quad \rightarrow$$



and traceless: $\mathrm{Tr}^{IJ}|R\rangle = 0 \;\rightarrow\; R^\mu{}_{\nu_1\ldots\mu\nu_s} = 0$

- The other independent constraint is a Bianchi-like equation

$$Q_I|R\rangle = 0 \;\rightarrow\; \partial_{[\mu} R_{\mu_1\nu_1]\ldots\mu_s\nu_s} = 0$$

R is an HS curvature that can be integrated in terms of an HS potential ϕ

R is an HS curvature that can be integrated in terms of an HS potential ϕ

- We solve Bianchi by introducing the HS gauge potential

$$|R\rangle = \frac{1}{s!}\epsilon^{I_1\dots I_s} Q_{I_1}\dots Q_{I_s}|\phi\rangle \quad \rightarrow \quad R_{\mu_1\nu_1\dots\mu_s\nu_s} = \partial_{\mu_1}\dots\partial_{\mu_s}\phi_{\nu_1\dots\nu_s}$$

a symmetric spin s tensor

R is an HS curvature that can be integrated in terms of an HS potential ϕ

- We solve Bianchi by introducing the HS gauge potential

$$|R\rangle = \frac{1}{s!}\epsilon^{I_1\dots I_s} Q_{I_1}\dots Q_{I_s}|\phi\rangle \quad \to \quad R_{\mu_1\nu_1\dots\mu_s\nu_s} = \partial_{\mu_1}\dots\partial_{\mu_s}\phi_{\nu_1\dots\nu_s}$$

  a symmetric spin s tensor

- Field equations are higher derivative $\to$ we introduce a compensator field to recast them as

$$\left(-2H + Q_I\bar{Q}^I + \frac{1}{2}\, Q_I Q_J \mathrm{Tr}^{IJ}\right)|\phi\rangle = Q_I Q_J Q_K|\rho^{IJK}\rangle$$

Bastianelli, Corradini, Latini; 2008

R is an HS curvature that can be integrated in terms of an HS potential ϕ

- We solve Bianchi by introducing the HS gauge potential

$$|R\rangle = \frac{1}{s!}\epsilon^{I_1\ldots I_s} Q_{I_1}\ldots Q_{I_s}|\phi\rangle \quad \rightarrow \quad R_{\mu_1\nu_1\ldots\mu_s\nu_s} = \partial_{\mu_1}\ldots\partial_{\mu_s}\phi_{\nu_1\ldots\nu_s}$$

  a symmetric spin s tensor

- Field equations are higher derivative $\rightarrow$ we introduce a compensator field to recast them as

$$\left(-2H + Q_I\bar{Q}^I + \frac{1}{2}\,Q_I Q_J \mathrm{Tr}^{IJ}\right)|\phi\rangle = Q_I Q_J Q_K|\rho^{IJK}\rangle$$

  Bastianelli, Corradini, Latini; 2008

- They are nothing but Fronsdal equations for spin s with compensators

$$\Box\phi_{(s)} - s\,\partial\partial\cdot\phi_{(s)} + \frac{s(s-1)}{2}\partial^2\,\mathrm{Tr}\,\phi_{(s)} = \partial^3\rho_{(s\text{-}3)}$$

Francia, Sagnotti; 2003

They are invariant under unconstrained gauge transformations

$$\delta\phi_{(s)} = \partial\Lambda_{(s-1)} \ , \quad \delta\rho_{(s-3)} \propto \mathrm{Tr}\,\Lambda_{(s-1)}$$

They are invariant under unconstrained gauge transformations

$$\delta\phi_{\text{(s)}} = \partial\Lambda_{\text{(s-1)}} \ , \quad \delta\rho_{\text{(s-3)}} \propto \text{Tr}\,\Lambda_{\text{(s-1)}}$$

Partially gauge fixing compensators to zero one gets usual Fronsdal equations

They are invariant under unconstrained gauge transformations

$$\delta\phi_{(s)} = \partial\Lambda_{(s\text{-}1)} \ , \quad \delta\rho_{(s\text{-}3)} \propto \mathrm{Tr}\,\Lambda_{(s\text{-}1)}$$

Partially gauge fixing compensators to zero one gets usual Fronsdal equations

$$\Box\phi_{(s)} - s\,\partial\partial\cdot\phi_{(s)} + \frac{s(s-1)}{2}\partial^2\,\mathrm{Tr}\,\phi_{(s)} = 0 \ ,$$

They are invariant under unconstrained gauge transformations

$$\delta\phi_{(s)} = \partial\Lambda_{(s\text{-}1)} \ , \quad \delta\rho_{(s\text{-}3)} \propto \mathrm{Tr}\,\Lambda_{(s\text{-}1)}$$

