The Cosmological side of Massive Gravity

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Based mainly on work in collaboration with D. Comelli, F. Nesti and M. Crisostomi

Why Massive Gravity ?

- Theoretical motivation

  Is GR gauge theory alike ?

  Broken (massive) gauge theory can be trusted up to $\Lambda = m\, g^{-1}$

  for GR: $\Lambda = m\, g^{-1} \sim \Lambda_2 = \left(m\, M_{pl}\right)^{1/2}$

- Cosmological motivation:

  If dark energy has an eq. of state with $w \neq -1$ $\qquad p = w\rho$

  $\Rightarrow$ no room for a cosmological constant

  Massive gravity with $m \sim H_0$ can give a pure gravity DE model

  No explanation of the coincidence and the CC problems

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Massive gravity, linearized level

- GR in the weak field limit

  $M_{pl}^2 E_{\mu\nu}^{(1)} = T_{\mu\nu}^{(1)}$ $\quad$ $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

  #DOF: $10 - 2 \times 4 = 2$ $\quad$ 4 gauge modes $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

- Linear mGR theory (Pauli and Fierz 1939)

  $L_{FP} = M_{pl}^2 L_{\text{grav}}^{(2)} + M_{pl}^2 m^2 \left(a\, h_{\mu\nu} h^{\mu\nu} + b\, h^2\right)$

  $E_{\mu\nu}^{(1)} - \frac{m^2}{4}\left(a\, h_{\mu\nu} + b\, h\, \eta_{\mu\nu}\right) = M_{pl}^{-2} T_{\mu\nu}^{(1)}$ $\quad$ $\partial^\nu E_{\mu\nu}^{(1)} = 0$

  4 constraints $\quad$ #DOF: $10 - 4 = 6 = 5 + 1$

- The sixth mode is a ghost (Boulware-Deser).
  Absent in flat space when $a + b = 0$ (FP theory)
- FP has no ghost but light bending is 25% off from the measured value
  The limit $m \to 0$ IS NOT smooth, (vDVZ) discontinuity
  the extra scalar degree of freedom does not decouple

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Avoiding the vDVZ discontinuity

- There is a way to avoid the vDVZ discontinuity: give up Lorentz invariance in the gravitational sector Rubakov 04, Dubovsky 04

$$\mathcal{L}_{\text{spin 2}} + \frac{1}{4}\left(m_0^2\, h_{00}^2 + 2m_1^2\, h_{0i}h_{0i} - m_2^2\, h_{ij}h_{ij} + m_3^2\, h_{ii}^2 - 2m_4^2\, h_{00}h_{ii}\right)$$

  different masses for each rotational invariant combination

  The vDVZ is deeply connected to the Lorentz invariant tensor structure. $SO(3)$ invariant mass terms give a smooth $m \to 0$ limit
- The matter coupling is standard, weak equivalence principle is preserved
- Weak field expansion works fine, the static gravitational Yukawa-like potential is

$$\Phi = \frac{M}{r}\left(A_1\, e^{-M_1 r} + A_2\, e^{-M_2 r}\right), \qquad A_1 + A_2 = 1$$

  $M_{1,2}^2 \sim m^2$, $A_{1,2}$ dependent on m_i^2 Comelli-Nesti-LP 13
  post Newtonian correction can be computed

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Linear Theory: Summary

- PF Lorentz invariant massive GR:

  either ruled out or norperturbative already at the solar system scale
- Lorentz breaking massive GR:

  consistent and in agreement with basic weak-field tests

- Non-local massive GR, see Michele Maggiore talk

What happens beyond the linear level ?

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Nonlinear Massive Gravity I

- Add to the GR Lagrangian an extra piece V that depends on the metric field (no derivatives allowed)

$$\sqrt{g}\left(R - m^2 V\right)$$

such that when $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\sqrt{g}\left(R - m^2 V\right) = \mathcal{L}_{\text{spin 2}} + \frac{1}{4}(m_0^2 h_{00}^2 + 2m_1^2 h_{0i}h_{0i} - m_2^2 h_{ij}h_{ij} + m_3^2 h_{ii}^2 - 2m_4^2 h_{00}h_{ii}) + \cdots$$

- Lorentz invariant mass term when V is such that:

$$m_0^2 = a + b\,, \quad m_1^2 = -b\,, \quad m_2^2 = -a\,, \quad m_4^2 = b\,, \quad m_3^2 = b$$

