Non Minimal Terms in Composite Higgs Model

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with Stefania De Curtis and Michele Redi (arXiv: 1403.3116/hep-ph)

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Cortona

Naturalness

$m_h \simeq 125\,\mathrm{GeV}$ SM may be valid up to M_{Pl}

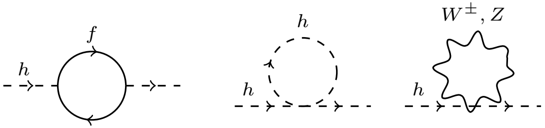
but

many reasons to expect new physics at a scale $\Lambda_{NP} > \Lambda_{SM}$

SM as an effective theory

$$\mathcal{L}_H = |D_\mu H|^2 + \mu^2 |H|^2 - \lambda(|H|^2)^2 - Y_{ij}\bar{\Psi}^i H \Psi^j + \text{h.c.}$$

$[H^2] = 2 \rightarrow$ naturalness problem



$$\delta m_h^2 \propto \frac{g_{SM}^2}{16\pi^2}\Lambda_{NP}^2$$

$$\Delta \equiv \left|\frac{\delta m_h^2}{m_h^2}\right| \sim \left(\frac{125\mathrm{GeV}}{m_h}\right)^2 \left(\frac{\Lambda_{NP}}{500\mathrm{GeV}}\right)^2$$

fine tuning $\sim 1/\Delta$

Composite Higgs Model: Higgs as pseudo-GB

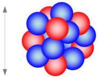
Symmetry can protect the higgs boson mass

- supersymmetry
- shift symmetry

Higgs as a composite pseudo Goldstone boson

D. B. Kaplan and H. Georgi ('80)

$l \sim 1/\Lambda_{com}$



$$h^{\hat{a}} \to h^{\hat{a}} + \alpha^{\hat{a}} + \cdots$$

no mass term at tree level

$$\delta m_h^2 \propto \frac{g_{SM}^2}{16\pi^2} \Lambda_{com}^2$$

elementary sector
$(g_0 < 1)$

$SU(2)_L \times U(1)_Y$

gauging of $SU(2)_L \times U(1)_Y$

$\Longleftrightarrow$

partial compositness

composite sector $(1 < g_\rho < 4\pi)$

SSB: $G \xrightarrow{f} H$

$m_\rho \sim g_\rho f$

Fun with group theory:

G	H	N_G	GBs rep[H]=rep[$SU(2) \times SU(2)$]
$SO(5)$	$SO(4)$	4	$\mathbf{4} = (\mathbf{2}, \mathbf{2})$
$SO(6)$	$SO(5)$	5	$\mathbf{5} = (\mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$
$SO(6)$	$SO(4) \times SO(2)$	8	$\mathbf{4}_{+2} + \bar{\mathbf{4}}_{-2} = 2 \times (\mathbf{2}, \mathbf{2})$
$SU(5)$	$SU(4) \times U(1)$	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2 \times (\mathbf{2}, \mathbf{2})$

MINIMAL MODEL with $SU(2)_C$

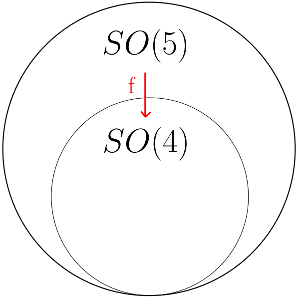
Agashe, Contino, Pomarol (hep-ph/0412089)

$SO(5) \to SO(4) \quad \to \quad$ GBs: $(\mathbf{2}, \mathbf{2})$

Higgs = pseudo-GB
$(m_h \ll m_\rho)$

EWSB like in the SM

$$v = f \sin \frac{\langle h \rangle}{f} \Longrightarrow \xi \equiv v^2/f^2$$



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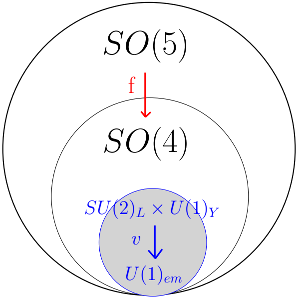

Explicit Models

- 5D CHM Agashe, Contino, Pomarol (hep-ph/0412089)