Partially gauge fixing compensators to zero one gets usual Fronsdal equations

$$\Box\phi_{(s)} - s\,\partial\partial\cdot\phi_{(s)} + \frac{s(s-1)}{2}\partial^2\,\mathrm{Tr}\,\phi_{(s)} = 0 \ ,$$

$$\delta\phi_{(s)} = \partial\Lambda_{(s\text{-}1)} \ , \quad \mathrm{Tr}\,\Lambda_{(s\text{-}1)} = 0 \ , \quad \mathrm{Tr}^2\,\phi_{(s)} = 0$$

They are invariant under unconstrained gauge transformations

$$\delta\phi_{(s)} = \partial\Lambda_{(s\text{-}1)} \ , \quad \delta\rho_{(s\text{-}3)} \propto \mathrm{Tr}\,\Lambda_{(s\text{-}1)}$$

Partially gauge fixing compensators to zero one gets usual Fronsdal equations

$$\Box\phi_{(s)} - s\,\partial\partial\cdot\phi_{(s)} + \frac{s(s-1)}{2}\partial^2\,\mathrm{Tr}\,\phi_{(s)} = 0 \ ,$$

$$\delta\phi_{(s)} = \partial\Lambda_{(s\text{-}1)} \ , \quad \mathrm{Tr}\,\Lambda_{(s\text{-}1)} = 0 \ , \quad \mathrm{Tr}^2\,\phi_{(s)} = 0$$

- For arbitrary even $D = 2k$ one gets Fronsdal-Labastida equations for gauge fields with rectangular $(k-1)\times s$ Young tableaux

They are invariant under unconstrained gauge transformations

$$\delta\phi_{\text{(s)}} = \partial\Lambda_{\text{(s-1)}} \ , \quad \delta\rho_{\text{(s-3)}} \propto \mathrm{Tr}\,\Lambda_{\text{(s-1)}}$$

Partially gauge fixing compensators to zero one gets usual Fronsdal equations

$$\Box\phi_{\text{(s)}} - s\,\partial\partial\cdot\phi_{\text{(s)}} + \frac{s(s-1)}{2}\partial^2\,\mathrm{Tr}\,\phi_{\text{(s)}} = 0 \ ,$$

$$\delta\phi_{\text{(s)}} = \partial\Lambda_{\text{(s-1)}} \ , \quad \mathrm{Tr}\,\Lambda_{\text{(s-1)}} = 0 \ , \quad \mathrm{Tr}^2\,\phi_{\text{(s)}} = 0$$

- For arbitrary even $D = 2k$ one gets Fronsdal-Labastida equations for gauge fields with rectangular $(k-1)\times s$ Young tableaux
- For odd N we have Fang-Fronsdal equations for fermionic HS fields

They are invariant under unconstrained gauge transformations

$$\delta\phi_{(s)} = \partial\Lambda_{(s-1)} \;, \quad \delta\rho_{(s-3)} \propto \mathrm{Tr}\,\Lambda_{(s-1)}$$

Partially gauge fixing compensators to zero one gets usual Fronsdal equations

$$\Box\phi_{(s)} - s\,\partial\partial\cdot\phi_{(s)} + \frac{s(s-1)}{2}\partial^2\,\mathrm{Tr}\,\phi_{(s)} = 0 \;,$$
$$\delta\phi_{(s)} = \partial\Lambda_{(s-1)} \;, \quad \mathrm{Tr}\,\Lambda_{(s-1)} = 0 \;, \quad \mathrm{Tr}^2\,\phi_{(s)} = 0$$

- For arbitrary even $D = 2k$ one gets Fronsdal-Labastida equations for gauge fields with rectangular $(k-1)\times s$ Young tableaux
- For odd N we have Fang-Fronsdal equations for fermionic HS fields
- In odd dimensions the model is empty

HS on (A)dS backgrounds

- Trying to couple the O(N) spinning particle to a curved space, the SUSY algebra is not first class, obstructed by target space curvature

HS on (A)dS backgrounds

- Trying to couple the O(N) spinning particle to a curved space, the SUSY algebra is not first class, obstructed by target space curvature
- On conformally flat spaces the algebra is first class, but with structure functions

Bastianelli, Corradini, Latini; 2008

HS on (A)dS backgrounds

- Trying to couple the O(N) spinning particle to a curved space, the SUSY algebra is not first class, obstructed by target space curvature
- On conformally flat spaces the algebra is first class, but with structure functions