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Nonlinear Massive Gravity II

- To build a nontrivial V we need extra stuff. There is no “scalar” function of the metric itself
- Option one: prior geometry
  Introduce an extra **non-dynamical metric** $\tilde{g}_{\mu\nu}$
- Option two: full dynamic theory
  Introduce an extra **dynamical metric** $\tilde{g}_{\mu\nu}$ with its own dynamics, we end up in a **bimetric theory**

  preferable but more complicated. NB: not an UV completion !
- A Lorentz invariant example: take $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$

  Scalars made out of $X^{\mu}_{\nu} = g^{\mu\alpha}\tilde{g}_{\alpha\nu}\,, \qquad \tau_n = \mathrm{Tr}(X^n)$

$$a\,(\tau_1 - 4)^2 + b\,(\tau_2 - 2\tau_1 + 4) = \left(a\,h_{\mu\nu}h^{\mu\nu} + b\,h^2\right) + O(h)^3$$

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Nonlinear Massive Gravity III

- Nondynamical $\tilde{g}$ breaks inevitably local Lorentz (grav. sector)

$$g_{ab} = \text{diag}(-1, 1, 1, 1)\,, \qquad \tilde{g}_{ab} = \text{diag}(-\alpha_0, \alpha_1, \alpha_2, \alpha_3)$$

  Accidental Lorentz symm. of V (unitary gauge) is different from local Lorentz symmetry of the equivalence principle

- Alternatively (Lorentz breaking) only global rotational invariance can be retained in the unitary gauge

  Helps with phenomenology

- Matter always couples with $g_{\mu\nu}$, weak equivalence principle is OK

Taming the zoo of Massive Gravity: non-linear analysis

Non-perturbative analysis needed

- to check what happens to the PF tuning at the non-linear level
- in general to constrain V which encodes the gravity modification

Canonical analysis is best suited $\begin{cases} \text{nonperturbative} \\ \text{background independent} \end{cases}$

ADM 3+1 splitting for the metric

lapse N, shift vector N^i and spatial metric γ_{ij}

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N^j \gamma_{ij} & \gamma_{ij} N^j \\ \gamma_{ij} N^j & \gamma_{ij} \end{pmatrix} \qquad N^A = (N, N^i)$$

$$\mathcal{V} = m^2 \sqrt{(g)}\, V = m^2\, N \,\gamma^{1/2}\, V(N^A, \gamma_{ij})$$

Taming the zoo of Massive Gravity

Results from Canonical Analysis

1. No condition on $V \Rightarrow$ 6 DoF propagate
   Around flat space : 2 tensors + 2 vectors + 1+ 1 scalars
   One of the scalars is a the Boulware-Deser ghost. No good
2. 5 DoFs propagate if and only if

   $$\det\left(\frac{\partial^2 V}{\partial N^A \partial N^B}\right) \equiv \det(V_{AB}) = 0 \quad \text{and} \quad \mathrm{rank}(V_{AB}) = 3$$ Monge-Ampere eq.

   plus another condition on V involving γ_{ij} not shown

3. $\mathrm{rank}(V_{AB}) < 3 \Rightarrow \text{DoF} \leq 3$
   broken rotational inv. and/or boring cosmology, strong coupling

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Taming the zoo of Massive Gravity: bottom line

Classification of **all** mGR theories with 5 DOF
infinetly many in terms of two functions !!

$$V(N, N^i, \gamma_{ij}) = \mathcal{U} + N^{-1}\left(\mathcal{E} + \mathcal{Q}^i\,\mathcal{U}_{\xi^i}\right)$$

$$\mathcal{U}(\mathcal{K}^{ij}), \quad \mathcal{E}(\gamma_{ij},\, \xi^i) \qquad \mathcal{K}^{ij} = \gamma^{ij} - \xi^i\,\xi^j$$

$$\xi^i \text{ is defined by } N^i - N\,\xi^i = \left(\frac{\partial^2\mathcal{U}}{\partial\xi^i\partial\xi^j}\right)^{-1}\frac{\partial\mathcal{E}}{\partial\xi^j} \equiv \mathcal{Q}^i(\gamma^{ij}, \xi^i)$$

- The known Lorentz invariant ghost free potential (de Rham-Gabadze-Tolley) plus some more