  5D gauge symmetry $\longrightarrow$ 4D global symmetry

- 4D effective descriptions

  - CCWZ low energy effective description of a GB Higgs
    Giudice, Grojean, Pomarol, Rattazzi (hep-ph/0703164)

  - DISCRETE MODELS
    Panico, Wulzer (hep-ph/1106.2719), De Curtis, Redi, Tesi (hep-ph/1110.1613)

SSB G/H → CCWZ formalism

$$U(\Pi) = \exp(i\Pi/f), \quad \Pi = \sqrt{2} h^{\hat{a}} T^{\hat{a}}, \quad U(\Pi) \to g\, U(\Pi)\, h^\dagger(\Pi, g)$$
$$iU^\dagger \partial_\mu U = e_\mu^a T^a + d_\mu^{\hat{a}} T^{\hat{a}} = e_\mu + d_\mu$$

$$\mathcal{L} = \frac{f^2}{4} \mathrm{Tr}[d_\mu d^\mu] = \frac{f^2}{2} (\partial^\mu \Phi)^T (\partial_\mu \Phi) \quad \text{for } SO(N)/SO(N-1)$$
$$\Phi = U(\Pi)\Phi_0, \quad (\Phi_0)_\alpha = \delta_{\alpha N}$$

Spin 1 resonances as gauge fields

$$SO(N)_L \times SO(N)_R \quad \times \quad SO(N)$$

$$\Omega_1 \to g_L \Omega_1 g_R^\dagger$$
$$\mathcal{L} \sim \mathrm{Tr}[\partial_\mu \Omega_1^\dagger \partial_\mu \Omega_1]$$

↓ ↓

$$\Omega_2 \to g\, \Omega_2 h^\dagger(\Pi, g)$$
$$\Phi_2 \equiv \Omega_2 \Phi_0$$
$$\mathcal{L} \sim (\partial^\mu \Phi_2)^T \partial_\mu \Phi_2$$

$$SO(N)_{L+R} \qquad SO(N-1)$$

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Spin 1 resonances as gauge fields

$$SO(N)_L \times \boxed{SO(N)_R \quad \times \quad SO(N)}$$

A, g

$$\Omega_1 \to g_L \Omega_1 g_R^{\dagger}$$
$$\mathcal{L} \sim \mathrm{Tr}[\partial_{\mu}\Omega_1^{\dagger}\partial_{\mu}\Omega_1]$$

$$SO(N)_L \times SO(N)_R \downarrow SO(N)_{L+R}$$

$$SO(N) \downarrow SO(N-1)$$

$$\Omega_2 \to g\, \Omega_2 h^{\dagger}(\Pi, g)$$
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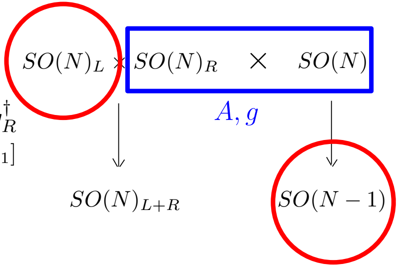
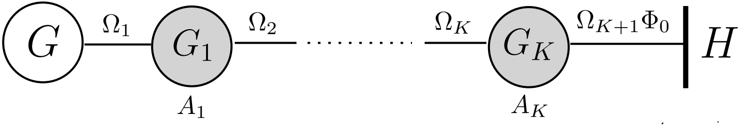


$$\Omega_2 \to g\, \Omega_2 h^{\dagger}(\Pi, g)$$
$$\Phi_2 \equiv \Omega_2 \Phi_0$$
$$\mathcal{L} \sim (\partial^\mu \Phi_2)^T \partial_\mu \Phi_2$$