  Bastianelli, Corradini, Latini; 2008
- On maximally symmetric spaces the algebra becomes quadratic in constraints $\rightarrow$ Simpler BRST quantization

HS on (A)dS backgrounds

- Trying to couple the O(N) spinning particle to a curved space, the SUSY algebra is not first class, obstructed by target space curvature
- On conformally flat spaces the algebra is first class, but with structure functions

  Bastianelli, Corradini, Latini; 2008

- On maximally symmetric spaces the algebra becomes quadratic in constraints $\to$ Simpler BRST quantization

$$R_{abcd} = b\,(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}) \quad \to$$

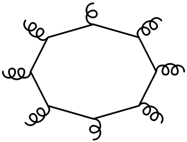
$$\{Q_i, Q_j\} = 2\delta_{ij} H - \frac{b}{2}\,\left(J_{ik}J_{jk} + J_{jk}J_{ik} - \delta_{ij}J_{kl}J_{kl}\right)$$

$$Q_i = \psi_i^a\, e_a^\mu\, \pi_\mu\;, \quad H = \frac{1}{2}\left(\pi^a\pi_a - i\omega^a{}_{ab}\pi^b\right) - \frac{b}{4}\,J_{ij}J_{ij} - bA(D,N)$$

HS Effective action on (A)dS

- One-loop effective action given by the worldline path integral

$$\Gamma[g] =$$



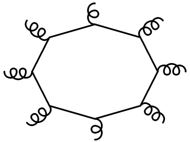
$$= \int_{S^1} \frac{\mathcal{D}x D\psi_i D e D\chi_i D a_{ij}}{\mathrm{Vol(Gauge)}} e^{-S}$$

Bastianelli, R.B., Corradini, Latini; 2012

HS Effective action on (A)dS

- One-loop effective action given by the worldline path integral

$$\Gamma[g] =$$



$$= \int_{S^1} \frac{\mathcal{D}x D\psi_i De D\chi_i Da_{ij}}{\mathrm{Vol(Gauge)}} e^{-S}$$

Bastianelli, R.B., Corradini, Latini; 2012

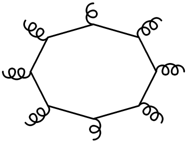
- After gauge fixing WL symmetries one has the Heat Kernel expansion

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^D x \sqrt{|g|}}{(2\pi T)^{D/2}} a_0 \left\langle\!\!\left\langle e^{-S_{\mathrm{int}}} \right\rangle\!\!\right\rangle$$

HS Effective action on (A)dS

- One-loop effective action given by the worldline path integral

$$\Gamma[g] =$$



$$= \int_{S^1} \frac{\mathcal{D}x D\psi_i DeD\chi_i Da_{ij}}{\mathrm{Vol(Gauge)}}\, e^{-S}$$

Bastianelli, R.B., Corradini, Latini; 2012

- After gauge fixing WL symmetries one has the Heat Kernel expansion

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^D x \sqrt{|g|}}{(2\pi T)^{D/2}} a_0 \left\langle\!\!\left\langle e^{-S_{\mathrm{int}}} \right\rangle\!\!\right\rangle$$

$$a_0 \left\langle\!\!\left\langle e^{-S_{\mathrm{int}}} \right\rangle\!\!\right\rangle = a_0 \left(1 + v_1\, R\, T + v_2\, R^2\, T^2 + \ldots\right)$$

Simplest case: $D = 4$ and even $N = 2s$

The SDW coefficients are defined as

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^4x\sqrt{|g|}}{(2\pi T)^2}\, a_0 \left(1 + v_1\, R\, T + v_2\, R^2\, T^2 + \ldots\right)$$

Simplest case: $D = 4$ and even $N = 2s$

The SDW coefficients are defined as

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^4x\sqrt{|g|}}{(2\pi T)^2}\, a_0 \left(1 + v_1\, R\, T + v_2\, R^2\, T^2 + ...\right)$$

and we get

$$a_0 = 2 - \delta_{s,0}\ , \quad v_1 = -\frac{s^2}{6}\ , \quad v_2 = -\frac{1}{8640} + \frac{s^2}{288} - \frac{s^4}{144}$$

Simplest case: $D = 4$ and even $N = 2s$

The SDW coefficients are defined as

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^4x\sqrt{|g|}}{(2\pi T)^2}\, a_0 \left(1 + v_1\, R\, T + v_2\, R^2\, T^2 + \ldots\right)$$

and we get

$$a_0 = 2 - \delta_{s,0}\ , \quad v_1 = -\frac{s^2}{6}\ , \quad v_2 = -\frac{1}{8640} + \frac{s^2}{288} - \frac{s^4}{144}$$