  $V \sim \mathrm{Tr}(X^{1/2}), \quad X^\mu_\nu = g^{\mu\alpha}\eta_{\alpha\nu}$

  Disappointing phenomenology: very low cutoff

  $\Lambda_3^{-1} = (m^2\,M_{pl})^{-1/3} \sim 10^3\ \mathrm{Km}^{-1}$ conflicts with Newton's law tests
- When *V* is not Lorentz invariant phenomenology is more interesting

  cutoff $\Lambda_2 = (m\,M_{pl})^{1/2} >> \Lambda_3$. OK with Newton's law tests

Massive Gravity Cosmology: general results I

- The most general ansatz compatible with homogeneity CMB isotropy frame ≡ massive GR preferred frame gives a FRW form for the metric

$$ds^2 = -N^2 dt^2 + a(t)^2 \, \delta_{ij} \, dx^i dx^j$$

zero spatial curvature for simplicity

- EMT: matter +"gravitational" fluid: $T^{\text{tot}}_{\mu\nu} = T_{\mu\nu} + (8\pi G)^{-1} \, \mathcal{T}_{\mu\nu}$

$$\mathcal{T}_{00} = m^2 \, \frac{N^2}{2} \, \mathcal{U} \equiv \rho_{\text{eff}} \, N^2 \qquad \text{where } \partial\mathcal{U}/\partial\gamma^{ij} \equiv \mathcal{U}' \, \gamma_{ij}$$

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EMT conservation for the gravitational fluid: $\nabla^\nu \mathcal{T}_{\mu\nu} = 0$

$$\partial_t \rho_{\rm eff} + 3H(\rho_{\rm eff} + p_{\rm eff}) = 0 \quad \Rightarrow \quad H\left(\mathcal{E}' - \frac{\mathcal{E}}{2}\right) = 0$$

$\mathcal{U}$ enters in the total Hamiltonian as $N\mathcal{U}$ likewise GR as required by time reparametrization $\Rightarrow \mathcal{U}$ part automatically conserved

Only the $\mathcal{E}$ part is constrained

- Either there is no cosmology or $\mathcal{E}$ suitably chosen (EMT cons.)
- for instance, $\mathcal{E}$ homogeneous of degree -3/2 in γ_{ij}
  Once the EMT is conserved, $\mathcal{E}$ does not enter anymore in the (background) equations
- The Lorentz invariant dRGT does not satisfy Bianchi

  no flat FRW cosmology

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- Once Bianchi is satisfied $\mathcal{E}$ disappears from the equation of motion

$$3H^2 = N^2 \left(\frac{m^2 \mathcal{U}}{2} + 8\pi G \rho_m \right) .$$

- the "gravitational" fluid mimics Dark Energy when $2\mathcal{U}'/\mathcal{U} < 1$

$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = -1 + \frac{2\mathcal{U}'}{\mathcal{U}}$$

- Studying perturbations one finds that:
  We loose control of the theory at the energy scale scale Λ_{eff}

$$\Lambda_{\text{eff}} = 4\pi\Lambda_2 \mathcal{U}'^{1/4} = 4\pi \left(mM_{pl}\right)^{1/2} \mathcal{U}'^{1/4}$$

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mGR Cosmology: perturbativity vs DE II

If FRW solutions exist strongly coupling in exact dS and/or Minkowski:
Perturbatively only 2 DOF (tensor modes) vs 5 nonperturbative DOF

- An almost dark energy dominated de Sitter Universe is fine
- How much a DE dominated Universe can differ from dS in mGr ?

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Ghost Free Bigravity Cosmology

Comelli-Crisostomi-LP

- All expected modes $2_S + 2_V + 2_T + 2_T = 8$ are present
- Scalar sector (long. gauge for simplicity)

$$ds^2 = -a^2\, dt^2 (1 + 2\Psi_1) + a^2\, d\vec{x}^2 (1 + 2\Phi_1)$$

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Conclusions and take-away messages

- A randomly picked mGR theory has 5+1 DOF;
  typically one is a ghost around Minkowski space

- Found nonperturbative construction of all mGR theories with 5 DoF. Very powerful from a phenomenological point of view

  1. In a predictive mGR theory Lorentz breaking seems to be unavoidable in the gravitational sector

     Besides the dGRT model, there are many ghost-free theories:

     - 5 DOF and no vDVZ discontinuity

     - cutoff $\Lambda_2 = (m\, M_{pl})^{1/2} \approx \left[10^{-2} mm\right]^{-1}$ the highest possible

     compare with dRGT cutoff: $\Lambda_3 = \left[10^3 Km\right]^{-1}$

  2. Connection between strong coupling scale and $w_{eff} + 1$ on a near dS background

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Cosmology in ghost free bigravity

Comelli-Crisostomi-Nesti-LP, see also Volkov and Strauss at al.