K copies $\longrightarrow$ moose model



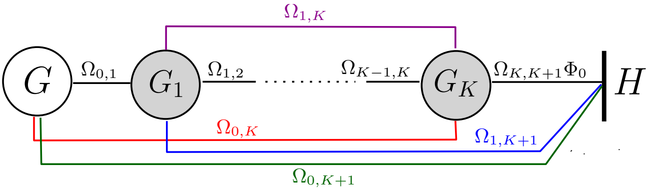
$$\Omega_i \to g_{i-1}\,\Omega_i\,g_i^\dagger\,, \quad \Omega_{K+1} \to g_K\,\Omega_{K+1}\,h^\dagger$$

$$\text{GBs}: \quad U(\Pi) = \Omega_1 \ldots \Omega_{K+1}$$

K massive spin-1 resonances $\quad + \quad$ $SO(N)/SO(N-1)$ GBs

nearest neighbour interactions : $\quad K \to \infty \quad \Rightarrow \quad$ 5D theory

Add non nearest neighbour interactions (also considered in low energy QCD)



link fields: $\Omega_{i,j} \equiv \prod_{k=i+1}^{j} \Omega_k$

$$\Omega_{i,j} \rightarrow g_i \Omega_{i,j} g_j^\dagger$$
$$\Omega_{i,K+1} \rightarrow g_i \Omega_{i,K+1} h^\dagger(\Pi, g_i)$$

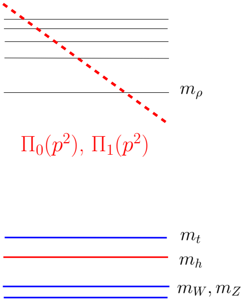
$$\mathcal{L} = \sum_{i,j=0}^{K} \frac{f_{ij}^2}{8}[(D_\mu \Omega_{i,j})^T D^\mu \Omega_{i,j}] + \sum_{i=0}^{K} \frac{f_{iK+1}^2}{2}(D^\mu \Omega_{i,K+1}\Phi_0)^T D^\mu \Omega_{i,K+1}\Phi_0 - \sum_{i=1}^{K} \frac{1}{4g_i^2}[A_i^{\mu\nu} A_{i\mu\nu}]$$

$$D_\mu \Omega_{i,j} = \partial_\mu \Omega_{i,j} - iA_\mu^i \Omega_{i,j} + i\Omega_{i,j} A_\mu^j \ , \qquad f_{ij} = f_{ji},\ \Omega_{j,i} \equiv (\Omega_{i,j})^T$$

The model reproduces the most general effective lagrangian up to two derivatives for a $SO(N)/SO(N-1)$ CHM

equivalent to CCWZ formulation (Marzocca, Serone, Shu (hep-ph/1205.0770))

- Interpolate between discrete models and low energy effective models
  calculability — generality
- Control deviations from extra dimensional theories
  - Higgs potential
  - S parameter



Integrate out the composite sector

Agashe, Contino, Pomarol (hep-ph/0412089)

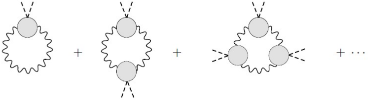
$$\Pi_0(p^2) = -\frac{p^2}{g_0^2} + \Pi_a(p^2)$$

$$\Pi_1(p^2) = 2[\Pi_{\hat{a}}(p^2) - \Pi_a(p^2)]$$

$$\Pi_a(p^2)(P_t)^{\mu\nu} \equiv \langle J_a^\mu(p) J_a^\nu(-p)\rangle$$

$$\Pi_{\hat{a}}(p^2)(P_t)^{\mu\nu} \equiv \langle J_{\hat{a}}^\mu(p) J_{\hat{a}}^\nu(-p)\rangle$$

Coleman Weinberg effective potential for the Higgs at one-loop order



$$V_{gauge}(h) \simeq \frac{9}{8}\int \frac{d^4p}{(2\pi)^4}\frac{\Pi_1(p^2)}{\Pi_0(p^2)} \sin^2\frac{h}{f}$$

Calculability of the potential

$$\int^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{\Pi_1(p^2)}{\Pi_0(p^2)}$$

$\Pi_0(p^2) \overset{p^2\to\infty}{\longrightarrow} p^2$

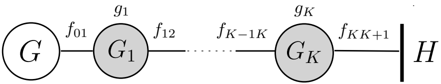
$\Pi_1(p^2) \overset{p^2\to\infty}{\longrightarrow} 0$ ~~Λ^2~~