- For $s = 0, 1$ known results for conformally improved scalar and Maxwell fields

Simplest case: $D = 4$ and even $N = 2s$
The SDW coefficients are defined as

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^4x\sqrt{|g|}}{(2\pi T)^2}\, a_0 \left(1 + v_1\, R\, T + v_2\, R^2\, T^2 + \ldots\right)$$

and we get

$$a_0 = 2 - \delta_{s,0}\ , \quad v_1 = -\frac{s^2}{6}\ , \quad v_2 = -\frac{1}{8640} + \frac{s^2}{288} - \frac{s^4}{144}$$

- For $s = 0, 1$ known results for conformally improved scalar and Maxwell fields
- For $s = 2$ there is a mismatch with known results

SO(N)	Christensen and Duff ('83)	Worldline spin 2 (2013)
$a_0 = 2$	$a_0 = 2$	$a_0 = 2$
$v_1 = -\frac{2}{3}$	$v_1 = ?$	$v_1 = -\frac{2}{3}$
$v_2 = -\frac{841}{8640}$	$v_2 = -\frac{571}{8640}$	$v_2 = -\frac{571}{8640}$

$$\text{i.e. } \Delta v_2 = \frac{1}{32}$$

- We conjecture this is a topological mismatch similar to those found for dual differential forms

SO(N)	Christensen and Duff ('83)	Worldline spin 2 (2013)
$a_0 = 2$	$a_0 = 2$	$a_0 = 2$
$v_1 = -\frac{2}{3}$	$v_1 = ?$	$v_1 = -\frac{2}{3}$
$v_2 = -\frac{841}{8640}$	$v_2 = -\frac{571}{8640}$	$v_2 = -\frac{571}{8640}$

$$\text{i.e. } \Delta v_2 = \frac{1}{32}$$

- We conjecture this is a topological mismatch similar to those found for dual differential forms
- It remains at higher spins (Giombi Klebanov, 2013)

SO(N)	Christensen and Duff ('83)	Worldline spin 2 (2013)
$a_0 = 2$	$a_0 = 2$	$a_0 = 2$
$v_1 = -\frac{2}{3}$	$v_1 = ?$	$v_1 = -\frac{2}{3}$
$v_2 = -\frac{841}{8640}$	$v_2 = -\frac{571}{8640}$	$v_2 = -\frac{571}{8640}$

$$\text{i.e. } \Delta v_2 = \frac{1}{32}$$

- We conjecture this is a topological mismatch similar to those found for dual differential forms
- It remains at higher spins (Giombi Klebanov, 2013)
- ζ-regulated sum over all spins vanish (agreement with GK for Vasiliev spectrum)

SO(N)	Christensen and Duff ('83)	Worldline spin 2 (2013)
$a_0 = 2$	$a_0 = 2$	$a_0 = 2$
$v_1 = -\frac{2}{3}$	$v_1 = ?$	$v_1 = -\frac{2}{3}$
$v_2 = -\frac{841}{8640}$	$v_2 = -\frac{571}{8640}$	$v_2 = -\frac{571}{8640}$

$$\text{i.e. } \Delta v_2 = \frac{1}{32}$$

- We conjecture this is a topological mismatch similar to those found for dual differential forms
- It remains at higher spins (Giombi Klebanov, 2013)
- ζ-regulated sum over all spins vanish (agreement with GK for Vasiliev spectrum)
- Coefficients obtained also for half-integer spins and conformal fields in all even dimensions

Bastianelli, RB, Corradini, Latini, JHEP 1212 (2012) 113 arXiv:1210.4649

Massive and massless HS in odd D

- $O(N)$ spinning particle is empty in odd dimensions $\rightarrow$ dimensional reduction from $D+1=2k$ with fixed momentum $p_5=m$

Massive and massless HS in odd D

- $O(N)$ spinning particle is empty in odd dimensions $\rightarrow$ dimensional reduction from $D+1=2k$ with fixed momentum $p_5=m$
- Split fermions as $\psi_i^M=(\psi_i^\mu,\theta_i)$, the $(D+1)$-dimensional superalgebra is unchanged.

