Magnetic Properties of the QCD Medium

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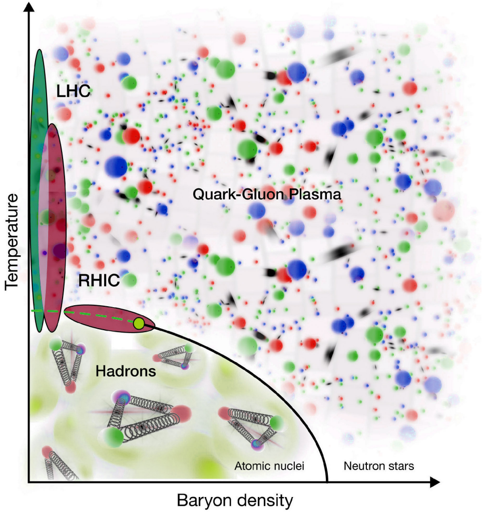
QCD phase diagram

QCD has a rich phase diagram in the $\rho - T$ plane, intensively studied in the recent years:

- $\rho = 0$: **analitic crossover** separets hadronic matter and the quark gluon plasma (QGP) (well established).
- Low T and high ρ: a **first order** transition may be found $\rightarrow$ Neutron stars (still open question).
- If a first order is present, one expect a **critical endpoint** with a second order transition.
- Higher ρ: exotic phases are expected (color superconductors).



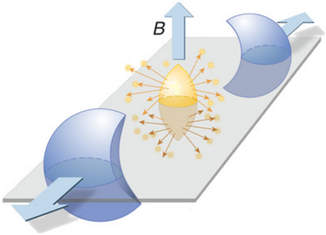
QCD with external B fields

QCD with B fields at the **strong scale**. Found in many phenomenolocical contests:

- Neutron stars and compact astrophysical objects, $\mathbf{B} \sim \mathbf{10^{10}}$ **T** [Duncan and Thompson, 1992]
- First phase of off-central heavy ion collisions, $\mathbf{B} \sim \mathbf{10^{15}}$ **T** [Skokov et al., 2009]
- Early universe, $\mathbf{B} \sim \mathbf{10^{16}}$ **T** [Vachaspati, 1991]

We consider the heavy-ion collision scenario:

- QGP formed after the the collision, $\tau \sim 0.5 \frac{\text{fm}}{c}$.
- Off-central collisions: ions generate magnetic fields, **ortogonal** to the reaction plane. Strength controlled by $\sqrt{s_{NN}}$ and the impact parameter.
- At LHC, B fields expected up to $eB \sim 15 m_\pi^2$



These magnetic fields can lead to relevant modification of the strong dynamics.

QCD with external B fields

Electromagnetic background interacts only with quarks, but loop effects can modify also the gluon dynamics.

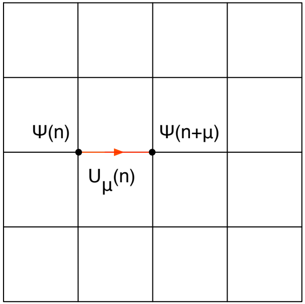
- Non perturbative effects lead to non trivial bahavior:
  - ▷ QCD phase diagram (location of the deconfinament cross over, ...)
  - ▷ QCD vacuum structure (chiral symmetry breaking, ...)
  - ▷ QCD equation of state (effect on the free energy of the QCD medium)

In this talk we discuss non perturbative magnetic effects on the **QCD equation of state**.

- QCD medium reacts as a **paramagnet** or a **diamagnet** to B fields?
- **Lattice QCD** $\rightarrow$ ideal tool to investigate susch issues from firt principles.

QCD on the lattice

- Start from path integral formulation of QCD in Euclidean space-time. Discretize the theory over a **finite space-time lattice**.



- $\begin{cases} \psi(n) & \text{quark fields} \\ U_\mu(n) = e^{iagA^a_\mu(n)} & \text{parallel transporters} \end{cases}$

- **Finite number** of integration variables
  $\rightarrow$ Monte-Carlo algorithms can be used.

- Sample configurations with the probability distribution: $\det M e^{-S[U]}$, then:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \det M e^{-S[U]} \mathcal{O}[U] \simeq \frac{1}{N} \sum_{i=0}^{N} \mathcal{O}[U^{(i)}] \ .$$

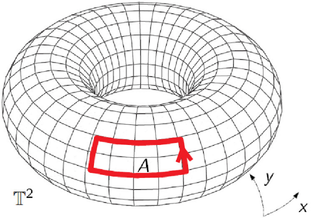
- Temperature of the statistical system: $T = \frac{1}{N_t a}$, with N_t temporal extension.

- Remember: i) check **finite size effects**, ii) perform **continuum limit**.