$p^2\Pi_1(p^2) \overset{p^2\to\infty}{\longrightarrow} 0$ ~~$\log\Lambda$~~

nearest neighbour interactions :

$$\Pi_1(p^2) = \frac{2m_\rho^4(m_{a_1}^2 - m_\rho^2)}{g_\rho^2(p^2+m_\rho^2)(p^2+m_{a_1}^2)} \qquad \checkmark$$

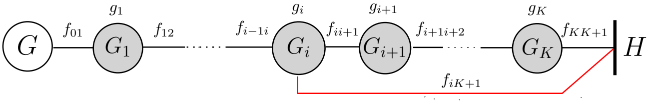
$$m_{a_1}^2 \to \infty \quad \Rightarrow \quad \log\Lambda$$



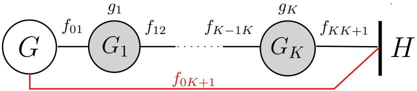
$$\Pi_1(p^2) = \frac{f_{KK+1}^2 \prod_{i=1}^{K} g_i^4 f_{i-1i}^4}{4^K \prod_{i=1}^{K}(p^2+m_{\rho_i}^2)(p^2+m_{a_i}^2)} \sim \frac{1}{(p^2)^{2K}}$$

Effect of non nearest neighbour interactions?

$$\Pi_1(p^2) = \frac{\mathcal{N}(p^2)}{\prod_{i=1}^{K}(p^2+m_{\rho_i}^2)(p^2+m_{a_i}^2)} + f_{0K+1}^2 \;\; ?$$



$$\Pi_1(p^2) \sim \frac{1}{(p^2)^d}\,, \quad d = 2i$$



$$\Pi_1(p^2) \sim f_{0K+1}^2 \quad \rightarrow \quad d = 0\,, \quad \Lambda^2$$

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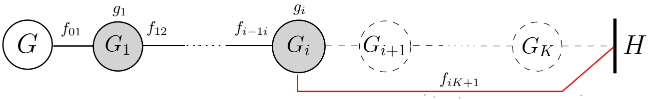

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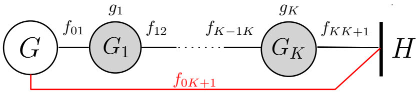

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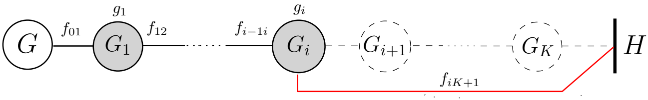

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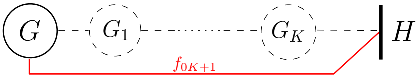


$$\Pi_1(p^2) \sim f_{0K+1}^2 \quad \rightarrow \quad d = 0\,, \quad \Lambda^2$$

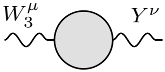
Rules for the calculability of the Higgs potential:

$$\Pi_1(p^2) \stackrel{p^2\to\infty}{\longrightarrow} 0 \quad \text{(I)}, \quad p^2\Pi_1(p^2) \stackrel{p^2\to\infty}{\longrightarrow} 0 \quad \text{(II)}$$

(I) violated by the term that directly connect the first site to the breaking $\Longrightarrow$ Λ^2

(II) can be violated with incomplete multiplet of resonances $\Longrightarrow$ $\log\Lambda$

Tree level contribution due to spin-1 resonances (see Grojean, Matsedonskyi, Panico hep-ph/1306.4655 for other contributions)



$$\Delta S = 4\pi v^2 \frac{d \ \log \Pi_1}{dp^2}\Big|_{p^2=0}$$

$$\Delta S = 4\pi v^2 \sum_{i=1}^{K} \left(\frac{1}{m_{\rho_i}^2} + \frac{1}{m_{a_i}^2} \right) + 4\pi v^2 \frac{d \log \ \mathcal{N}}{dp^2}\Big|_{p^2=0}$$

nearest neighbour interactions $\longrightarrow$ $\Delta S \gtrsim \frac{4\pi v^2}{m_\rho^2}$ ($\sim$ ED theories Barbieri, Pomarol, Rattazzi hep-ph/0310285)