Massive and massless HS in odd D

- $O(N)$ spinning particle is empty in odd dimensions $\to$ dimensional reduction from $D+1=2k$ with fixed momentum $p_5 = m$
- Split fermions as $\psi_i^M = (\psi_i^\mu, \theta_i)$, the $(D+1)$-dimensional superalgebra is unchanged.
- For $N = 2s$ use complex combinations of fermions $(\psi_I^\mu, \bar{\psi}^{\mu I})$, $(\theta_I, \bar{\theta}^I)$ with manifest $U(s)$ covariance. Relevant constraints:

$$\mathcal{J}_I^J = J_I^J + \theta_I \frac{\partial}{\partial \theta_J} - k\,\delta_I^J\,, \quad \mathcal{K}^{IJ} = \mathrm{Tr}^{IJ} + \frac{\partial^2}{\partial\theta_I \partial\theta_J}\,, \quad \mathcal{Q}_I = Q_I + m\,\theta_I$$

Bastianelli, R.B., Corradini, Latini, to appear

Massive and massless HS in odd D

- $O(N)$ spinning particle is empty in odd dimensions $\rightarrow$ dimensional reduction from $D+1=2k$ with fixed momentum $p_5 = m$
- Split fermions as $\psi_i^M = (\psi_i^\mu, \theta_i)$, the $(D+1)$-dimensional superalgebra is unchanged.
- For $N=2s$ use complex combinations of fermions $(\psi_I^\mu, \bar\psi^{\mu I})$, $(\theta_I, \bar\theta^I)$ with manifest $U(s)$ covariance. Relevant constraints:

$$\mathcal{J}_I^J = J_I^J + \theta_I \frac{\partial}{\partial\theta_J} - k\,\delta_I^J\,, \quad \mathcal{K}^{IJ} = \mathrm{Tr}^{IJ} + \frac{\partial^2}{\partial\theta_I\partial\theta_J}\,, \quad \mathcal{Q}_I = Q_I + m\,\theta_I$$

- Generic state is a sum of Lorentz tensors with θ-expansion

$$\mathcal{R}(x,\psi,\theta) = \sum_{n=0}^{s} \tfrac{1}{n!}\, R^{I_1\dots I_n}(x,\psi)\,\theta_{I_1}\dots\theta_{I_n}$$

Bastianelli, R.B., Corradini, Latini, to appear

Constraints

- $\mathcal{J}_I^J$ constraints impose $GL(D)$ irreducibility. At fixed n the states $R^{l_1\ldots l_n}$ consist of a single Lorentz tensor with Young tableau ($D = 2k-1$)

$R^{l_1\ldots l_n} \sim$

1					s
k-1					
1	...	s-n			

$$R^{l_1\ldots l_n}(x,\psi) = R^{l_1\ldots l_n}_{\mu^1_1..\mu^1_k,..,\mu^{l_i}_1..\mu^{l_i}_{k-1},..,\mu^s_1..\mu^s_k}(x)\,\psi_1^{\mu^1_1}..\psi_1^{\mu^1_k}..\psi_{l_i}^{\mu^{l_i}_1}..\psi_{l_i}^{\mu^{l_i}_{k-1}}..\psi_s^{\mu^s_1}..\psi_s^{\mu^s_k}$$

Constraints

- $\mathcal{J}_I^{\ J}$ constraints impose $GL(D)$ irreducibility. At fixed n the states $R^{I_1\ldots I_n}$ consist of a single Lorentz tensor with Young tableau ($D = 2k - 1$)

$$R^{I_1\ldots I_n} \sim \begin{array}{|c|c|c|c|c|c|} \hline 1 & & & & & s \\ \hline & & & & & \\ \hline \text{k-1} & & & & & \\ \hline 1 & \ldots & \text{s-n} \\ \cline{1-3} \end{array}$$

$$R^{I_1\ldots I_n}(x,\psi) = R^{I_1\ldots I_n}_{\mu^1_1..\mu^1_k,..,\mu^{I_i}_1..\mu^{I_i}_{k-1},..,\mu^s_1..\mu^s_k}(x)\psi_1^{\mu^1_1}..\psi_1^{\mu^1_k}..\psi_{I_i}^{\mu^{I_i}_1}..\psi_{I_i}^{\mu^{I_i}_{k-1}}..\psi_s^{\mu^s_1}..\psi_s^{\mu^s_k}$$