- FRW solutions exist fon any κ
- Cosmological ansatz for the ghost free massive gravity ($\kappa = 0$)

$$ds^2 = a^2(t)\left(-dt^2 + dr^2 + r^2\, d\Omega^2\right)$$
$$\tilde{ds}^2 = \omega^2(t)\left[-c^2(t)\, dt^2 + dr^2 + r^2\, d\Omega^2\right] .$$

- Two branches depending on Q conservation: E = T + Q

$$\nabla^\nu Q^\mu \nu = 0 \Rightarrow m^2\ \left(6a_3\,\xi^2 + 4a_2\,\xi + a_1\right)\left(c\,\omega\, a' - a\,\omega'\right) = 0$$
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Massive Gravity Cosmology: Branch one

$$\left(6a_3\,\xi^2 + 4a_2\,\xi + a_1\right) = 0\,, \quad \xi = const. \qquad H = a'/a^2 \equiv H_\omega = \omega'/\omega^2$$

Mass deformation induce an effective CC, late time acceleration except for that very similar to GR

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{m^2}{3}\left[a_0 - 6\bar{\xi}^2\left(2a_3\bar{\xi} + a_2\right)\right]$$
$$\Lambda_{eff} = \frac{m^2}{8\pi G}\left[a_0 - 6\bar{\xi}^2\left(2a_3\bar{\xi} + a_2\right)\right]$$

Eqs for $\tilde{g}$ gives an algebraic equation for $c(t)$

However cosm. perturbations are strongly coupled ! Missing DoF

$2_T + 2_T + 0_V + 1_S = 5$ Found

$2_T + 2_T + 2_V + 2_S = 8$ Expected

Massive Gravity Cosmology: Branch two

$$c = \frac{H_\omega}{H}\,\xi \qquad \xi \text{ not constant}$$

$$3\,\kappa\,\xi\,H^2 = m^2\left[6\,\xi\left(4\,a_4\,\xi^2 + 3\,a_3\,\xi + a_2\right) + a_1\right] \qquad \text{Hubble dynamics}$$

$$m^2\left[\frac{a_1}{3\,\kappa\,\xi} - 2\,a_3\,\xi^3 + \xi^2\left(\frac{8\,a_4}{\kappa} - 2\,a_2\right) + \xi\left(\frac{6\,a_3}{\kappa} - a_1\right) + \frac{2\,a_2}{\kappa} - \frac{a_0}{3}\right] = \frac{8\pi G\,\rho_m}{3}\,.$$

Early time: Standar matter dominates (NB $a_1 > 0$)

$$\xi \sim \frac{a_1\;m^2}{8\,\pi\,G\,\kappa\,\rho_m} \qquad \rho_m + \rho_g = \rho_m \qquad w_{eff} = w$$

Late time: ρ_m is diluted, universe settles down in a dS phase with $\xi = const$ and $c = 1$, fixed point of the ξ dynamics

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Nonlinear Massive Gravity II

- Diff restored by four Stuckelberg fields ϕ^A Arkani-Hamed-Georgi-Schwartz 03

$$\tilde{g}_{\mu\nu} = \partial_\mu \phi^A \partial_\nu \phi^B \eta_{AB}$$

The Stuckelberg are "coordinates" of a fictitious flat space and $e^A = d\phi^A$ are the tetrads with $de^A = 0$

- Adapting the coordinates such that $\tilde{g}_{\mu\nu}$ is the Minkowski metric

(unitary gauge) $\partial_\mu \phi^A = \delta^A_\mu$

- Unitary gauge coordinates represents a preferred frame to be specified

For instance: the frame where the sun is at rest or where the CMB is almost isotropic

- Similar Stuckelberg construction in the Lorentz breaking case

Dubovsky 04, Comelli-Nesti-Pilo '13

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Perturbing the FRW geometry: 5 DoF expected: 2T+3V+1S

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$$L^{(s)} = \frac{a^4\,\Lambda_2^4\,\mathcal{U}'}{2\,k^2}\,\Sigma'^2 - \frac{a^4\,M_{\rm pl}^2(m_2^2 - m_3^2)}{2}\,\Sigma^2 + \cdots \qquad \Sigma = k^2\,\sigma$$

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