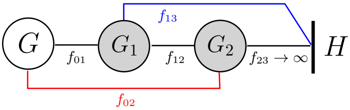
non nearest neighbour interactions $\longrightarrow$ $\mathcal{N}(p^2)$

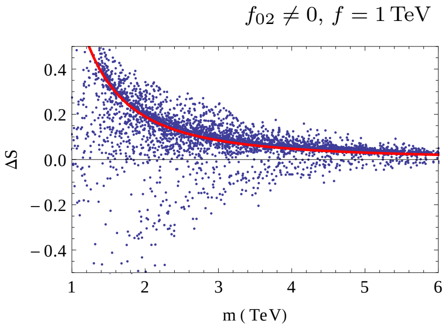
$$f_{0K+1} \ : \quad \Delta S = 4\pi v^2 \sum_{i=1}^{N} \left(\frac{1}{m_{\rho_i}^2} + \frac{1}{m_{a_i}^2} \right) \frac{f^2 - f_{0K+1}^2}{f^2}$$

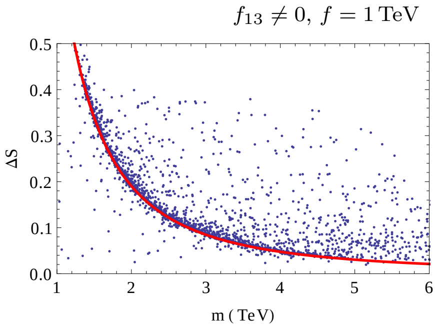
smaller contribution to S but the potential depends quadratically on the cut-off

Simple model: ρ_1, ρ_2, a_1

$$V(h) \sim \log\Lambda$$







$$\Delta S < \frac{4\pi v^2}{m_\rho^2}$$

non minimal terms can lower the tree level contribution to the S parameter

Effective model for hadrons interactions Son, Stephanov (hep-ph/0304182)

Bando, Kugo, Uehara, Yamawaki, Yanagida (PRL 54 (1985) 1215)

$$\frac{SO(4)}{SO(3)} \simeq \frac{SU(2)_L \times SU(2)_R}{SU(2)_{L+R}}$$

pions $\longleftrightarrow$ pGB Higgs

$m^2_{\pi^+} - m^2_{\pi^0} \longleftrightarrow V(h)$

$$m^2_{\pi^+} - m^2_{\pi^0} \simeq \frac{3\alpha_{EM}}{4\pi} \frac{m^2_\rho m^2_{a_1}}{m^2_{a_1} - m^2_\rho} \log\left(\frac{m^2_{a_1}}{m^2_\rho}\right) \quad \surd$$

$$m^2_\rho \neq 2\, g^2_{\rho\pi\pi} f^2_\pi \quad \text{(KSRF)} \quad \times$$

nearest neighbour interactions $\longrightarrow$ $\frac{m^2_\rho}{g^2_{\rho\pi\pi} f^2_\pi} > 3$

Becciolini, Redi, Wulzer (hep-ph/0906.4562)

non minimal terms : possible to reproduce both KSRF and δm^2_π

- we generalized a 4D CHM where the Higgs is a pGB
- the calculability of the Higgs potential is under control
- non minimal terms may lower the tree level contribution to the S parameter
- in an effective model of hadrons non minimal terms may help to reproduce phenomenology

Thank you for your attention!

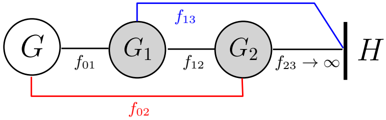
Explicit model

$\rho(770)$, $a_1(1260)$: only the extreme non minimal term is possible

KSRF can be reproduced but $m^2_{\pi^+} - m^2_{\pi^0} \sim \Lambda^2$

Effective model of $\rho(770)$, $a_1(1260)$, $\rho'(1450)$

$$m^2_{\pi^+} - m^2_{\pi^0} \sim \log \Lambda$$



KSRF + δm^2_π

$$\mathcal{L}_{eff} = \frac{1}{2} P_t^{\mu\nu} \left[\Pi_0(Q^2)\, \delta_{AB} + \Pi_1(Q^2)\, \Phi^T T_A T_B \Phi \right] A_0^{A\mu} A_0^{B\nu}$$