- $\mathcal{K}^{IJ}$ constraints relate traces of higher rank tensors to lower rank ones

$$\mathrm{Tr}^{KL} R^{I_1\ldots I_n} - R^{KLI_1\ldots I_n} = 0$$

and will enforce dynamical field equations

Constraints

- $\mathcal{J}_I^{\ J}$ constraints impose $GL(D)$ irreducibility. At fixed n the states $R^{I_1 \dots I_n}$ consist of a single Lorentz tensor with Young tableau ($D = 2k - 1$)

$$R^{I_1 \dots I_n} \sim \begin{array}{|c|c|c|c|c|c|} \hline 1 & & & & & s \\ \hline & & & & & \\ \hline \text{k-1} & & & & & \\ \hline 1 & \dots & \text{s-n} \\ \cline{1-3} \end{array}$$

$$R^{I_1 \dots I_n}(x, \psi) = R^{I_1 \dots I_n}_{\mu_1^1 .. \mu_k^1, .., \mu_1^{I_i} .. \mu_{k-1}^{I_i}, .., \mu_1^s .. \mu_k^s}(x) \psi_1^{\mu_1^1} .. \psi_1^{\mu_k^1} .. \psi_{I_i}^{\mu_1^{I_i}} .. \psi_{I_i}^{\mu_{k-1}^{I_i}} .. \psi_s^{\mu_1^s} .. \psi_s^{\mu_k^s}$$

- $\mathcal{K}^{IJ}$ constraints relate traces of higher rank tensors to lower rank ones

$$\mathrm{Tr}^{KL} R^{I_1 \dots I_n} - R^{KLI_1 \dots I_n} = 0$$

  and will enforce dynamical field equations

- $\mathcal{Q}_I$ constraints provide integrability conditions and relations between tensors of different ranks

$$Q_K R^{I_1 \dots I_n} = m \, (-)^{ks+n} n \, \delta_K^{[I_1} R^{I_2 \dots I_n]}$$

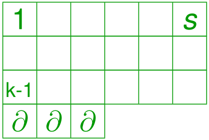
Massive case: Pauli-Fierz

- In the massive case, only independent field is $R^{I_1 \ldots I_s} = \epsilon^{I_1 \ldots I_s} \phi$

Massive case: Pauli-Fierz

- In the massive case, only independent field is $R^{l_1 \ldots l_s} = \epsilon^{l_1 \ldots l_s} \phi$
- Other fields expressed as derivatives on ϕ

$$R^{l_1 \ldots l_n} = \frac{(-)^{(s-n)(ks+1)}}{m^{s-n}(s-n)!} \, \epsilon^{l_1 \ldots l_s} \, Q_{l_{n+1}} \ldots Q_{l_s} \, \phi \sim$$



Massive case: Pauli-Fierz

- In the massive case, only independent field is $R^{l_1 \dots l_s} = \epsilon^{l_1 \dots l_s} \phi$
- Other fields expressed as derivatives on ϕ

$$R^{l_1 \dots l_n} = \frac{(-)^{(s-n)(ks+1)}}{m^{s-n}(s-n)!}\, \epsilon^{l_1 \dots l_s}\, Q_{l_{n+1}} \dots Q_{l_s}\, \phi \sim$$

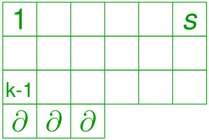
1					s
k-1					
∂	∂	∂			

- Relevant trace constraints are the last three, others giving further derivatives of the e.o.m.

Massive case: Pauli-Fierz

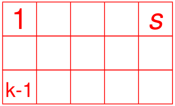
- In the massive case, only independent field is $R^{I_1 \dots I_s} = \epsilon^{I_1 \dots I_s} \phi$
- Other fields expressed as derivatives on ϕ

$$R^{I_1 \dots I_n} = \frac{(-)^{(s-n)(ks+1)}}{m^{s-n}(s-n)!} \epsilon^{I_1 \dots I_s} Q_{I_{n+1}} \dots Q_{I_s} \phi \sim$$



- Relevant trace constraints are the last three, others giving further derivatives of the e.o.m.
- In terms of ϕ they are the triplet of Pauli-Fierz conditions for the massive field

$$\phi_{\mu_1^1 \dots \mu_{k-1}^1, \dots, \mu_1^s \dots \mu_{k-1}^s} \sim$$



$$\mathrm{Tr}^{IJ} \phi = 0 \,, \quad \bar{Q}^I \phi = 0 \,, \quad \left(\Box - m^2 \right) \phi = 0$$

Massless limit: Fronsdal-Labastida multiplet

- Letting $m \to 0$ the tensor structures of $R^{l_1 \cdots l_n}$ remain the same, as well as trace constraints

Massless limit: Fronsdal-Labastida multiplet

- Letting $m \to 0$ the tensor structures of $R^{I_1 \cdots I_n}$ remain the same, as well as trace constraints
- Integrability constraints now decouple and are Bianchi equations for $s + 1$ independent fields $Q_K R^{I_1 \cdots I_n} = 0$