Magnetic fields on the Lattice

- Add proper $U(1)$ phases to the parallel transports: $U_\mu(n) \to U_\mu(n) u_\mu(n) \qquad u_\mu = \exp\left(iqa_\mu(n)\right)$

- Periodic boundary conditions to reduce finite size effects $\to$ Compact manifold with no boundary.



- Charge moving along a closed path with $B\hat{z}$ orthogonal and homogeneous. Phase gained:

$$\oint a_\mu dx_\mu = AB \quad \text{or} \quad \oint a_\mu dx_\mu = (A - l_x l_y)B$$

Quantization condition:

$$e^{iqBA} = e^{iqB(A-l_x l_y)} \to qB = \frac{2\pi b}{l_x l_y} = \frac{2\pi b}{a^2 L_x L_y} \,, \quad b \in \mathbb{Z}$$

- E.g. $B\hat{z}$ on the lattice: discretize $a_y = Bx$:

$$u_y^{(q)}(n) = e^{ia^2 qBn_x} \quad u_x^{(q)}(n)|_{n_x = L_x} = e^{-i\ a^2 qL_x Bn_y} \quad \text{otherwise} \quad u_\nu(n) = 1$$

Constant flux $a^2 B$ in all x-y plaquettes, exluded one plaquette at the corner, with flux $(1 - L_x L_y)a^2 B \to$ Dirac string.

Our method

- For "small" magnetic fields: $f(T,B) = f(T,0) + \frac{1}{2}c_2(T)B^2 + \mathcal{O}(B^3)$
  Then $\chi \propto c_2(T) = \left.\frac{\partial^2 f(T,B)}{\partial B^2}\right|_{B=0}$... **But** $\frac{\partial}{\partial B}$ not defined on the lattice!

- Analitic extension of $f(T,B)$ (defined only for $B = b \in \mathbb{Z}$) to non-integer B.

- Calculate on the lattice $M(T,B) = \frac{\partial f(T,B)}{\partial B}$ (this is **not** the magnetization!).

- Numerical integration of M to determine :

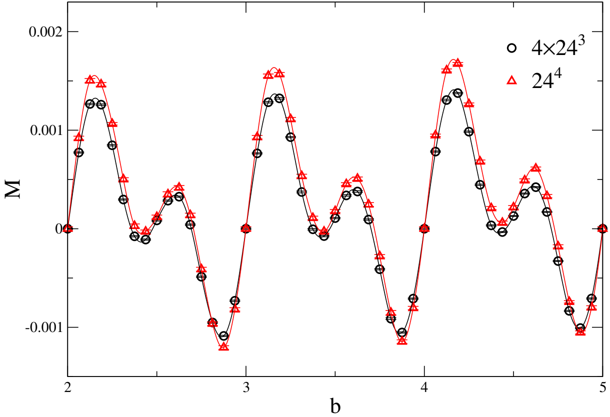
$$\Delta f(T,b) = f(T,b) - f(T,0) = \int_0^b M(B,T)dB \quad b \in \mathbb{Z}.$$

- B-dependent additive renormalizations are removed using:

$$\Delta f_r(T,b) = \Delta f(T,b) - \Delta f(0,b) \ .$$

M

M computed at $T = 0$ (red line) and $T \approx 225$ MeV (black line)



- Unphysical oscillations$\rightarrow$ B no more quantized
- To evaluate Δf, perform numerical integration over M spline interpolations.

Magnetic susceptibility

We need to estimate $c_2(T)$ defined by $\Delta f(B_k, T) \approx \frac{1}{2} c_2(T) B_k^2$. To minimize error propagation in the integration we fit:

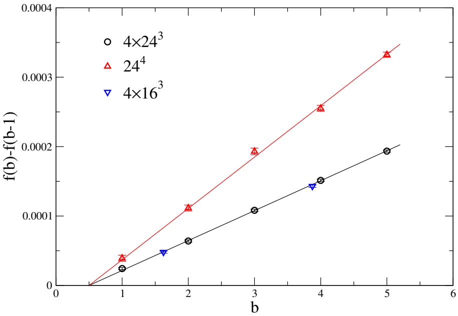
$$f(b,T) - f(b-1,T) = \int_{b-1}^{b} M(B,T) dB$$

with the function $\frac{1}{2} c_2(T)[b^2 - (b-1)^2] = \frac{1}{2} c_2(T)(2b-1)$.

- $c_2(T)$ determined from linear fit coefficient. Then:

$$\tilde{\chi}(T) = -\frac{e^2 \mu_0 c}{18 \hbar \pi^2} L^4 c_2(T)$$

- Blue points to check finite size effects→ Good.



Our method

- For small field and a linear, homogeneous, isotropic medium, the magnetization is proportional to the field:

$$\mathbf{M} = \tilde{\chi}\frac{\mathbf{B}}{\mu_0} = \chi\mathbf{H}$$

where $\mathbf{B}$ total field, $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ external field, and $\chi = \frac{\tilde{\chi}}{1-\tilde{\chi}}$.