calculating the path integral over the composite fields on a constant GB background

$$\mathcal{L}_{eff} = \frac{1}{2} A^A_{0\mu} \Sigma^{\mu\nu}_{AB} A^B_{0\nu}$$

$$\Sigma^{\mu\nu}_{AB} = -\frac{g_1^2 f_{01}^4}{4} \langle A^\mu_{1A} A^\nu_{1B} \rangle + \frac{f_{01}^2}{2} \eta^{\mu\nu} \delta_{AB}$$

rotate the GBs on the last site ($\Phi_{K,K+1} = \Phi\,,\, \Omega_{i-1,i} = \mathcal{I}$)

in constant GB background the only term that contains Φ is

$$\frac{g_K^2 f_{K\,K+1}^2}{2} A^A_{K\mu} A^{B\mu}_K\, \Phi^T T_A T_B \Phi \quad \longrightarrow$$

the UV leading contribution to $\Pi_1(Q^2)$ comes from the diagram

$$\Pi_1(Q^2) \sim \frac{1}{(Q^2)^d}$$

$$d = 2K$$

Backup: Spin 1/2 resonances

Dirac fields Ψ_r^i at each site (r is a representation of $SO(5)$)

dependence on the choice of the representation $\rightarrow \underbrace{\mathbf{5}}_{SO(5)} = \underbrace{\mathbf{4} \oplus \mathbf{1}}_{SO(4)}$

the link fields $\Omega_{i,j}$ allow fermions at site i and j to interact

$$\mathcal{L} = \sum_{i=1}^{K} \bar{\Psi}^i i \not{D}^{A_i} \Psi^i$$

$$- \sum_{i,j=0}^{K} \left[M_{ij} \bar{\Psi}_L^i \Omega_{i,j} \Psi_R^j + Y_{ij} \bar{\Psi}_L^i \Omega_{i,K+1} \Phi_0 \Phi_0^T \Omega_{K+1,j} \Psi_R^j + h.c. \right]$$

$$M_{ii} = m_i$$

$$\mathcal{L} = \bar{q}_L \not{p} \left(\Pi_0^q(p^2) + \frac{1}{2}\sin^2\frac{h}{f}\,\Pi_1^q(p^2)\hat{H}_c\hat{H}_c^\dagger \right) q_L + \bar{t}_R \not{p} \left(\Pi_0^t(p^2) + \frac{1}{2}\sin^2\frac{h}{f}\,\Pi_1^t(p^2) \right) t_R$$
$$+ \frac{1}{\sqrt{2}}\sin\frac{h}{f}\cos\frac{h}{f}\, M_1^t(p^2)\; \bar{q}_L \hat{H}_c t_R + \text{h.c.}$$

$$V(h)_{top} \simeq -N_c \int \frac{d^4Q}{(2\pi)^4}\left[\left(\frac{\Pi_1^q(Q^2)}{\Pi_0^q(Q^2)} + \frac{\Pi_1^t(Q^2)}{\Pi_0^t(Q^2)}\right)\sin^2\frac{h}{f} + \frac{|M_1^t(Q^2)|^2}{Q^2\Pi_0^q(Q^2)\Pi_0^t(Q^2)}\sin^2\frac{h}{f}\cos^2\frac{h}{f}\right]$$

$$Q^4\Pi_1^q(Q^2) \overset{Q^2\to\infty}{\longrightarrow} 0 \quad , \quad Q^4\Pi_1^t(Q^2) \overset{Q^2\to\infty}{\longrightarrow} 0$$
$$Q^2|M_1^t(Q^2)|^2 \overset{Q^2\to\infty}{\longrightarrow} 0\,.$$

$$\begin{aligned} \Pi_1^q(Q^2) &\sim \frac{1}{Q^{d_{LL}+1}} \\ \Pi_1^t(Q^2) &\sim \frac{1}{Q^{d_{RR}+1}} \\ M_1^t(Q^2) &\sim \frac{1}{Q^{d_{LR}}} \end{aligned} \qquad \Longrightarrow \quad d_{LL,RR} > 3 \quad d_{LR} > 1$$