Massless limit: Fronsdal-Labastida multiplet

- Letting $m \to 0$ the tensor structures of $R^{I_1 \cdots I_n}$ remain the same, as well as trace constraints
- Integrability constraints now decouple and are Bianchi equations for $s+1$ independent fields $Q_K R^{I_1 \cdots I_n} = 0$
- Integrate curvatures $R^{I_1 \cdots I_n}$ in terms of gauge potentials $\varphi^{I_1 \ldots I_n}$

$$R^{I_1 \ldots I_n} = q\,\varphi^{I_1 \ldots I_n}\ , \quad q = \frac{1}{s!}\,\epsilon^{I_1 \ldots I_s} Q_{I_1} \ldots Q_{I_s}$$

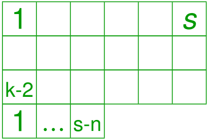
Massless limit: Fronsdal-Labastida multiplet

- Letting $m \to 0$ the tensor structures of $R^{I_1 \cdots I_n}$ remain the same, as well as trace constraints
- Integrability constraints now decouple and are Bianchi equations for $s+1$ independent fields $Q_K R^{I_1 \cdots I_n} = 0$
- Integrate curvatures $R^{I_1 \cdots I_n}$ in terms of gauge potentials $\varphi^{I_1 \ldots I_n}$

$$R^{I_1 \ldots I_n} = q\, \varphi^{I_1 \ldots I_n} \,, \qquad q = \frac{1}{s!}\, \epsilon^{I_1 \ldots I_s} Q_{I_1} \ldots Q_{I_s}$$

- $s+1$ different gauge fields with Young tableaux

$\varphi^{I_1 \ldots I_n} \sim$



- In $D=3$ we have a multiplet of symmetric tensors ranging from spin zero to s $\to$ dof of (truncated) Vasiliev theory

Massless limit: Fronsdal-Labastida multiplet

- trace constraints give e.o.m. (after a linear field redefinition of φ's) as Fronsdal-Labastida equations with compensators for a multiplet of $s+1$ mixed symmetry tensors

$$\left(-2H + Q_I \bar{Q}^I + \tfrac{1}{2}\, Q_I Q_J \,\mathrm{Tr}^{IJ}\right) \varphi^{I_1 \dots I_n} = Q_I Q_J Q_K \,\rho^{IJK|I_1..I_n}$$

$$\left(\Box - \partial_I \left(\partial^I \cdot\right) - \tfrac{1}{2}\, \partial_I \partial_J \,\mathrm{Tr}^{IJ}\right) \varphi^{I_1 \dots I_n} = \partial_I \partial_J \partial_K \,\rho^{IJK|I_1..I_n}$$

Massless limit: Fronsdal-Labastida multiplet

- trace constraints give e.o.m. (after a linear field redefinition of φ's) as Fronsdal-Labastida equations with compensators for a multiplet of $s+1$ mixed symmetry tensors

$$\left(-2H + Q_I\bar{Q}^I + \tfrac{1}{2}\,Q_IQ_J\,\mathrm{Tr}^{IJ}\right)\varphi^{I_1\dots I_n} = Q_IQ_JQ_K\,\rho^{IJK|I_1..I_n}$$

$$\left(\Box - \partial_I\left(\partial^I\cdot\right) - \tfrac{1}{2}\,\partial_I\partial_J\,\mathrm{Tr}^{IJ}\right)\varphi^{I_1\dots I_n} = \partial_I\partial_J\partial_K\,\rho^{IJK|I_1..I_n}$$

- Unconstrained gauge transformations

$$\delta\varphi^{I_1\dots I_n} = Q_K\,\Lambda^{K|I_1\dots I_n}\,, \quad \delta\rho^{IJK|I_1\dots I_n} = \tfrac{1}{2}\,\mathrm{tr}^{[IJ}\Lambda^{K]|I_1\dots I_n}$$

Massless limit: Fronsdal-Labastida multiplet

- trace constraints give e.o.m. (after a linear field redefinition of φ's) as Fronsdal-Labastida equations with compensators for a multiplet of $s+1$ mixed symmetry tensors

$$\left(-2H + Q_I \bar{Q}^I + \tfrac{1}{2}\, Q_I Q_J \,\mathrm{Tr}^{IJ}\right) \varphi^{I_1 \dots I_n} = Q_I Q_J Q_K \, \rho^{IJK|I_1 .. I_n}$$

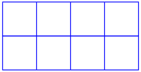
$$\left(\Box - \partial_I \left(\partial^I \cdot\right) - \tfrac{1}{2}\, \partial_I \partial_J \,\mathrm{Tr}^{IJ}\right) \varphi^{I_1 \dots I_n} = \partial_I \partial_J \partial_K \, \rho^{IJK|I_1 .. I_n}$$