- In the small field limit we can use:

$$\Delta f = \int \mathbf{H} d\mathbf{B} \ \rightarrow \Delta f_r = -\int \mathbf{M} d\mathbf{B} \approx -\frac{\tilde{\chi}}{\mu_0}\int \mathbf{B} d\mathbf{B} = -\frac{\tilde{\chi}}{2\mu_0}\mathbf{B}^2$$

- Our simulations are QED quenched, no backreaction from the medium $\rightarrow$ $\mathbf{B}$ coincides with the external field added to the Dirac operator.

- QED quench does not affect the $\tilde{\chi}$ measure. However, adding the backreaction of the medium increase Δf_R by a factor $1/(1-\tilde{\chi})^2$ $\rightarrow$ Irrelevant a posteriori.

Results

Continuum extrapolation of $\tilde{\chi}$ from our lattice results.

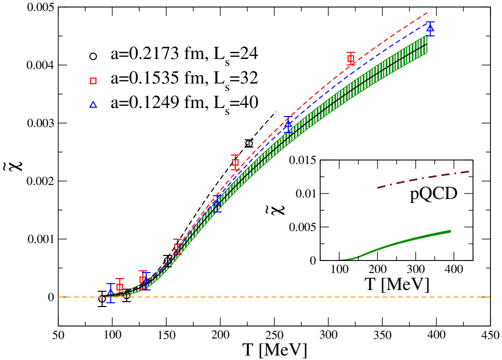
- The QCD medium is a **paramegnet** in all the explored temperature.
- Sharp increase of $\tilde{\chi}$ above $T_C \sim 150 - 160$ MeV.
- Agreement at low T with HRG behavior:

$$\tilde{\chi}(T) = A\exp(-M/T)$$

- Agreement at high T with pQCD behavior:

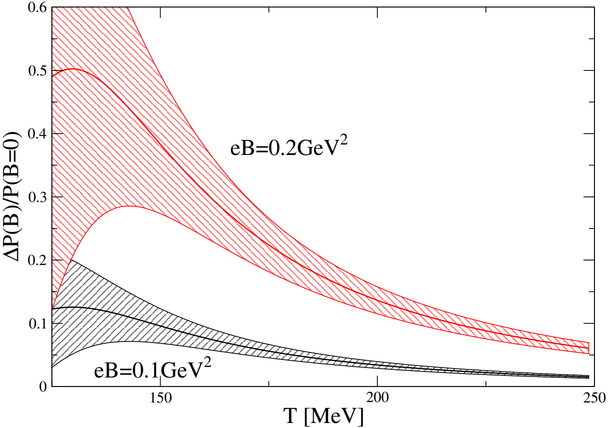
$$\tilde{\chi}(T) = A\log(T/M)$$

- We observed a linear response up to $eB \approx 0.2$ GeV^2.



Pressure contribution

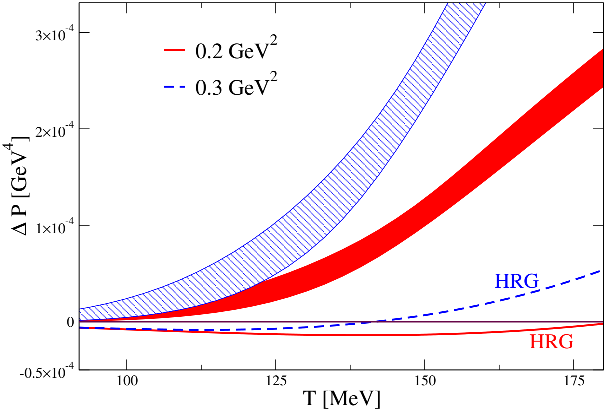
Magnetic contribution to the pressure: $\Delta P(B) = -\Delta f = \frac{1}{2}\tilde{\chi}(eB)^2$.



Of the order 10% for 0.1 GeV^2, 50% for 0.2 GeV^2.

HRG

Low T: check with the hadron resonance model predictions→ Effective model with hadrons as fundamental $d.o.f.$



HRG predicts **diamagnetic** behavior below T_c [Endrodi, 2013]

→ Something is missing in the model?

→ Need more statistics at low T?

Conclusions

- The QCD medium medium behaves as a paramagnet in all the explored temperatures.
- Weak magnetic activity in the confined phase, while the magnetic susceptibility increase sharply across $T_c \approx 150 - 160$ MeV.
- The QCD medium has linear response up to $eB \approx 0.2$ GeV^2.
- The magnetic contribution to the preassure is $10 - 50\%$ in the range of fields expected at LHC, $0.1 - 0.2$ GeV^2.

Future studies:

- Determination of higher order terms $\rightarrow$ relevant for cosmological models, where $eB \sim 1$ GeV^2.
- c quark contributions can be relevant at higher temperatures.