- Unconstrained gauge transformations

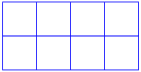
$$\delta\varphi^{I_1 \dots I_n} = Q_K \, \Lambda^{K|I_1 \dots I_n} \,, \quad \delta\rho^{IJK|I_1 \dots I_n} = \tfrac{1}{2}\, \mathrm{tr}^{[IJ} \Lambda^{K]|I_1 \dots I_n}$$

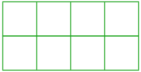
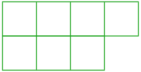
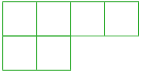
- The model appear to be deformable to (A)dS backgrounds (nonlinear constraint algebra), work in progress

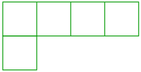
Example $D = 5$, $s = 4$

- massive field $\rightarrow \phi \sim$ 

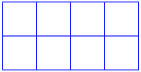
Example $D = 5$, $s = 4$

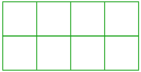
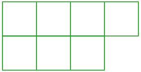
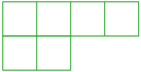
- massive field $\rightarrow \phi \sim$ 
- massless limit $\rightarrow$

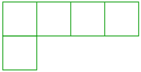
$\varphi \sim$  , $\varphi^{I} \sim$  , $\varphi^{IJ} \sim$  ,

$\varphi^{IJK} \sim$  , $\varphi^{IJKL} \sim$ 

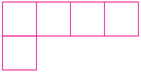
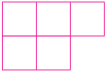
Example $D = 5$, $s = 4$

- massive field $\rightarrow \phi \sim$ 
- massless limit $\rightarrow$

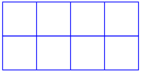
$\varphi \sim$  , $\varphi^{I} \sim$  , $\varphi^{IJ} \sim$  ,

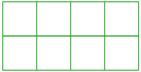
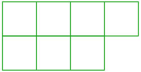
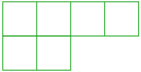
$\varphi^{IJK} \sim$  , $\varphi^{IJKL} \sim$ 

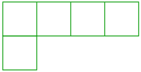
- For instance $\delta\varphi^{IJ} = Q_K \Lambda^{K|IJ}$, gauge parameters

$\Lambda^{K|IJ} \sim$  $K \notin \{I, J\}$ or  $K \in \{I, J\}$

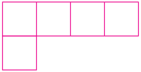
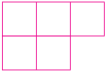
Example $D = 5$, $s = 4$

- massive field $\rightarrow \phi \sim$ 
- massless limit $\rightarrow$

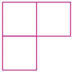
$\varphi \sim$  , $\varphi^{I} \sim$  , $\varphi^{IJ} \sim$  ,

$\varphi^{IJK} \sim$  , $\varphi^{IJKL} \sim$ 

- For instance $\delta\varphi^{IJ} = Q_K \Lambda^{K|IJ}$, gauge parameters

$\Lambda^{K|IJ} \sim$  $K \notin \{I, J\}$ or  $K \in \{I, J\}$

- Its compensators $\rho^{KLM|IJ}$

$\rho^{KLM|IJ} \sim$  or 

Outlook

- $O(N)$ spinning particle quantized on conformally flat?

Outlook

- $O(N)$ spinning particle quantized on conformally flat?
- Dimensional reduction of several dimensions $\rightarrow$ patterns of massless multiplets with mixed symmetry in even and odd dimensions

Outlook

- $O(N)$ spinning particle quantized on conformally flat?
- Dimensional reduction of several dimensions $\rightarrow$ patterns of massless multiplets with mixed symmetry in even and odd dimensions
- Attempt to compute exact effective actions in $(A)dS$ $\rightarrow$ zeta function methods, log radial reduction from flat space?

Outlook

- $O(N)$ spinning particle quantized on conformally flat?
- Dimensional reduction of several dimensions $\rightarrow$ patterns of massless multiplets with mixed symmetry in even and odd dimensions
- Attempt to compute exact effective actions in $(A)dS$ $\rightarrow$ zeta function methods, log radial reduction from flat space?
- Possible applications to Vasiliev theory in $D = 3$

Outlook

- $O(N)$ spinning particle quantized on conformally flat?
- Dimensional reduction of several dimensions $\to$ patterns of massless multiplets with mixed symmetry in even and odd dimensions
- Attempt to compute exact effective actions in $(A)dS$ $\to$ zeta function methods, log radial reduction from flat space?
- Possible applications to Vasiliev theory in $D = 3$
- Construct a “Vasiliev spinning particle” $\to$ wishful thinking $\to$ one loop results with HS background

THANKS FOR YOU ATTENTION!